April 7, 1936.

M. K. ZINN

2,036,381

IMPEDANCE EQUALIZING SYSTEM

Filed Sept. 12, 1934

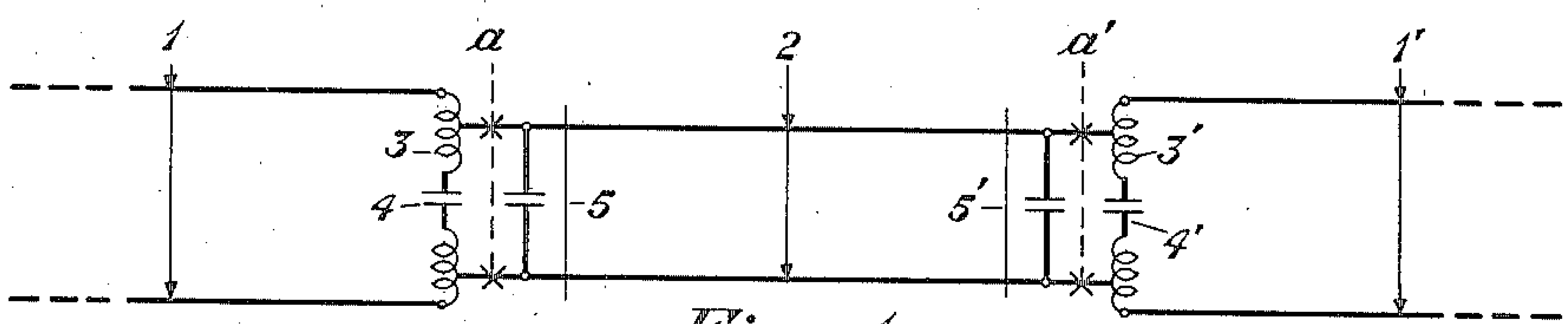

Fig. 1

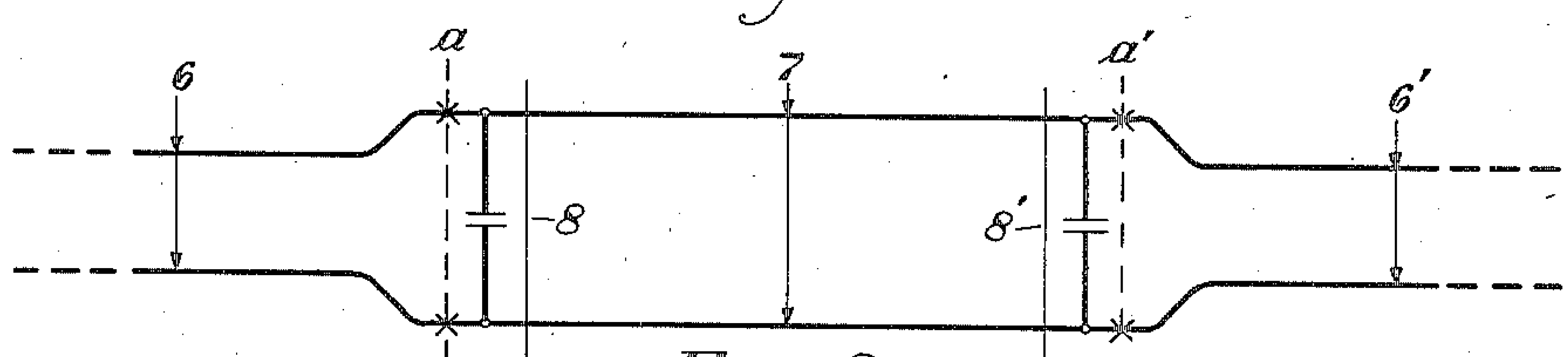

Fig. 2

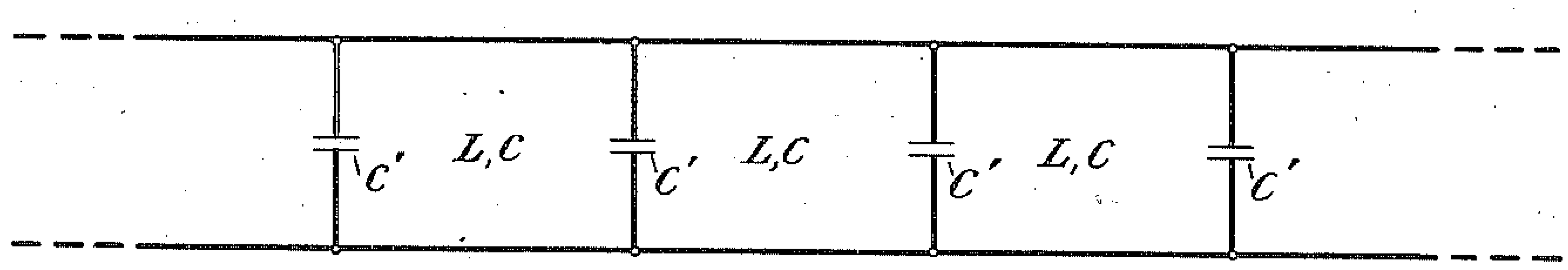

Fig. 3

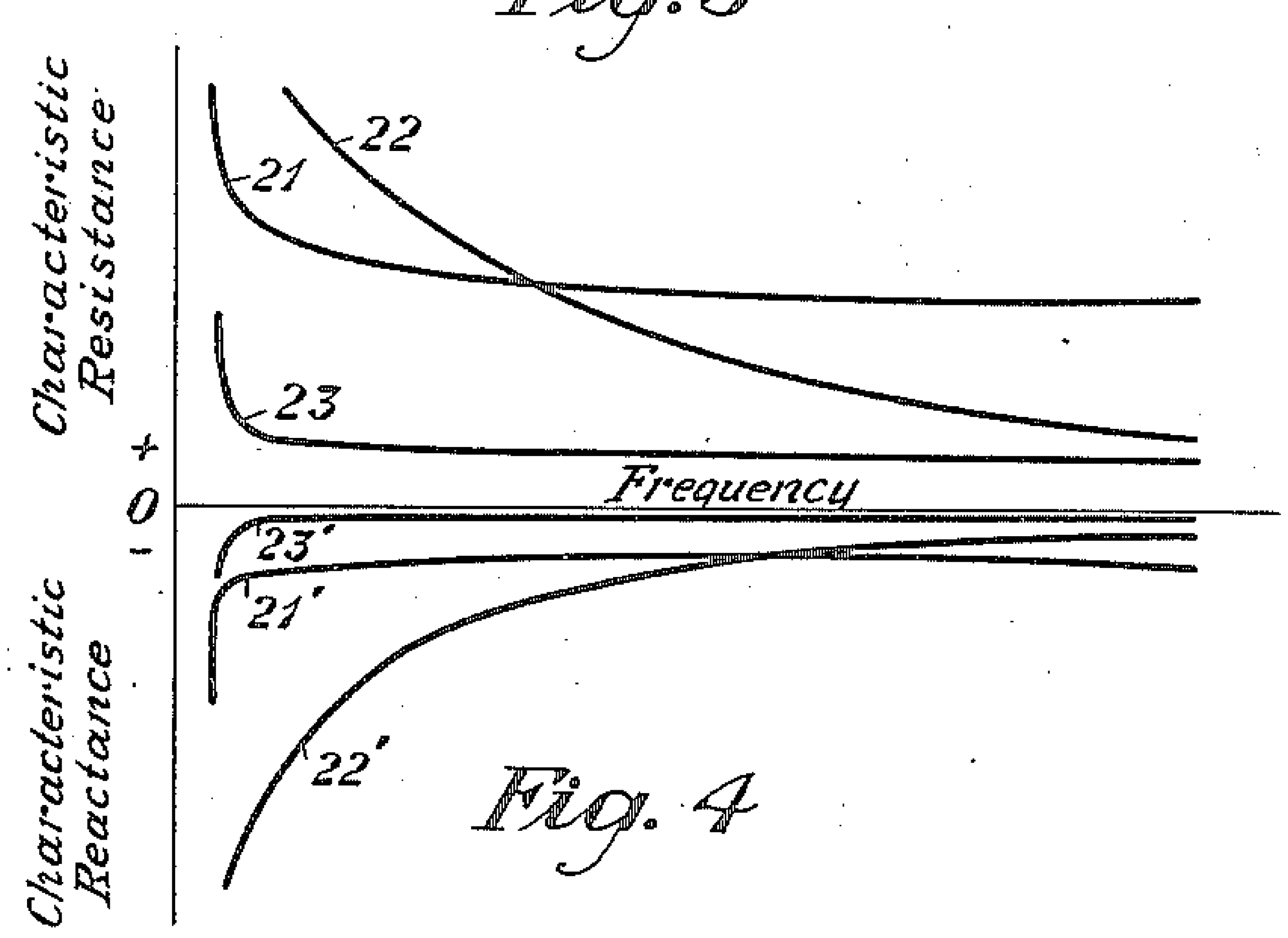

Fig. 4

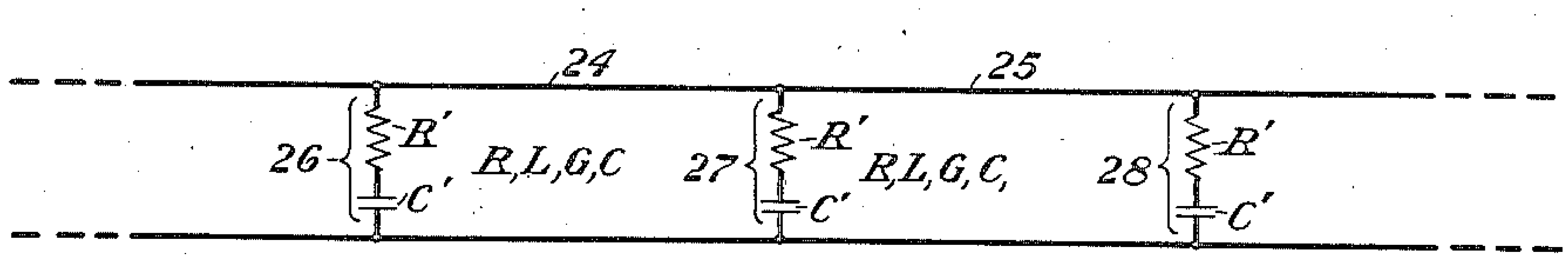

Fig. 5

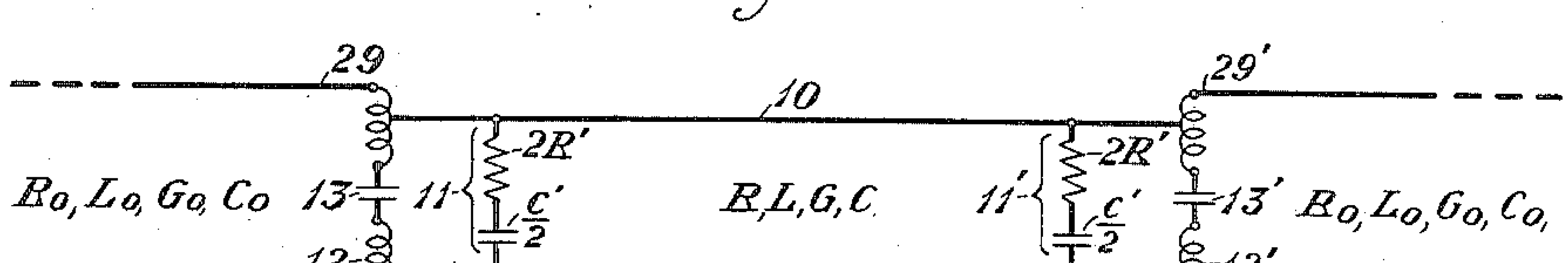

Fig. 6

INVENTOR
M. K. Zinn
BY
[signature]
ATTORNEY

Patented Apr. 7, 1936

2,036,381

UNITED STATES PATENT OFFICE

2,036,381

IMPEDANCE EQUALIZING SYSTEM

Manvel Keeport Zinn, Brooklyn, N. Y., assignor to American Telephone and Telegraph Company, a corporation of New York

Application September 12, 1934, Serial No. 743,765

6 Claims. (Cl. 178—44)

My invention relates, in general, to transmission lines, such as a pair of wires in a cable used for telephone communication, and more particularly to apparatus and a method for obtaining certain desirable properties in such a line by introducing additional capacitance and/or other impedance elements in shunt between the wires.

One object of my invention is to decrease the impedance of the line by a predetermined amount in order to reduce, or substantially eliminate, the reflection of electric waves at the junctions of the line with other types of line, or with apparatus having on impedance lower than that of the unmodified line.

Another object of my invention is to simplify the practical application of transformers, autotransformers or other reflection reducing apparatus by permitting such apparatus to be constructed with a single impedance ratio, or, at the most, with only a few impedance ratios, the remaining impedance difference between particular lines, or lines and apparatus, then being minimized by the use of shunted capacitances in the manner hereinafter described.

Another object of my invention is to reduce, or substantially eliminate, the reflection of low frequency electric waves at the junction of two transmission lines by adjusting the low frequency impedance of one line to make it closely equal to that of the other.

Another object of my invention is to accomplish a saving in the cost of reducing the low frequency impedance of a transmission line by providing means that are less expensive than other means that might be used, such as increasing the size of the copper line conductors.

Another object of my invention is to facilitate the practical application of transformers or autotransformers as devices for reducing the impedance irregularity at the junction of two dissimilar lines, such as, for example, a cable pair and an open wire pair.

