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E. I. GREEN ET AL

2,029,421

CONCENTRIC CONDUCTOR TRANSMISSION SYSTEM

Filed Feb. 2, 1932

2 Sheets-Sheet 1

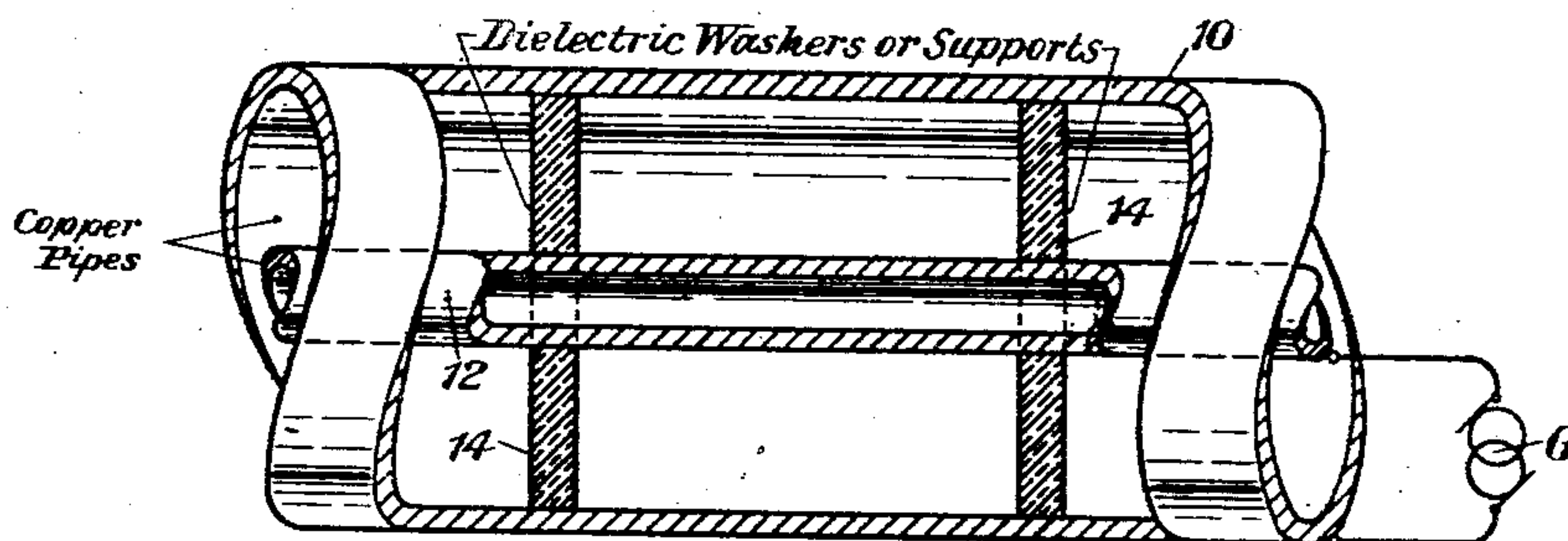


Fig. 1, Longitudinal Section

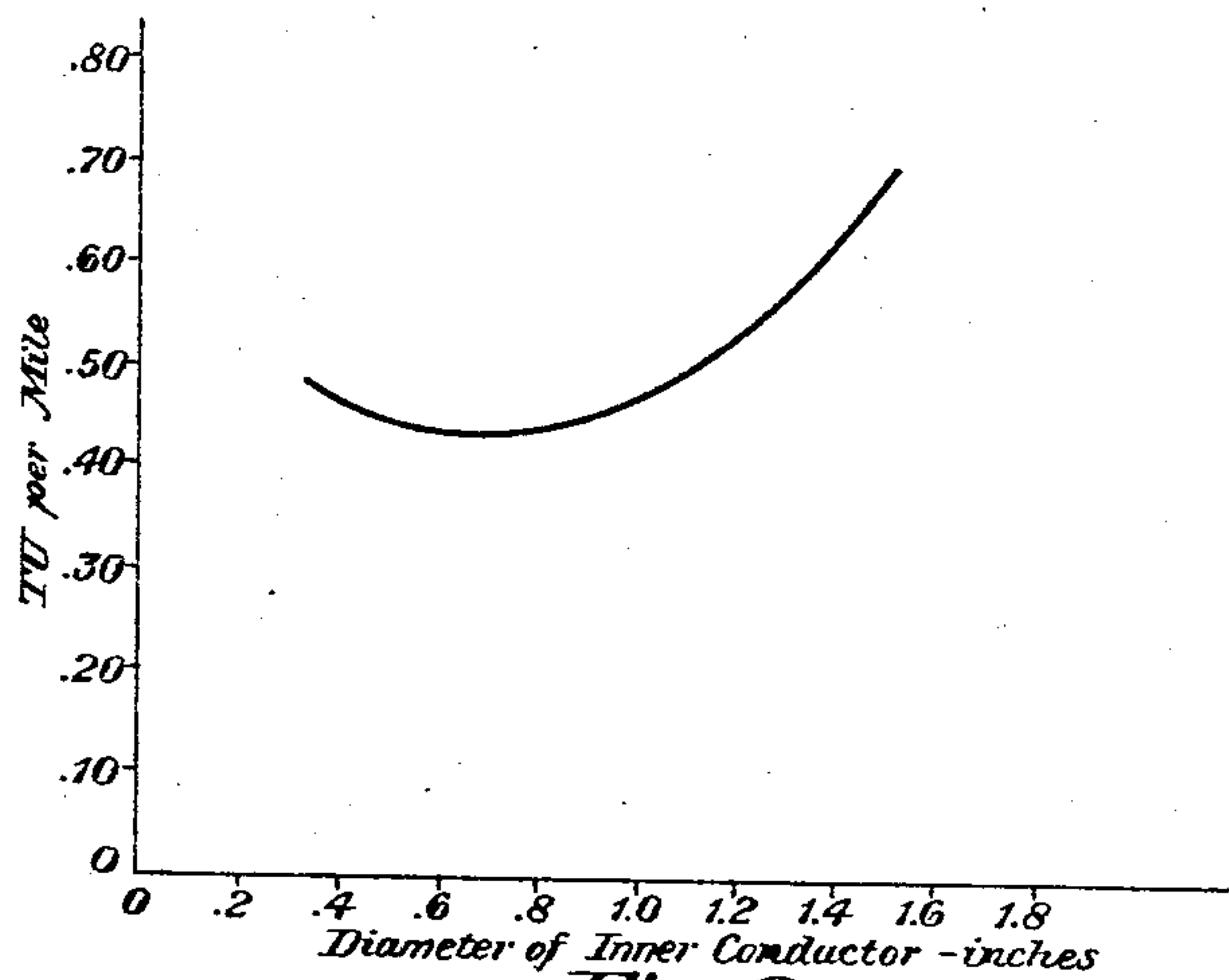


Fig. 2

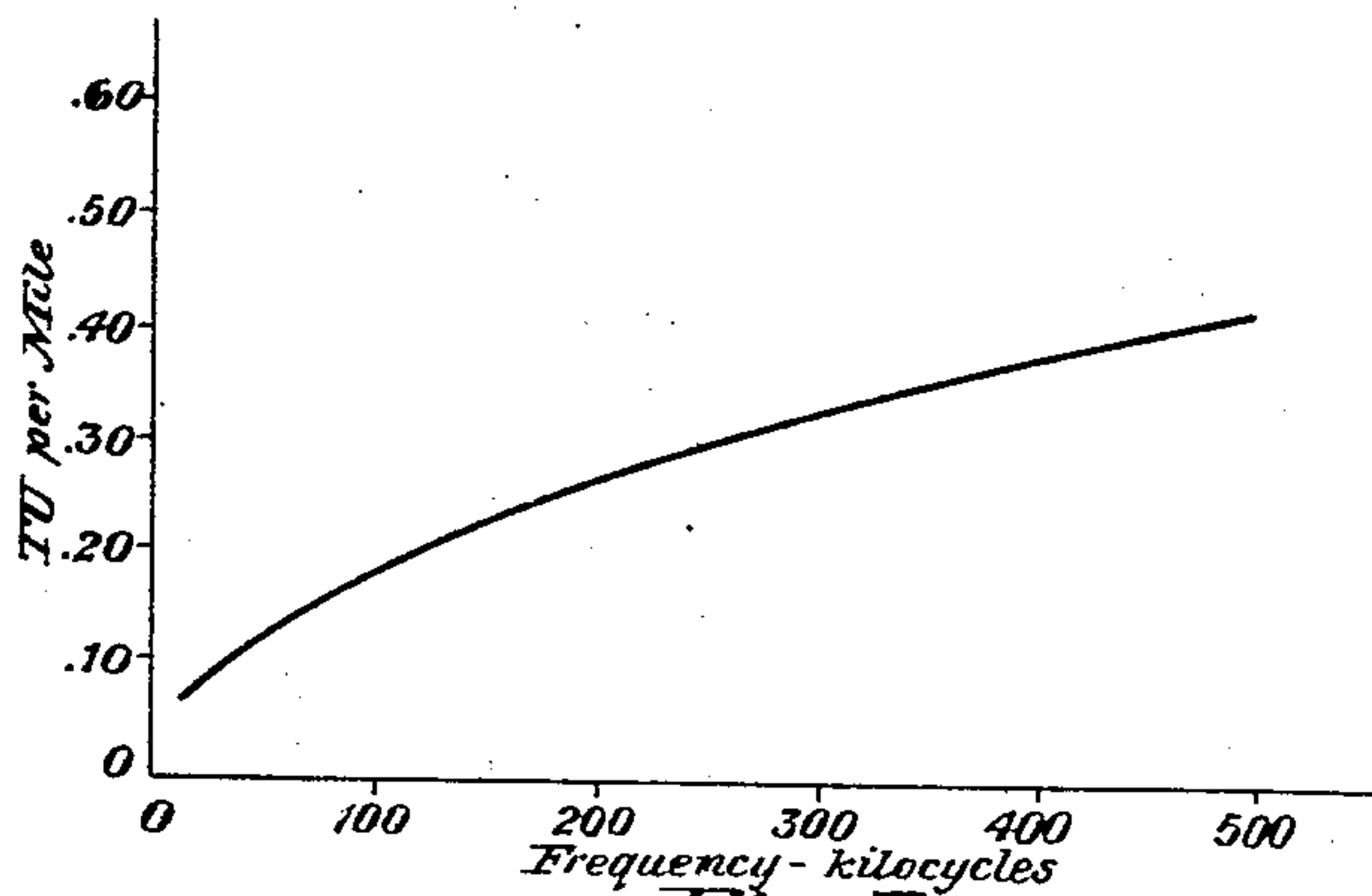


