

Nov. 23, 1937.

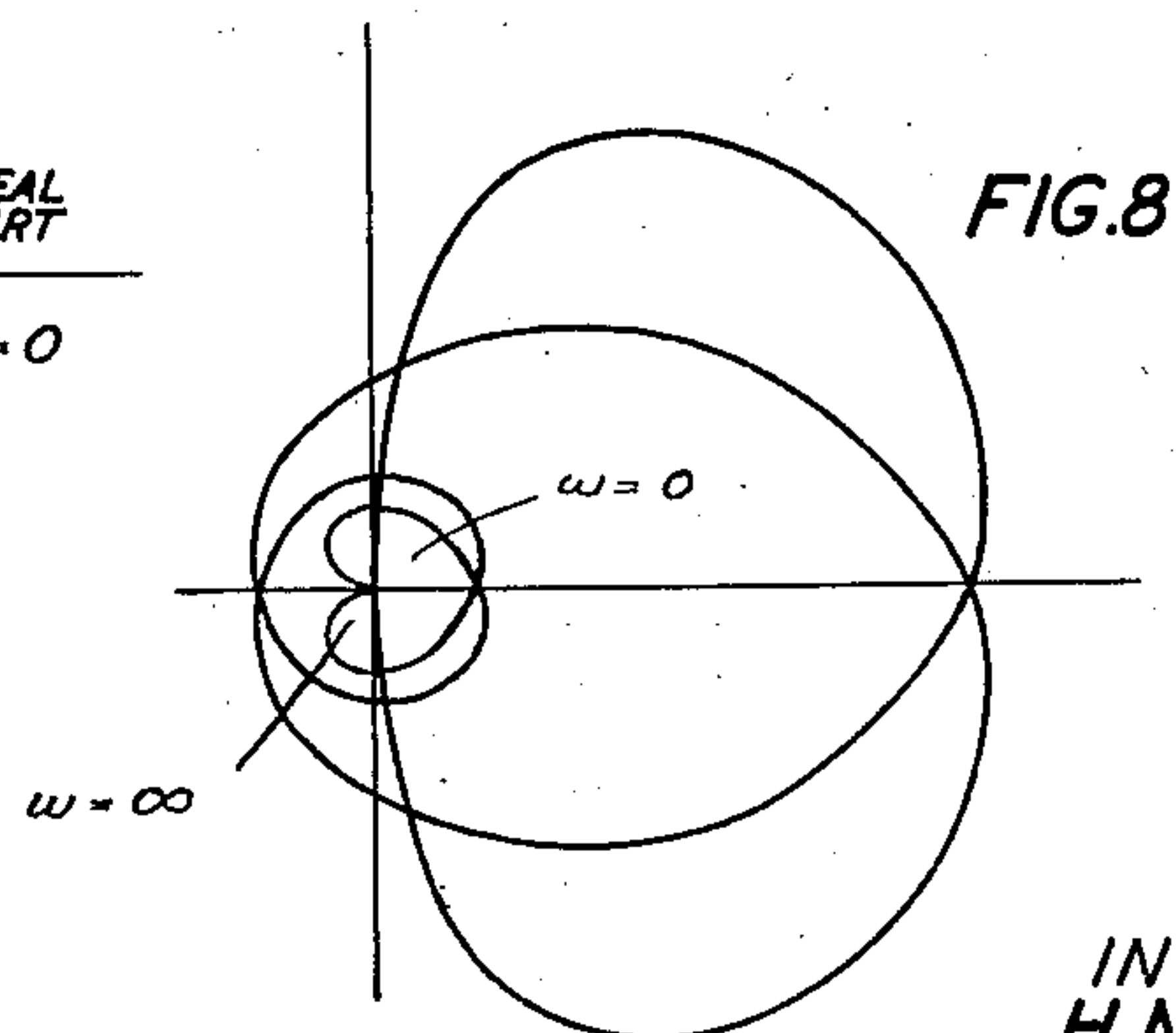
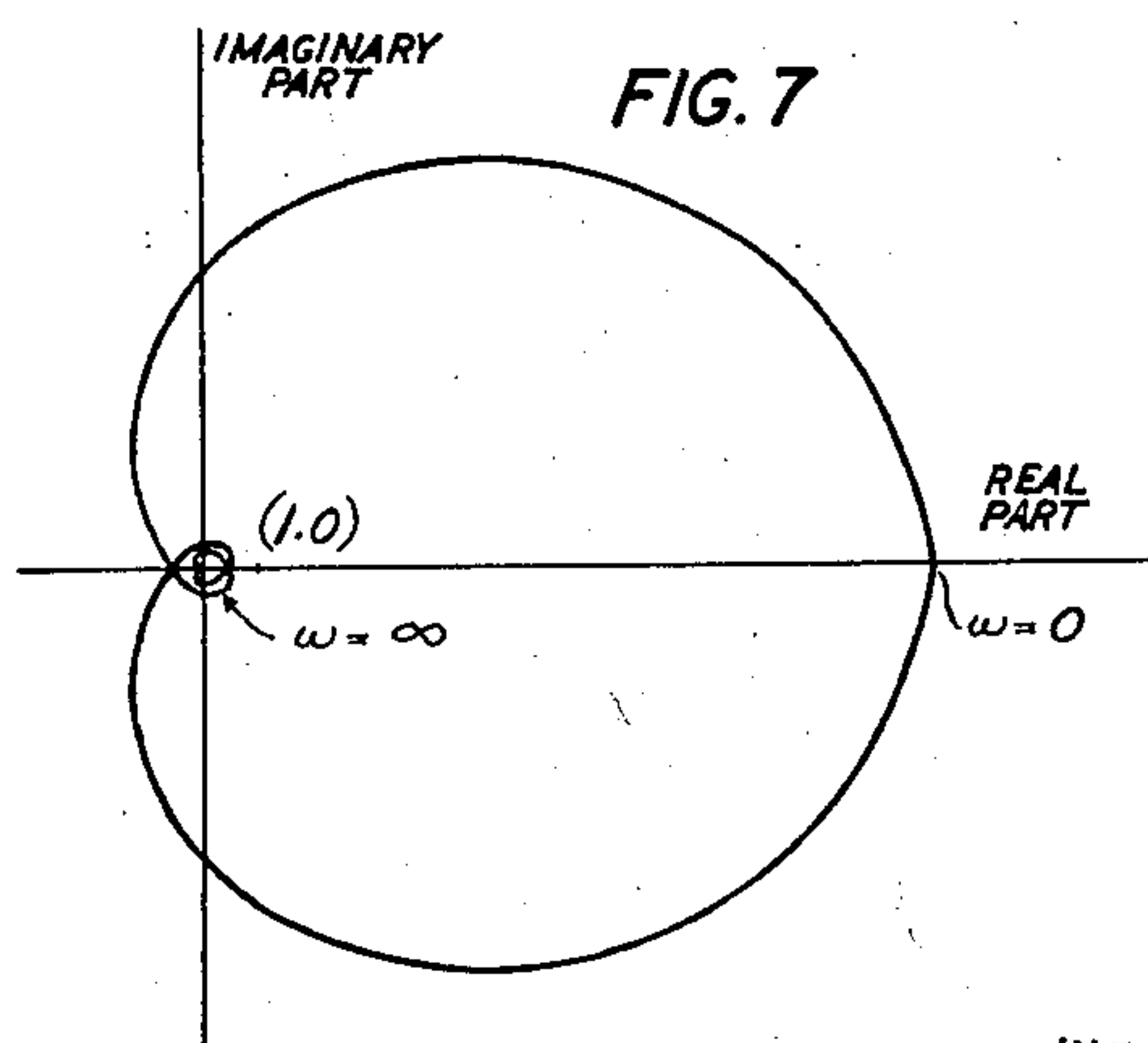
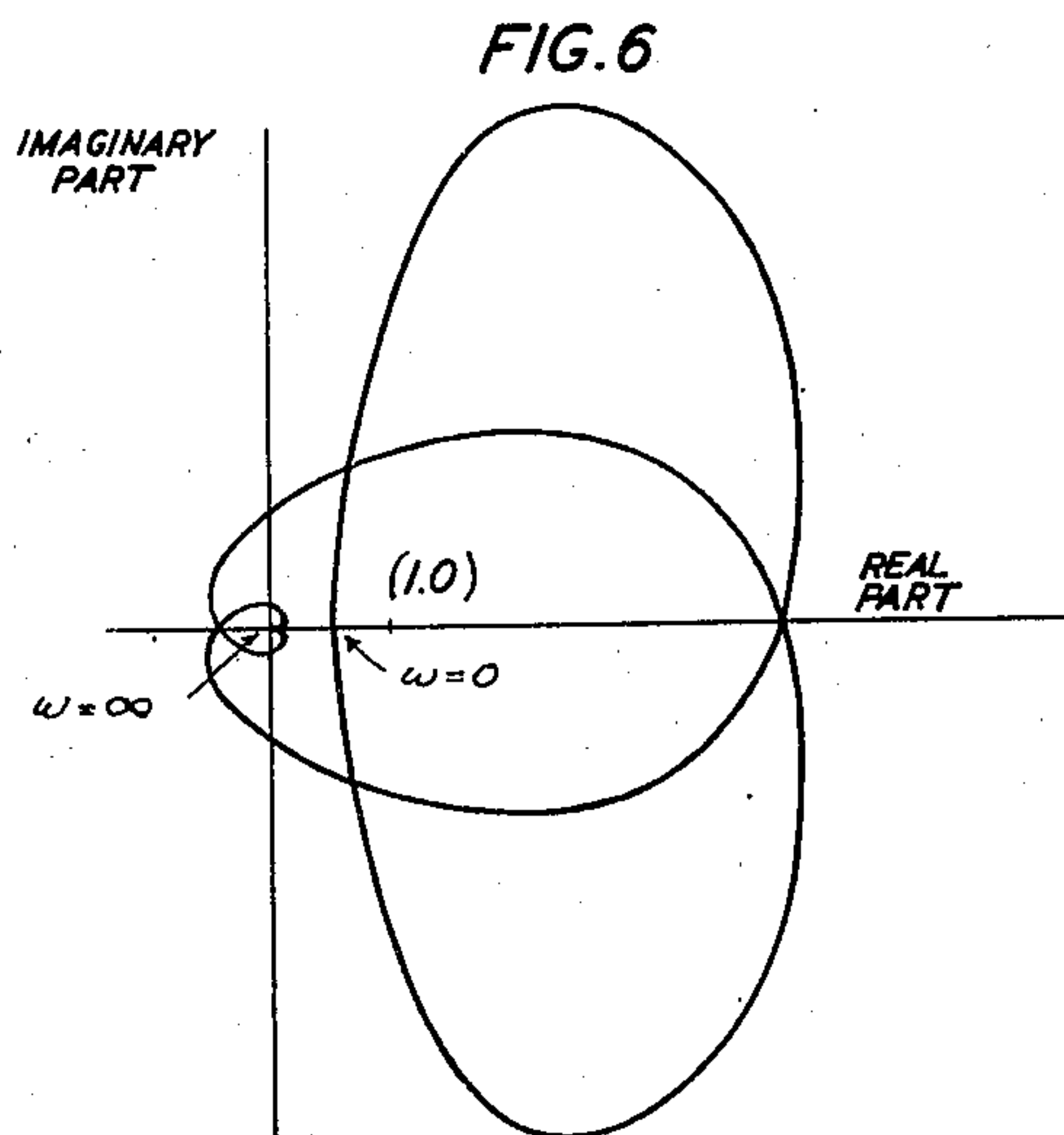
