March 17, 1936. E. I. GREEN ET AL 2,034,032

SHIELDED PAIR OF WIRES

Filed June 7, 1933 3 Sheets-Sheet 1

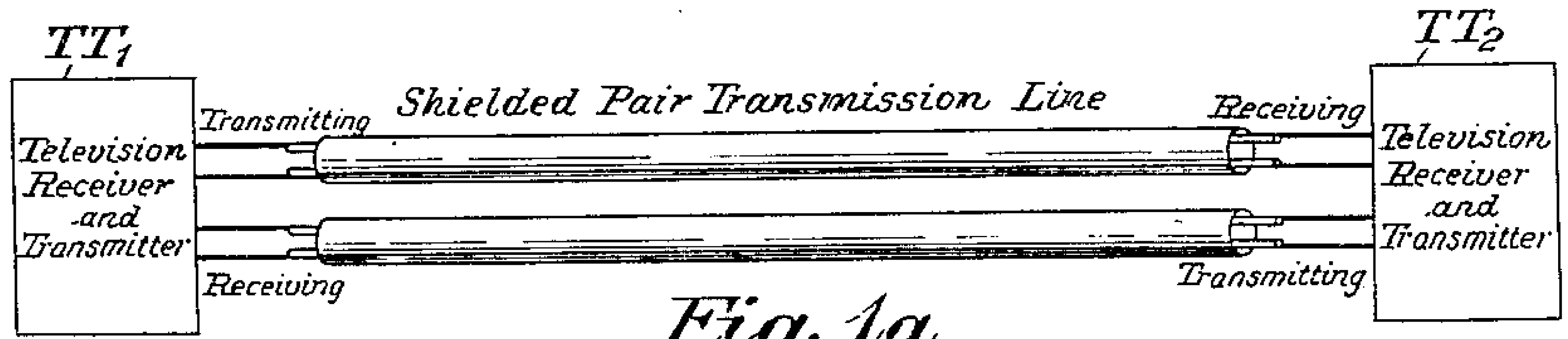

Fig. 1a

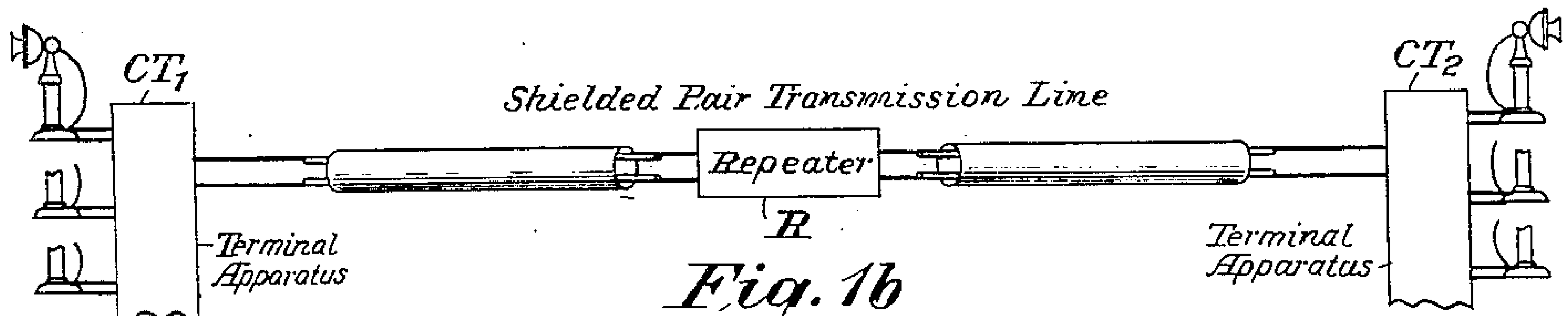

Fig. 1b

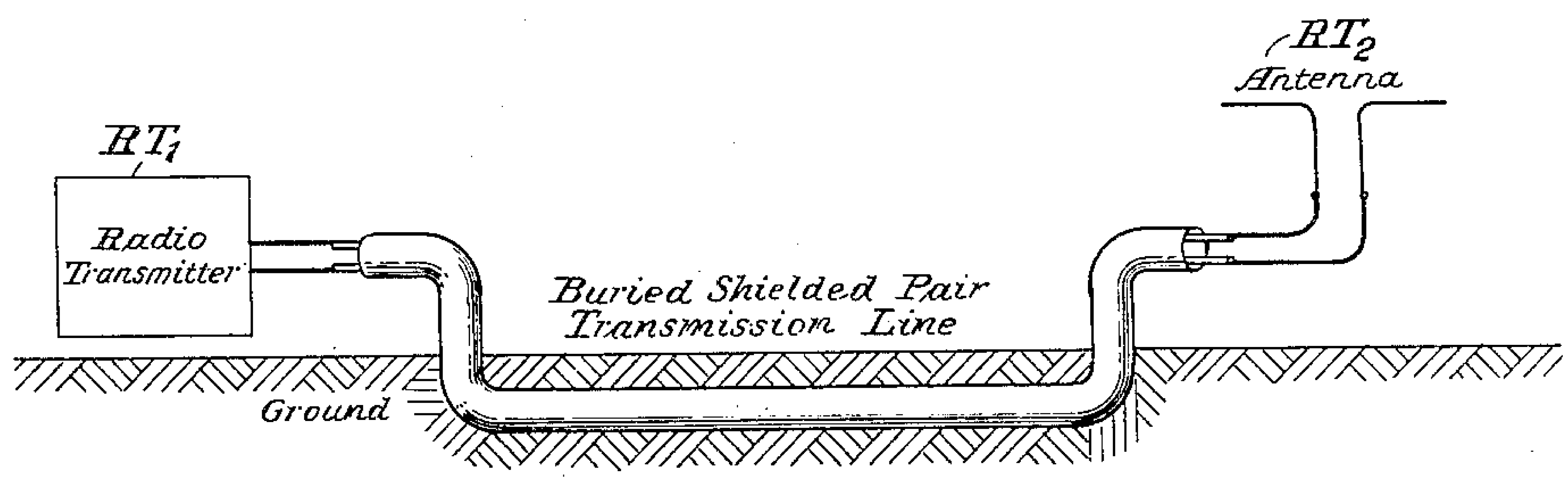

Fig. 1c