Fig. 3

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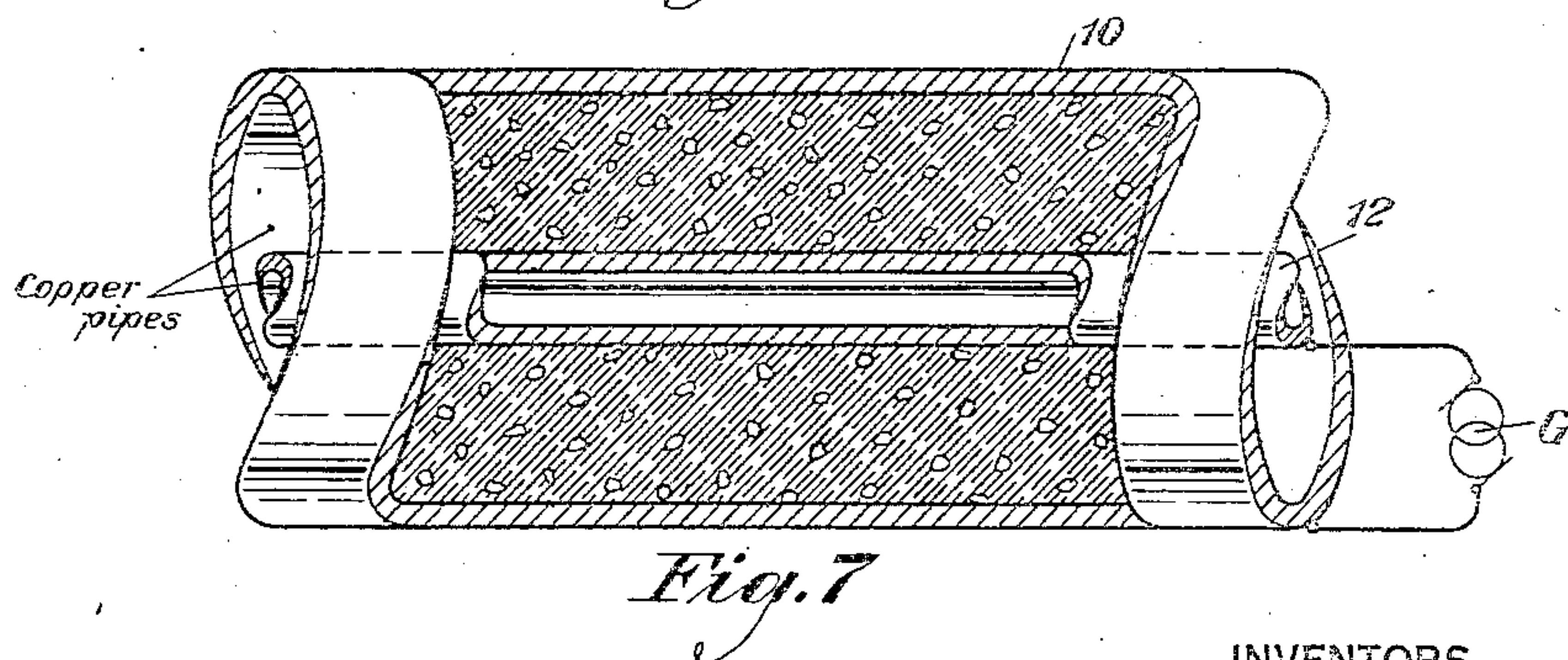
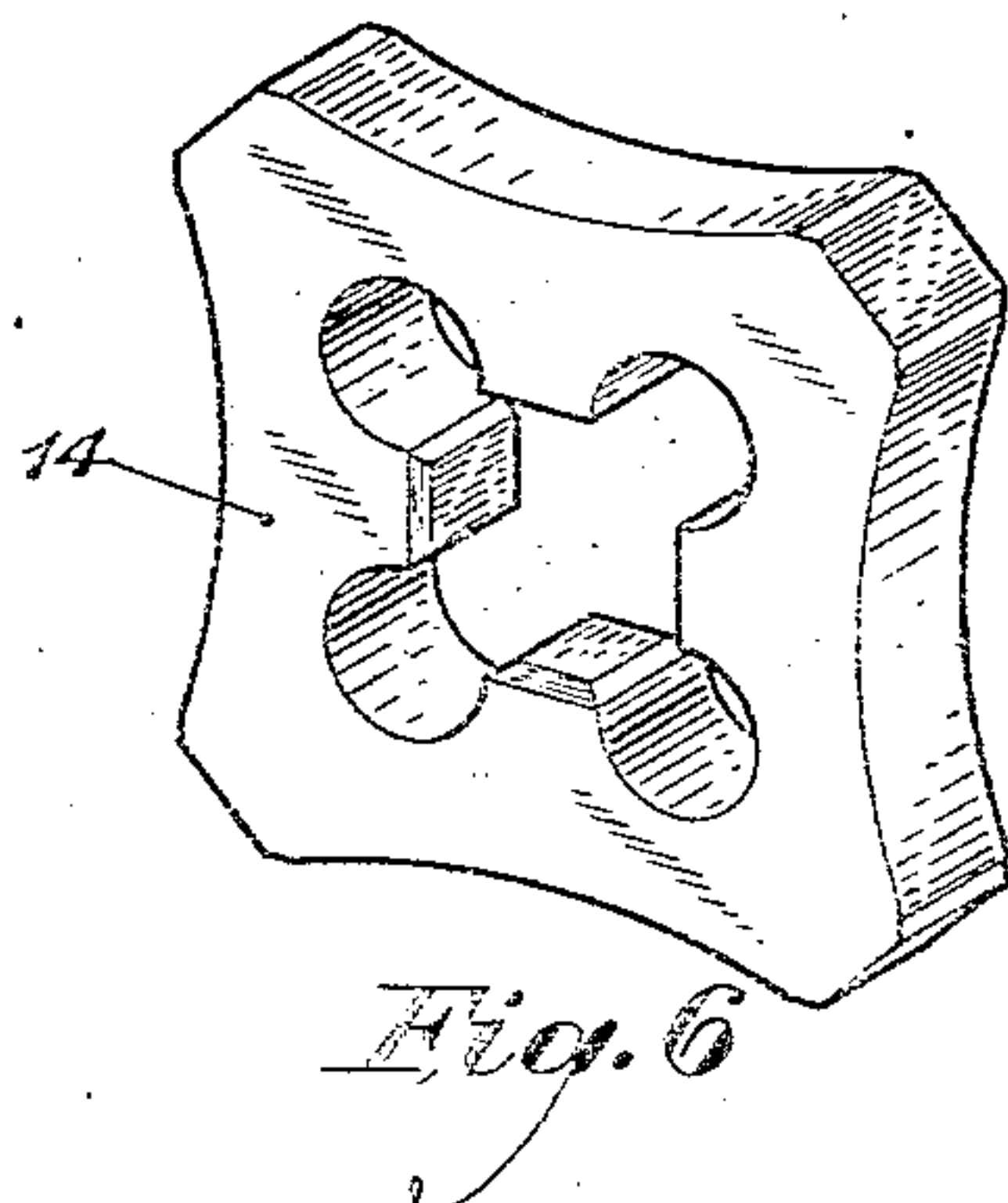