The objects of my invention and the method of attaining them may be more fully understood after reading the following description of certain telephone transmission problems to which the invention affords a convenient solution.

Open wire lines frequently contain short lengths of cable, particularly where the circuits come into cities and where rivers must be crossed. These unavoidable pieces of cable give rise to difficult problems in connection with reducing the impedance irregularities at the points where they join the open wire line. It is a familiar fact in the telephone art that the transmission gain that can be obtained with a two-way repeater depends upon the accuracy with which the impedance of a network can be made to simulate the line impedance over the desired frequency range. If an open-wire line, for example, contains irregularities, such as those introduced by the presence of a piece of cable having a characteristic impedance lower than that of the open wire line itself, reflections will occur at the irregularity, and these reflections will cause the sending end impedance of the line at the repeater to rise and fall in a succession of waves as the frequency is varied. Such a wavy impedance characteristic cannot be matched with any simple standard form of balancing network. For carrier frequency transmission in which two-way operation on the same pair of wires is not practiced, no such network balance problems are involved, but it is nevertheless important that the line be comparatively free from irregularities, because the reflection of unwanted interfering currents at the irregularities would cause the interfering currents traveling in the same direction as the signal currents to be augmented by the reflected portion of those originally traveling in the opposite direction.

One way of altering the impedance of a cable to make it match that of the open-wire line, in order to eliminate the irregularity, is to load it; this is common practice. The required impedance, together with the necessity of keeping the cutoff frequency of the loading well above the highest frequency to be transmitted, determines the inductance of the loading coils and their spacing. The higher the frequencies to be transmitted, the shorter must be the coil spacing. For example, if only voice frequencies are to be transmitted, the loading coils can be placed as far as two miles apart, while for carrier frequencies (up to about 30 kilocycles) the spacing cannot exceed