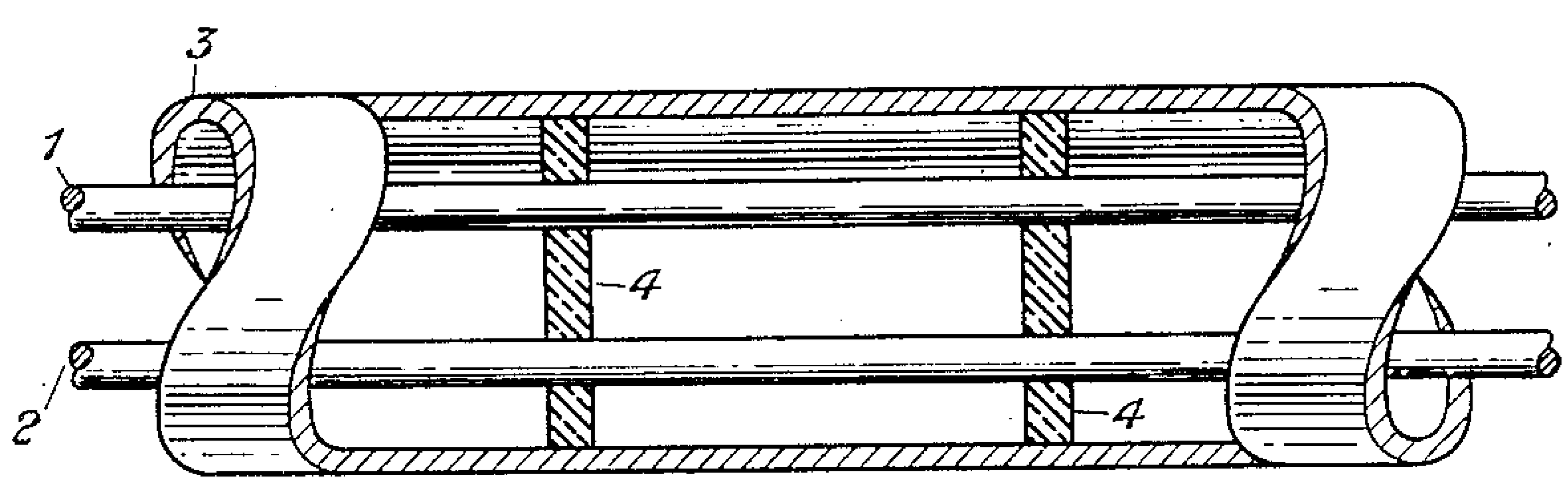

Fig. 2

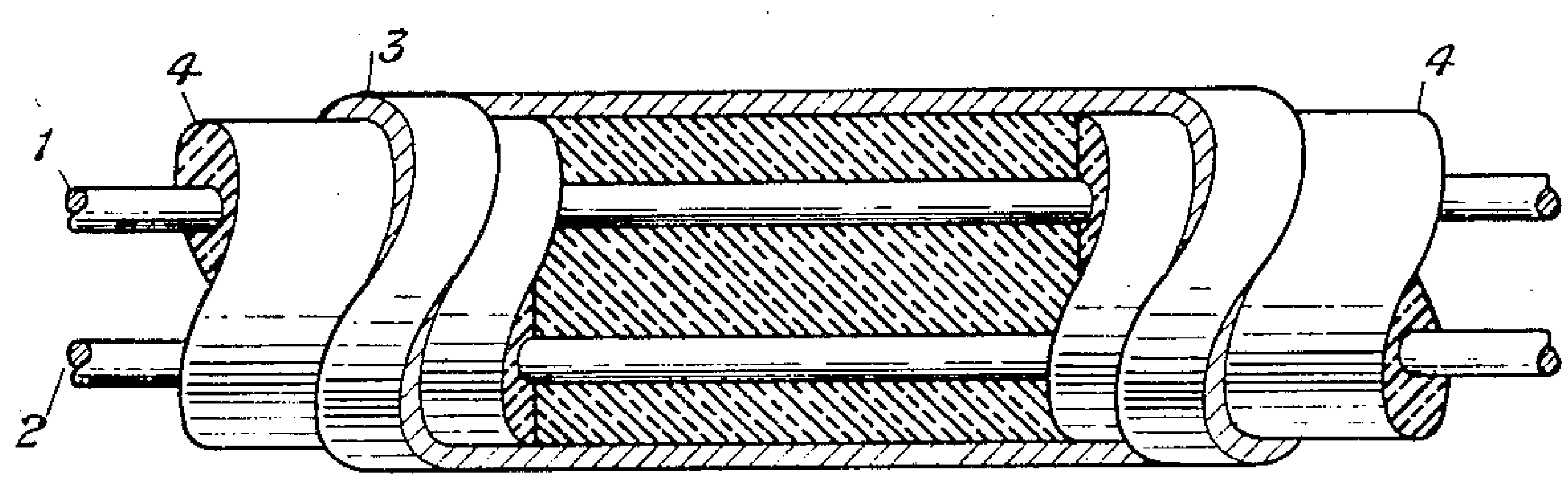

Fig. 3

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SHIELDED PAIR OF WIRES

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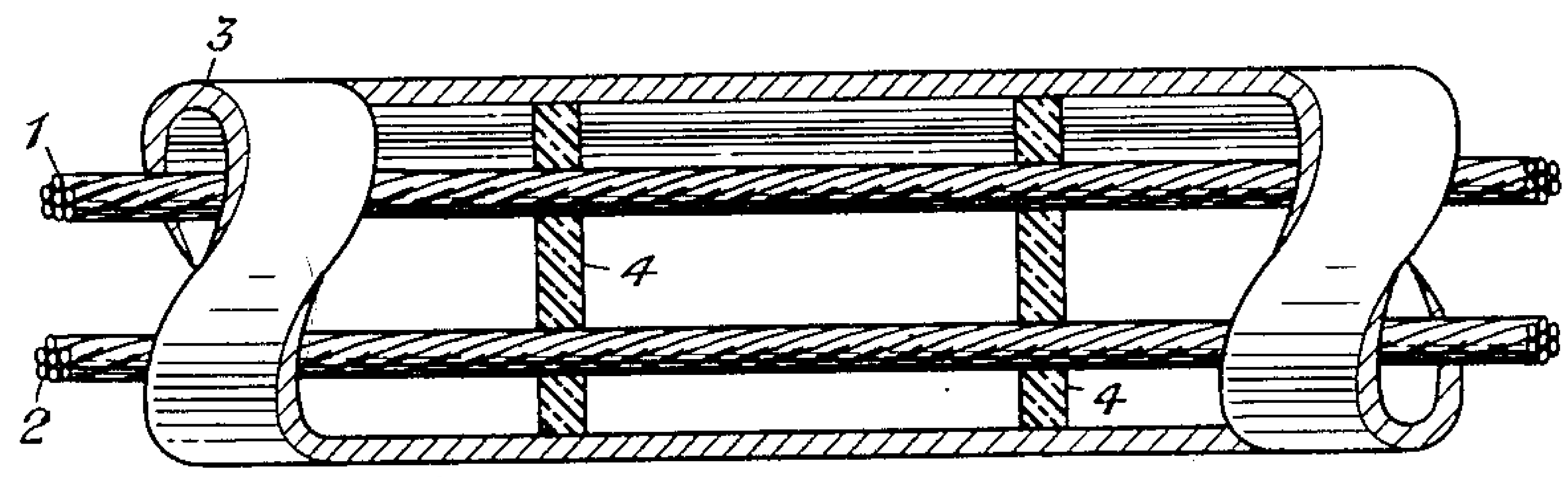