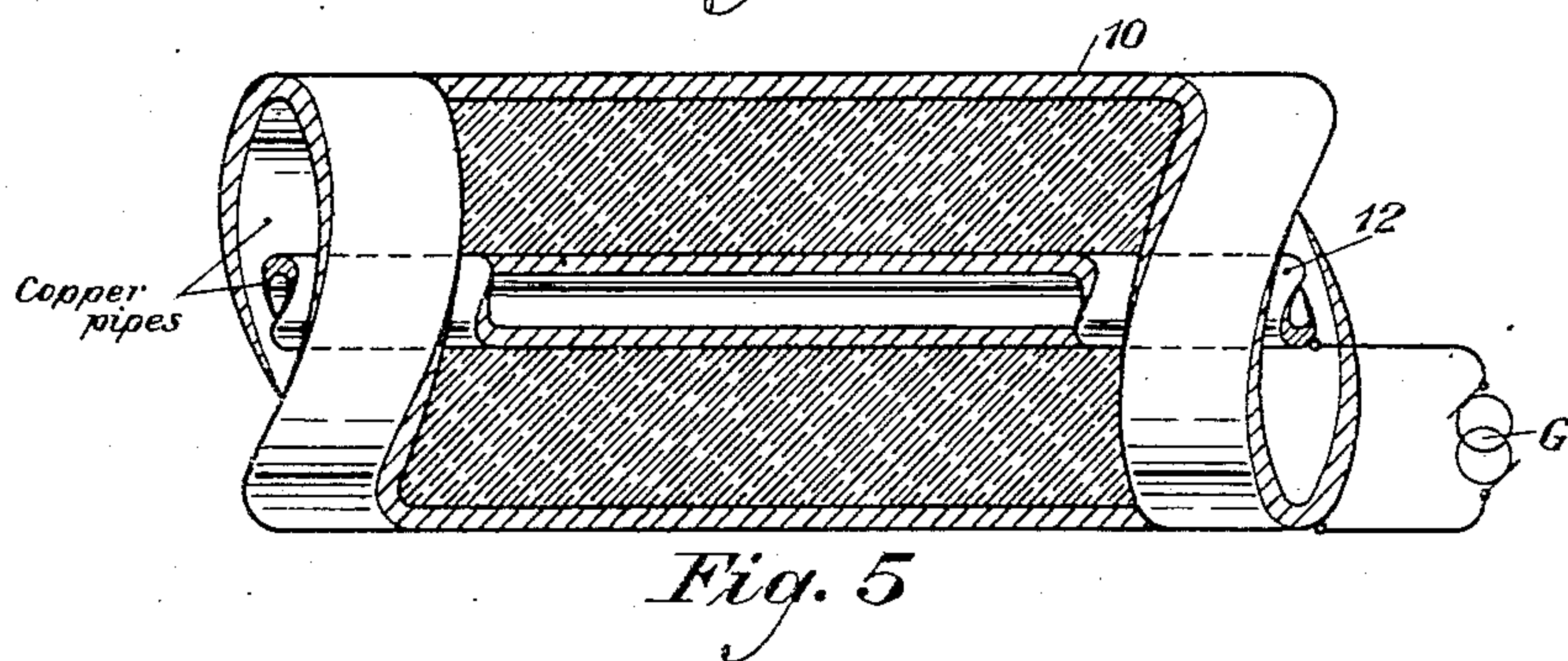
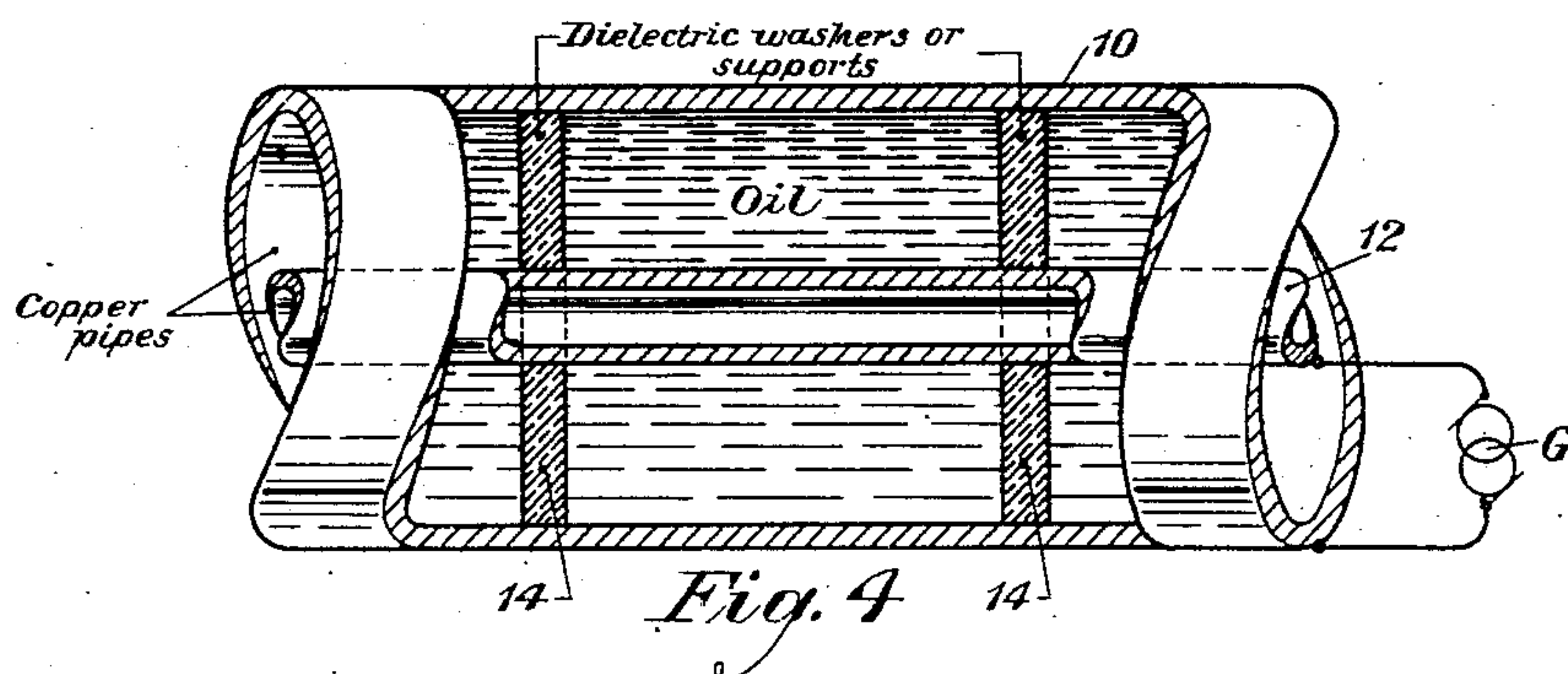
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CONCENTRIC CONDUCTOR TRANSMISSION SYSTEM

