

Nov. 12, 1935.

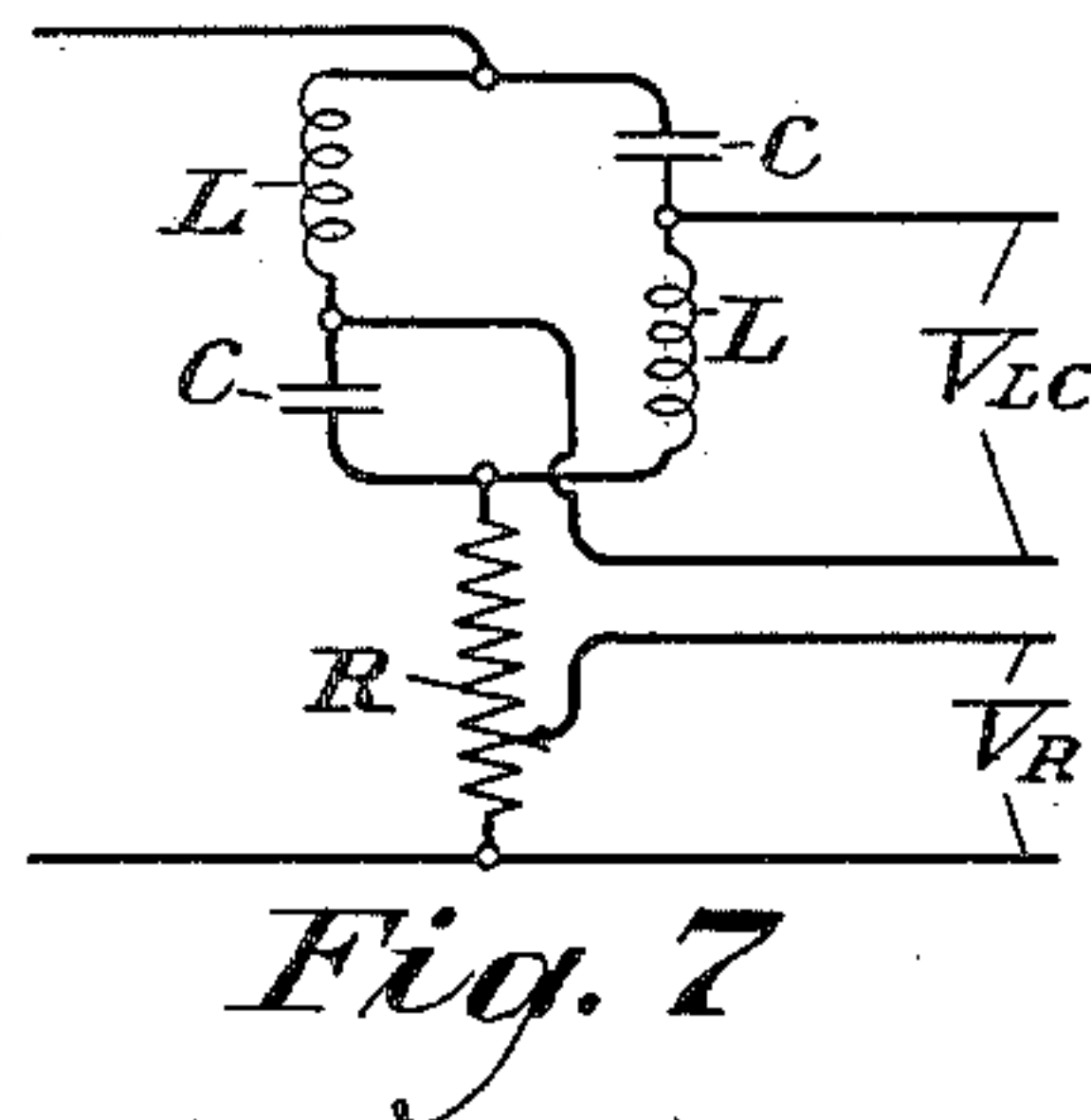