about 1000 feet. In a number of cases, however, submarine crossings are encountered where the length of cable required exceeds the maximum permissible loading spacing, and where the depth of the water or other conditions do not permit submarine loading coil cases to be used. Autotransformers are then resorted to for the purpose of stepping up the characteristic impedance of the cable to equal that of the open wire. The use of autotransformers is generally less desirable than the use of loading, owing to the greater attenuation of the non-loaded cable plus the losses in the autotransformers, and to the impairment of d. c. telegraph transmission. But in many cases the conditions are such as to make the use of autotransformers, or transformers, necessary for the geographical reasons mentioned above, or desirable, at least, for other reasons.

The problem of providing autotransformers for entrance and intermediate cables is a difficult one from the practical standpoint because, although cable impedances do not vary appreciably, the open-wire impedances have various values depending upon the gauge of the wire and the spacing between wires. Since the demand for autotransformers is relatively small, it is expensive and inconvenient to standardize a series of autotransformers covering a sufficient number of winding ratios to accommodate all the open-wire cable combinations that may be encountered in various installations. By the combined use of autotransformers and capacitances shunted across the cable, however, it is possible to standardize a single autotransformer having a ratio of the right value to connect the open-wire line of highest impedance with a cable. This autotransformer can then be used with other types of open-wire line having lower impedances by the use of shunt capacitances to reduce the cable impedance to the point where the ratio is correct again. This procedure involves a small increase in attenuation loss on most of the cable open-wire situations in exchange for the practical advantage of having a single type of autotransformer. Shunting the cable with capacitance at its ends can be done conveniently by the use of building-out stub cable, which is now in general use for other purposes. Since the stub cable is external to the case containing the autotransformers, it can be adjusted at will to suit the requirements of any particular installation, and does not, therefore, introduce any complication of standards to offset the simplification obtained by using a single type of autotransformer for all jobs.