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2 Sheets-Sheet 2



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CONCENTRIC CONDUCTOR TRANSMISSION
SYSTEM

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Application February 2, 1932, Serial No. 590,496

11 Claims. (Cl. 178—44)

This invention relates to a novel form of conductor structure employing concentric cylindrical conductors for the transmission of a wide band of frequencies with relatively low attenuation. The invention principally relates to such proportioning of the relative dimensions of the conductors as to produce minimum attenuation. This application is a continuation in part of our application Serial Number 365,517, filed May 23, 1929.

If a solid cylindrical conductor or a hollow cylindrical conductor is provided with a return conductor comprising a hollow cylindrical conductor concentrically arranged with respect to the first conductor, and the two conductors are separated by a dielectric consisting largely of air or other gaseous medium, the transmission line thus formed will have a number of desirable characteristics. Its attenuation at all frequencies will be quite low as compared with the corresponding attenuation of open wire lines and cable circuits such as are now commonly used for telephone transmission. Such a transmission circuit may, therefore, be employed for the transmission of a much wider band of frequencies than has been possible with types of transmission circuits heretofore used. It also has the advantage that it is substantially free from interference from neighboring conductor systems and in itself tends to produce but little interference into adjacent transmission circuits.

The present invention is, however, more particularly concerned with the discovery that if the diameter of one of the concentric conductors is assigned a predetermined value, there is an optimum diameter for the other conductor for which the attenuation of the system will be a minimum.

It is a characteristic of a transmission system of the type herein considered that at high frequencies the current tends to flow at the outer surface of the inner conductor and at the inner surface of the outer conductor. Therefore, the outer radius of the inner conductor and the inner radius of the outer conductor are of importance from an attenuation standpoint. In accordance with the present invention, it has been found that if the ratio of the inner radius of the outer conductor to the outer radius of the inner conductor is approximately 3.6, the attenuation will be a minimum at all frequencies for any given predetermined diameter of one of the conductors. The attenuation will be decreased, however, by increasing the diameters of the two conductors so long as the optimum ratio of about 3.6 is maintained.

The invention will now be more fully understood from the following description when read in connection with the accompanying drawings in which Figure 1 is a symbolic representation of a concentric conductor system; Fig. 2 is a curve showing how the attenuation has a minimum value for a certain diameter of the inner conductor when the diameter of the outer conductor is fixed, Fig. 3 is a curve showing the variation of the attenuation with frequency at the minimum value indicated by the curve of Fig. 2 and Figs. 4, 5, 6 and 7 show modified forms of the dielectric arrangement between conductors.

Referring to Fig. 1 of the drawings, 10 designates an outer conductor in the form of a hollow cylinder of suitable conducting material. A second cylindrical conductor 12 is mounted concentrically with the outer conductor 10. One of the conductors acts as a return for the other and not as a mere shield, this fact being indicated by the conventional representation of a source of electromotive force *G* with its terminals connected to the two conductors.

In order that the attenuation may be small at high frequencies, the leakage loss between the conductors should be as small as possible. As the leakage loss is due to the nature of the dielectric interposed between the conductors, the dielectric should be principally of air, since air introduces no leakage loss. Accordingly, the two conductors may be held in proper concentric relation and out of electrical contact with each other by means of spaced dielectric washers 14. These washers should be separated from each other a suitable distance and should be made as thin as possible consistent with the required mechanical strength. They should also be composed of some dielectric of small loss angle and low dielectric constant, since if these conditions are obtained, the leakage loss (which in the ordinary open wire system comprises a large part of the attenuation) may be made so small as to be practically negligible. For example, hard rubber or preferably pyrex glass or other good insulating material, may be used for the insulating washers 14. In this connection, it should be noted that as the outer conductor may be made watertight, the insulating washers may be maintained dry and free from dirt or contamination, so that the leakage loss will not increase or change with time but will be maintained at the low value which is characteristic of insulators in the dry and clean condition in which they come from the factory.