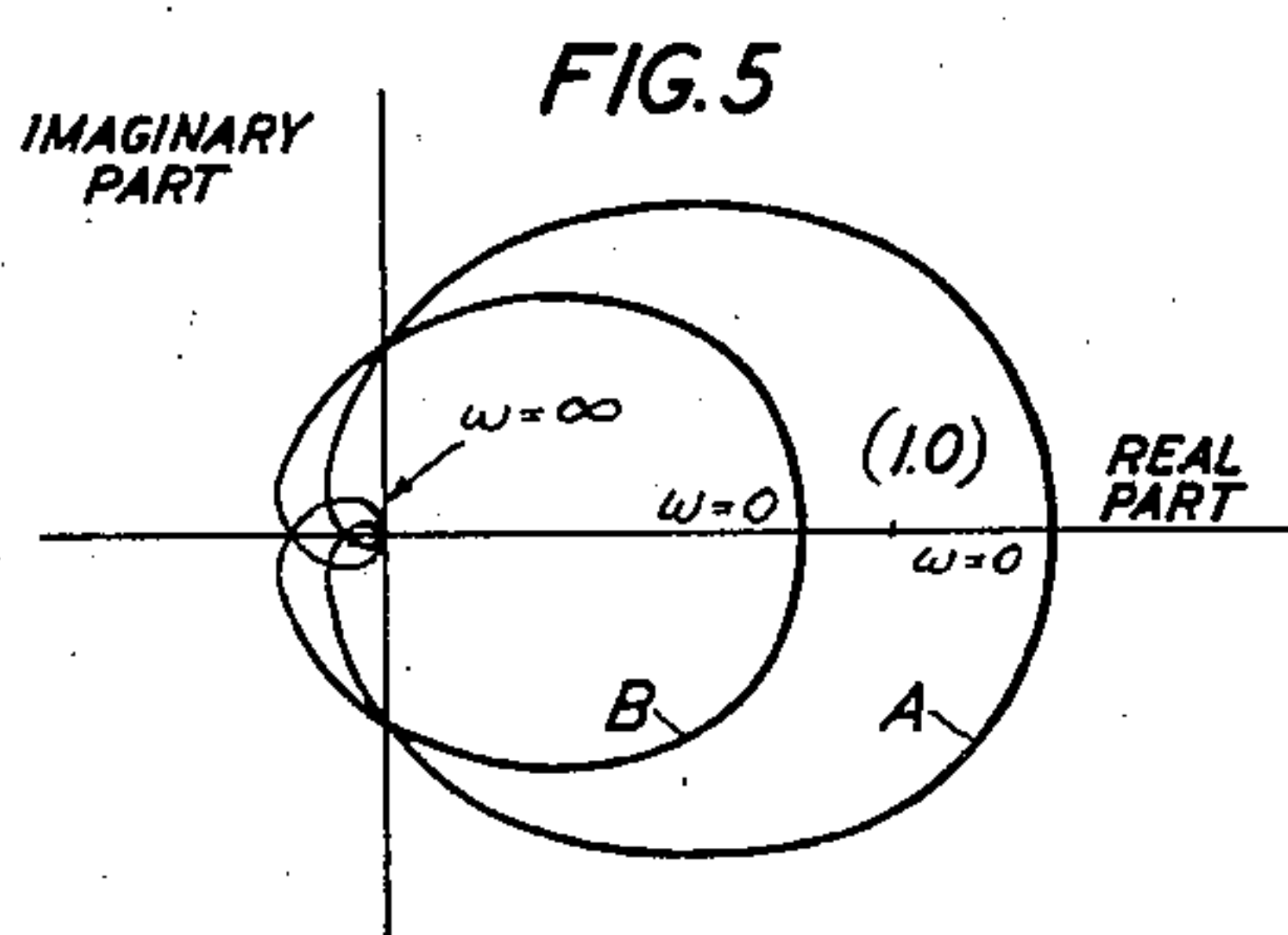
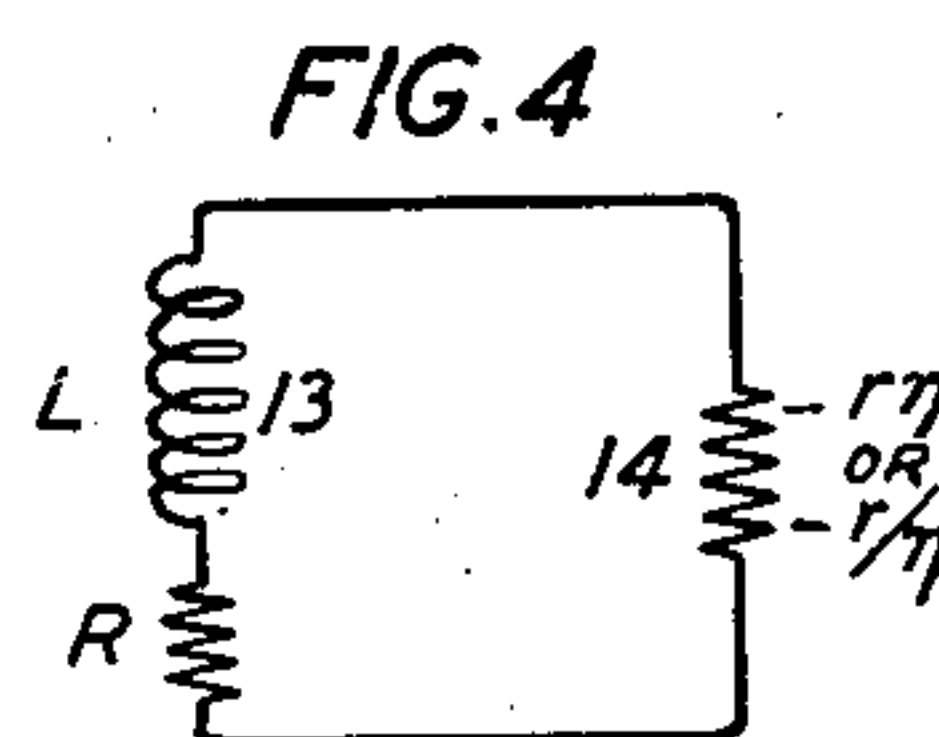
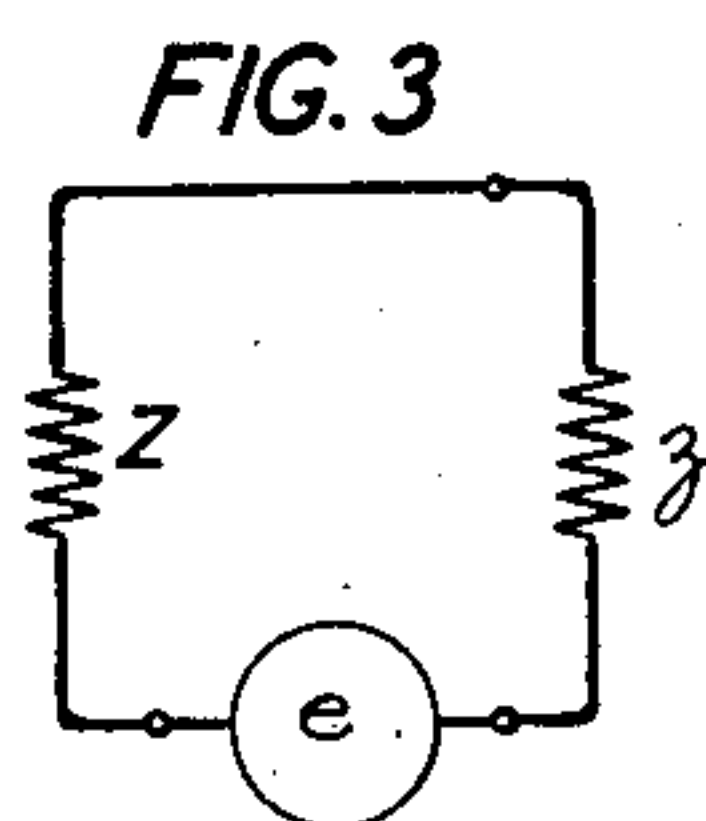