Fig. 4

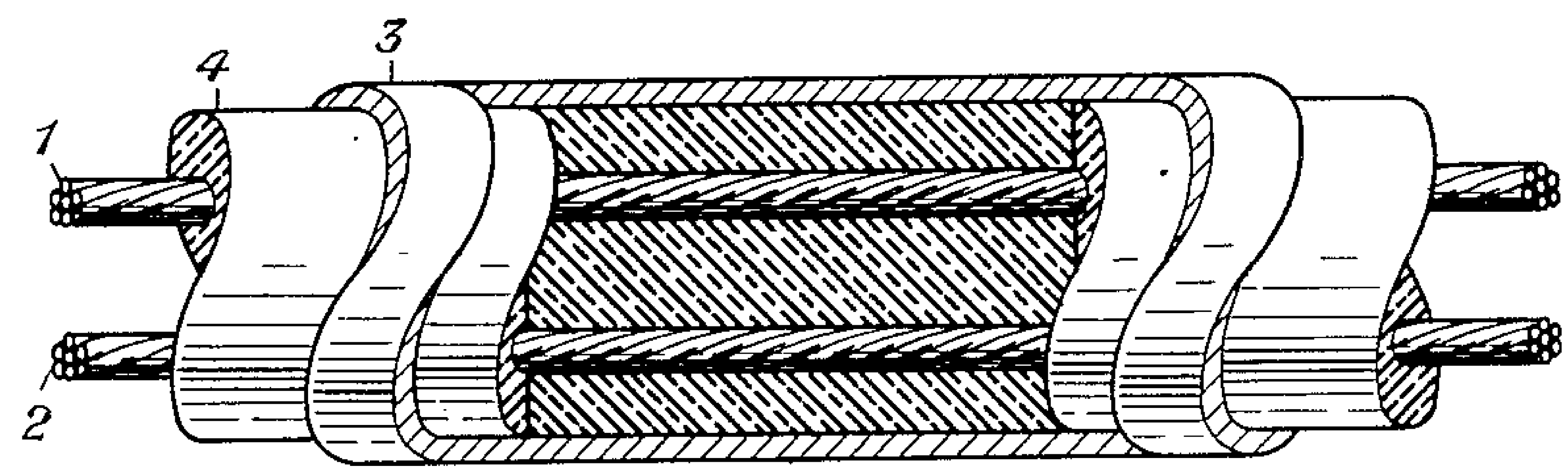

Fig. 5

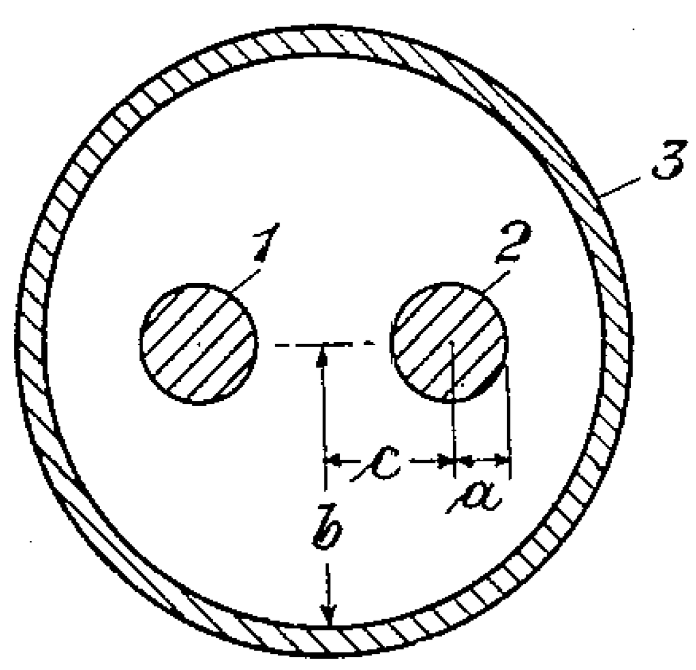

Fig. 6

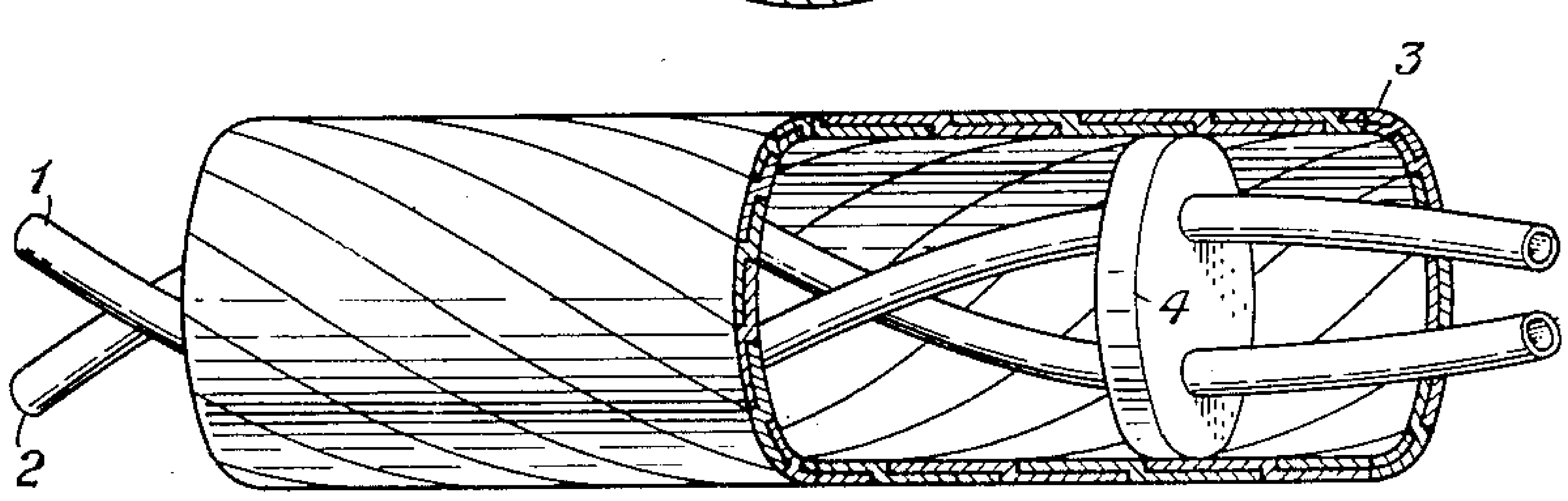

Fig. 7

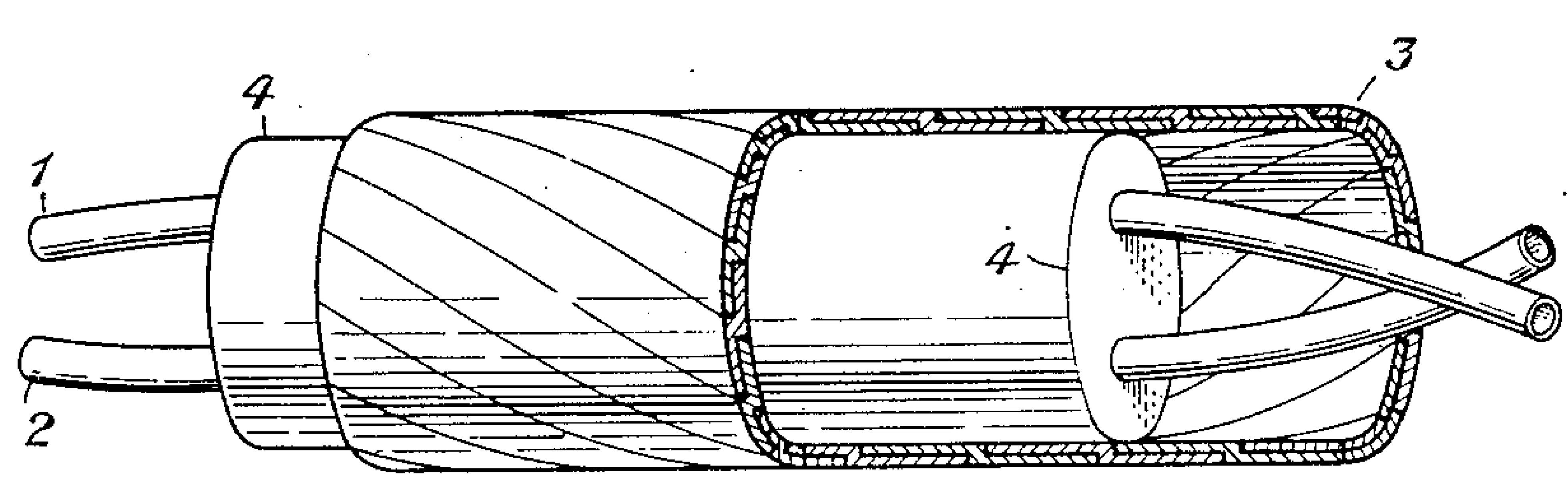

Fig. 8

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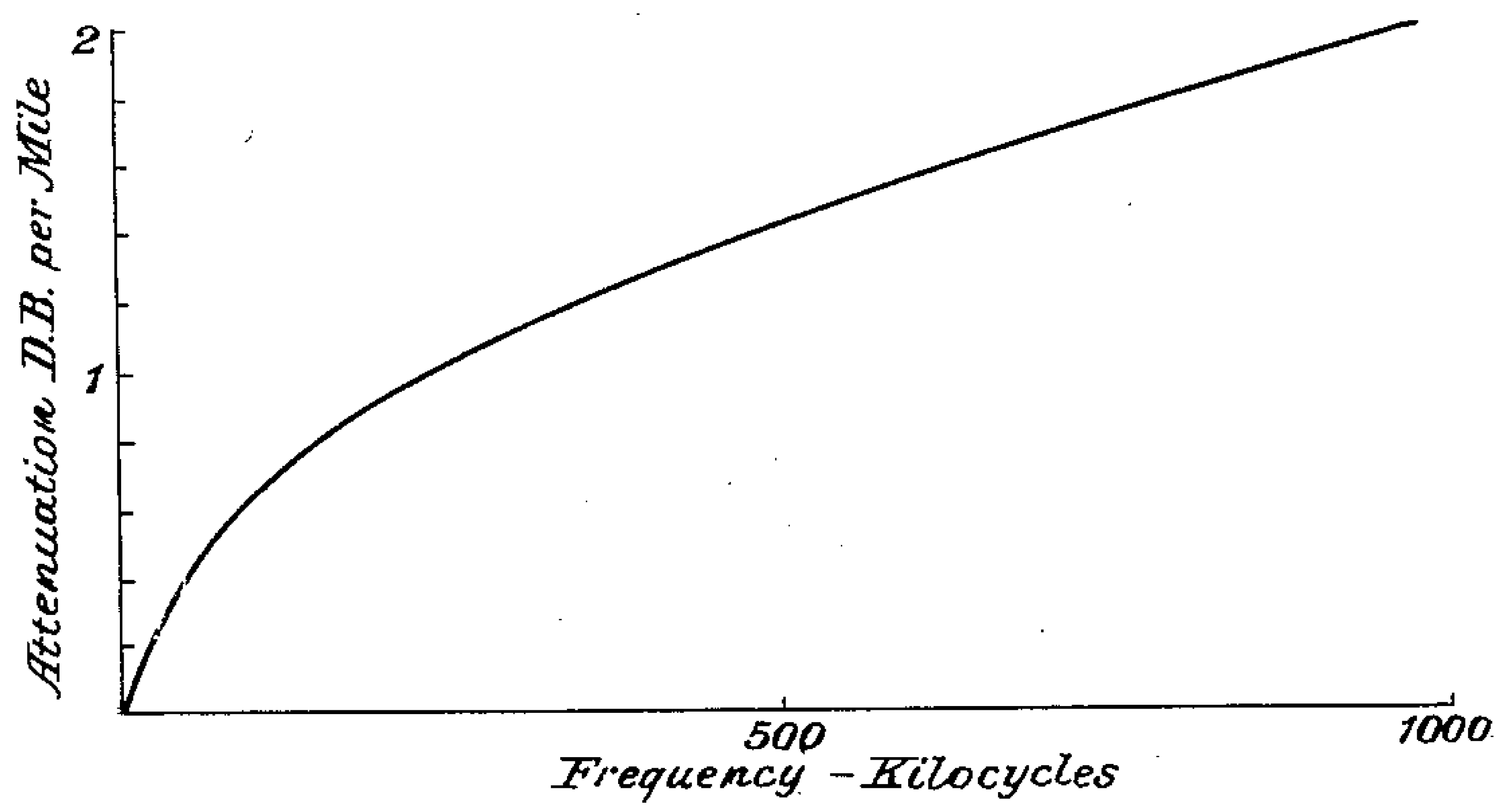