In order that a conducting system such as herein disclosed may have as small attenuation as

possible at high frequencies, the diameters of the two concentric conductors should be made as large as possible. However, due to practical considerations it may be desirable that the conductor should be of such character that it might be used in existing cable ducts or in connection with present aerial cable construction. For these reasons, in practice, it may be convenient to make the diameter of the external conductor not much greater than about two and five-eighths inches, if the conductor is to be used in the existing telephone plant. For economical reasons the thickness of the conductors should be made as small as is consistent with securing proper electrical characteristics and mechanical strength. In general, it has been found that if the conducting shells are made of proper thickness to satisfy the mechanical requirements, the thickness is not a limiting factor in the attenuation at high frequencies. This is due to the skin effect which, as previously described, causes the current to crowd to the outer surface of the inner conductor and the inner surface of the outer conductor as the frequency increases, thereby rendering the remaining cross-sectional area of little utility for carrying current.

As the outer conductor is, or at least can be made watertight, the leakage losses can be reduced to very low values by the use of pyrex or other insulation where mechanical support is necessary, with the largest possible air space between the two conductors. Under these circumstances the leakage loss will not change with weather conditions. For zero leakage (a condition which would be approximately obtained) the attenuation equals

$$\frac{R}{2} \sqrt{\frac{C}{L}}$$

at high frequencies, where R represents the resistance, C the capacity and L the inductance. From this expression it is evident that the values of R and C should be as small as possible. At high frequencies R is inversely proportional to the diameter of the conductor, and hence the attenuation will be smaller at any given frequency the larger the diameter of the conductor. The capacity C also is an inverse function of the diameter and decreases as the difference between the diameters of the inner and outer conductors increases. Consequently, if the diameter of the outer conductor is fixed, as the diameter of the inner conductor increases from some small value the resistance of the conducting system decreases, while at the same time the capacity increases. The decrease in resistance tends to reduce the attenuation, while the increase in capacity tends to increase the attenuation. For a given diameter of the inner conductor these two effects balance and the attenuation becomes a minimum.

Fig. 2 is a curve showing how the attenuation varies with the diameter of the inner conductor at 500,000 cycles, with the inner diameter of the outer conductor fixed at two and one-half inches (a size which corresponds to the size of the ordinary cable employed in the telephone plant). This curve shows a minimum attenuation of .43 transmission units per mile for an inner conductor diameter of about .7 inch. As will be clear from the curve, either an increase or a decrease of the diameter of the inner conductor from the foregoing value results in an increase in the attenuation. It will also be noted that the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor for the

case plotted in the curve of Fig. 2 (which curve was determined empirically) is about 3.6 which corresponds to the theoretical optimum ratio for a minimum attenuation as determined theoretically hereinafter.

In Fig. 3 is shown a curve of the attenuation at various frequencies of a concentric conductor system whose outer conductor has a diameter of two and one-half inches and the inner conductor has the optimum diameter of about .7 inch. It will be observed from this curve that while the attenuation increases with frequency, the slope of the curve is not steep and the increase in attenuation is very much less than would be the case for an open wire line.

A mathematical analysis of the concentric type of conductor system will now be given to show the optimum ratio of the diameters of the two conductors which corresponds to minimum attenuation for the system.

Diameter relations—Size of outer conductor fixed

Unless the walls of the coaxial conductors are made extremely thin, the attenuation at high frequencies will be practically independent of the thickness of wall. This is because of the large skin effect which makes the current flow in a very thin wall on the outside of the inner tube and on the inside of the outer tube. Consequently, the thickness of the conductor walls will ordinarily be determined by mechanical considerations. Under such conditions, the following formulas may be used at high frequencies (above the voice range).

Resistance of inner conductor=

$$R_1 = K_0 \frac{\sqrt{f}}{b} \quad (1)$$

Resistance of outer conductor=

$$R_0 = K_0 \frac{\sqrt{f}}{c} \quad (2)$$

Linear inductance=

$$L = K_1 \log \frac{c}{b} \quad (3)$$

also

Linear capacity=

$$C = \frac{K_2}{\log \frac{c}{b}} \quad (4)$$

where K_0 , K_1 and K_2 are constants, f is the frequency, c is the inner radius of the outer conductor, and b is the outer radius of the inner conductor. It will be noted that the outer radius of the outer conductor and the inner radius of the inner conductor do not appear in the formulas.

The attenuation at high frequencies is

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} \quad (5)$$

where R , L , C and G are the linear resistance, inductance, capacity, and leakage conductance, respectively. Let us assume first that air insulation is employed between the two conductors, and $G=0$. We have then

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{R_1 + R_0}{2} \sqrt{\frac{C}{L}} \quad (6)$$

Substituting from Equations (1) to (4), we obtain

$$\alpha = \frac{K_0 \sqrt{f}}{2} \left(\frac{1}{b} + \frac{1}{c} \right) \sqrt{\frac{K_2}{K_1} \frac{1}{\left(\log \frac{c}{b} \right)^3}} \quad (7)$$

Letting

$$K_2 = \frac{K_0}{2} \sqrt{\frac{K_1}{K_2}} \text{ and } x = \frac{c}{b},$$

we have

$$\alpha = K_2 \sqrt{\frac{1}{c}} (1+x) \frac{1}{\log x} \quad (8)$$

Whence, c being assumed fixed

$$\frac{d\alpha}{dx} = K_2 \sqrt{\frac{1}{c}} \left[\frac{1}{\log x} - \frac{x+1}{x(\log x)^2} \right] \quad (9)$$

Setting

$$\frac{d\alpha}{dx} = 0, \quad (10)$$

we obtain

$$\log x = \frac{x+1}{x}$$

The logarithms in expression (10) and in the following discussion are natural logarithms (base e). From (10) we find that

$$x = \frac{c}{b} = 3.59 \quad (11)$$

This is the condition for minimum attenuation for the concentric arrangement when the inner diameter of the outer conductor is fixed. It is interesting to note that it is independent of the frequency, the size of the conductors, and other variables. It will be obvious that the attenuation can be reduced by increasing the size of both conductors (leaving the ratio

$$\frac{c}{b}$$

fixed). In a given case, however, the maximum diameter for the outer conductor will doubtless be fixed by practical considerations. The optimum outer diameter for the inner conductor can then be determined from the relation given in (11).