My invention may now be more fully understood by means of the following description when read in connection with the accompanying drawing.

In this drawing, Figure 1 is a diagram of a transmission line with an interposed relatively low impedance section connected according to my invention. Fig. 2 is a diagram of a transmission line with an interposed relatively high impedance section connected according to my invention. Fig. 3 is a diagram of an ideal line to which reference will be made in explaining the principles of my invention. Fig. 4 is a curve diagram showing certain impedance components as functions of frequency. Fig. 5 is a diagram of a line embodying elements of my invention in a form specially adapted for low frequencies, and Fig. 6 is a diagram showing my invention in a form adapted for both high and low frequencies.

The sketch designated Fig. 1 shows an intermediate cable connected to an open-wire line by means of autotransformers. On the sketch, **1** and **1′** are the two portions of a line of relatively high impedance, such as an open-wire line. The two portions of the line, **1**—**1′**, are connected by a line, **2**, of relatively low impedance, such as a cable pair. **3** and **3′** are autotransformers which have been introduced to step up the impedance of the line **2** approximately to equal that of the line **1**—**1′**. In the particular situation where the invention is applicable, **3** and **3′** have an impedance ratio somewhat too high to equalize exactly the impedance difference between the two kinds of line. Therefore, the lumped capacitances **5** and **5′** are connected across the ends of the line **2** to reduce, or substantially eliminate, the remaining impedance dissimilarity. **4** and **4′** are merely condensers commonly introduced at the electrical midpoints of the autotransformers for the purpose of avoiding short circuits on the line with respect to direct currents used for testing purposes or for telegraph transmission. Other apparatus, such as shunt impedance elements to improve the impedance ratio of the autotransformer at low voice frequencies and composite sets to by-pass the telegraph circuits through a separate cable pair, is frequently associated with the autotransformers.

For the amounts of impedance correction involved in entrance or intermediate cable problems, it usually will be sufficient to place the capacitance lumps only at the ends of the cable. In the case of long cables, however, it may be necessary, in accordance with certain theoretical principles, to be explained later, to divide the total capacitance required into finer lumps, connecting some of the lumps at additional points along the cable. Where the conditions in submarine cables, for example, are not favorable to the use of building-out stub cables at underwater points, the desired addition of capacitance at submarine points can be obtained conveniently by connecting in series with the ordinary cable short lengths of high capacitance cable.

The sketch designated Fig. 2 illustrates another situation where my invention provides a practical solution. **6** and **6′** are the two portions of a line of relatively low impedance, such as a cable pair or a concentric main; while **7** is an intervening length of a line of relatively high impedance, such as an open-wire line. The undesirable impedance irregularities that exist at the two junctions of **7** with **6**—**6′** can then be reduced by connecting the capacitances **8** and **8′** in shunt across the junction points, as shown, or by connecting at more than two points finer lumps of capacitance having a total value the same as that of **8**+**8′**.

It will be appreciated that the invention also would be embodied in arrangements the reverse of those shown in Figs. 1 and 2, namely, if the lines of relatively low and high impedances were interchanged so that the capacitance lumps were distributed along **1** and **1′** in Fig. 1 or along **6** and **6′** in Fig. 2. It also will be appreciated that the invention would be embodied in arrangements whereby the lines **1**—**1′** or **6**—**6′** were replaced by terminal apparatus of the same impedance as these lines. It further will be appreciated that the invention would be embodied in situations where autotransformers or transformers were involved in the case shown in Fig. 2.

In order to obtain the advantages of the invention it is necessary to know how to calculate the size of capacitances required to accomplish the desired impedance correction for any given case. Also it is necessary to know how to calculate the maximum permissible spacing of the capacitances in order to determine whether it will be sufficient, in a given case, to connect them across the intervening line at its terminals only, or whether it will be necessary further to divide the total required capacitance into finer lumps connecting one or more lumps across the line at points intermediate between its terminals. It is therefore necessary to have a theoretical knowledge of the principles involved and these principles, now to be disclosed, are an integral part of the specification of my invention.

The requirement for eliminating reflection effects in the arrangement shown in Fig. 1 is that the characteristic impedance of an infinite periodic line formed by repeating the line 2 with its shunt capacitance between points a and a', when multiplied by the impedance ratio of the autotransformer, must equal the characteristic impedance of the line 1—1'. Similarly, for the arrangement shown in Fig. 2, the requirement is that the characteristic impedance of an infinite periodic line formed by repeating the intervening line 7 with its shunt capacitances between the points a and a' must equal the characteristic impedance of the line 6—6'.

To determine the proper values for the capacitances 5—5' or 8—8', and to examine their effect upon the attenuation and impedance and upon the variation of these quantities with frequency, we have to approach the problem, therefore, by examining the properties of an infinitely long line across which are connected capacitances at equal intervals. It will be found that connecting capacitances across a line causes the originally infinite transmission band of the unmodified line to be interrupted by an infinite succession of attenuation bands. These effects are similar to the frequency discrimination properties of a line having distributed inductance and capacitance when inductance coils are serially inserted at periodic intervals, as described in U. S. Patent No. 1,602,491, to R. S. Hoyt.

