

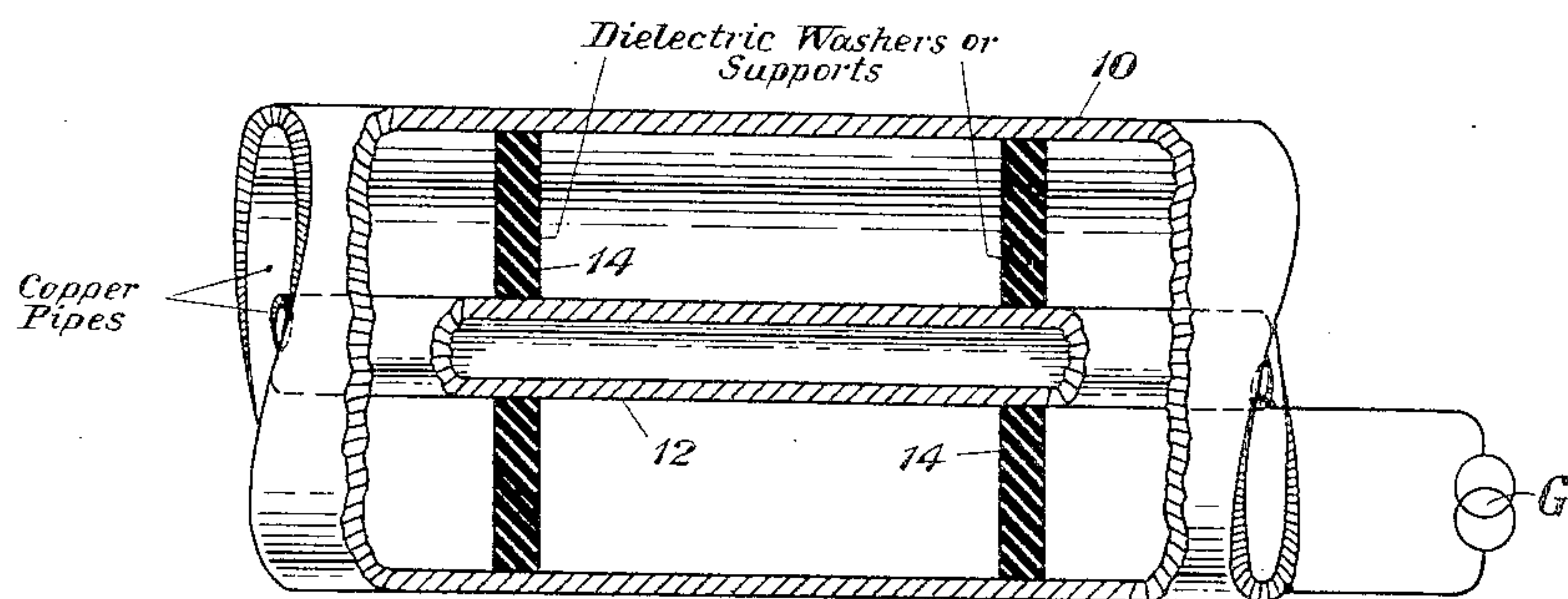
Feb. 4, 1936.

E. I. GREEN

2,029,420

CONCENTRIC CONDUCTOR TRANSMISSION SYSTEM

Filed May 23, 1929



INVENTOR
E. I. Green

BY

[Signature]
ATTORNEY

UNITED STATES PATENT OFFICE

2,029,420

CONCENTRIC CONDUCTOR TRANSMISSION
SYSTEM

Estill I. Green, East Orange, N. J., assignor to
American Telephone and Telegraph Company,
a corporation of New York

Application May 23, 1929, Serial No. 365,518

10 Claims. (Cl. 178—44)

This invention relates to a novel form of conductor structure employing concentric cylindrical conductors for the transmission of a wide band of frequencies with relatively low attenuation. The invention principally relates to such proportioning of the relative dimensions of the conductors as to produce minimum attenuation.