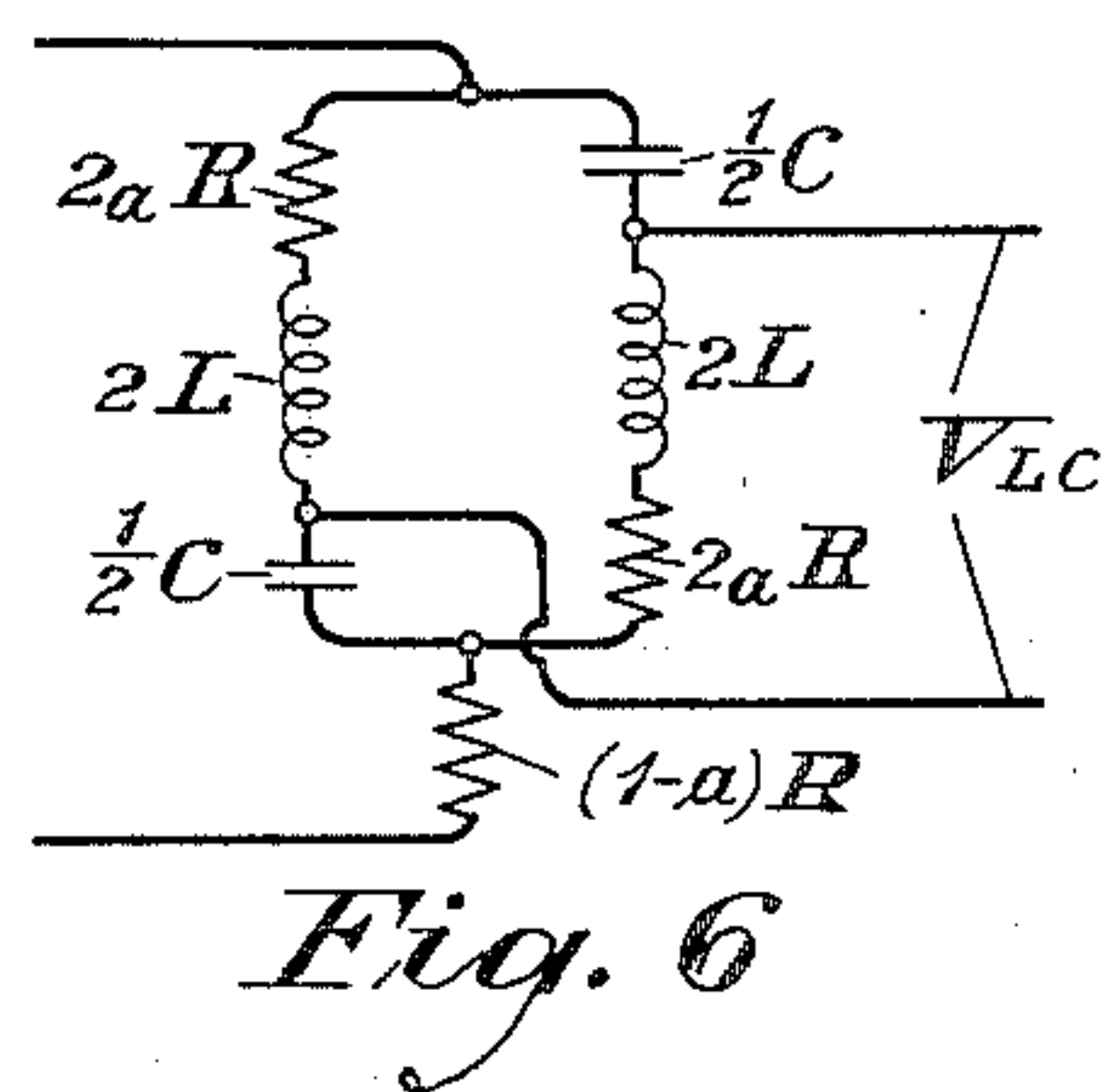