Fig. 9

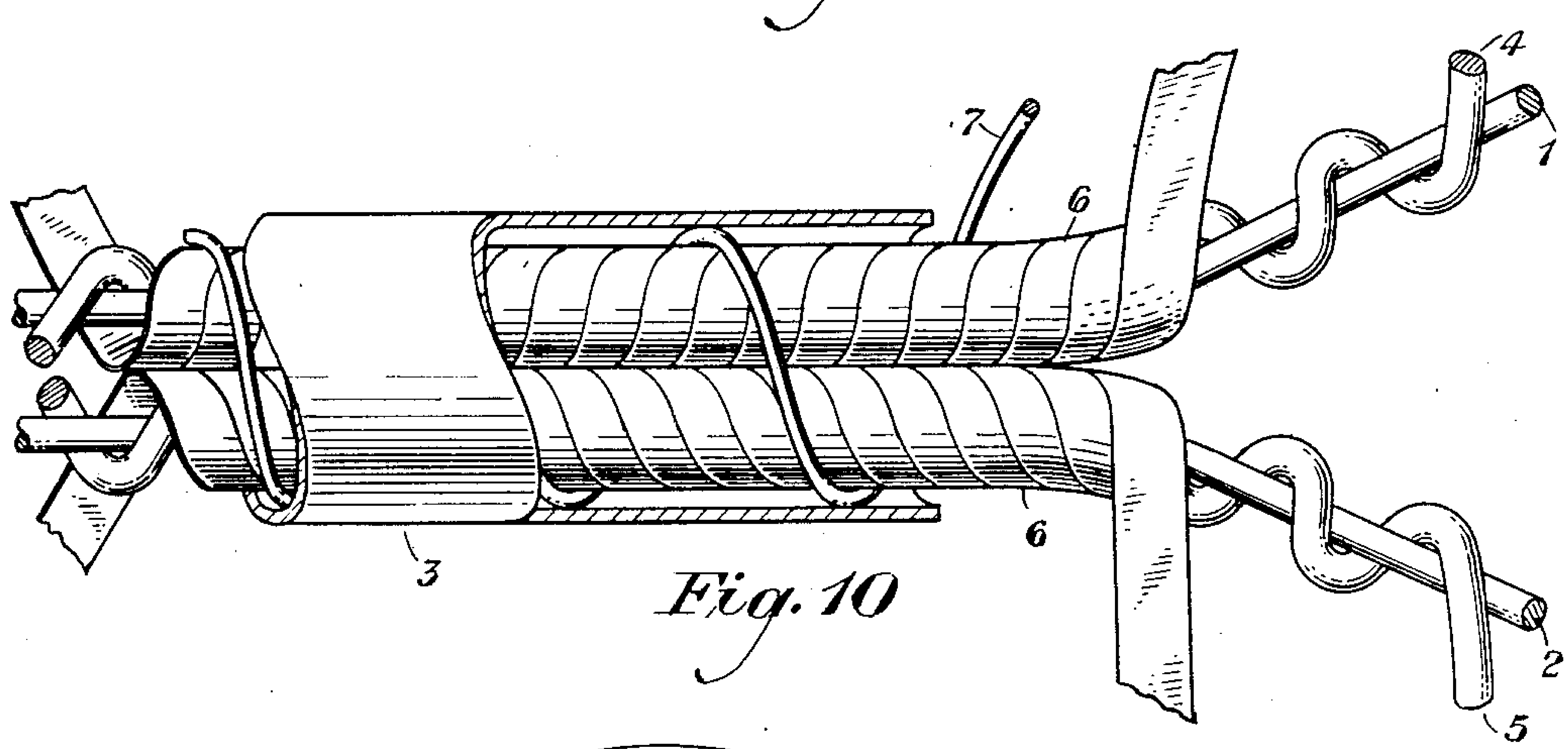

Fig. 10

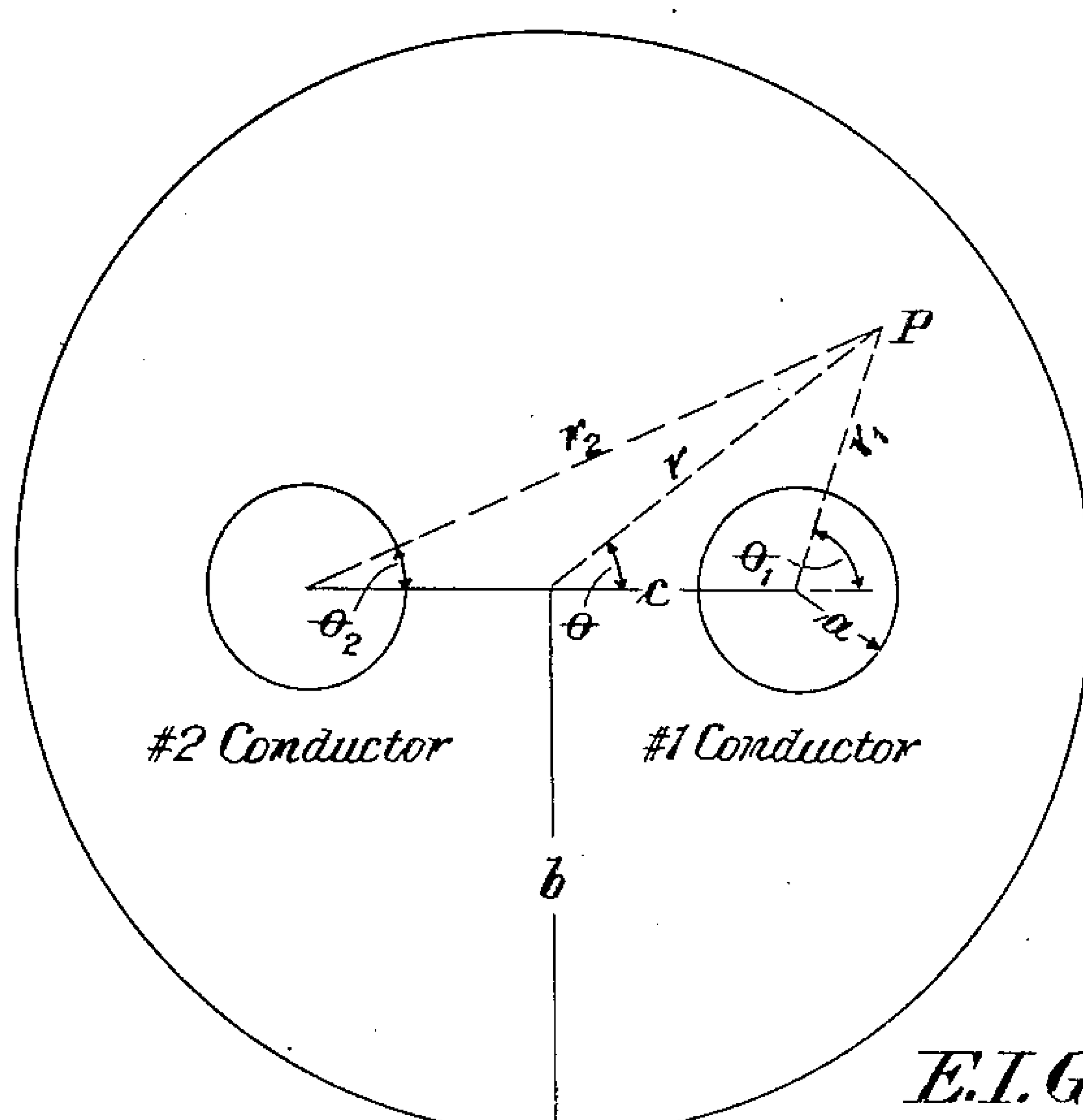

Fig. 11

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UNITED STATES PATENT OFFICE

2,034,032

SHIELDED PAIR OF WIRES

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Application June 7, 1933, Serial No. 674,762

23 Claims. (Cl. 178—44)

This invention relates to electrical transmission circuits and is concerned especially with circuits comprising a pair of conductors surrounded by an individual shield. A particular object of this invention is to obtain an individually shielded circuit which has the properties of low attenuation and substantial freedom from external induction throughout a wide range of frequencies. Another object of the invention is to obtain a circuit of such characteristics which is balanced with respect to ground.

The frequency range which may be transmitted over a circuit consisting of an ordinary unshielded pair of conductors is limited both by the increasing susceptibility of the circuit to crosstalk from nearby conductors and interference from external sources as the frequency is increased, and also, in many instances, by the large high frequency attenuation which results from the use of solid dielectric material. In accordance with the invention it is proposed to enclose a pair of conductors in a conducting shield which acts to prevent external electromagnetic or electrostatic disturbances from causing disturbances in the pair, and conversely to prevent the currents transmitted over the pair from causing disturbances in external circuits. Moreover, since the shielding effect of such an enclosing shield decreases as the frequency decreases, it is proposed in accordance with the invention, to twist the conductors of the pair helically around the axis of the shield or otherwise transpose them in order to annul any interference which may pass through the shield at low frequencies.

In order to reduce the high frequency attenuation of the shielded pair it is proposed in one embodiment of the invention to employ a substantially gaseous dielectric between the conductors of the pair and between these conductors and the surrounding sheath. The invention comprehends also, however, the use of non-gaseous dielectric material to insulate the conductors from one another and from the sheath. The invention has to do especially with individually shielded pairs of conductors in which the conductors are either solid, tubular, or composed of a number of uninsulated strands. Pairs of conductors which are surrounded by shields of substantially circular cross-section are a particular subject of this invention.

A particular object of the invention is to so proportion the ratio of the inner diameter of the shield to the diameter of the conductors and the ratio of the interaxial separation of the conductors to the inner diameter of the shield that the high frequency attenuation will be a minimum for a predetermined size of the shield.

More broadly, the invention is concerned with systems for utilizing individually shielded balanced pairs for the transmission of high frequencies or wide bands of frequencies. The satisfactory transmission of television images with good definition requires the transmission of a frequency band whch may extend from zero frequency to hundreds or even thousands of kilocycles. If, for example, it is desired to transmit with a total of 24 reproductions per second an image containing 40,000 picture elements, there is required a frequency band of approximately 500 kilocycles in width. Still wider bands may be necessary for representing with adequate detail such scenes as a theatrical performance or an athletic event. A shielded transposed pair designed in accordance with the principles of the invention is especially suited for the transmission of such television bands because it may be given comparatively low attenuation and relative freedom from interference over the entire band.

Moreover, by the application of multiplexing the wide frequency bands obtained from a shielded twisted pair may be used to provide substantial numbers of narrower frequency bands suitable for other uses as for example, for telephone circuits which may require bands of about 2500 cycles in width, for high quality program circuits which may require bands extending up to 10,000 cycles or higher, for high speed facsimile transmission or for other purposes.

Inasmuch as the two conductors are symmetrical at all points with respect to the shield, the potential between each conductor and the shield would be equal. Therefore, if such a shielded pair were buried so that the shield makes electrical contact with the ground, or if the shield were electrically connected to ground at frequent intervals, the two conductors would form a balanced-to-ground circuit. Even if the shield were not connected by wires to ground or buried, it would be effectively connected to ground due to the electrical capacity between it and ground. Hence, the two conductors would always form a balanced-to-ground circuit. Such a balanced-to-ground circuit would be very useful for interconnecting electrical elements which are themselves balanced to ground.

For instance, it is frequently desirable in the radio art to employ an antenna which is balanced with respect to ground rather than to connect the transmitting or receiving apparatus between antenna and ground. Such, for example, is the case when using a diamond antenna or a horizontal dipole antenna. A shielded transposed pair of the type described herein is peculiarly adapted for connecting such balanced antennas with radio transmitting or receiving apparatus, inasmuch as such a pair may be balanced to ground and may be designed to have low attenuation and substantial immunity from external interference at the frequency or frequencies employed for radio transmission.

These and other objects and features of the invention will be more readily understood from the following description when read in connection with the accompanying drawings, in which Figs. 1A, 1B, and 1C represent various transmission systems utilizing shielded pairs; Figs. 2, 3, 4 and 5 are longitudinal sections of various shielded pairs; Fig. 6 is a cross-sectional view of a shielded pair; Figs. 7 and 8 illustrate another possible form of construction for a shielded pair; Fig. 9 represents an attenuation-frequency curve for an optimum design of shielded pair circuit having a shield whose inner diameter is one inch; Fig. 10 illustrates a form of construction of a shielded pair in which the conductors and shield are held in their proper position by means of insulating strings twisted around the conductors; and Fig. 11 represents another cross-sectional view of a pair with circular shield.