In the line having shunt capacitances, it is essential that the capacitances be so spaced that the first cutoff frequency at the lower margin of the first attenuation band shall be considerably higher than the highest frequency to be transmitted. This is for the reason that, at frequencies in a certain range considerably below this first cutoff frequency, the characteristic impedance of the capacitance-shunted line is substantially a constant resistance that can be adjusted to any desired value lower than that of the cable without the added capacitances. As the first cutoff frequency is approached, however, the impedance varies rapidly with the frequency. The cutoff frequency will be higher the closer the spacing of the capacitances. There is, therefore, a design technique involved in determining the proper spacing of the capacitances in such a line, just as there is a design technique in spacing loading coils properly in a coil-loaded line.

An exposition of the attenuation characteristics of any transmission system having prominent selective effects with respect to the frequency of the current transmitted is most conveniently carried out by considering first the characteristics of an ideal non-dissipative system, that is, one in which all the elements are pure reactances in which energy can be stored but not dissipated. Approaching such problems from the standpoint of the non-dissipative case reduces the number of variables to a point where the principal features of the transmission characteristics can be set forth in a clear and instructive manner. Furthermore, for proper working, the amount of dissipation must be small, so that most of the characteristics of an actual dissipative system usually approach those of the theoretical non-dissipative system quite closely.

In the sketch designated as Fig. 3 is shown a smooth non-dissipative line divided into sections each having a distributed inductance L and a distributed capacitance C. Such a line has a pure resistance characteristic impedance at all frequencies and no attenuation at any frequency. If, now, shunt capacitances C' be introduced across each section (L, C) of the line as shown, its infinite transmission band is replaced by an infinite series of transmission bands separated by attenuation bands. In the limiting case, where the normal distributed capacitance of the line between loads is negligible compared with the added shunt capacitance, the line will have only one transmission band extending up to a certain cutoff frequency, and one attenuation band extending from that frequency to infinity. The attenuation characteristics of a capacitance-shunted line may, therefore, be described in an alternative manner with reference to this limiting case having negligible distributed capacitance by saying that the effect of the distributed capacitance of the line between the capacitance lumps is to break up the infinite attenuation band of the limiting line into an infinite series of finite transmission and attenuation bands. It will be recognized that a capacitance-shunted line having negligible distributed capacitance is the same thing structurally as an inductance-loaded line having negligible distributed inductance.

The disposition of the transmission and attenuation bands of the capacitance-shunted line can be visualized most readily by introducing the conception of a "compound band" comprising a transmission band and the succeeding attenuation band, in accordance with the procedure followed by Hoyt in Patent No. 1,602,491. It can then be stated, first of all, that the widths of all the compound bands are equal; the width of each in terms of the phase constant of a section of line between two successive capacitance lumps is equal to π. On the frequency scale, the width of each compound band is

$$1/2\sqrt{LC}$$

If the lumped capacitance is large compared to the distributed capacitance, the attenuation bands will all be wider than the transmission bands. On the other hand, if the lumped capacitance is small compared to the distributed capacitance, the lower transmission bands will be wider than their succeeding attenuation bands. In either case, however, the attenuation bands become wider, and the transmission bands narrower, as the frequency is increased.

The first cutoff frequency at the transition point between the first transmission band and the first attenuation band is the cutoff of major interest. This cutoff point can be determined by a simple calculation in the case of a capacitance-shunted line having negligible distributed capacitance, being given by the formula,

$$f_c = 1/\pi\sqrt{LC'}$$

When the distributed capacitance or inductance is considerable, however, as compared to the corresponding lumped quantities, the upper limits of transmission bands occur where either

$$w \tan w = q \text{ or } w \cot w = -q$$

and the lower limits of transmission bands, which are the margins of the compound bands above referred to, occur where $w = n\pi/2$, where $n = 0, 1, 2, 3$—etc. The symbols in these formulas are defined in terms of L, C and C' by

$$w = \frac{\omega}{2}\sqrt{LC}$$

(=half the phase constant of a line interval between successive capacitance lumps), $\omega=2\pi$ frequency and $q=C/C'$.

A quantity of principal interest in the design of a capacitance-shunted line is the nominal impedance of the line, which is the value that the impedance of the non-dissipative line approaches at low frequencies. The nominal impedance is given by

$$\sqrt{L/(C'+C)}$$

It will be seen, therefore, that the nominal impedance of a condenser-loaded line can be arrived at merely by considering the distributed line capacitance as being added directly to the lumped capacitance, just as, in the case of an inductance-loaded line, the distributed inductance may be considered as being added directly to the lumped inductance.

In an actual dissipative capacitance-shunted line at low frequencies, not exceeding about one-fifth of the first cutoff frequency, the characteristic impedance of the line is given by the following approximate formula:

$$K \doteq \sqrt{\frac{L}{C}}\sqrt{\frac{q}{1+q}}\sqrt{\frac{1+R/i\omega L}{1+(G+G')/i\omega(C+C')}}$$

where R and G are the line resistance and leakance per line interval between successive capacitance lumps, G' is the leakance of one of the capacitance lumps, and the other quantities are as already defined.

In this approximate formula, the first factor is the nominal impedance of the unmodified non-dissipative line; the second factor is the factor by which this nominal impedance is modified by the addition of the capacitance lumps. The first two factors multiplied together give the nominal impedance of the capacitance-shunted line. The third factor gives the modifying effect of dissipation.