It is also interesting to note that the fixed ratio of c to b gives fixed values of inductance, capacity and characteristic impedance for the concentric arrangement, regardless of the actual dimensions of the conductors. By analogy to Formula (55), page 109 of "Calculation of Alternating Current Problems" by L. Cohen, the capacity of the structure herein considered, bearing in mind that the dielectric is equivalent to air will have the value

$$C = \frac{0.0894}{\log \frac{c}{b}} \times 10^{-8} \text{ farads per mile.}$$

Likewise from Formula (56) on page 72 of the Cohen publication the inductance in abhenries per centimeter has the value

$$L = 2 \log \frac{c}{b}$$

Reducing this to henries per mile

$$L = \frac{2 \times 1.60935 \times 10^9}{10^3} \log \frac{c}{b} = .32187 \times 10^{-3} \log \frac{c}{b}$$

The value of the nominal characteristic impedance corresponding to

$$\frac{c}{b} = 3.59$$

then becomes

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{.32187 \times 10^{-3}}{.0894 \times 10^{-8}}} \times \log \frac{c}{b} = 60 \log \frac{c}{b} =$$

$$76.7 \text{ ohms} \quad (12)$$

The values of inductance and capacity corresponding to the values of 3.59 for

$$\frac{c}{b}$$

are .411 mh. per mile and .0700 mf. per mile, respectively.

It will be recalled that in the derivation of the ratio

$$\frac{c}{b}$$

for minimum attenuation it was assumed that the value of leakage conductance G is zero. In practice, it will, of course, be necessary to use spacers of some solid dielectric material in order to keep the two conductors apart. These spacers or washers will produce a slight increase in the capacity between the two conductors and will give a small leakage conductance. By using a good dielectric material (with small dielectric constant and low power factor), by making the spacers thin, and by placing them far apart the effect of the spacers upon the constants of the concentric system could be made negligible. Thus the desired value of 3.6 for the ratio

$$\frac{c}{b}$$

would remain unchanged.

Furthermore, it can be shown that even if the capacity and leakage conductance introduced by the insulating spacers are appreciable the component of attenuation due to leakage is substantially independent of the conductor dimensions, while the component due to resistance is approximately the same function of

$$\frac{c}{b}$$

as before. Hence the desired value of

$$\frac{c}{b}$$

is 3.6 regardless of the spacers. In order to illustrate this point let us consider the case where the space between the two conductors is filled with a substantially uniform non-gaseous dielectric material having a dielectric constant ϵ . Such an arrangement would be obtained, for example, if the insulators 14 of Fig. 1 were made thick enough to touch one another, or if the space between the insulators were filled with oil as shown in Fig. 4, or if the conductors were separated by a continuous rubber insulation as in Fig. 5.

If the leakage conductance of the insulating material cannot be neglected, Formula (5) gives the correct expression for the attenuation. The expressions for the resistances of the inner and outer conductors will be the same as before (Formulas 1 and 2), as will the expression for the inductance (Formula 3). The expression for the capacity, however, now becomes

$$C = \frac{K_2 \epsilon}{\log \frac{c}{b}} \quad (13)$$

while the leakage conductance is given by

$$G = P \omega C = \frac{P \omega K_2 \epsilon}{\log \frac{c}{b}} \quad (14)$$

where P is the power factor of the insulation and

$\omega=2\pi f$. Substituting these values (13) and (14) in formula (5) we obtain

$$\alpha = K_3 \sqrt{f\epsilon} \frac{1}{c \log x} + \frac{\sqrt{K_1 K_2 P \omega \sqrt{\epsilon}}}{2} \quad (15)$$

Since f and ϵ are not functions of x , and since c is assumed fixed, this may be written for purposes of minimization as

$$\alpha = K_4 \frac{x+1}{\log x} + K_5 \quad (16)$$

where K_4 and K_5 are constants.

Upon differentiating with respect to x the condition for minimum attenuation is found to be, as before,

$$\log x = \frac{x+1}{x} \quad (17)$$

The high frequency impedance in this case becomes

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{60 \log x}{\sqrt{\epsilon}} \quad (17)$$

Now let us consider a case where the space between the conductors consists of a combination of gaseous and non-gaseous dielectrics, as for example, when insulating discs or washers of thickness t (and of the type illustrated in Fig. 1) are used with a spacing d between corresponding surfaces and are made of a material having a dielectric constant ϵ . The capacity now becomes

$$C = \frac{K_2}{\log \frac{c}{b}} \left[1 + \frac{(\epsilon-1)t}{d} \right] \quad (18)$$

while the leakage is

$$G = \frac{t}{d} \times \frac{P \omega K_2 \epsilon}{\log \frac{c}{b}} \quad (19)$$