Over-all systems embodying the inventions are schematically illustrated in Figs. 1A, 1B and 1C. In these figures shielded pair transmission lines are shown associated with various kinds of apparatus at the terminals. Thus in Fig. 1A is shown a pair of wide band television transmitters and receivers indicated as TT_1 and TT_2 respectively, connected to the terminals of two shielded pairs. Fig. 1B shows a shielded pair utilized in a carrier telephone system. The terminal apparatus CT_1 and CT_2 connected to the shielded pair may consist of modulators, demodulators, filters, and amplifiers. A repeater R is shown between two sections of the shielded pair line. Fig. 1C shows a shielded pair transmission line used to connect a radio transmitter RT_1 to a balanced-to-ground radio antenna RT_2, for example, a horizontal dipole. If desired, the shield can be buried as shown in Fig. 1C.

Referring to Fig. 2, **1** and **2** represent two solid conductors which are held in proper relation and out of electrical contact with each other and with the circular conducting shield **3** by means of spaced dielectric washers **4**. These washers should be separated from each other a suitable distance and should be made as thin as possible consistent with the required mechanical strength. They should also be composed of some dielectric of small loss angle and low dielectric constant, since if these conditions are obtained, the leakage loss may be made so small as to be practically negligible. Fig. 3 shows a longitudinal section through a shielded pair having solid dielectric. **1** and **2** are the two solid conductors; **3** is a circular shield of conducting material such as a copper pipe. **4** is the solid dielectric by means of which the conductors and shield are held in proper relation. This solid dielectric should have a small loss angle and low dielectric constant in order to make the attenuation as low as possible. For example, paragutta or other similar insulating material may be used for the dielectric **4**. Fig. 4 also represents a longitudinal section of shielded pair. **1** and **2** are conductors composed of a number of uninsulated strands. **3** represents the circular conducting shield and **4** insulating spacers as in Fig. 2. Fig. 5 is also longitudinal section of a shielded pair. **1** and **2** are conductors composed of a number of uninsulated strands. **3** represents the circular conducting shield and **4** the solid dielectric as in Fig. 3. Fig. 6 shows a cross-section of a shielded pair. **1** and **2** are the solid conductors and **3** is the circular conducting shield. b is the inner radius of the shield, a is the radius of the conductor, and c is the distance from the axis of the shield to the axis of either conductor.

The conductors may be of such a type that currents of frequencies well above the audible range travel substantially on the outer surface of the conductors. For example, they may be either solid or tubular. In the latter case, the thickness of the walls of the tubes may be small compared to the diameter. The conductors may consist of a cylindrical assembly of conducting strips, tapes, ribbons, uninsulated wires or the like. The latter form of construction might be particularly desirable if the conductors are large and if a flexible structure is desired.

The two conductors may be either parallel or transposed at frequent intervals. In either case the conductors may be considered sensibly parallel for mathematical purposes and the optimum diameter and spacing ratios will be the same in either case. One method of accomplishing this result is helically twisting the conductors around the axis of the shield.

Any of various forms and shapes may be employed for the insulation between the two conductors and between conductors and sheath. One possible arrangement would be to use a continuous spirally applied string or strip of dielectric material around each conductor and another spirally applied string to separate them from the shield. Generally, it will be desirable that the amount of insulating material employed be a minimum, in order that the dielectric between the two conductors may be largely gaseous. In some cases, however, it may be found advantageous to use a dielectric which is partly or wholly non-gaseous as, for example, rubber insulation.

The shield surrounding the two conductors, instead of being formed of a single tube might consist of a cylindrical assembly of conducting strips, tapes, ribbons, wires or the like. Such forms of construction might be particularly advantageous where a flexible structure is desired.

Figs. 7 and 8 illustrate some possible variations in the structure of the shielded pair. In these cases it will be seen that the conductors **1** and **2** consist of conducting tubes such as copper pipes. The insulation **4** in Fig. 7 takes the form of thin discs of insulating material spaced relatively far apart; in Fig. 8 the insulation takes the form of a solid dielectric material such as paragutta or similar insulating material. In both Figs. 7 and 8 the shield **3** is formed of tapes arranged in the form of a cylinder.

In connection with the shield it may be noted that in addition to performing an electrical function by protecting the circuit from external induction, it may be useful in affording mechanical protection to the circuit and thereby permitting the use of an air dielectric to a very considerable extent. Due to skin effect the high frequency currents will penetrate very little into the shield and conductors so that the electrical requirements are satisfied by a very thin shield and thin wall conductors. Consequently, the thickness of the shield and the thickness of the walls of the conductors will ordinarily be determined by mechanical considerations. The thickness of the shield will usually be such that it does not enter in the problem of determining the optimum configuration of conductors and shield.

The use of the shield will ordinarily make it possible where desired to allow the signals transmitted over the pair to drop down to a minimum level determined by the noise due to thermal agitation of electricity in the conductors. Hence, the use of the shield may facilitate the spacing of intermediate amplifiers in the circuit at wider intervals than would otherwise be possible.

In Fig. 10, 1 and 2 represent two solid conductors and 3 a copper tube as a shield. 4 and 5 are strings of paper or of other suitable insulating materials twisted around the conductors. 6 is a tube of suitable insulating material covering each conductor and its associated string. The strings 4 and 5 can be of such size that the conductors can be separated by any desired amount. 7 is another insulating string wrapped around the two insulated conductors holding them in position with respect to the shield.

Referring now to Fig. 11, the ratio of the inner diameter of the shield to the diameter of the conductors and the ratio of the interaxial separation of the conductors to the inner diameter of the shield, which makes the high frequency attenuation a minimum for any given inner diameter of shield, will be determined. This will be done by first developing a general expression for the attenuation of such a system as a function of these ratios and then determining the values of these ratios which will make the attenuation a minimum for any predetermined size of shield.

A formula will be derived for the attenuation of a wave of high frequency (above the audible range) propagated in such a system consisting of a pair of long straight parallel wires of circular cross-section enclosed symmetrically in a hollow metallic sheath of circular cross-section whose axis is parallel to and coplanar with the axes of the wires. From the cross-section of the arrangement shown in Fig. 11 the notation for coordinate systems and dimensions will be clear. For convenience, the sheath is assumed of infinite extent, its thickness being of no significance in the present problem. The conductors are assumed non-magnetic and the leakage conductance is assumed to be zero.

It is well known that the attenuation α, per unit length of a conducting system, when the leakage is zero and the frequency is so high that ω^2L^2 is large compared with R^2, is given by

$$\alpha=\frac{R}{2}\sqrt{\frac{C}{L}} \qquad (1)$$

where R, L and C are the effective resistance, inductance and capacity of the system per unit length. Denoting by Z, the impedance of the system per unit length, we have

$$Z=R+i\omega L=2Z_1+4i\omega\log 2c/a+\Delta Z \qquad (2)$$

Z_1 being the internal impedance of the wire per unit length with concentric return and ΔZ the proximity effect correction due to the presence of the sheath and the reaction of the wires upon each other. The inductance L may be written

$$L=L_0+L_{12}$$

where L_0 is the internal inductance of wires and sheath and L_{12} the external inductance of the system. By external inductance is meant that portion of the inductance which is independent of the conductivity of the conductors and which may, therefore, be determined by assuming infinite conductivity. Hence, we have

$$L_{12}C=\mu k$$

where μ and k are, respectively, the permeability and specific inductive capacity of the dielectric. Thus putting $\mu=1$,

$$C=k/L_{12} \qquad (3)$$

Hence, when the impedance Z has been determined, the attenuation readily follows from Equations, (1), (2) and (3).

The immediate object then is to derive the formula for the impedance Z; namely,

$$Z=R+i\omega L=2Z_1[1+2\lambda^2(1+2\lambda^2)(1-4\epsilon^2)]$$
$$+8\epsilon^2Z_3[1+\epsilon^4-2\lambda^2(1+\lambda^2)]+$$
$$4i\omega[\log\frac{1}{\lambda}\frac{1-\epsilon^2}{1+\epsilon^2}-\lambda^2(1+\lambda^2)(1-4\epsilon^2)] \qquad (4)$$

the notation being as follows:

$Z_1=R_1+i\omega L_1$
=internal impedance of wire with its return,
$Z_3=R_3+i\omega L_3$,
=internal impedance of sheath with its return,
$\lambda=a/2c$
$\epsilon=c/b$
a=radius of wire
b=radius of sheath
$2c$=interaxial separation of wires.

This is an approximate formula based upon a first order correction for proximity effect.

In the dielectric between wires and sheath the magnetic force H is expressible as three symmetrical waves centered on the axes of the three conductors, respectively. Also, the vector potential, $F/i\omega$, is given by the relation

$$i\omega H=\text{curl } F=-\frac{\delta F}{\delta r};\text{ where }\frac{\omega}{2\pi}=f=\text{frequency}$$

We assume unit permeability in the dielectric and in the conductors. Thus, we write, at any point P in the dielectric with coordinates (r_1, θ_1), (r_2, θ_2) and (r, θ) with respect to the first wire, the second wire and the sheath, respectively,

$$F=-A_0\log\frac{r_2}{r_1}+\sum_1^\infty{}_nA_n\left(\frac{\cos n\theta_1}{r_1^n}-(-1)^n\frac{\cos n\theta_2}{r_2^n}\right)$$
$$-\sum_1^\infty{}_nB_{2n-1}r^{2n-1}\cos(2n-1)\theta \qquad (5)$$

Denoting by I the current in the wire, we have

$$4\pi I=\int_0^{2\pi}aHd\theta\text{ at }r_1=a$$

or

$$A_0=-2i\omega I \qquad (6)$$

Inside the conductors the axial electric forces may be written, respectively,

$$E=Z_1I\frac{J_0(r_1\alpha_1)}{J_0(a\alpha_1)}+\sum_1^\infty{}_ng_nJ_n(r_1\alpha_1)\cos n\theta_1,$$
$$0\leqq r_1\leqq a \qquad (7)$$

$$E=-Z_1I\frac{J_0(r_2\alpha_1)}{J_0(a\alpha_1)}-\sum_1^\infty{}_n(-1)^ng_nJ_n(r_2\alpha_1)\cos n\theta_2,$$
$$0\leqq r_2\leqq a \qquad (8)$$

$$E=\sum_1^\infty{}_nh_{2n-1}K_{2n-1}(r\alpha_3)\cos(2n-1)\theta, b\leqq r\leqq\infty \qquad (9)$$

where

$$J_n(r\alpha_k)\text{ and }K_n(r\alpha_k)=$$

Bessel functions of first and second kinds, respectively, of order n and complex argument

rak

$$\alpha_1 = i\sqrt{i}\,\sqrt{4\pi\sigma_1\omega}, \alpha_3 = i\sqrt{i}\,\sqrt{4\pi\sigma_3\omega}.$$

σ_1 and σ_3=conductivity of wire and sheath, respectively.