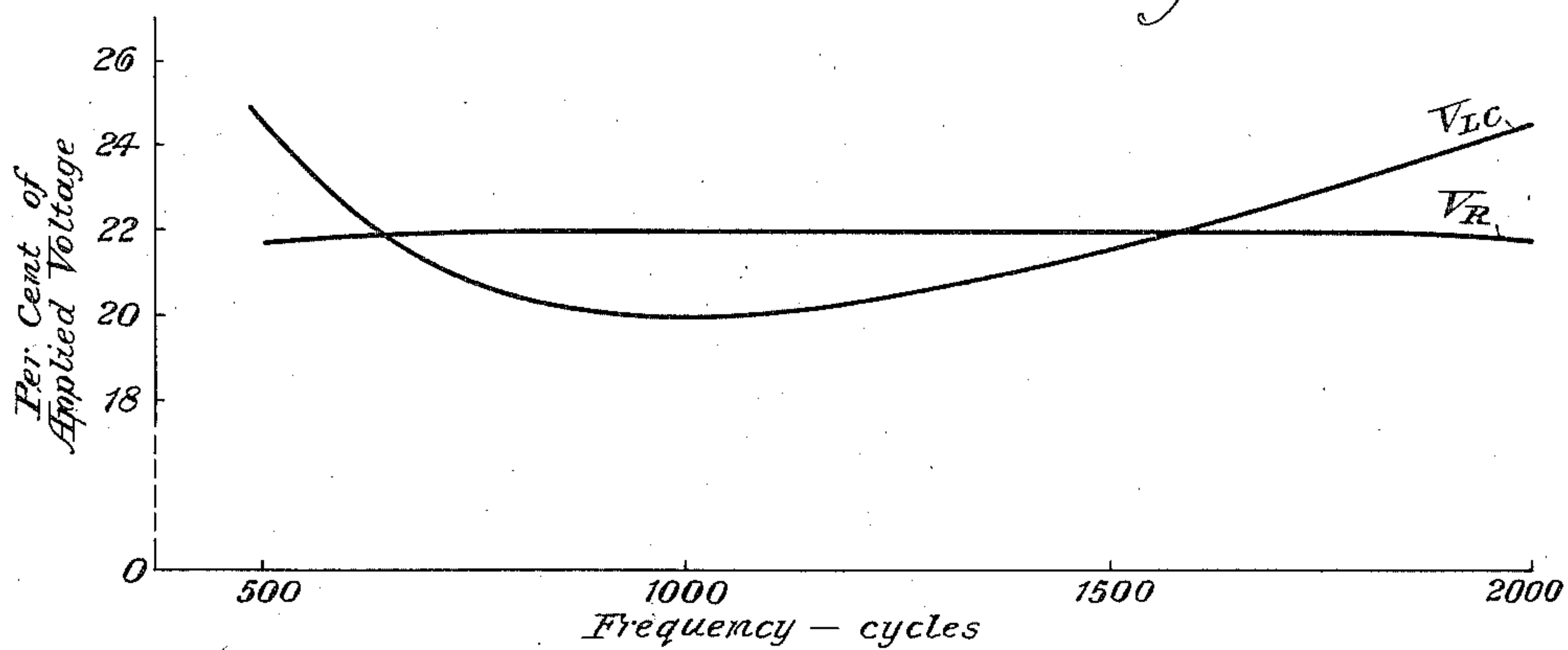
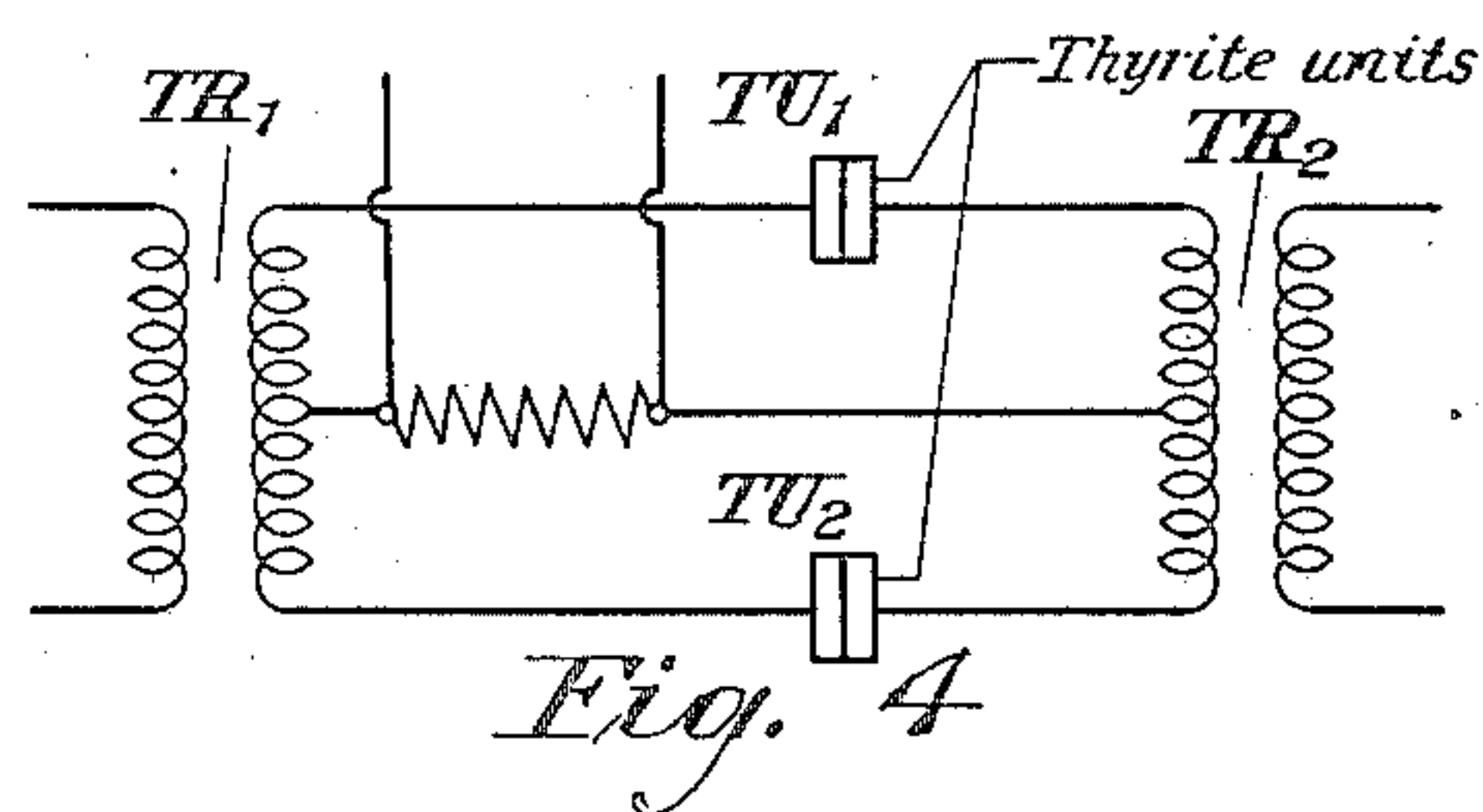