It will be seen that, as the frequency increases, the third factor, that giving the effect of dissipation, approaches unity, so that the impedance approaches the pure resistance nominal impedance. If the lines were smooth, the impedance would approach the nominal impedance more and more closely as the frequency is further increased. This is not true of the line with lumped capacitances, however, because the above approximate formula does not hold for the lumpy line as the first cutoff frequency is approached. For frequencies above about .6 of this cutoff, the impedance depends upon the part of a loading section in which the line is terminated, and upon the relative distribution of the smooth and lumped constants of the line, and comparatively upon the dissipation. For a frequency, f, in the range from about .6 to about .9 of the first cutoff frequency f_0, the impedance at the center of one of the capacitance lumps is given by

$$\sqrt{L/(C+C')}/\sqrt{1-(f/f_0)^2}$$

and at the center of a line section by

$$\sqrt{L/(C+C')}.(\sqrt{1-(f/f_0)^2}$$

approximately. When the other line or apparatus connected to the capacitance-shunted line has an impedance approaching a pure resistance at the highest frequency to be transmitted, as is usually the case, the capacitance lumps across the intervening line must be placed at sufficiently short intervals to make this frequency not greater than about .6 of the first cutoff frequency of the capacitance-shunted line.

An example of the design of a capacitance-shunted line now be given to contribute to a clear understanding of the application of the principles just explained.

Suppose it were desired to use a 1200-foot cable for bringing in circuits from a radio antenna to the receiving apparatus. The high frequency impedances of the antenna and of the apparatus are both 75 ohms, while the cable circuit whose impedance will most nearly match this value is a phantom circuit having an impedance of 80 ohms. It is therefore desired to reduce the impedance of the phantom circuit from 80 to 75 ohms. It is required to know how much lumped capacitance must be added to the phantom to accomplish this result. Also it is required to determine at how many points in the cable the capacitances must be added in order to keep the first cutoff frequency well above the frequencies to be transmitted (50 to 70 kilocycles). In particular, it is required to determine if the cutoff frequency will be high enough when the required capacitances are added only at the two ends of the cable.

The frequencies to be transmitted are so high that dissipation has a very small effect upon the cable impedance. Also, care will be taken to distribute whatever capacitance is required over a sufficient number of points in the cable to keep the first cutoff well above the transmitted frequencies; the impedance will not be affected, therefore, by the aforementioned "lumpiness effects" that arise as the cutoff is approached. Therefore, the impedance of the capacitance-shunted cable may be regarded, within an order of accuracy appropriate to the case, as substantially a pure resistance equal to the nominal impedance. It is desired to reduce the nominal impedance of the cable phantom circuit (80 ohms) to 75 ohms. Therefore,

$$\frac{C+C'}{C}=1+\frac{1}{q}=\frac{(80)^2}{(75)}=1.137$$

and $q=7.3$.

The total capacitance of a phantom circuit 1200 feet in length is 22,700 micro-microfarads. Therefore, we require a total added capacitance of 22,700/7.3=3,110 micro-microfarads.

The next thing to determine is the spacing of the capacitance lumps in order to satisfy the cutoff requirements. We may consider first the two simplest schemes, namely, dividing the 3,110 micro-microfarads into two equal lumps to be connected across each end of the cable, or placing the whole 3,110 micro-microfarads in one lump at the center of the cable. It will be apparent that both of these schemes give the same cutoff frequency, because the same infinite structure would be arrived at by repeating the cable modified according to either scheme. To find the cutoff for either of these conditions, the equation $w \tan w=q$ may be solved for the value of $q=7.3$. The solution is most easily found graphically by plotting $w \tan w$ for various values of w to find where the graph crosses the value 7.3. The solution is found to be $w=1.38$.

Then

$$1.38=\frac{\omega_c}{2}\sqrt{(.0007)(.10)(.227)^2(10^{-6})}$$

where .0007=inductance of phantom per mile in henries.

.10 =capacitance of phantom per mile in microfarads.

.227 =length of cable in miles (1200/5280)

From this $\omega_c=1.45\ (10^6)$

or, cutoff$=\omega_c/2\pi=232$ kilocycles.

Inasmuch as the highest frequency to be transmitted (70 kilocycles) will then be only about .3 of the cutoff, it is certain that the impedance in the desired range, 50–70 kilocycles, will be satisfactorily constant and closely equal to the required value of 75 ohms.

The capacitance to be added to the cable phantoms may be provided by "building-out", with the use of stub cable, to a total value of 22,700+3,110=25,810 micro-microfarads. The "building-out" required for each quad may be provided by two stub cables placed at the two ends of the main cable; it may be provided by a single stub cable placed at the approximate center of the main cable; or, again, it may be provided by inserting in series with the main cable suitable lengths of high capacitance cable at either the center or at the two ends of the main cable.

On Fig. 4 are illustrated typical impedance characteristics of two uniform transmission lines. The curves 21 and 21′ represent the characteristic resistance and reactance, respectively, of one line, while the curves 22 and 22′ represent the corresponding quantities for the other line. At high frequencies, the characteristic impedances of both lines approach constant, pure resistances, but at low frequencies the resistance and reactance components rise as the frequency is reduced, those represented by curves 22—22′ for one line rising more rapidly than those represented by curves 21—21′ for the other line.

The pairs of curves 21—21′ and 22—22′ can be taken to illustrate, in a relative way, the characteristic impedances of an open-wire pair and of a cable pair, respectively. If two such lines are joined in a telephone connection reflection of electric waves will occur at the junction by reason of the dissimilarity in impedance of the two lines. The waves reflections are undesirable because they limit the gain that can be obtained from repeaters on two-way voice frequency circuits or give rise to crosstalk in one-way carrier frequency circuits. It becomes necessary, therefore, to equalize the impedances of the two lines. In the case illustrated, this could be done, in so far as relatively high frequencies are concerned, by the use of transformers or autotransformers at the junction point to step up the impedance of the cable pair. It is apparent, however, that this procedure would only increase the impedance dissimilarity at lower frequencies, because a transformer or autotransformer has a substantially constant impedance ratio at all frequencies and the characteristic impedance curves of the two kinds of line have different shapes. My invention, however, removes this difficulty. By the methods disclosed below, the impedance of one line is first modified so that the curves representing it have the same shape as those representing the impedance of the other line, that is, the two impedances are made to differ only by a constant factor at all frequencies. The curves 23—23′ represent the impedance that results from modifying the line whose original impedance is represented by 22—22′. If then a transformer or autotransformer be introduced between the two lines, the impedance irregularity is finally eliminated (except at extremely low and unimportant speech frequencies, when the impedance ratio of such devices becomes variable).