If a solid cylindrical conductor or a hollow cylindrical conductor is provided with a return conductor comprising a second hollow cylindrical conductor concentrically arranged with respect to the first conductor, and the two conductors are separated by a dielectric consisting largely of air or other gaseous medium, the transmission line thus formed will have a number of desirable characteristics. Its attenuation at all frequencies will be quite low as compared with the corresponding attenuation of open wire lines and cable circuits such as are now commonly used for telephone transmission. Such a transmission circuit may, therefore, be employed for the transmission of a much wider band of frequencies than has been possible with types of transmission circuits heretofore used. It also has the advantage that it is substantially free from interference from neighboring conductor systems and in itself tends to produce but little interference into adjacent transmission circuits.

The present invention is, however, more particularly concerned with the discovery that if the diameter of one of the concentric conductors is assigned a predetermined value, there is an optimum diameter for the other conductor for which the attenuation of the system will be a minimum.

It is a characteristic of a transmission system of the type herein considered that at high frequencies the current tends to flow at the outer surface of the inner conductor and at the inner surface of the outer conductor. Therefore, the outer radius of the inner conductor and the inner radius of the outer conductor are of importance from an attenuation standpoint. Furthermore, if the walls of the conductors be made quite thin, the attenuation will be equalized with respect to frequency over a range up to a frequency bearing a certain relation to the thickness of the walls. In accordance with the present invention, it has been found that if the walls of the conductors are sufficiently thin to produce equalization in the manner just referred to, the attenuation will be a minimum at all frequencies for any given predetermined diameter of the conductors when the ratio of the inner radius of the outer conductor to the outer radius of the inner conductor is approximately 3.6. The at-

tenuation will be decreased, however, by increasing the diameters of the two conductors so long as the optimum ratio of about 3.6 is maintained.

The invention will now be more fully understood from the following description when read in connection with the accompanying drawing in which the figure is a symbolic representation of a concentric conductor system.

Referring to the figure of the drawing, 10 designates an outer conductor in the form of a hollow cylinder of suitable conducting material. A second cylindrical conductor 12 is mounted concentrically with the outer conductor 10. One of the conductors acts as a return for the other and not as a mere shield, this fact being indicated by the conventional representation of a source of electromotive force *G* with its terminals connected to the two conductors.

In order that the attenuation may be small at high frequencies, the leakage loss between the conductors should be as small as possible. As the leakage loss is due to the nature of the dielectric interposed between the conductors, the dielectric should be principally of air, since air introduces no leakage loss. Accordingly, the two conductors may be held in proper concentric relation and out of electrical contact with each other by means of spaced dielectric washers 14. These washers should be separated from each other a suitable distance and should be made as thin as possible consistent with the required mechanical strength. They should also be composed of some dielectric of small loss angle and low dielectric constant, since if these conditions are obtained, the leakage loss (which in the ordinary open wire system comprises a large part of the attenuation) may be made so small as to be practically negligible. For example, hard rubber or preferably Pyrex glass or other good insulating material, may be used for the insulating washers 14. In this connection, it should be noted that as the outer conductor may be made watertight, the insulating washers may be maintained dry and free from dirt or contamination, so that the leakage loss will not increase or change with time but will be maintained at the low value which is characteristic of insulators in the dry and clean condition in which they come from the factory.

Having in mind the foregoing brief description of the type of system to which the invention relates, a mathematical discussion will now be given to show that the condition of minimum attenuation at all frequencies will be attained for a given optimum ratio of the inner diameter

of the outer conductor to the outer diameter of the inner conductor.