The continuity of tangential magnetic force and normal magnetic induction gives the boundary relations at $r_1=a$,

$$\frac{\partial F}{\partial \theta_1} = -\frac{\partial}{\partial \theta_1}E \tag{10}$$

$$\frac{\partial F}{\partial r_1} = -\frac{\partial}{\partial r_1}E \tag{11}$$

and, at $r=b$,

$$\frac{\partial F}{\partial \theta} = -\frac{\partial E}{\partial \theta} \tag{12}$$

$$\frac{\partial F}{\partial r} = -\frac{\partial}{\partial r}E \tag{13}$$

Relations (10)-(13) are sufficient to determine the arbitrary constants A_n, B_n, g_n and h_n of Equations (5), (7), (8) and (9).

Assume for the moment that these are known. Now denoting by E_1 and E_2, F_1 and F_2 and V_1 and V_2 the values of axial electrical force, vector potential and scalar potential at the surfaces $r_1=a$ and $r_2=a$, respectively, we have the additional relations

$$E_1 = -F_1 - \frac{\partial}{\partial z}V_1$$

and

$$E_2 = -F_2 - \frac{\partial}{\partial z}V_2$$

Then

$$-\frac{\partial}{\partial z}(V_1 - V_2) = (E_1+F_1) - (E_2+F_2) = 2(E_1+F_1) \tag{14}$$

It is assumed herein, as is usual in transmission problems, that the wave and all vector components vary with the time t and the axial coordinate z as exp $(i\omega t - \gamma z)$, γ being the propagation constant per unit length. Also, it is well known that

$$V = V_1 - V_2 = \frac{\gamma}{i\omega C}I \tag{15}$$

and since

$$\gamma^2 = i\omega C(R + i\omega L) \tag{16}$$

we have, from Equations (10), (11) and (12),

$$(R + i\omega L)I = 2(E_1 + F_1) \tag{17}$$

The formal solution of the problem is then complete. We proceed now to determine A_n, B_n and g_n which are required in the expressions for E_1 and F_1 in Equation (17).

To apply the boundary conditions at $r_1=a$, transform (5) to the coordinate system $(r_1\theta_1)$. We shall use the relations

$$\log r_2 = \log 2c + \frac{r_1}{2c}\cos\theta_1 - \frac{1}{2}\left(\frac{r_1}{2c}\right)^2 \cos 2\theta_1 + \frac{1}{3}\left(\frac{r_1}{2c}\right)^3 \cos 3\theta - \cdots$$

$$\frac{\cos n\theta_2}{r_2^n} = \left(\frac{1}{2c}\right)^n\left(1 - \frac{n}{1!}\left(\frac{r_1}{2c}\right)\cos\theta_1 + \frac{n(n+1)}{2!}\left(\frac{r_1}{2c}\right)^2\cos 2\theta_1 - \cdots\right)$$

$$r^n \cos n\theta = c^n\left(1 + \frac{n}{1!}\frac{r_1}{c}\cos\theta_1 + \frac{n(n-1)}{2!}\left(\frac{r_1}{c}\right)^2\cos 2\theta_1 + \cdots\right)$$

Introducing these in (5) we write for F in the neighborhood of $r_1=a$,

$$F = -A_0 \log\frac{2c}{r_1} - \sum_1^\infty n\left(-\frac{1}{2c}\right)^n A_n - \sum_1^\infty n c^{2n-1} B_{2n-1} + \sum_1^\infty n \cos n\theta_1 \left[\left(-\frac{r_1}{2c}\right) n\left(\frac{A_0}{n} + S_A\right) - \left(\frac{r_1}{c}\right)^n \sum\nolimits_B + \frac{A_n}{r_1^n}\right] \tag{18}$$

where

$$S_A = \frac{A_1}{2c} - (n+1)\frac{A_2}{(2c)^2} + \frac{(n+1)(n+2)}{2!}\frac{A_3}{(2c)^3} - \cdots$$

$$\sum\nolimits_B = c^n B_n + (n+1)c^{n+1}B_{n+1} + \frac{(n+1)(n+2)}{2!}c^{n+2}B_{n+2} + \cdots$$

but

$$B_0 = B_2 = \ldots = B_{2n} = \ldots = 0$$

Similarly, to apply the boundary conditions at the surface $r=b$ we transform Equation (5) to the coordinate system (r,θ) by means of the relations

$$\log r_1 = \log r - \frac{c}{r}\cos\theta - \frac{1}{2}\left(\frac{c}{r}\right)^2\cos 2\theta - \frac{1}{3}\left(\frac{c}{r}\right)^3\cos 3\theta - \cdots$$

$$\frac{\cos n\theta_1}{r_1^n} = \frac{1}{r^n}\left(\cos n\theta + n\frac{c}{r}\cos(n+1)\theta + \frac{n(n+1)}{2!}\left(\frac{c}{r}\right)^2\cos(n+2)\theta + \cdots\right)$$

$$\log r_2 = \log r + \frac{c}{r}\cos\theta - \frac{1}{2}\left(\frac{c}{r}\right)^2\cos 2\theta + \frac{1}{3}\left(\frac{c}{r}\right)^3\cos 3\theta - \cdots$$

$$\frac{\cos n\theta_2}{r_2^n} = \frac{1}{r^n}\left(\cos n\theta - \frac{n}{1!}\frac{c}{r}\cos(n+1)\theta + \frac{n(n+1)}{2!}\left(\frac{c}{r}\right)^2\cos(n+2)\theta - \cdots\right)$$

Thus

$$\log\frac{r_2}{r_1} = 2\left(\frac{c}{r}\cos\theta + \frac{1}{3}\left(\frac{c}{r}\right)^3\cos 3\theta + \frac{1}{5}\left(\frac{c}{r}\right)^5\cos 5\theta + \cdots\right)$$

$$\frac{\cos n\theta_1}{r_1^n} - (-1)^n\frac{\cos n\theta_2}{r_2^n} = \frac{2}{r^n}\left(\cos n\theta + \frac{n(n+1)}{2!}\left(\frac{c}{r}\right)^2\cos(n+2)\theta + \frac{n(n+1)(n+2)(n+3)}{4!}\left(\frac{c}{r}\right)_4 \cos(n+4)\theta + \cdots\right)$$

Hence, the expression for F in the neighborhood of $r=b$ may be written,

$$F = \sum_1^\infty n\cos(2n-1)\theta\left(-2A_0\frac{(c/r)^{2n-1}}{2n-1} + \frac{2}{r^{2n-1}}\sum\nolimits_A - r^{2n-1}B_{2n-1}\right) \tag{19}$$

where

$$\sum\nolimits_A = A_{2n-1} + \frac{2n-2}{1!}cA_{2n-2} + \frac{(2n-2)(2n-3)}{2!}c^2A_{2n-3} + \ldots + c^{2n-2}A_1$$

It is possible now to express $R+i\omega L$ as given by Equation (17) in terms of A_n and B_n alone. Introduce (7) and (18) in boundary condition (10) and equate harmonic coefficients. Writing

$$a\alpha = z_1,\ \text{and}\ J_n(a\alpha_1)=J_n$$

this gives

$$J_n g_n = -\frac{A_n}{a^n} - \left(-\frac{a}{2c}\right)^n\left(\frac{A_0}{n}+S_A\right)+\left(\frac{a}{c}\right)\sum\nolimits_B$$

Evidently the harmonic terms in (17) vanish giving

$$R+i\omega L = 2Z_1 + 4i\omega\log\frac{2c}{a}+\Delta Z \qquad (20)$$

with

$$\Delta Z = -\frac{2}{1}\sum_1^\infty n\left(-\frac{1}{2c}\right)^n A_n - \frac{2}{1}\sum_1^\infty n\, c^{2n-1}B_{2n-1}$$

Since we require only a first order correction corresponding to small values of λ and ϵ^2, we employ the following method of attack, a process of successive approximations.

(1) Determine $B_n^{(0)}$ by conditions at $r=b$, neglecting the summation in the A's. This is equivalent to assuming uniform distribution of current over the surface of the wires or, what amounts to the same thing, the current concentrated in filaments at the centers of the wires.

(2) Determine $A_1^{(0)}$ in terms of $B_1^{(0)}$ by conditions at $r_1=a$, representing the series S_A and Σ_B by their leading terms.

(3) Determine $B_1^{(1)}$ in terms of $B_1^{(0)}$ and $A_1^{(0)}$ by conditions at $r=b$.