Again, the two lines may have the same impedance at high frequencies, as in the case of a loaded cable pair connected to a non-loaded open-wire pair, but the impedances may differ at low frequencies, because the relative amount of energy dissipated in the series resistance and shunt leakance is different for the two lines. The low-frequency impedance irregularity could be eliminated by decreasing the resistance of the line having the higher impedance, but this would require larger line conductors, or, in some cases, larger wire on loading coil windings and would be expensive. The impedance can be reduced more economically by the use of my invention whereby shunt elements having a certain predetermined amount of energy dissipation are introduced across the line of higher impedance.

Fig. 5 shows a transmission line divided into sections 24, 25, etc., each of which has the primary constants, R, L, G and C. According to my invention the shunt elements 26, 27, 28, etc., are, introduced across each line section. These periodically recurring shunts each comprise a resistance R′ in series with a capacitance C′, the amount of resistance and capacitance required being predetermined in accordance with the theoretical principles disclosed further on.

Fig. 6 illustrates the application of my invention to the case first cited, namely, where transformers or autotransformers are used to eliminate the impedance irregularity at high frequencies. 29 and 29′ are two sections of a main line that are connected together by an intermediate line 10 of another kind. In a typical case, 29—29′ would be an open-wire line and 10 would be a non-loaded cable that is introduced of necessity arising from a river crossing or other geographical condition. The line 29—29′ has primary constants R_0, L_0, G_0 and C_0 per unit length, while the length of intermediate line 10 has the constants R, L, G and C. To correct the steeply rising impedance curve of the line 10 at low frequencies, the dissipative shunts 11 and 11′ are introduced across its two ends. Thus the section of line 10 is the same as one section of the line shown in Fig. 2, the section being terminated at the centers of the shunts, making the resistance of each shunt 2R′ and the capacitance C′/2. In so far as low frequency impedance correction is concerned, the same result would be obtained by placing a single shunt R′, C′ at the center of the cable 10, but usually it would be more convenient to locate the shunts at its ends. 12 and 12′ are autotransformers employed to eliminate the impedance difference between the two lines at high frequencies. 13 and 13′ are merely condensers commonly introduced at the midpoints of autotransformers to avoid interference between grounded telegraph circuits on the two line wires and to avoid short-circuiting direct currents sent over the line for testing purposes; these condensers have no bearing upon the novelty of my invention.

It will be appreciated that the invention would be embodied in an arrangement whereby the lines 29—29′ and 10 were interchanged so that the shunts would be connected at periodic intervals along the line 29—29′ instead of along the line 10. It also will be appreciated that the invention would be embodied in an arrangement whereby the line 10 was connected directly to the line 29—29′ without transformers or autotransformers. It also will be appreciated that the invention would be embodied in an arrangement where the line sections 29—29′ were replaced by terminal apparatus having an impedance characteristic similar to that of the line 29—29′. It also will be appreciated that the invention would be embodied in an arrangement where each of the shunts 11 and 11′ was divided into two equal parts with a ground connection at the midpoint of each shunt, as may sometimes be desirable to reduce noise interference on the line.

In order properly to proportion the elements R′ and C′ of the shunts, it is necessary to understand certain theoretical principles involved.

Let the characteristic impedances of the two lines concerned be given by

$$Z_0=\sqrt{\frac{R_0+i\omega L_0}{G+i\omega C_0}} \quad (1)$$

and

$$K_0=\sqrt{\frac{R+i\omega L}{G+i\omega C}} \quad (2)$$

where R_0, L_0, G_0, C_0 and R, L, G, C are the primary constants of the two lines in any chosen length. If the lines are smooth, these formulas apply exactly. If one of the lines is loaded, the formula still applies closely enough for our purpose at the relatively low frequencies where the shunts have the desired effect in modifying the line impedance.

If shunts each having a conductance G_s and a capacitance C_s are added to the second line, its low frequency impedance becomes

$$K_{os}=\sqrt{\frac{R+i\omega L}{(G+G_s)+i\omega(C+C_s)}} \quad (3)$$

It will now be shown how the values of R′ and C′ of the shunts may be calculated to obtain the desired object. If r is the impedance ratio of a transformer or autotransformer that may be introduced to correct the impedance irregularity at high frequencies, then the requirement for a perfect match at low frequencies is that $Z_0=rK_0$, r being a constant factor independent of the frequency. Then from (1) and (3)

$$\frac{R_0+i\omega L_0}{G_0+i\omega L_0}=r^2\frac{R+i\omega L}{G+i\omega C+(G_s+i\omega C_s)} \quad (4)$$

From this we obtain

$$G_s+i\omega C_s=r^2\frac{R+i\omega L}{R_0+i\omega L_0}(G_0+i\omega C_0)-(G+i\omega C) \quad (5)$$

In all practical telephone lines, the leakance is negligible compared to the capacitance susceptance, within the order of accuracy which is required for engineering design purposes, so that we take, instead of (5),

$$G_s+i\omega C_s=i\omega\left[r^2\frac{R+i\omega L}{R_0+i\omega L_0}C_0-C\right] \quad (6)$$