1. Diameter relations—size of outer conductor fixed

Unless the walls of the coaxial conductors are made extremely thin, the attenuation at high frequencies will be practically independent of the thickness of wall. This is because of the large skin effect, which makes the current flow in a very thin wall on the outside of the inner tube and on the inside of the outer tube. Consequently, the thickness of the conductor walls will ordinarily be determined by mechanical considerations. Under such conditions, the following formulas may be used at high frequencies (above the voice range).

$$\text{Resistance of inner conductor} = R_i = K_0 \sqrt{f} / b \quad (1)$$

$$\text{Resistance of outer conductor} = R_o = K_0 \sqrt{f} / c \quad (2)$$

$$\text{Linear inductance} = L = K_1 \log \frac{c}{b} \quad (3)$$

Also

$$\text{Linear capacity} = C = \frac{K_2}{\log \frac{c}{b}} \quad (4)$$

where K_0 , K_1 and K_2 are constants, f is the frequency, c is the inner radius of the outer conductor, and b is the outer radius of the inner conductor. It will be noted that the outer radius of the outer conductor and the inner radius of the inner conductor do not appear in the formulas. The attenuation at high frequencies is

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} \quad (5)$$

where R , L , C and G are the linear resistance, inductance, capacity, and leakage conductance, respectively. As authority for this equation reference is made to equation (25) and context on page 147 of the book by K. S. Johnson, "Transmission circuits", published 1924 and thereafter by the Van Nostrand Company, New York. Let us assume first that air insulation is employed between the two conductors, and $G=0$. We have then

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{R_i + R_o}{2} \sqrt{\frac{C}{L}} \quad (6)$$

Substituting from Equations (1) to (4), we obtain

$$\alpha = \frac{K_0 \sqrt{f}}{2} \left(\frac{1}{b} + \frac{1}{c} \right) \sqrt{\frac{K_2}{K_1} \frac{1}{\left(\log \frac{c}{b} \right)^2}} \quad (7)$$

Letting $K_3 = \frac{K_0}{2} \sqrt{\frac{K_2}{K_1}}$ and $x = \frac{c}{b}$, we have

$$\alpha = K_3 \sqrt{f} \frac{1}{c} (1+x) \frac{1}{\log x} \quad (8)$$

Whence, c being assumed fixed

$$\frac{d\alpha}{dx} = K_3 \sqrt{f} \frac{1}{c} \left[\frac{1}{\log x} - \frac{x+1}{x (\log x)^2} \right] \quad (9)$$

Setting $\frac{d\alpha}{dx} = 0$, we obtain

$$\log x = \frac{x+1}{x} \quad (10)$$

The logarithms in expression (10) and the fol-

lowing discussion are natural logarithms. From (10) we find that

$$x = \frac{c}{b} = 3.59 \quad (11)$$

This is the condition for minimum attenuation for the concentric arrangement when the inner diameter of the outer conductor is fixed. It is interesting to note that it is independent of the frequency, the size of the conductors, and other variables. It will be obvious that the attenuation can be reduced by increasing the size of both conductors (leaving the ratio

$$\frac{c}{b}$$

fixed). In a given case, however, the maximum diameter for the outer conductor will be fixed by practical considerations. The optimum outer diameter for the inner conductor can then be determined from the relation given in (11).

It is also interesting to note that the fixed ratio of c to b gives fixed values of inductance, capacity and characteristic impedance for the concentric arrangement, regardless of the actual dimensions of the conductors. By analogy to Formula (55), page 109 of "Calculation of Alternating Current Problems" by L. Cohen, the capacity of the structure herein considered, bearing in mind that the dielectric is equivalent to air, will have the value

$$C = \frac{0.0894}{\log \frac{c}{b}} \times 10^{-6} \text{ farads per mile.}$$

Likewise from Formula (56) on page 72 of the Cohen publication the inductance in abhenries per cm. has the value

$$L = 2 \log \frac{c}{b}$$

Reducing this to henries per mile

$$L = \frac{2 \times 1.60935 \times 10^3}{10^9} \log \frac{c}{b} = .32187 \times 10^{-3} \log \frac{c}{b}$$

The value of the nominal characteristic impedance corresponding to

$$\frac{c}{b} = 3.59$$

then becomes

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{.32187 \times 10^{-3}}{.0894 \times 10^{-6}}} \times \log \frac{c}{b} =$$

$$60 \log \frac{c}{b} = 76.7 \text{ ohms} \quad (12)$$

The values of inductance and capacity corresponding to the value of 3.59 for

$$\frac{c}{b}$$

are .411 mh. per mile and .0700 mf. per mile, respectively.