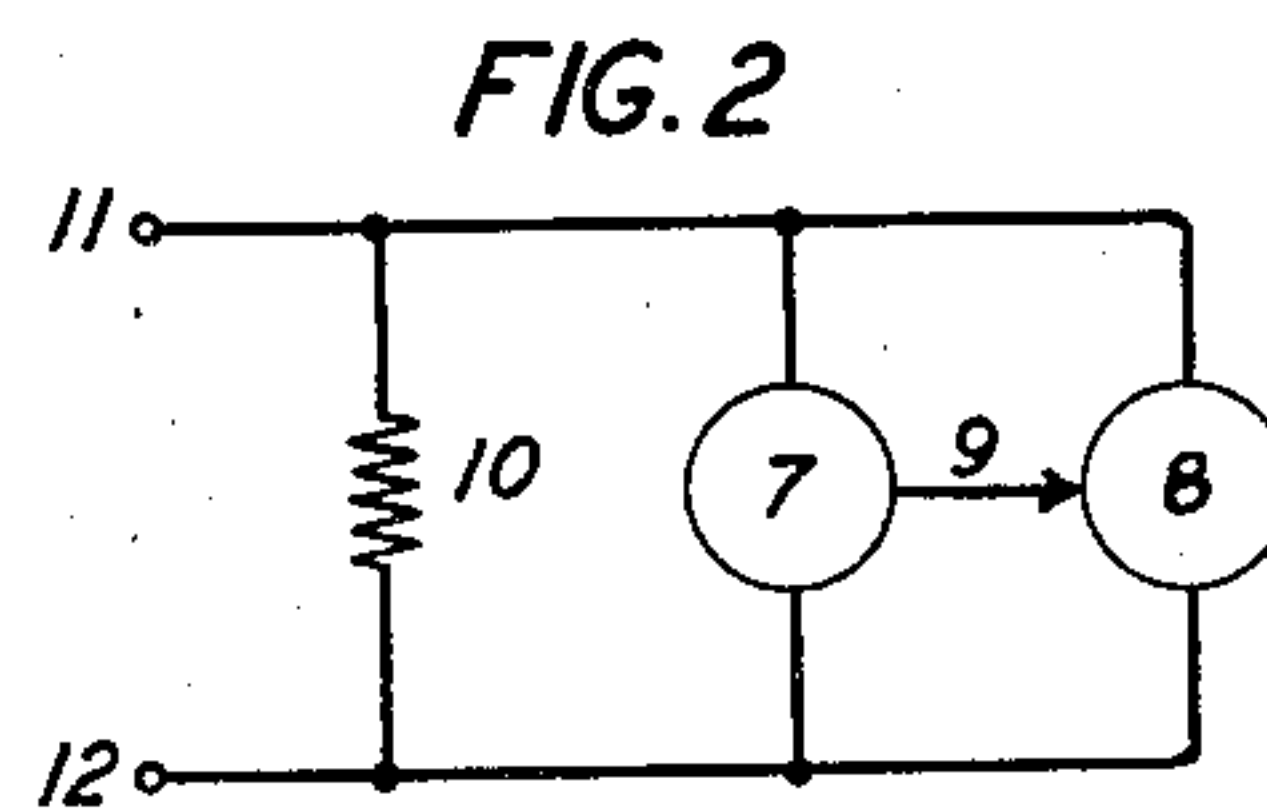
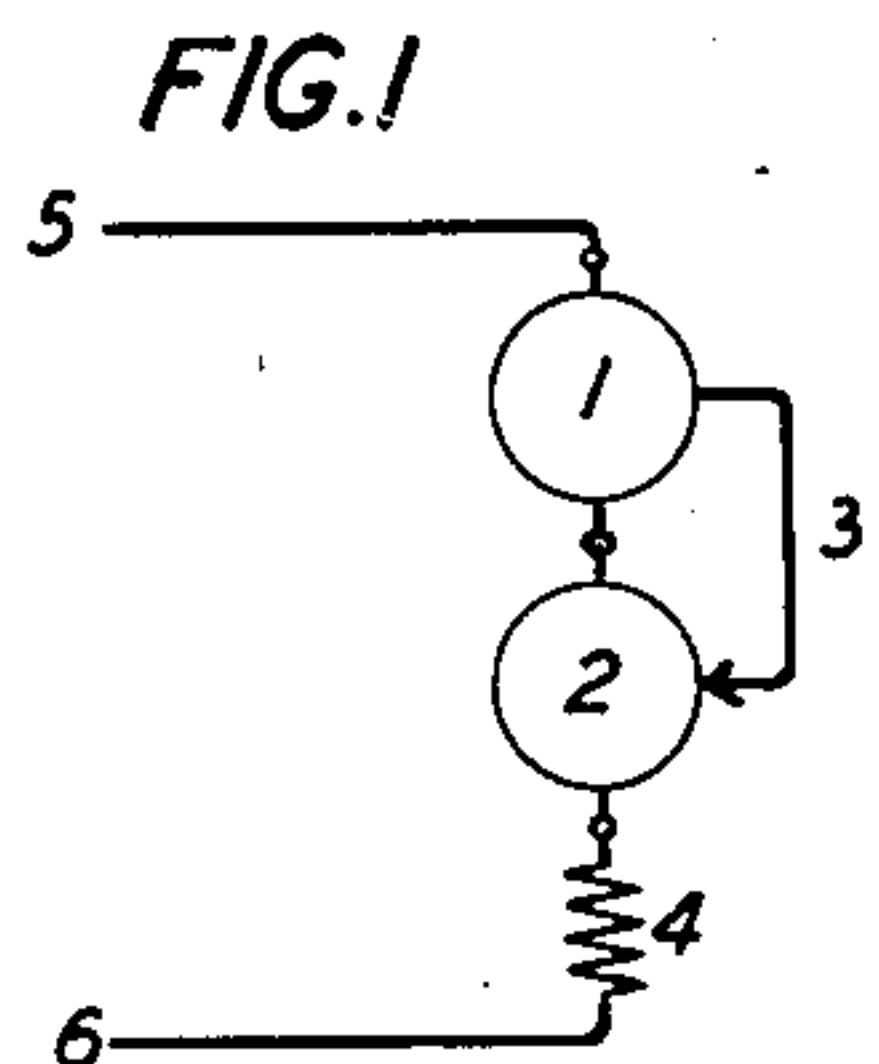
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2,099,769