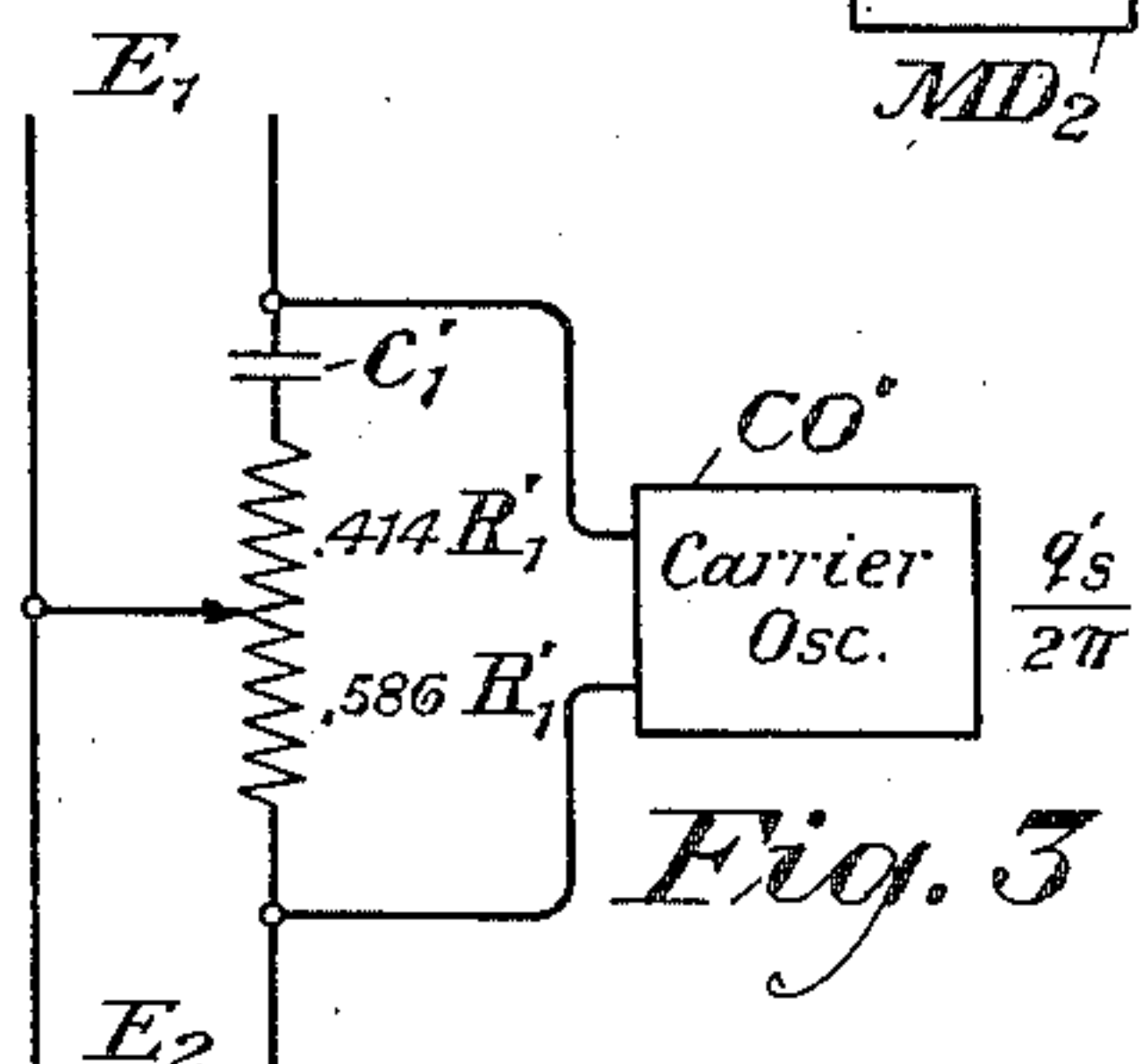