It will be recalled that in the derivation of the ratio

$$\frac{c}{b}$$

for minimum attenuation it was assumed that the value of leakage conductance G is zero. In practice, it will, of course, be necessary to use spacers of some solid dielectric material in order to keep the two conductors apart. These spacers or washers will produce a slight increase in the capacity between the two conductors and will give a small leakage conductance. By using a good dielectric material (with small dielectric constant and low power factor), by making the

spacers thin, and by placing them far apart the effect of the spacers upon the constants of the concentric system could be made negligible. Thus the desired value of 3.6 for the ratio

$$\frac{c}{b}$$

would remain unchanged.

Furthermore, it can be shown that even if the capacity and leakage conductance introduced by the insulating spacers are appreciable the component of attenuation due to leakage is substantially independent of the conductor dimensions, while the component due to resistance is approximately the same function of

$$\frac{c}{b}$$

as before. Hence the desired value of

$$\frac{c}{b}$$

is 3.6 regardless of the spacers. The theoretically desirable characteristic impedance of 77 ohms will be slightly reduced by the use of spacers but in practice this effect should be negligible.

2. Use of thin walls to secure constant attenuation

If the conductor walls are made sufficiently thin the attenuation of the coaxial arrangement can be made substantially constant at all frequencies below a certain value. In general, the attenuation will be practically constant for frequencies below about

$$\frac{25}{t^2},$$

where t is the thickness of the conductor wall in centimeters. The maximum frequency for any desired degree of constancy of attenuation and for any assigned value of wall thickness can be determined by plotting a curve of attenuation versus the product

$$t\sqrt{f}.$$

3. Diameter relations with thin walls—size of outer conductor fixed

The previous discussion of diameter relations dealt entirely with walls of appreciable thickness. Assuming that the walls of the conductors are made very thin in order to avoid skin effect and secure constant attenuation, the question arises as to the proper diameter relations for minimum attenuation under such conditions. If a equals the inner radius of the inner conductor and d equals the outer radius of the outer conductor, we have the following equations:

$$R_i = \frac{K_8}{b^2 - a^2} \quad (13)$$

and

$$R_o = \frac{K_8}{d^2 - c^2} \quad (14)$$

where K_8 is constant. Note that since the walls are so thin that the attenuation and the resistance are uniform over the frequency range considered, the factor f does not appear in Equations (13) and (14). L and C are the same as in Equations (3) and (4). Consequently, substituting from Equations (13), (14), (3) and (4) in Equation (6)

$$\alpha = K_8 \left[\frac{1}{b^2 - a^2} + \frac{1}{d^2 - c^2} \right] \sqrt{\frac{K_2}{K_1}} \frac{1}{\log \frac{c}{b}} \quad (15)$$

Let us assume now that the thickness of the two conductor walls has the same value, which may be designated t . In other words:

$$b - a = d - c = t \quad (16)$$

It will be found that for small values of wall thickness α is approximately given by the following:

$$\alpha = K_8 \sqrt{\frac{K_2}{K_1}} \left(\frac{1}{2bt} + \frac{1}{2ct} \right) \frac{1}{\log \frac{c}{b}} \quad (17)$$

Or, letting

$$\frac{c}{b} = x, \text{ and } \frac{K_8}{2} \sqrt{\frac{K_2}{K_1}} = K_9$$

$$\alpha = K_9 \frac{1}{tc} (x+1) \frac{1}{\log x} \quad (18)$$

Differentiation shows that the minimum value of α will be obtained when:

$$\log x = \frac{x+1}{x} \quad (19)$$

Consequently the diameter relation for minimum attenuation is, as before:

$$x = \frac{c}{b} = 3.59 \quad (20)$$

If we assume that the thickness of the two walls is not the same (both, however, being thin enough to avoid skin effect) and that the thickness of the inner conductor is $mt = b - a$, and that of the outer conductor is $t = d - c$, then by substituting the corresponding values in Equation (15), and differentiating, it will be found that the condition for minimum attenuation is:

$$\log x = \frac{x+m}{x} \quad (21)$$

The values of x which satisfy this relation are given in the following table for various values of m :

Table 1

$m \left(= \frac{b-a}{d-c} \right)$	$x \left(= \frac{c}{b} \right)$
.25	2.96
.33	3.02
.5	3.18
1.	3.59
2.	4.32
3.	4.97
4.	5.57

The value of x for intermediate values of m can be determined graphically.