CIRCUITS EMBODYING NEGATIVE IMPEDANCE

Filed June 23, 1936

2 Sheets-Sheet 1



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CIRCUITS EMBODYING NEGATIVE IMPEDANCE

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2 Sheets-Sheet 2

FIG. 9

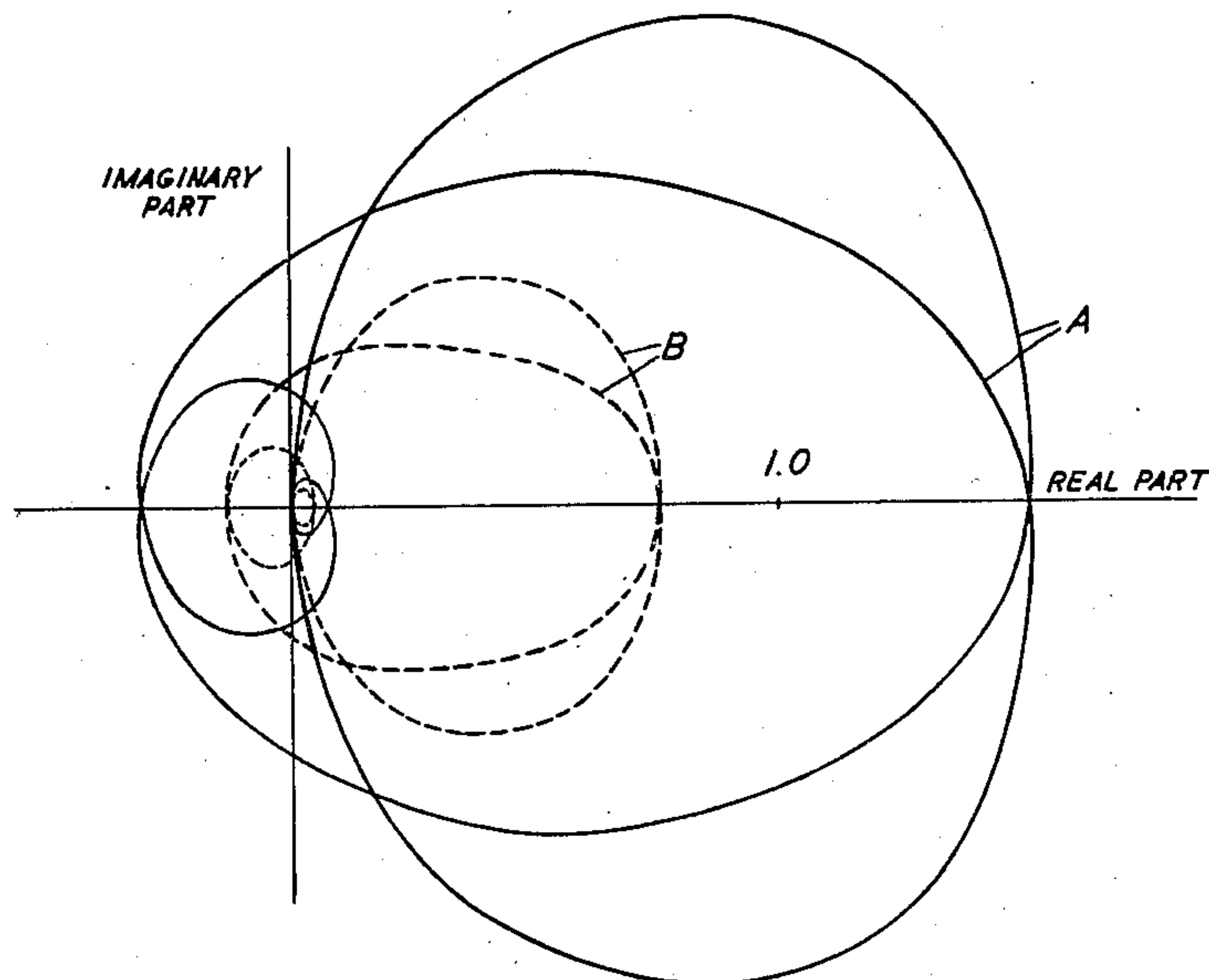


FIG. 10

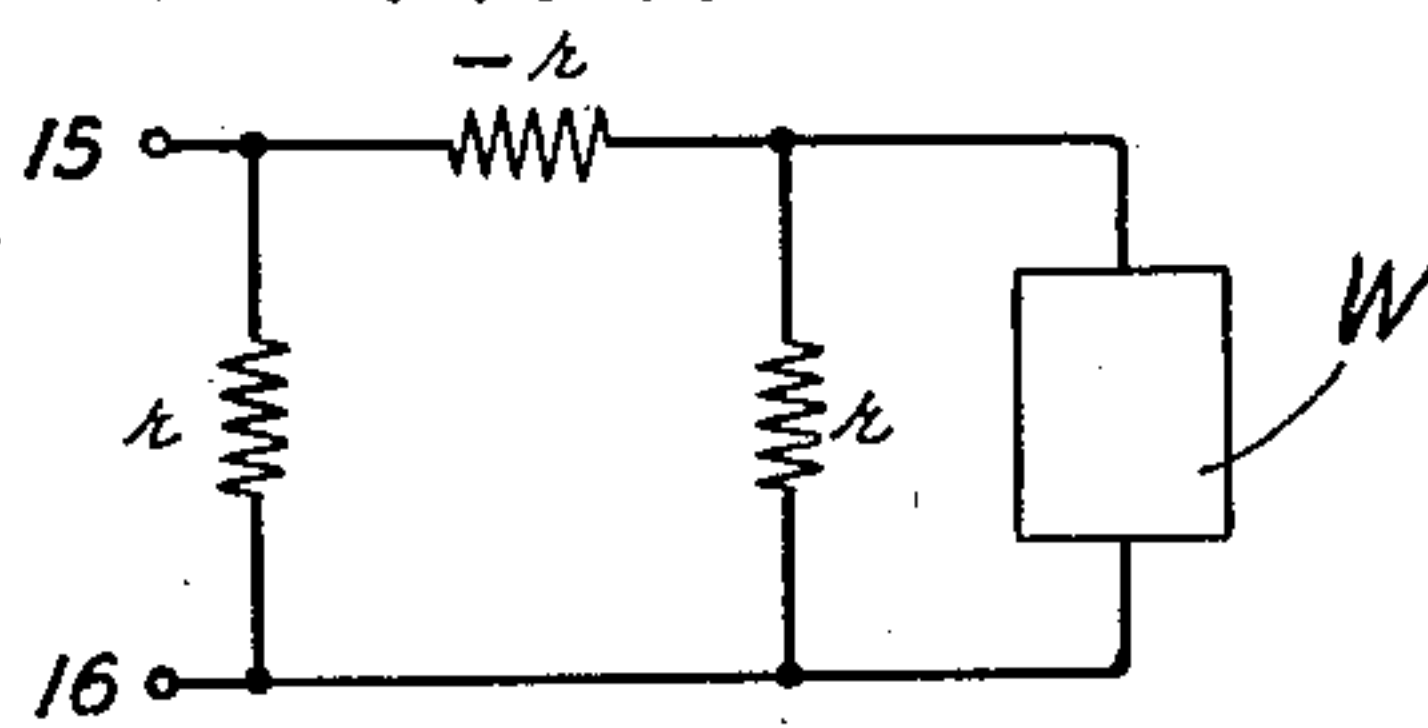


FIG. 11

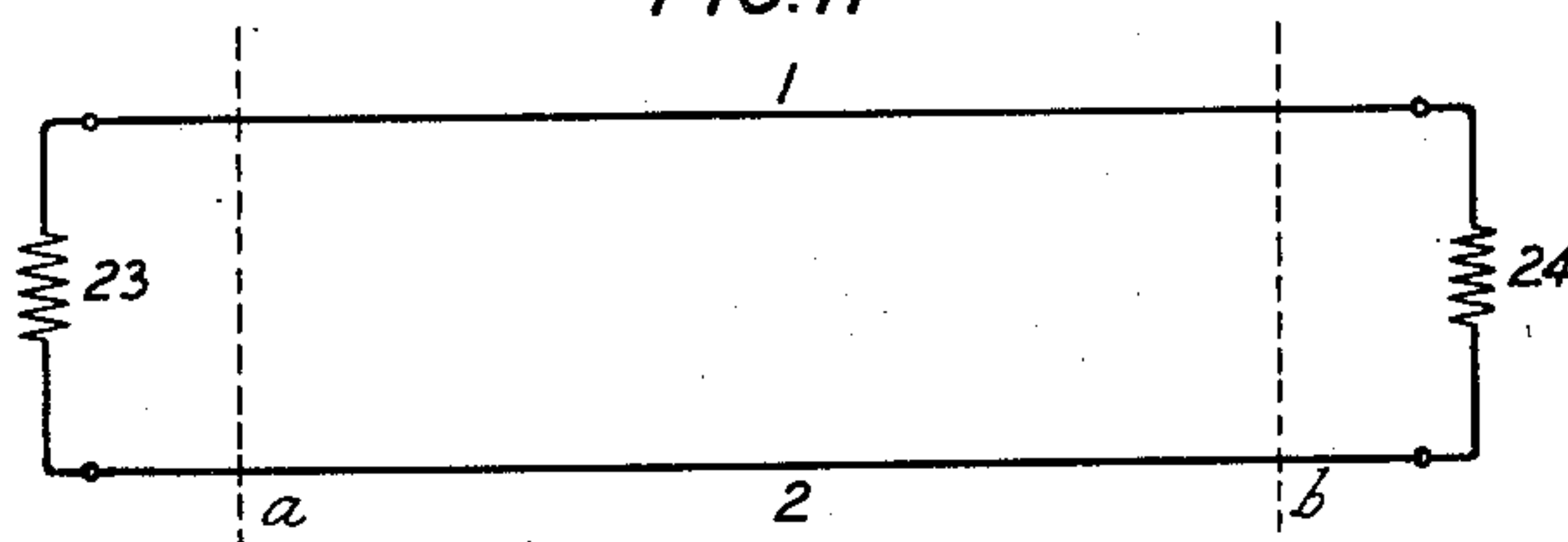


FIG. 12

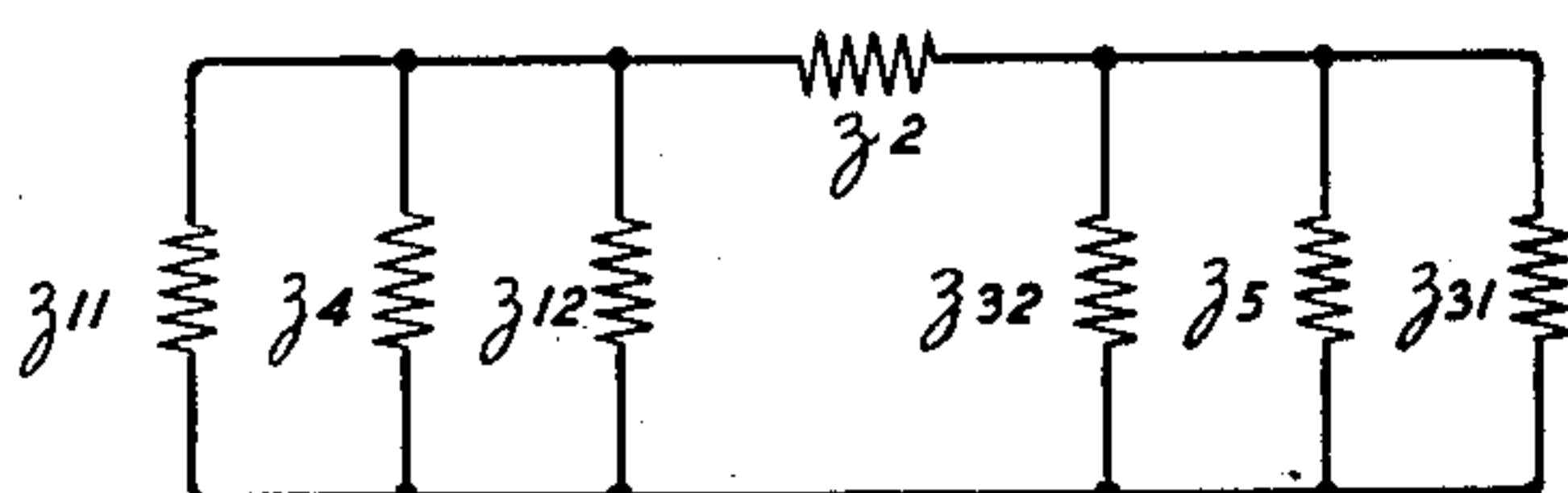
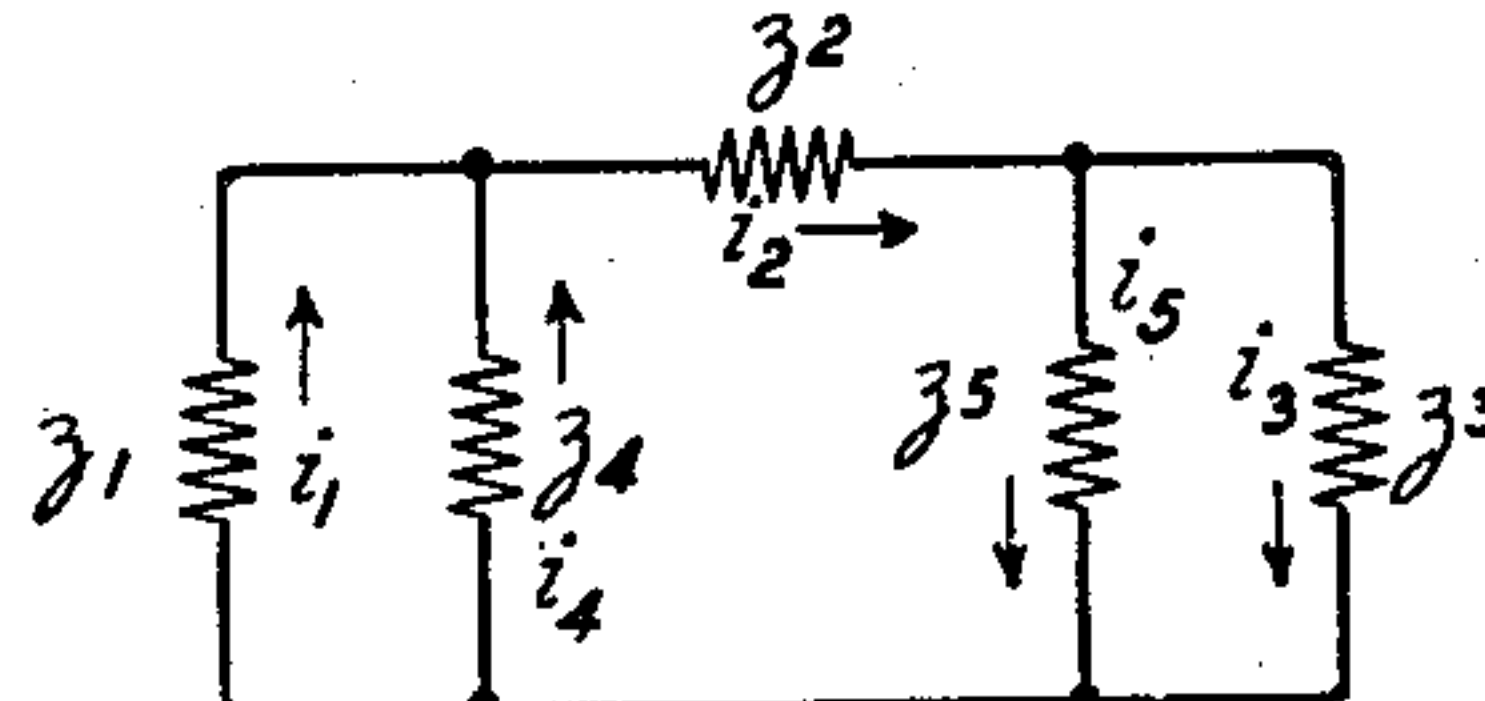


FIG. 13



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CIRCUITS EMBODYING NEGATIVE IMPEDANCE

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Application June 23, 1936, Serial No. 86,882

4 Claims. (Cl. 178—44)

The present invention relates to circuits which consist in part of negative impedance elements.

It is an object to so design and proportion the negative impedance elements that the network as a whole shall be stable.

It is a further object of the invention to adjoin to a given positive impedance network having undesirable impedance characteristics such a negative impedance as will neutralize all or a part of the undesirable characteristics.

It is well known by those familiar with the properties of negative impedances that the combination of negative impedance elements and positive impedance elements in a common network may result in a condition of instability. It is also known that negative impedances may be classified into two types, the series type and the shunt type. It is further well understood that when a positive resistance element and a negative resistance are connected at both terminals the conditions for stability are that the positive impedance should be numerically greater than the negative impedance if the latter is of the series type and should be numerically smaller than the negative impedance if the latter is of the shunt type. For a fuller statement of these facts the reader may be referred to an article "Negative Impedance and the Twin 21-Type Repeater" by George Crisson published in the Bell System Technical Journal, volume 10, page 485, July 1931.