The first order correction in the impedance is then given by

$$\Delta Z = -\frac{2}{1}\left[\sum_1^\infty n\left(-\frac{1}{2c}\right)^n (A_n^{(1)}+A_n^{(2)}+\ \ldots)+\sum_1^\infty n\, c^{2n-1}(B^{(0)}_{2n-1}+B^{(1)}_{2n-1}+\ \ldots)\right] \qquad (21)$$

$$\approx \Delta_1 Z = \frac{2}{1}\left[\frac{1}{2c}(A_1^{(1)}+A_1^{(2)}+\ \ldots) - \sum_1^\infty n\, c^{2n-1}B^{(0)}_{2n-1} - c(B_1^{(1)}+B_1^{(2)}+\ \ldots)\right]$$

The application follows:

(1) Putting $r=b$ in Equations (9) and (19) and neglecting the summation in the A's, boundary conditions (12) and (13) give

$$z_3K'_{2n-1}h^{(0)}_{2n-1} = (2n-1)\left[-\frac{2A_0\epsilon^{2n-1}}{2n-1}+b^{2n-1}B^{(0)}_{2n-1}\right]$$

$$(2n-1)K_{2n-1}h^{(0)}_{2n-1} = 2A_0\epsilon^{2n-1}+b^{2n-1}B^{(0)}_{2n-1}$$

where

$$z_3=b\alpha_3,\ K_n=K_n(b\alpha_3),\ K'_n=K'_n(b\alpha_3)$$

Solving simultaneously and making use of the relations (see Jahnke u. Ende "Funktionentafeln", page 165),

$$zK'_n+nK_n=zK_{n-1}$$

$$zK'_n-nK_n=zK_{n+1}$$

gives

$$c^{2n-1}B^{(0)}_{2n-1} = -2A_0(\epsilon^2)^{2n-1}\frac{\eta_{2n-1}}{2n-1} \qquad (22)$$

where

$$\eta_n=\frac{K_{n-1}}{K_{n+1}}=1-\frac{2n}{z_3}\frac{K_n}{K_{n+1}}$$

Since

$$\eta_n \to 1-i\frac{2n}{z_3}\ \text{when}\ \alpha_3\to\infty$$

and

$$Z_3=2i\omega K_0/z_3K_0'\ \text{(from boundary conditions)}$$

$$Z_3\to -2\omega/z_3\ \text{when}\ \alpha_3\to\infty$$

we have

$$-\frac{2}{1}\sum c^{2n-1}B_{2n-1}^{(0)} = 8\epsilon^2(1+\epsilon^4)Z_3+4i\omega\log\frac{1-\epsilon^2}{1+\epsilon^2} \qquad (23)$$

(2) Putting $r_1=a$ in Equations (7) and (18), boundary conditions (10) and (11) give,

$$z_1J'_1g_1=\frac{A_1^{(1)}}{a}+\lambda\left(A_0+\frac{A_1^{(1)}}{2c}\right)+aB_1^{(0)}$$

$$J'_1g_1=-\frac{A_1^{(1)}}{a}+\lambda\left(A_0+\frac{A_1^{(1)}}{2c}\right)+aB_1^{(0)}$$

where

$$z_1=a\alpha_1,\ J_n=J_n\ (a\alpha_1),\ J_n'=J_n'\ (a\alpha_1)$$

Solving simultaneously and using the relations

$$zJ_n'+nJ_n=zJ_{n-1}$$

$$zJ_n'-nJ_n=-zJ_{n+1}$$

gives

$$\frac{A_1^{(1)}}{2c}=\frac{\lambda^2/\tau}{1-\lambda^2/\tau}\ (A_0+2cB_1^{(0)}) \qquad (24)$$

where

$$\tau_1=-\frac{J_0}{J_2}=1-\frac{2}{z_1}\frac{J_1}{J_2}$$

But

$$\tau_1=1+i\frac{2}{z_1}\ \text{when}\ \alpha_1\to\infty$$

and

$$Z_1=2i\omega J_0/z_1J_0'\ \text{(from boundary relations)}$$

$$\to -2\omega/z_1\ \text{when}\ \alpha_1\to\infty$$

Thus, neglecting terms of second order or higher in $2/z_1$, we have

$$\frac{2}{1}\frac{A_1^{(1)}}{2c}=2Z_1\frac{2\lambda^2(1-4\epsilon^2)}{(1-\lambda^2)^2}-16Z_3\frac{\epsilon^2\lambda^2}{1-\lambda^2}-4i\omega\frac{\lambda^2(1-4\epsilon^2)}{1-\lambda^2} \qquad (25)$$

Four our purpose it is unnecessary to proceed further. Since λ^2 is assumed small with respect to unity, we write

$$\frac{1}{1-\lambda^2}=1+\lambda^2\ \text{and}\ \frac{1}{(1-\lambda^2)^2}=1+2\lambda^2$$

and get from (21), (23) and (25),

$$\Delta Z=2Z_1[2\lambda^2(1+2\lambda^2)(1-4\epsilon^2)]+8Z_3\epsilon^2[1+\epsilon^4=2\lambda^2(1+\lambda^2)]+4i\omega\left[\log\frac{1-\epsilon^2}{1+\epsilon^2}-\lambda^2(1+\lambda^2)(1-4\epsilon^2)\right] \qquad (26)$$

Introducing this expression for ΔZ in (20) gives Equation (4) immediately. Also, from (20), (26) and (3)

$$C=k/4\left[\log\frac{1}{\lambda}\frac{1-\epsilon^2}{1+\epsilon^2}-\lambda^2(1+\lambda^2)(1-4\epsilon^2)\right] \qquad (27)$$

Since

$$Z_1=\frac{1}{a}\sqrt{\frac{f}{\sigma_1}}(1+i)\ \text{and}\ Z_3=\frac{1}{b}\sqrt{\frac{f}{\sigma_3}}(1+i)$$

expression (4) immediately becomes

$$R+i\omega L=2\frac{1}{a}\sqrt{\frac{f}{\sigma_1}}[1+2\lambda^2(1+2\lambda^2)(1-4\epsilon^2)](1+i)+8\epsilon^2\frac{1}{b}\sqrt{\frac{f}{\sigma_3}}[1+\epsilon^4-2\lambda^2(1+\lambda^2)](1+i)+4i\omega\left[\log_e\frac{1}{\lambda}\frac{1-\epsilon^2}{1+\epsilon^2}-\lambda^2(1+\lambda^2)(1-4\epsilon^2)\right] \qquad (28)$$

From relations (1), (3) and (28) assuming σ_1 and

σ_3 are equal, we have, since L_0/L_{12} is negligible compared with unity,

$$\alpha=\frac{\left[\frac{1}{a}[1+2\lambda^2(1+2\lambda^2)(1-4\epsilon^2)]+4\epsilon^2\frac{1}{b}[1+\epsilon^4-2\lambda^2(1+\lambda^2)]\right]\sqrt{\frac{kf}{\sigma}}}{4\left[\log_\epsilon\frac{1}{\lambda}\frac{1-\epsilon^2}{1+\epsilon^2}-\lambda^2(1+\lambda^2)(1-4\epsilon^2)\right]}$$

$$=\frac{\left[\frac{1}{2\epsilon\lambda}[1+2\lambda^2(1+2\lambda^2)(1-4\epsilon^2)]+4\epsilon^2[1+\epsilon^4-2\lambda^2(1+\lambda^2)]\right]\frac{1}{4b}\sqrt{\frac{kf}{\sigma}}}{\left[\log_\epsilon\frac{1}{\lambda}\frac{1-\epsilon^2}{1+\epsilon^2}-\lambda^2(1+\lambda^2)(1-4\epsilon^2)\right]} \quad (29)$$

The values of ϵ and λ which make this a minimum can be determined in various ways. For instance, the inner radius b being assumed a constant, the diameter and the interaxial separation of the conductors for minimum attenuation can be obtained by determining the values of a and c which satisfy the equations

$$\frac{\partial\alpha}{\partial a}=0 \text{ and } \frac{\partial\alpha}{\partial c}=0$$

An alternate way is to substitute various pairs of values of

$$\frac{b}{a}$$

and

$$\frac{c}{b}$$

in expression (29) and determine graphically or otherwise the pair which makes (29) a minimum. This pair of values is approximately:

$$\frac{b}{a}=5.4 \quad (30)$$

$$\frac{c}{b}=.46 \quad (31)$$

(In later work we have carried farther the process of successive approximations, which is the method of solution leading to the foregoing results in Equations (30) and (31); thereby we have obtained a more exact formula for the shielded pair attenuation. Computations on the basis of this formula give the somewhat more closely approximate values, $c/b=0.44$ and $b/a=5.6$, for the optimum separation and diameter ratios, respectively.)

This is the condition for minimum attenuation for the shielded pair system when the inner diameter of the outer conductor is fixed. It is interesting to note that the optimum proportioning ratios are independent of the frequency, the size of the conductors, and other variables. It will be obvious that the attenuation of the system can be reduced by increasing the size of the shield, keeping the ratios

$$\frac{b}{a} \text{ and } \frac{c}{b}$$

fixed. The diameter of the shield would probably be determined by such considerations as the maximum frequency to be transmitted over the system and the maximum allowable attenuation at that frequency.

The attenuation-frequency curve for a shielded pair of wires designed in accordance with the above condition, and having an inner diameter of shield equal to one inch, is shown in Fig. 9. In this case it is assumed that the leakage conductance is negligible. It will be noted that the attenuation at a frequency of 1000 kc. is approximately 2.0 db per mile. Hence, if repeaters having a gain of 60 db each were employed with such a circuit, these could be spaced at intervals of about 30 miles.

It is also interesting to note that the fixed ratios give fixed values of inductance, capacity and characteristic impedance regardless of the actual size of the system. Thus, by substituting the ratios

$$\frac{b}{a} \text{ and } \frac{c}{b}$$

as given in (30) and (31) into expressions (28) and (3), and changing to practical units the inductance and capacity can be obtained. They are, respectively, L=.724 millihenry per mile and C=.0397 microfarad per mile. The value of the high frequency characteristic impedance for the optimum configuration becomes

$$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{.724\times10^{-3}}{.0397\times10^{-6}}}=135 \text{ ohms}$$

(The foregoing values for L and C were computed from an approximate formula. A later computation by means of a more nearly exact formula, gives the result, L=0.755 millihenry per mile and C=0.0381 microfarad per mile. With these values the resulting value for Z_0 is 140 ohms.)