The relationship expressed in Equation (21) in terms of conductor dimensions and wall thickness may be expressed directly in terms of conductor resistance. Bearing in mind we are interested in the ratio of the high frequency resistance of the outer conductor to that of the inner conductor, it is clear from Equations (1) and (2) that

$$\frac{R_o}{R_i} = \frac{b}{c} = \frac{1}{x} \quad (22)$$

Hence

$$\frac{R_o}{R_i}$$

varies inversely with x . If the conductors are of similar material and the walls are thin enough to avoid skin effect and have constant attenuation at high frequencies, then if, as stated in connection with Equation (21), the thickness of the outer conductor is represented by t and that

of the inner conductor by mt , it follows that the high frequency resistance depends upon the conductor thickness so that R_0 varies with

$$\frac{1}{t}$$

and R_i varies with

$$\frac{1}{mt}$$

In other words, referring to Equation (14) and recognizing that $d-c$ equals t and that $d+c$ is approximately equal to $2c$, we may write

$$R_0 = \frac{K_8}{2ct}$$

Similarly, since $b-a$ equals mt and $b+a$ is approximately equal to $2b$, we may write

$$R_i = \frac{K_8}{2bmt}$$

We may then express the high frequency resistance ratio

$$\frac{R_0}{R_i} = \frac{m}{x}$$

or

$$m = x \frac{R_0}{R_i}$$

If we write r in place of

$$x \frac{R_0}{R_i}$$

in the above expression, then Equation (21) may be written

$$\log x = \frac{x+r}{x} \quad (21a)$$

where r is equal to x times the ratio of the high frequency resistance of the outer conductor to the high frequency resistance of the inner conductor.

4. Diameter relations for different conductivities—size of outer conductor fixed

If the conductivity of the two conductors is not the same, the conductivity of the inner conductor being n times that of the outer one, the condition for minimum attenuation is:

$$\log x = \frac{x + \sqrt{n}}{x} \quad (22)$$

This applies when the walls are of appreciable thickness. When the walls are equal in thickness and are very thin the condition for minimum attenuation is

$$\log x = \frac{x + n}{x} \quad (22')$$

The reason that in Equation (22)

$$\sqrt{n}$$

appears is that the resistance at high frequencies is inversely proportional to the square root of the conductivity. See Equation 105, page 547 of Philosophical Magazine, vol. XVII (sixth series), 1909. In the case of Equation (22'), however, since the walls are so thin that the current is distributed uniformly over the cross-section of the conductor the resistance is obviously inversely proportional to the conductivity and hence n (the ratio of the conductivities of the two conductors) appears. The values of x which satisfy conditions (22) and (22') may be obtained from Table 1

using the figures in the first column as values of

$$\sqrt{n}$$

and n respectively.

From the definition of n as given in connection with Equation (22) and from Equations (13) and (14) it follows that

$$\frac{R_0}{R_i} = \sqrt{n} \cdot \frac{b}{c}$$

We may then express the high frequency resistance ratio thus:

$$\frac{R_0}{R_i} = \frac{\sqrt{n}}{x}$$

or

$$\sqrt{n} = x \frac{R_0}{R_i}$$

Again writing r in place of

$$x \frac{R_0}{R_i}$$

in the foregoing expression, we have from Equation (22)

$$\log x = \frac{x+r}{x} \quad (22a)$$

where r is x times the ratio of the high frequency resistance of the two conductors.

Similarly, in the case represented by Equation (22'), it is obvious that

$$n = x \frac{R_0}{R_i}$$

so that again we may write

$$\log x = \frac{x+r}{x}$$

If the conductivity of the two conductors is not the same and the wall thickness is not the same (both walls being thin enough for no skin effect) the minimum attenuation occurs when

$$\log x = \frac{x + mn}{x} \quad (23)$$

m and n being defined as before. Table 1 can be used to find the values of x which satisfy (28), the figures in the first column being used as values of mn .