In addition to the negative resistance elements there may be reactive negative impedance elements and negative elements which are partly resistive and partly reactive. Moreover, the positive networks to which they may be connected may be partly or wholly reactive. In order to make use of negative impedance elements in combination with positive elements it is therefore necessary to know how to proportion the negative elements for more general combinations than those hitherto studied and to know whether the series or shunt type should be used in any specific case.

In order to make the instructions for design intelligible a brief theoretical discussion of negative impedances will first be given. This discussion will then be set forth in rules which will enable those skilled in the art of circuit design to include negative impedance elements without incurring instability.

Reference is now made to the accompanying drawings in which:

Figure 1 is a schematic representation of a negative impedance of the series type.

Figure 2 is a schematic representation of a negative impedance of the shunt type.

Fig. 3 represents the series connection of two impedances and a source of electromotive force.

Fig. 4 represents the series connection of a positive inductance, a positive resistance and a negative resistance.

Figs. 5, 6, 7, 8 and 9 represent mathematical curves to be fully explained hereinafter.

Fig. 10 represents an electrical network having one negative impedance member and

Figs. 11, 12 and 13 illustrate the use of negative impedances in connection with a transmission line.

A negative impedance is characterized by having an internal source of energy. This source of energy may be of different form in different impedance elements but it must be such that it is controlled from the terminals. This control may be of either of two kinds. Either the impressed current may control the internal source of energy so as to produce an electromotive force, in which case the negative impedance is said to be of the series type, or the impressed electromotive force may control the internal source of energy to produce a current at the terminals in which case the negative impedance is said to be of the shunt type.

Referring now more particularly to Fig. 1, which represents schematically a negative impedance of the series type 1 is a current actuated device, and 2 is an internal electromotive force whose value is controlled by the current actuated device, the control being indicated by the arrow 3. The elements 1 and 2 are to be considered free of internal impedance and any actual impedance which they have may be considered lumped in series with both as at 4. The figure may be made clearer by considering a specific approximation to a negative impedance namely a series wound dynamo generator running at a uniform speed. Then 1 is the field winding and 2 is the armature and commutator. The arrow 3 indicates the well known property of such machines whereby the current in the winding determines the electromotive force appearing at the commutator terminals. Finally, 4 is the sum total of the internal impedance of the machine, which is positive but normally numerically small in comparison with the negative impedance of the machine.

In a similar manner Fig. 2 represents schematically a negative impedance of the shunt type. The element 7 is an electromotive force actuated device and 8 is a source of internal current

whose magnitude is determined by the electromotive force across 7. The control is indicated by the arrow 9. Both 7 and 8 are to be considered free of internal admittance and any internal admittance which they may have may be considered lumped as at 10.

If the mechanism of the impedance is understood then a comparison with Figs. 1 and 2 will establish whether any given negative impedance is of the series or the shunt type. It may, however, be necessary to deal with negative impedance whose mode of operation is not well understood and a more general criterion for type is necessary. An inspection of Figs. 1 and 2 will show that the series type impedance is stable when open circuited (or connected to a sufficiently great positive impedance) and is unstable when short-circuited (or connected to a sufficiently small positive impedance), and that the shunt type impedance is stable when short-circuited and unstable when open circuited. These properties may then be used as criteria of types.

In any known system the control indicated by 3 in Fig. 1 or by 9 in Fig. 2 is not strictly instantaneous. As the frequency increases indefinitely the control ultimately dwindles and disappears. In Fig. 1 this means that the ratio of generated electromotive force to controlling current approaches zero. In Fig. 2 it means that the ratio of generated current to controlling electromotive force approaches zero. This property may be expressed by saying that a negative impedance of the series type contains a factor η which is in the nature of a positive transfer admittance, which is substantially unity at frequencies of practical interest and which approaches zero as the frequency increases indefinitely, and that a negative impedance of the shunt type contains a factor $1/\eta$, where η is defined as before.

We are now in a position to define negative impedances of both types quantitatively. An impedance z_1 will be said to be negative and of the series type if $z_1 = -Z\eta$ and it is stable when open circuited and unstable when short circuited; and an impedance z_2 will be said to be negative and of the shunt type if it satisfies the condition $z_2 = -Z/\eta$ and is stable when short circuited and unstable when open circuited. Z is a positive impedance and η is a factor of the form discussed above. In what follows η will be limited to the form

$$\eta = e^{-\sqrt{i\omega\epsilon}}$$

where e is the base of natural logarithm,

$$i = \sqrt{-1}$$

and ϵ is a positive number, which may be taken arbitrarily small. This restriction is sufficient for practical purposes. It is not necessary here to prove that negative impedances are necessarily such as to be unstable either when open-circuited or when short circuited. It is however, desirable to point out that all impedances which approach to the property of being negative, are of one or the other type as far as existing literature discloses.

Referring now more particularly to Fig. 3, the diagram represents a network of impedances Z and z connected in series with the electromotive force $e(t)$. $e(t)$ varies with time so as to approach zero as t increases indefinitely. In order to investigate the stability of the network it is necessary and sufficient to determine whether under these conditions the current $i(t)$ arising in

the network as a response to $e(t)$ also approaches zero as t increases indefinitely. The case will first be considered when Z is a positive impedance and z is a negative impedance of the series type. Since z is stable when open circuited and unstable when short circuited it follows that there is no meaning in inquiring what current will flow in z in response to a given electromotive force but there is meaning in inquiring what electromotive force E will appear across the terminals as a result of given current. The necessary analysis will be carried out as a series of successive approximations which avoids the former inquiry. It will be assumed that the reader is familiar with my article "Regeneration Theory" published in Bell System Technical Journal, volume 11, page 126, January 1932. The notation below will be the same as the one there used except that μ is used instead of z .

Let E be defined by

$$e(t) = \frac{1}{2\pi i} \int_{s+} E(\mu) e^{\mu t} d\mu \quad (1)$$

As a first approximation we will compute the current that would flow if z were made equal to zero. It is

$$i_0(t) = \frac{1}{2\pi i} \int_{s+} E(\mu) e^{\mu t} d\mu / Z(\mu) \quad (2)$$

This procedure for computing the current for a given voltage is justified, for example, in the article "The Practical Application of the Fourier Integral" by G. A. Campbell in B. S. T. J. for October 1928. The effect $E(g)$ is my $i_0(t)$; the cause $C(g)$ is my $e(t)$. The function Y is to be taken as $1/Z$ since the cause is a voltage and the effect is a current. Particular reference is made to the equations near the top of page 656, together with pairs 101 and 102 on page 663. Reference is also made to applicant's article referred to, particularly Equation (11) for the relationship between the integral here used and the one treated by Campbell. Next we may assume this current flowing in the circuit and inquire what electromotive force it produces in z . This electromotive force can be applied to Z and a correction term for the current can thus be computed. This can be computed by the same processes as were used in Equation (2) with the distinction that the current is now to be taken as the cause and the voltage as the effect, and therefore the appropriate function to use for Y is z . We have for the drop resulting from $i_0(t)$

$$e_1(t) = \frac{1}{2\pi i} \int_{s+} E z e^{\mu t} d\mu / Z \quad (3)$$

and for the resulting correction term to the current

$$i_1(t) = \frac{1}{2\pi i} \int_{s+} E z e^{\mu t} d\mu / Z^2 \quad (4)$$

Similarly this correction term gives rise to an electromotive force across z and this gives rise to the current

$$i_2 = \frac{1}{2\pi i} \int_{s+} E z^2 e^{\mu t} d\mu / Z^3 \quad (5)$$

By repeated application we have for the n th correction term

$$i_n = \frac{1}{2\pi i} \int_{s+} E z^n e^{\mu t} d\mu / Z^{n+1} \quad (6)$$

The counterelectromotive force resulting when current flows through an impedance is of opposite sign to the electromotive force which causes that current to flow. Hence in adding these current components alternate components have their sign reversed. The total current up to and including the n th correction term is:

$$i_0(t) - i_1(t) + i_2(t) \dots + (-1)^n i_n(t) = \frac{1}{2\pi i} \int_{s+} \frac{E}{Z} \left(1 - \frac{z}{Z} + \frac{z^2}{Z^2} \dots + (-1)^n \frac{z^n}{Z^n} \right) e^{\mu t} d\mu = \frac{1}{2\pi i} \int_{s+} \frac{E}{Z(1+z/Z)} e^{\mu t} d\mu + (-1)^n \int_{s+} \frac{E(z/Z)^n}{Z(1+z/Z)} e^{\mu t} d\mu \quad (7)$$

The presence of the factor

$$e^{-\sqrt{i\omega\epsilon}}$$

in z insures that the last of these integrals approaches zero as n increases. We have then for the total current the value which the series approaches as n is increased indefinitely, namely:

$$i(t) = \frac{1}{2\pi i} \int_{s+} \frac{E}{Z \left(1 + \frac{z}{Z} \right)} e^{\mu t} d\mu \quad (8)$$

Now comparing (8) with Equation (33) in the article on "Regeneration Theory" referred to we arrive at the following stability rule:

I. If Z is a positive impedance and z is a negative impedance of the series type their combination is stable or unstable according to whether or not the point (1,0) lies outside the locus of $(-z/Z)$.