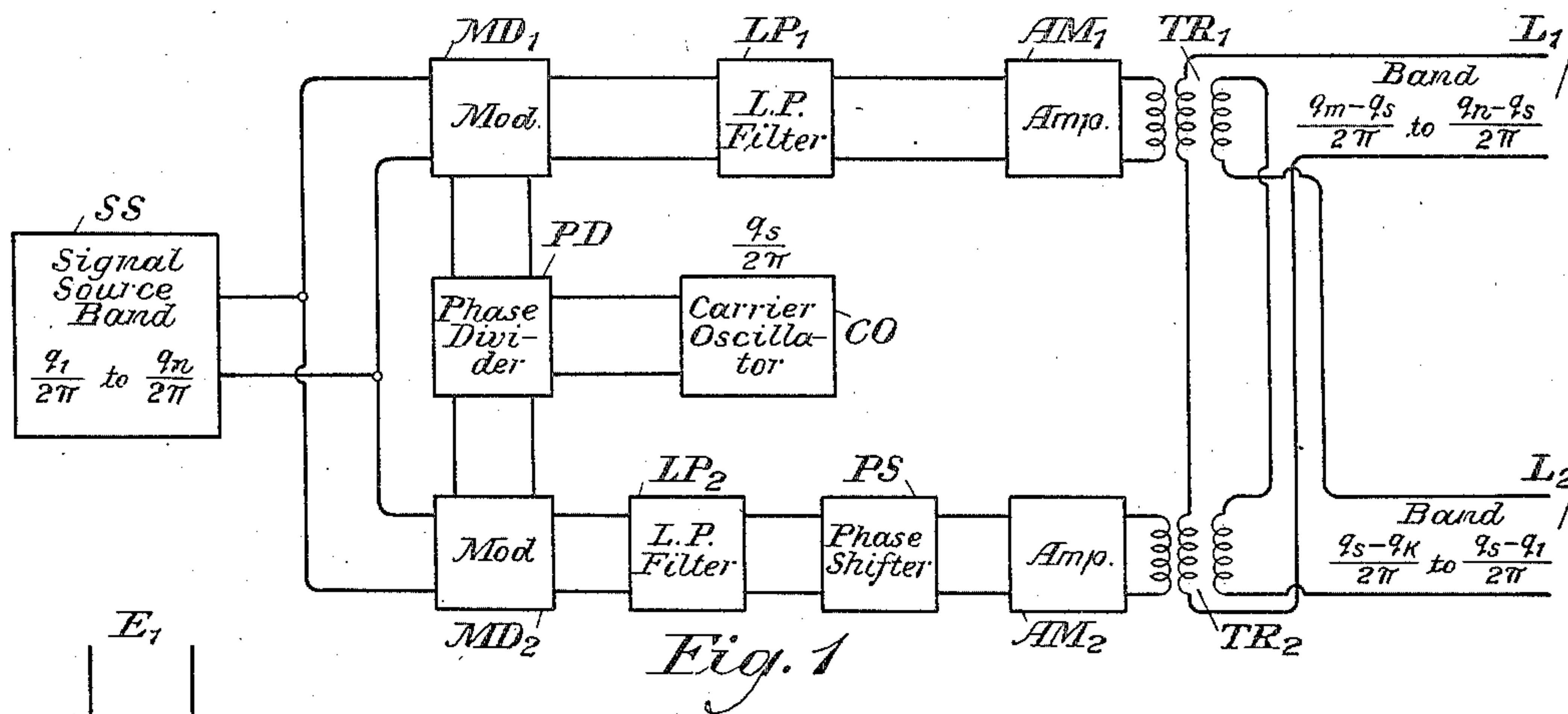
E. I. GREEN