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. A concentric conductor system comprising two concentrically arranged cylindrical shells of conductive material connected one as a return for the other and having walls so thin that the attenuation over a wide range of frequencies extending up to a high frequency many times the limit of audibility is substantially uniform, the inner diameter of the outer conductor and the outer diameter of the inner conductor bearing such ratio to each other that the attenuation of the system will be a minimum.

2. A concentric conductor system comprising two concentrically arranged cylindrical shells of conductive material connected one as a return for the other and having walls of equal thickness so thin that the attenuation over a wide range of frequencies extending up to a high frequency many times the limit of audibility is substantially uniform, the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor being approximately 3.6.

3. A concentric conductor system comprising two concentrically arranged cylindrical shells of conductive material connected one as a return for the other and having walls of equal thickness so thin that the attenuation over a wide range of frequencies extending up to a high frequency many times the limit of audibility is substantially uniform, the dimensions of the conductors being such that where x is the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor, the relation

$$\log x = \frac{x+1}{x}$$

will approximately hold.

4. A concentric conductor system comprising two concentrically arranged cylindrical shells of conductive material having walls so thin that the attenuation over a wide range of frequencies extending up to a high frequency well above audibility is substantially uniform, the dimensions of the conductors being such that where x is the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor, and m is the ratio of the thickness of the wall of the inner conductor to that of the outer wall, the relation

$$\log x = \frac{x+m}{x}$$

will approximately hold.

5. A concentric conductor system comprising two concentrically arranged cylindrical shells of conductive material having walls so thin that the attenuation over a wide range of frequencies extending up to a high frequency well above audibility is substantially uniform, the dimensions and conductivity of the conductors being such that where x is the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor, m is the ratio of the thickness of wall of the inner conductor to that of the outer wall, and n is the ratio of conductivity of inner conductor to outer conductor, the relation

$$\log x = \frac{x+mn}{x}$$

will approximately hold.

6. A concentric conductor system comprising two concentrically arranged cylindrical shells of conductive material connected one as a return for the other and having walls of equal thickness so thin that the attenuation over a wide range of frequencies extending up to a high frequency many times the limit of audibility is substantially uniform, said cylindrical conductors being separated by a dielectric which is for the most part

gaseous, the inner diameter of the outer conductor and the outer diameter of the inner conductor bearing such ratio to each other that at frequencies above the voice range the characteristic impedance will be approximately 77 ohms.

7. A concentric conductor system comprising two concentrically arranged cylindrical conductors for transmitting a wide range of frequencies extending up to a high frequency well above audibility, the dimensions of the conductors being such that where x is the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor, and n has a value other than unity and is the ratio of the conductivity of the inner conductor to that of the outer conductor, the relation

$$\log x = \frac{x+\sqrt{n}}{x}$$

will approximately hold.

8. A transmission circuit comprising two conductors arranged coaxially and connected one as a return for the other for transmitting a wide range of frequencies extending up to a frequency many times the upper limit of audibility, the dimensions of the conductors being such as to satisfy the relation:

$$\log x = \frac{x+r}{x},$$

where x is the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor, and where r has a value other than unity and is x times the ratio of the resistance of the outer conductor to the resistance of the inner conductor at approximately the highest frequency of the transmitted range.

9. A coaxial conductor transmission system comprising inner and outer conductors of different effective resistivities, the ratio of the internal diameter of the outer conductor to the external diameter of the inner conductor being the optimum for low attenuation of waves lying near the top of the frequency band transmitted.

10. A concentric conductor system comprising two concentrically arranged cylindrical conductors for transmitting a wide range of frequencies extending up to a high frequency well above audibility, the ratio of the effective conductivity of the inner conductor to that of the outer conductor being different from unity and the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor having such value in relation to said ratio of the conductivities that the attenuation of the system will be a minimum.

ESTILL I. GREEN.