By the locus of any function $f(\omega)$ is here to be understood the curve obtained by plotting plus and minus the imaginary part of $f(\omega)$ as a function of the real part for all real and positive values of ω .

By a similar process I have found that when z is negative and of the shunt type the following rule holds good:

II. If Z is a positive impedance and z is a negative impedance of the shunt type, then their combination is stable or unstable according to whether or not the point (1,0) lies outside the locus of $(-Z/z)$.

I have also found by comparing these two rules that the following rule is true:

III. If a positive impedance Z and a negative impedance z form a stable combination then the change of type of z results in an unstable combination.

This rule emphasizes the necessity of specifying the type of impedance as well as its numerical value. If the type is left unspecified one at least of the two possibilities will be certain to be unstable.

It is sometimes desirable to make up networks containing a plurality of positive or negative impedances. This case is not covered by the rules just given. By applying the methods given above to this more general case I have found a rule for this case also which will now be stated.

Let the general network consist of h impedance branches and $g-1$ independent junction points and let

$z_1 z_2 \dots z_k$ be negative, series type
 $z_{k+1}, z_{k+2} \dots z_n$ be negative, shunt type
 $z_{n+1}, z_{n+2} \dots z_h$ be component impedances of the positive part of the net work.

Write $g-1$ equations of the form

$$\sum_g a_{pq} i_q = 0, 0 \leq p < g \quad (IVa)$$

expressing the fact that the total current flowing toward a junction point equals zero. Select $h-g+1$ independent meshes and write the equations

$$\sum_q a_{pq} i_q = 0, g \leq p < h \quad (IVb)$$

expressing the fact that the electromotive force around any mesh equals zero. Form the determinant Δ of the a 's and write

$$D = \Delta / z_{k+1} z_{k+2} \dots z_n$$

The stability rule then is:

IV. If the origin lies outside of the locus of D , the circuit is stable. If not, it is unstable.

It sometimes happens when some of the impedance elements are pure reactances that the quantity to be plotted, $-z/Z$, $-Z/g$, or D takes on infinite values and thus cannot be plotted. I have found that the plots can be completed and that they lead to correct results if a small resistance or a small conductance is included with the reactance. For example, a capacitance has an infinite impedance a at zero frequency and therefore an expression which has this impedance in the numerator cannot be plotted. If an arbitrarily small conductance is bridged around the condenser, the result can be plotted. Similarly, a pure inductance has zero impedance at zero frequency and an expression having it as a factor in the denominator cannot be plotted. By adding a small series resistance this difficulty can be overcome.

The application of the rules enumerated can be made clearer by few specific examples. Referring to Fig. 4, 13 is a coil whose impedance is sufficiently well represented by an inductance L and a resistance R , both positive, connected in series. Now, suppose it has been decided that R has been found greater than desirable and it has been proposed to reduce it by connecting the negative resistance, 14, in series therewith. By negative resistance is to be understood an impedance having the value $-r\eta$ or $-r/\eta$ as the case may be, r being real and positive and η being as described hereinbefore.

Referring to Fig. 5 curve A represents the locus for the case where the negative resistance is of the series type and r is numerically greater than R . The point (1,0) lies inside of this curve and therefore the combination is unstable (Rule I, above). Curve B represents the curve for the case when the negative resistance is of the series type and R is numerically greater than r . The point (1,0) lies outside of this curve and the combination is stable (Rule I).

Fig. 6 represents the locus when the negative impedance is of the shunt type (Rule II). Here the point (1,0) is seen to lie inside of the locus regardless of whether r is great or small in comparison with R . With a negative impedance of the shunt type, then, the proposed circuit is always unstable. I have thus found that a negative resistance for the purpose contemplated must be numerically smaller than the positive resistance, which might have been suspected from the known facts when a negative and a positive resistance both without reactance are connected together. I have furthermore found, however, that the negative impedance must not be of the shunt type, no matter what its value, which would not be suspected from the known factor about connecting resistances together.

The question might be under consideration of connecting negative capacitance across a tele-

phone line to neutralize some or all of the natural line capacitance. In order to exemplify the methods herein described this case may now be considered. As a first step, let the line be distortionless; that is, let its line constants, L , C , R and G be uniformly distributed and so related that $L/C = R/G$. It is well known that such a line has a characteristic impedance which is a pure resistance, K , independent of the frequency. If a negative capacitance is to be connected about such a line, the question that must be answered is: Under what conditions may a negative capacity be bridged across a pure resistance?

If the negative capacitance is of the series type, Rule I may be applied. We have

$$Z = \frac{K}{2}$$

the characteristic impedance being K , and the line east and the line west being effectively in parallel, and $z = -[1/i\omega C]\eta$. The quantity to be plotted is then $2\eta/i\omega CK$. This expression is infinite for $\omega = 0$, so that according to the instructions given above we may add the extremely small conductance $-g$ across the capacitance making the quantity to be plotted $(2\eta/K)(i\omega C + g)$. Referring now to Fig. 7, the plot for this quantity shows that (1,0) lies inside and the circuit is unstable for all values of C . Therefore, no negative capacitance of the series type may be connected across a positive resistance.

If next the negative capacitance is of the shunt type, Rule II applies and we have

$$Z = \frac{K}{2}$$

$Z = -1/i\omega C\eta$ and the quantity to be plotted is $-Z/z = i\omega CK\eta/2$. This plot is shown in Fig. 8 and again shows the point (1,0) on the inside for any value of C . Hence, no negative capacitance of the shunt type may be connected across a positive resistance.

The proposal to connect a negative capacitance across a line which is already distortionless has then been found fruitless because both types of negative capacitance lead to an unstable condition regardless of how small or how large the value. The next step to consider then is the case where the line is not distortionless to begin with but has an excess of capacity. To illustrate the principles, it is sufficient to answer to the question: Under what conditions may a negative capacitance be bridged across a positive impedance consisting of a resistance and a capacitance in parallel? The consideration of the case in which the capacitance is of the series type is not essentially different from the considerations which led to Fig. 7, and the conclusion is the same as there arrived at, namely, that no negative capacitance of the series type is permissible. If the negative capacitance is of the shunt type, we have for the application of Rule II that $Z = 1(i\omega C + g)$, $z = -1/i\omega C\eta$ and the quantity to be plotted is $-Z/z = i\omega C\eta/(i\omega C + g)$. The resultant plots are shown in Fig. 9 for two conditions. Curve A represents the case in which $C/c > 1$, and shows an unstable condition. Curve B represents the case in which $C/c < 1$, and shows a stable condition. Hence, a negative capacitance of the shunt type may be used, provided it is numerically smaller than the positive capacitance. To sum up: Across a positive network consisting of a resistance and a capacitance in parallel, there can be connected no negative capacitance

of the series type, but a capacitance of the shunt type can be connected, provided it is numerically less than the positive capacity.

Similarly, it may be proposed to bridge a negative capacitance across a positive network consisting of a resistance and a capacitance in series. The analysis is sufficiently like that given so that it need not be gone through with in this case. The result is: Across a positive network consisting of a resistance and a capacitance in series there can be connected no negative capacitance of the shunt type, but a negative capacitance of the series type can be connected, provided it is numerically greater than the positive capacitance.

Similarly, by a straightforward application of the rules stated above, those familiar with networks will be readily able to construct a locus for any proposed network and determine whether or not it is stable. It should be specifically pointed out that Rule IV permits the inclusion of a plurality of negative impedances of either or both types.

Most of the negative impedances described in the literature on the subject are such that the designer can determine their type by a comparison with Figs. 1 and 2, described above. There are cases, however, where the negative impedances are constructed out of a negative resistance and positive impedance elements. A discussion of such a case will be given with the two-fold object of enabling the designer to determine the type in similar cases and of giving one more illustration of the stability rules.