2,020,409

BAND SEPARATION SYSTEM

Filed Aug. 15, 1933

2 Sheets-Sheet 1



INVENTOR  
E. I. Green  
BY *g. 704*  
ATTORNEY



Nov. 12, 1935.

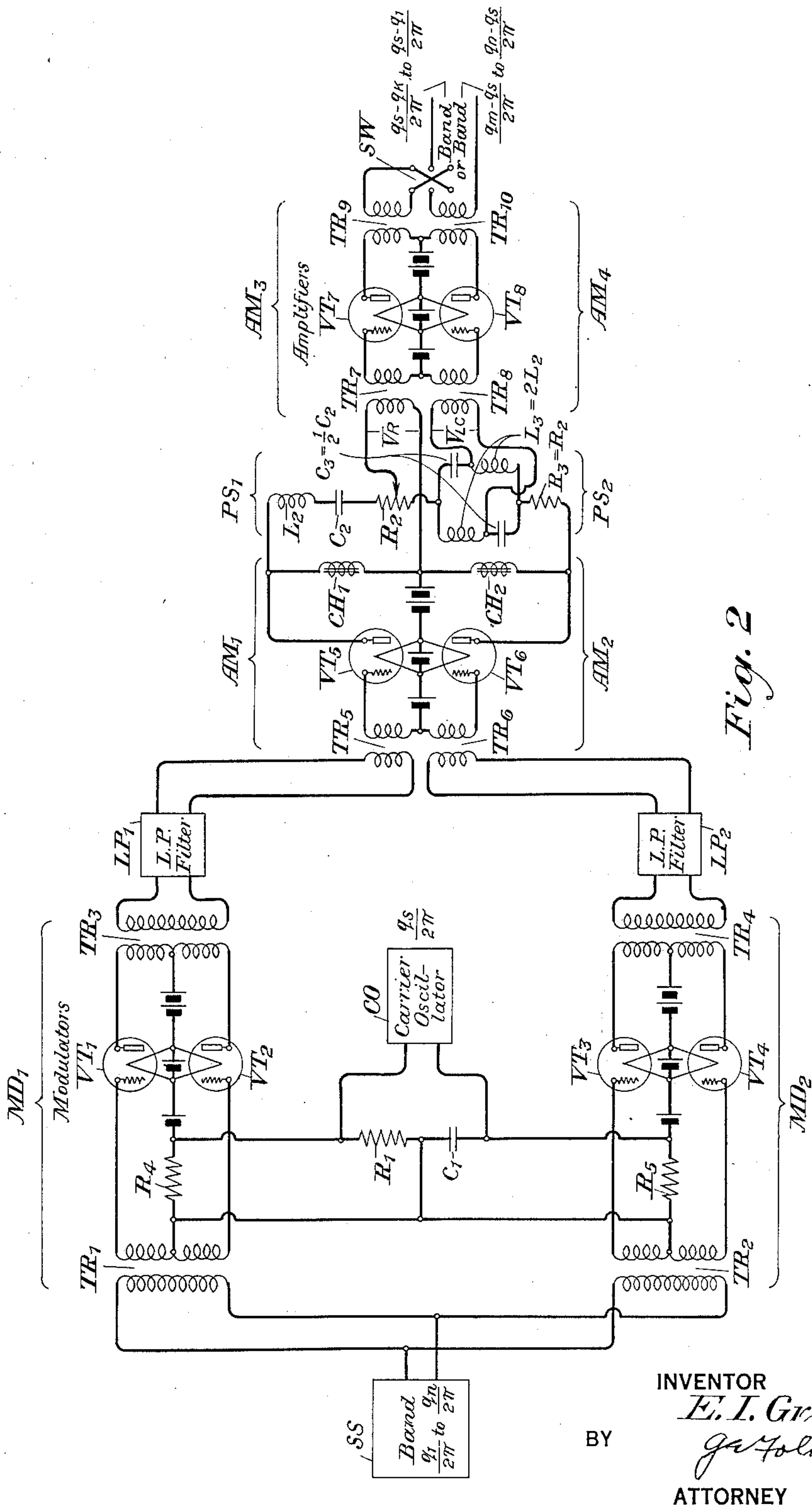
E. I. GREEN

2,020,409

BAND SEPARATION SYSTEM

Filed Aug. 15, 1933

2 Sheets-Sheet 2



INVENTOR  
E. I. Green  
BY  
J. F. Folger  
ATTORNEY



## UNITED STATES PATENT OFFICE

2,020,409

## BAND SEPARATION SYSTEM

Estill I. Green, East Orange, N. J., assignor to  
American Telephone and Telegraph Company,  
a corporation of New York

Application August 15, 1933, Serial No. 685,273

20 Claims. (Cl. 178—44)

This invention relates to a method for dividing a band of frequencies and more especially to a method for dividing a frequency band by the use of phase discrimination without the use of electrical filter selection.

The usual method of selecting certain parts of a band of frequencies is to use electrical filters. If, for example, it is desired to divide a band of frequencies into two parts, a combination of high and low-pass filters may be used. In the present invention a method is described for dividing a band of frequencies which does not essentially involve the use of electrical filters. This method of dividing a band of frequencies is based upon the general principle of separately modulating differently phased components of the same carrier frequency with the band to be divided. The different phase relations between the frequencies resulting from the two modulations are used as a basis for eliminating the band of frequencies either above or below the carrier frequency. The invention, therefore, provides a device to accomplish functions similar to those of wave filters and suited to a wide variety of applications.

The features of the invention will now be better understood from the detailed exposition which follows, when read in connection with the accompanying diagrams, Figures 1 to 5. Figure 1 shows an arrangement which might be used to divide a band of frequencies by phase discrimination. Fig. 2 illustrates in greater detail a possible embodiment of the arrangement shown in Fig. 1. Fig. 3 shows a phase divider for the carrier frequency to be used with third order modulation. Fig. 4 illustrates a third order modulating circuit. Fig. 5 indicates the relative amplitudes of the voltages over a band of frequencies which can be obtained by means of the phase shifting networks shown in Fig. 2, or in Fig. 7. Figs. 6 and 7 illustrate band phase shifting networks.

The essential principles of the invention will now be demonstrated in connection with Fig. 1. Consider a band of frequencies from some source of signals such as SS. This band may be represented as

$$\sin q_1 t, \sin q_2 t, \dots, \sin q_k t, \sin q_m t, \dots, \sin q_n t \quad (1)$$

Since the amplitudes of the frequencies composing the band do not affect the operation of the method, the amplitude indices have been omitted in expression (1).