On substituting these values in Formula (5) the following expression results

$$\alpha = K_3 \sqrt{\left[1 + \frac{(\epsilon-1)t}{d} \right] \frac{1}{c \log x} + \frac{\sqrt{K_1 K_2 P \omega \epsilon t}}{2d} \sqrt{\frac{d}{d + (\epsilon-1)t}}} \quad (20)$$

which for purposes of differentiation in this case may be written

$$\alpha = K_6 \frac{x+1}{\log x} + K_7 \quad (21)$$

hence as before, the condition for minimum attenuation is

$$\log x = \frac{x+1}{x} \quad (22)$$

The high frequency impedance in this case, however, is

$$Z_0 = \frac{60 \log x}{\sqrt{1 + \frac{(\epsilon-1)t}{d}}} \quad (22)$$

Noting that the quantity under the square root sign in the denominator of this expression is the weighted average dielectric constant of the insulating medium, and letting this quantity equal ϵ' , we have

$$Z_0 = \frac{60 \log x}{\sqrt{\epsilon'}} \quad (23)$$

The case just considered was based on the condition that the gaseous and non-gaseous materials are separated from each other by planes perpendicular to the axis of the conductors, or in other words, that a line of dielectric flux passes

through only one kind of material. When consideration is given to cases wherein the boundaries between the different materials are other than the aforementioned planes, it is found that a line of dielectric flux may pass through more than one kind of material in going from one conductor to the other. Such a condition may be due to a number of circumstances; for example, the insulators 14 of Fig. 1 might be made with parts cut away to form a webbed or spoked shape, as shown in Fig. 6, or the insulators might have such dimensions that there would be considerable clearance between themselves and the conductors. Another possibility is the use of a solid dielectric in which bubbles of gas were entrapped during manufacture, giving the effect of a number of alternate layers of solid and gaseous material through which the dielectric flux passes as shown in Fig. 7. A somewhat similar effect would be obtained if the dielectric consists of a round strip of solid insulation spiraled around the inner conductor to support it centrally within the outer conductor, the diameter of the strip of insulating material being approximately equal to the inner radius of the outer conductor minus the outer radius of the inner conductor.

In such cases where a line of dielectric flux passes through more than one kind of material, it becomes extremely difficult to obtain a mathematical solution for the diameter ratio which results in minimum attenuation for the circuit, as it involves a three-dimensional field problem. Furthermore, a solution for one arrangement of solid and gaseous dielectrics will not apply to another arrangement, and it would be necessary to consider each arrangement individually. However, in cases where the dielectric is made of solid and non-solid materials arranged in any desirable manner, the optimum diameter ratio apparently will not differ much from 3.59. It has been shown that for either a gaseous or a non-gaseous dielectric the optimum diameter ratio is 3.59, and that the same value holds when a combination of materials is used and their boundaries are planes perpendicular to the axis of the conductors. Another condition which gives a value of 3.59 occurs when we have a mixture of two dielectrics such as a solid with entrapped bubbles of gas, the bubbles being relatively small and uniformly distributed so that essentially a homogeneous substance is produced. It thus seems evident that the optimum diameter ratio must be reasonably close to 3.59 for a dielectric made of any desirable arrangement of materials.

Such types of construction as those described above, involving the use of a partly or wholly non-gaseous dielectric, may be desirable for either mechanical or electrical reasons.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated without departing from the spirit of the invention as defined in the following claims.

We claim:

1. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other and insulated from each other by a substantially non-gaseous dielectric, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being such that the attenuation of the circuit will be a minimum for frequencies above the voice range.

2. A high frequency transmission circuit com-

prising two concentrically arranged cylindrical conductors connected one as a return for the other, said conductors having walls of substantial thickness as compared with their diameters and being insulated from each other by a substantially non-gaseous dielectric, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being such that the attenuation of the circuit will be a minimum for frequencies above the voice range.

3. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other and insulated from each other by a substantially non-gaseous dielectric, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being approximately 3.6 so that the attenuation will be a minimum for frequencies above the voice range.

4. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other, said conductors having walls of substantial thickness as compared with their diameters and being insulated from each other by a substantially non-gaseous dielectric, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being approximately 3.6 so that the attenuation will be a minimum for frequencies above the voice range.

5. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other, said conductors being separated from one another by means of solid dielectric material so arranged that said solid dielectric material occupies a substantial proportion of the space between said conductors, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being approximately 3.6 so that the attenuation will be a minimum for frequencies above the voice range.

6. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other, said conductors having walls of substantial thickness in comparison with their diameters and being separated from one another by means of solid dielectric material so arranged that said solid dielectric material occupies a substantial proportion of the space between said conductors, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being approximately 3.6 so that the attenuation will be a minimum for frequencies above the voice range.

7. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other and insulated from each other by a substantially solid dielectric, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being approximately 3.6 so that the attenuation will be a minimum for frequencies above the voice range.

8. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other and insulated from one another by a dielectric which is partly gaseous but principally non-gaseous, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being approximately 3.6 so that the attenuation will be a minimum for frequencies above the voice range.

9. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other and insulated from each other by a substantially non-gaseous dielectric, the proportioning of the dimensions of the conductors being such that at frequencies above the voice range the characteristic impedance in ohms will be approximately 77 divided by the square root of the effective dielectric constant of the insulating material.

10. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other, and insulated from each other by a substantially non-gaseous dielectric, said conductors having walls of substantial thickness as compared with their diameters, the proportioning of the dimensions of the conductors being such that at frequencies above the voice range the characteristic impedance in ohms will be approximately 77 divided by the square root of the effective dielectric constant of the insulating material.

11. A high frequency transmission circuit comprising two concentrically arranged cylindrical conductors connected one as a return for the other and insulated from each other by a dielectric at least half non-gaseous, the proportioning of the dimensions of the conductors being such that at frequencies above the audible range the characteristic impedance in ohms will be approximately 77 divided by the square root of the effective dielectric constant of the insulating media.

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