Referring now to Fig. 10, numerals 15 and 16 represent the terminals of a network made up of a positive impedance W , two positive resistances r and a negative resistance $-r$ (strictly $-r\eta$ or $-r/\eta$). It can be shown by straightforward calculations that the impedance of such a network is $-r/W$. It is therefore a negative impedance. To determine its type assume first that the resistance $-r$ is of the series type. With the terminals 15, 16 short circuited the network consists of $(-r)$, r , and W in parallel. Applying Rule I it is apparent that $-z/Z$ is greater than +1 for $\omega = 0$ and $-z/Z = 0$ for $\omega = \infty$. The point (1,0) is therefore inside the locus and there is instability. Similarly when the terminals are open circuited $-z/Z$ is not positive and greater than unity for any value of ω . The circuit is therefore stable. The whole network being stable when open circuited and unstable when short-circuited is of the series type. Similarly when the resistance element $(-r)$ is of the shunt type the whole network is of the shunt type.

An example will now be given of an application of Rule IV. It will be understood that the application of this rule will generally result in rather lengthy calculations. In order to reduce the discussion the case taken is simplified by being limited to two negative impedances and in other respects. It does not however omit anything that is essential in the study of more complex cases.

In Fig. 11 let 1 and 2 be the conductors of a uniform line and let 23 and 24 be impedances terminating it. Let a and b be sections of the line situated at distances equal to one-fourth of the entire distance from the ends and let it be required to determine what negative impedances may be bridged at a and b without causing instability. For simplicity let it be agreed that the line is such that $RC > LG$ and that the negative impedances should be of the shunt type.

As a first step find a π network which is equivalent to the portion of the line between a and b and find impedances equivalent to the portion to the left of a and similarly to the right of b including the terminating impedances. The method of making such computations are given for example in the book "Transmission Circuits for Telephone Communication" by K. S. Johnson. After these computations have been made the problem under consideration is equivalent to the problem of the network shown in Fig. 12, where z_{11} and z_{31} are equivalent to the portions of the given circuit to the left of a and to the right of b respectively, z_{12} , z_2 , z_{32} are equivalent to the portion between a and b , and z_4 and z_5 are the negative impedances of the problem. Next compute the impedances z_1 and z_3 which are equivalent to z_{11} and z_{12} and z_{31} and z_{32} in parallel respectively. The new equivalent network is shown in Fig. 13.

At the junction of z_1 , z_4 and z_2 the following equation expresses the fact that the net current converging there is zero

$$i_1 + i_4 - i_2 = 0$$

Similarly for the junction of z_2 , z_5 , and z_3

$$-i_2 + i_5 + i_3 = 0$$

The following three equations express the fact that the net electromotive force around the three meshes shown is zero in each case.

$$z_1 i_1 - z_4 i_4 = 0$$

$$z_4 i_4 + z_2 i_2 + z_5 i_5 = 0$$

$$z_3 i_3 - z_5 i_5 = 0$$

The determinant of these five equations is

$$\begin{vmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \\ z_1 & 0 & 0 & -z_4 & 0 \\ 0 & z_2 & 0 & z_4 & z_5 \\ 0 & 0 & z_3 & 0 & -z_5 \end{vmatrix}$$

which in accordance with a well known rule of algebra may be written:

$$z_1 z_2 z_3 + z_1 z_2 z_5 + z_1 z_3 z_4 + z_1 z_3 z_5 + z_1 z_4 z_5 + z_2 z_3 z_4 + z_2 z_4 z_5 + z_3 z_4 z_5$$

In order to obtain the quantity D to be plotted this expression is to be divided by $z_4 z_5$ since both the negative impedances are to be of the shunt type. Also on account of the assumed symmetry we have $z_1 = z_3$ and $z_4 = z_5$ we then have

$$D = \frac{z_1^2 z_2}{z_4^2} + \frac{2z_1(z_1 + z_2)}{z_4} + (2z_1 + z_2)$$

In general the next step is to substitute the values of z_1 and z_2 as computed for the given line and to substitute such values for z_4 as it is proposed to investigate. This gives a numerical value for D for each frequency and the real and imaginary parts are plotted as indicated previously. The location of the resulting curve with respect to the origin determines whether or not the resultant circuit is stable.

In this particular case, however, a simpler procedure may be followed. The expression for D may be factored:

$$D = \left(\frac{z_1}{z_4} + 1 \right) \left(\frac{z_1 z_2}{z_4} + 2z_1 + z_2 \right)$$

If neither factor when plotted results in a curve which encloses the origin it is clear that the product also does not result in a curve which encloses

the origin. Conversely it can be shown that if either factor (or both) gives a curve which encloses the origin so does the product. Now, under the conditions assumed the reactive part of z_1 does not vary as rapidly as the inverse first power of the frequency. If then z_4 be made a negative capacitance there will be a frequency for which the real part of $-z_4/z_4$ is greater than unity and therefore the curve includes the origin and the circuit is unstable. This is true even if the negative capacitance be taken numerically small, the frequency for which the real part of $-z_1/z_4$ becomes greater than unity being correspondingly higher. It may be concluded that a negative capacity cannot be used.

Now if in Fig. 11 the terminating resistances z_{23} and z_{24} be made equal to the characteristic impedance of the line K the impedances z_{11} and z_{31} in Fig. 12 are also equal to K . The impedances z_{12} and z_{32} are equal to

$$K / \tanh \frac{P}{2}$$

and the impedance Z_2 is equal to $K \sinh P$ where P is the propagation constant of the line section between a and b . See for example page 281 of K. S. Johnson's book. If we now attempt to make z_4 equal to $-MK/\eta$ where M is a positive constant the expression for D reduces to the form

$$\left(-\frac{\eta}{M \left(1 + \tanh \frac{P}{2} \right)} + 1 \right) \left(-\frac{\eta}{M \left(1 + \coth \frac{P}{2} \right)} + 1 \right)$$

except for a factor made up of positive impedances which does not affect the question of whether the origin is included. Now it can be seen either by drawing the curves or by inspection in view of the known properties of $\tanh$

$$\frac{P}{2}$$

that the circuit is stable provided M is taken so large that

$$M \left(1 + \tanh \frac{A}{2} \right) < 1$$

for all frequencies, where A is the real part of P .

It thus appears that while negative capacitance cannot be employed for the purpose contemplated it is nevertheless possible to employ negative impedances when they are suitably chosen. It may be observed that when the midsection is short so that A approaches zero the permissible value of M approaches 1, but when the section is long

$$\frac{A}{2}$$

so that A is great (and $\tanh A$ is near unity) the permissible value of M approaches $1/2$. For intermediate lengths intermediate values are permissible. It should also be pointed out that M may be given values greater (by any amount) than these limiting values. Finally it should be remarked that impedance of forms other than $-MK/\eta$ are permissible, the criterion for any proposed impedance being whether or not the resulting curve of D encloses the origin.

It will be particularly understood that the various cases discussed hereinbefore are examples and in no way limit the scope of the invention, which is set forth in the appended claims.

What is claimed is:

1. The method of stabilizing a circuit including positive impedance elements and a negative impedance which consists in adjusting the constants of said circuit for a given frequency and

measuring the positive impedance and the negative impedance, adjusting the circuit for another frequency and again measuring the positive and negative impedances, and continuing until the shape of the plot of such measurements can be obtained for all frequencies from zero to infinity, plotting the locus of the real part of the ratio of the positive to the negative impedance against the imaginary part of such ratio at said frequencies, and adjusting the constants of the circuit until the said locus is a closed curve that excludes the point for which the real part of the ratio is unity and the imaginary part is zero.

2. The method of stabilizing a circuit which includes one or more positive impedances and one or more negative impedances which comprises adjusting the constants of said circuit for a given frequency and measuring the impedances to determine the quantity D as defined herein, adjusting the constants of said circuit for each of a number of other frequencies and measuring the positive and negative impedances necessary to determine the quantity D over a sufficient range of frequencies so that the shape of the plot of

such measurements can be obtained for all frequencies from zero to infinity, plotting the locus of the real part against the imaginary part of the quantity D at said frequencies, and adjusting the constants of the circuit until the said locus is a closed curve which does not include the origin.

3. A series type negative impedance circuit having a pair of terminals, a positive impedance connected to the end of said circuit opposite said terminals, a network of Π configuration connected in the circuit between said terminals and said impedance, the series arm of said network being a negative impedance of the series type, and the shunt arms being positive impedances.

4. A shunt type negative impedance circuit having a pair of terminals, a positive impedance connected to the end of said circuit opposite said terminals, a network of Π configuration connected in the circuit between said impedance and said terminals, the series arm of said network being a negative impedance of the shunt type, and the shunt arms being positive impedances.

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