Now it will be noted from (1) and (3) that at high frequencies Z_0 and K_0 approach the values,

$$Z_0=\sqrt{L_0/C_0}, K_0=\sqrt{L/C}$$

Therefore, the transformer or autotransformer ratio must be

$$r=\sqrt{L_0/C_0}/\sqrt{L/C}$$

By substituting for r^2 its value, L_0C/LC_0, we obtain

$$G_s+i\omega C_s=i\omega C\frac{\frac{L_0}{R_0}\frac{R}{L}-1}{1+i\omega\frac{L_0}{R_0}} \quad (7)$$

The quantities L_0/R_0 and L/R are the "time constants" of the two lines. It is convenient to designate these T_0 and T, respectively. Then

$$G_s+i\omega C_s=i\omega C\frac{T_0/T-1}{1+i\omega T_0} \quad (8)$$

We are not so much interested in the admittance $G_s+i\omega C_s$ as we are in its reciprocal, $R'+1/i\omega C'$, which is the required shunt impedance. It is

$$R'+1/i\omega C'=1/(G_s+i\omega C_s)=\frac{T_0}{((T_0/T)-1)C}+\frac{1}{i\omega C((T_0/T)-1)} \quad (9)$$

It will be seen from (9) that the required shunt impedance element comprises a simple series combination of a resistance and a condenser having the values

$$R'=\frac{1}{C}\frac{T_0}{(T_0/T)-1} \quad (10)$$

$$C'=C((T_0/T)-1) \quad (11)$$

The spacing of the shunts usually will be dictated by geographical conditions, as in the case of a submarine cable. Owing to the highly dissipative property of the shunts, their introduction across a line does not give rise to any sharply defined selective transmission effect with respect to frequency and their spacing is not, therefore, a matter of primary interest. Nevertheless, it may be desirable, in certain cases, particularly where more than two shunts are employed, to determine the effect of the spacing upon the attenuation loss of the line.

If Γ and γ are the propagation constants per periodic interval of the line with and without shunts, respectively, and if $S=R'+1/i\omega C'$ is the impedance of each shunt the propagation constant of the shunted line is given exactly at all frequencies by

$$\cosh\Gamma=\cosh\gamma+\frac{K_0}{2S}\sinh\gamma \quad (12)$$

It can be shown from this equation that at low frequencies the attenuation per periodic interval of the line is increased approximately by the ratio

$$A/\alpha\sqrt{\frac{C+C_a}{C}\left(1+\frac{1}{2}\left(\frac{G_s}{\omega C_s}\right)^2\right)} \quad (13)$$

if, as is usually the case, $G/\omega C$ is small compared to unity and to $G_s/\omega C_s$.

At high frequencies, such as carrier frequencies that may be transmitted over the same line, the shunts give rise to "humps" in the attenuation curve, these "humps" not being sufficiently large, however, to react seriously on transmission. It can be shown from Equation (12) that the excess attenuation loss per periodic interval represented by each "hump" has the approximate value, $K_0/2S$, and that the spacing of the "humps" on the frequency scale is

$$\tfrac{1}{2}\sqrt{LC}$$

The first "hump" occurs at a frequency,

$$\tfrac{1}{4}\sqrt{LC}$$

From this it will be seen that spacing the shunts farther apart makes the "humps" recur at a closer frequency interval and increases the height of each hump. For example, in a practical case where the shunts were spaced one mile apart on a cable it was found that the first "hump" occurred at 32 kilocycles; the succeeding "humps", which would have occurred every 64 kilocycles, were not of any practical interest. The height of the hump was .6 decibel. If the shunts had been spaced two miles apart the first hump would have occurred at 16 kilocycles and its height would have been 1.2 decibels.

I claim:

1. A transmission line comprising two like parts having a certain characteristic impedance

value and an intermediate section of lower value, respective autotransformers at the two junction places, their ratios being alike and somewhat greater than the ratio of the two impedance values, and two shunt condensers across the intermediate section, one near each end, each condenser having its capacity adjusted to the value which makes the impedances approximately the same looking each way from a point between that condenser and the corresponding autotransformer.

2. A transmission line comprising two like parts having a certain characteristic impedance value and an intermediate section of different value in combination with two shunt condensers across the intermediate section, one near each end, each condenser having its capacity adjusted to the value which makes the impedances approximately the same looking both ways from a point near the condenser on the side thereof away from the said intermediate section.

3. A transmission line comprising two like parts of a certain characteristic impedance value and an intermediate section of lower value, respective autotransformers at the two junction places, their ratios being alike and somewhat greater than the ratio of the two impedance values, and two shunts across the intermediate section, one near each end, each said shunt consisting of a condenser and a resistance in series, the capacity of said condenser and the measure of said resistance being adjusted to make the impedances approximately the same looking each way from a point between the said shunt and the corresponding autotransformer.

4. A transmission line comprising two like parts of a certain characteristic impedance value and an intermediate section of lower value, respective autotransformers at the two junction places, their ratios being alike and somewhat greater than the ratio of the two impedance values, each autotransformer comprising a condenser interposed at the middle of its winding, and two shunt condensers across the intermediate section of the line, one near each end, each condenser having its capacity adjusted to the value which makes the impedances approximately the same looking each way from a point between that condenser and its corresponding autotransformer.

5. In combination, two lines of unequal characteristic impedance, an autotransformer of nearly but not exactly the correct ratio connecting them, and a condenser across the line on one side of the autotransformer adjusted to a capacity value to make the impedances the same looking both ways from a point between said condenser and the corresponding autotransformer.

6. In combination, two lines of unequal impedance, an autotransformer connecting them, said autotransformer having its ratio nearly but not exactly right for the connection, and a reactance shunt across a line on one side of the autotransformer, said reactance shunt having its value adjusted to equalize the impedance looking both ways from a point between it and the associated autotransformer.

MANVEL KEEPORT ZINN.