In the determination of the configuration for minimum attenuation it was assumed that the value of leakage conductance was zero. However, in practice it will be necessary to support the two conductors and to keep them in the desired relative position with respect to the shield. This will require the use of a certain amount of dielectric material inside the shield which will consequently introduce a certain amount of leakage conductance, and increase the average dielectric constant of the system.

By making such supports out of materials having low loss and low dielectric constant and by spacing them as far apart as possible, it is possible to make the effect on the attenuation and consequently on the configuration for minimum attenuation negligible.

However, if for any reason it is desired to support the conductors with respect to the shield in a manner which introduces a relatively large amount of leakage loss, it is still possible in many cases to determine the values of

$$\frac{b}{a} \text{ and } \frac{c}{b}$$

for minimum attenuation. Thus it will be shown that the configuration which results in minimum attenuation is independent of the dielectric loss and the average dielectric constant, provided that the value of the average dielectric constant is independent of the configuration of the conductors and sheath.

At frequencies high enough so that ω^2L^2 is large compared with R^2 and ω^2C^2 is large compared with G^2, the attenuation of a transmission system in which the leakage loss is not negligible is

$$\frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}} \quad (32)$$

where G the leakage loss$=2\pi f p C$, p being the power factor and

$$C=\frac{k}{L}$$

Therefore, the high frequency attenuation is

$$\frac{R}{2}\sqrt{\frac{C}{L}}+\pi p f\sqrt{k} \qquad (33)$$

The last term in (33) is independent of the size and spacing of the conductors and sheath provided that the average dielectric constant k is independent of changes in configuration of the circuit. If this condition is fulfilled, the ratio of the inner diameter of the shield to the diameter of the conductors and the ratio of the interaxial separation of the conductors to the inner diameter of the shield for minimum attenuation for any given inner diameter of shield is independent of the dielectric loss. The high frequency characteristic impedance Z_0 is given closely by

$$\sqrt{\frac{L}{C}}$$

where C is the capacity per unit length of the system. This also is substantially independent of the dielectric loss.

These conditions would be met by completely filling the space between the conductors and shield with a material, such as oil or a solid rubber insulation or else by using insulators made of thin flat discs of an appropriate insulation, such as porcelain or glass as shown in Fig. 2. A suitable insulator might also consist of a solid dielectric, such as rubber in which air has been entrapped. If the air bubbles are small and evenly distributed through the non-gaseous portion of the dielectric, the average dielectric constant and the dielectric loss can be made nearly independent of the diameter and spacing ratios of the system.

If the value of the average dielectric constant varies with changes in the diameter and spacing ratios of the conductors and shield, it becomes extremely difficult to obtain a mathematical solution for the diameter and spacing ratios which give minimum attenuation for any fixed diameter of shield. However, when gaseous and non-gaseous dielectrics are arranged in any desirable manner, the ratio of the inner diameter of the shield to the diameter of the conductors will not differ much from 5.4 and the ratio of the interaxial spacing of the conductors to the inner diameter of the shield will not differ much from .46. If both non-gaseous and gaseous dielectrics are used, and are disposed in such a manner that the boundary surfaces between different dielectrics lie along paths followed by the flux in a homogeneous dielectric, the optimum configuration will apparently be the same as for a gaseous dielectric.

Such types of construction as those described above, involving the use of a partly or wholly non-gaseous dielectric may be desirable for either mechanical or electrical reasons.

The conditions that must be satisfied for maximum high frequency characteristic impedance for a shielded solid pair can also be determined. The high frequency characteristic impedance of a shielded solid pair when the conductors are small compared to the shield is:

$$Z_0=\frac{4}{\sqrt{K}}\log\frac{1}{\lambda}\frac{1-\epsilon^2}{1+\epsilon^2} \qquad (34)$$

For any given ratio of inner diameter of shield to diameter of conductor, the characteristic impedance becomes:

$$Z_0=\frac{4}{\sqrt{K}}\log(\text{constant})+\frac{4}{\sqrt{K}}\log(\epsilon)\frac{(1-\epsilon^2)}{(1+\epsilon^2)} \qquad (35)$$

For maximum characteristic impedance the term

$$\epsilon\frac{1-\epsilon^2}{1+\epsilon^2}$$

must be maximized. This can be accomplished by taking its derivative with respect to ϵ and putting it equal to zero. Thus we find that $\epsilon=.486$ for maximum characteristic impedance for any ratio of inner diameter of shield to diameter of conductor, providing the latter ratio is large.

If, however, the conductors are large with respect to the shield, Equation (34) no longer holds. However, the position of the conductors with respect to the shield must be such as to minimize the capacity, since the capacity and high frequency characteristic impedance are inversely proportional to one another. It can be seen that as the diameters of the conductors approach in size the inner radius of the shield, ϵ approaches .5 for minimum capacity and hence maximum high frequency characteristic impedance. Thus, for any given ratio of inner diameter of shield to diameter of conductor, the ratio of the interaxial separation of the conductors to the inner diameter of the shield should be between the limits .486 and .500. For practical purposes a value of about .49 may generally be used to secure maximum impedance.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

2. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another by a substantially gaseous dielectric, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said di-

mensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

3. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, said conductors being helically twisted around the axis of said shield, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attentuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

4. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attentuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

5. A transmission circuit comprising two solid cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

6. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, said conductors consisting of a plurality of non-insulated conducting strands, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attentuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

7. A transmission circuit comprising two hollow cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, said conductors having walls of substantial thickness as compared with their diameters, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attentuation of said circuit as (dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

8. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another by a substantially gaseous dielectric, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

9. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less

than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, said conductors and shield being insulated from one another by a substantially non-gaseous dielectric, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

10. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, said conductors being helically twisted around the axis of said shield, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

11. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, said conductors being transposed at frequent intervals, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, the said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the relative dimensions and spacings of said conductors and shield being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

12. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, the ratio of the interaxial separation of said conductors to the inner diameter of said shield and the ratio of the inner diameter of said shield to the diameter of each of said conductors being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

13. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another by a substantially gaseous dielectric, the ratio of the interaxial separation of said conductors to the inner diameter of said shield and the ratio of the inner diameter of said shield to the diameter of each of said conductors being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C and G are, respectively, the linear resistance, inductance, capacity and leakage.

14. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the ratio of the interaxial separation of said conductors to the inner diameter of said shield and the ratio of the inner diameter of said shield to the diameter of each of said conductors being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C, and G are, respectively, the linear resistance, inductance, capacity and leakage.

15. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diame-

ter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another by a substantially gaseous dielectric, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the ratio of the interaxial separation of said conductors to the inner diameter of said shield and the ratio of the inner diameter of said shield to the diameter of each of said conductors being such that for a given cross-sectional area comprised within said shield, the attenuation of said circuit (as dependent upon said dimensions and spacings) will be a minimum at frequencies so high that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C, and G are, respectively, the linear resistance, inductance, capacity and leakage.

16. A transmission circuit comprising two cylindrical conductors, a cylindrical conducting shield surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, the ratio of the interaxial separation of said conductors to the inner diameter of said shield being approximately .46 and the ratio of the inner diameter of said shield to the diameter of each of said conductors being approximately 5.4.

17. A transmission circuit comprising two cylindrical conductors, a cylindrical conducting shield surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another by a substantially gaseous dielectric, the ratio of the interaxial separation of said conductors to the inner diameter of said shield being approximately .46 and the ratio of the inner diameter of said shield to the diameter of each of said conductors being approximately 5.4.

18. A transmission circuit comprising two cylindrical conductors, a cylindrical conducting shield surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the ratio of the interaxial separation of said conductors to the inner diameter of said shield being approximately .46 and the ratio of the inner diameter of said shield to the diameter of each of said conductors being approximately 5.4.

19. A transmission circuit comprising two cylindrical conductors, a cylindrical conducting shield surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another by a substantially gaseous dielectric, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the ratio of the interaxial separation of said conductors to the inner diameter of said shield being approximately .46 and the ratio of the inner diameter of said shield to the diameter of each of said conductors being approximately 5.4.

20. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield of predetermined diameter surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another, the ratio of the interaxial separation of said conductors to the inner diameter of said shield being such that for a given cross-sectional area comprised within said shield, the characteristic impedance of said circuit will be a maximum for frequencies sufficiently high so that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C, and G are, respectively, the linear resistance, inductance, capacity and leakage.

21. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, one of said conductors being connected as a return for the other, a cylindrical conducting shield of predetermined diameter surrounding said conductors, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, said conductors and shield being insulated from one another, the ratio of the interaxial separation of said conductors to the inner diameter of said shield being such that for a given cross-sectional area comprised within said shield, the characteristic impedance of said circuit will be a maximum for frequencies sufficiently high so that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C, and G are, respectively, the linear resistance, inductance, capacity and leakage.

22. A transmission circuit comprising two cylindrical conductors of the same size and arranged side by side but having an interaxial separation greater than their diameters, the diameter of the conductors being substantially less than their interaxial separation, a cylindrical conducting shield surrounding said conductors, one of said conductors being connected as a return for the other, said conductors and shield being insulated from one another by a substantially gaseous dielectric, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, the ratio of the interaxial separation of said conductors to the inner diameter of said shield being such that for a given cross-sectional area comprised within said shield, the characteristic impedance of said circuit will be a maximum for frequencies sufficiently high so that ω^2L^2 and ω^2C^2 are large as compared with R^2 and G^2, respectively, where ω is 2π times the frequency, and R, L, C, and G are, respectively, the linear resistance, inductance, capacity and leakage.

23. A transmission circuit comprising two cylindrical conductors, a cylindrical conducting shield surrounding said conductors, one of said

conductors being connected as a return for the other, said conductors being of such a type that conduction of currents whose frequencies are substantially above the audible range takes place substantially on the surface of said conductors, said conductors and shield being insulated from one another, the ratio of the interaxial separation of said conductors to the inner diameter of said shield lying between 0.486 and 0.50.

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