The frequency intervals between successive components of the band may be of any width

and may be equal or unequal. The band (1) may be located anywhere in the frequency spectrum.

However, in order to avoid difficulties in obtaining the desired lower side band in the output of the modulators, it may be convenient to have the band (1) located so that

$$\left( \frac{q_n - q_s}{2\pi} \right)$$

is less than

$$\frac{q_1}{2\pi} \text{ and } \frac{q_s}{2\pi}$$

is less than twice

$$\frac{q_1}{2\pi}, \frac{q_s}{2\pi}$$

being the frequency at which division is desired and

$$\frac{q_1}{2\pi} \text{ and } \frac{q_n}{2\pi}$$

being the lower and upper frequencies of the band. This avoids any overlapping of the input frequencies of the modulator with those of the lower sideband output. It is not essential, however, that this condition be obtained.

Assume that it is desired to separate the band (1) into two parts, the point of division lying between two of its component frequencies

$$\frac{q_k}{2\pi} \text{ and } \frac{q_m}{2\pi}$$

There is generated in the carrier frequency oscillator CO the frequency

$$\frac{q_s}{2\pi}$$

such that

$$q_k < q_s < q_m \quad (2)$$

Two components of this frequency differing in phase by 90 electrical degrees are obtained in any desired manner by means of the phase divider PD. These components of the carrier frequency may be designated  $\sin q_s t$  and  $\cos q_s t$ . Each of these components is then modulated with the frequency band (1) extending from  $\sin q_1 t$  to  $\sin q_n t$ , in the modulators MD<sub>1</sub> and MD<sub>2</sub>, which may be of any suitable second order type. Only the lower sideband of the modulation process is retained in each case. The upper sideband may be suppressed by filters such as LP<sub>1</sub> and LP<sub>2</sub> of Fig. 1, or tuned circuits, or by phasing out in a single sideband modulator.

Assume, for the present, that second order modulators are used. As a result of the modula-



tion of  $\sin q_s t$ , the following band of frequencies is obtained in the output of modulator MD<sub>1</sub>:

$$\begin{aligned} & \cos (q_s - q_1) t \\ & \cos (q_s - q_2) t \\ & \text{-----} \\ & \cos (q_s - q_k) t \\ & \cos (q_m - q_s) t \\ & \text{-----} \\ & \cos (q_n - q_s) t \end{aligned} \quad (3)$$

while through modulation of  $\cos q_s t$ , there is obtained from modulator MD<sub>2</sub>:

$$\begin{aligned} & -\sin (q_s - q_1) t \\ & -\sin (q_s - q_2) t \\ & \text{-----} \\ & -\sin (q_s - q_k) t \\ & +\sin (q_m - q_s) t \\ & \text{-----} \\ & +\sin (q_n - q_s) t \end{aligned} \quad (4)$$

It will be noted that the sign of the terms in the second series changes at the point where the division is to be accomplished. Next, as indicated in Fig. 1, the phases of all components from modulator MD<sub>2</sub>, series (4), are shifted 90 degrees by the phase shifter PS without distortion of amplitudes relative to those of the other series. Thus there would be obtained a new series as follows:

$$\begin{aligned} & -\cos (q_s - q_1) t \\ & -\cos (q_s - q_2) t \\ & \text{-----} \\ & -\cos (q_s - q_k) t \\ & +\cos (q_m - q_s) t \\ & \text{-----} \\ & +\cos (q_n - q_s) t \end{aligned}$$

Amplification is introduced by the amplifiers AM<sub>1</sub> and AM<sub>2</sub> which have identical gain-frequency characteristics and introduce no phase distortion. These amplifiers supply the primaries of transformers TR<sub>1</sub> and TR<sub>2</sub>. Two of the output windings, one from each coil, are connected series aiding and the other two are connected series opposing. Thus the output into line L<sub>1</sub> may be obtained by adding together the groups of frequencies represented by Equations (3) and (5) which will give the band of frequencies from

$$\frac{q_m - q_s}{2\pi} \text{ to } \frac{q_n - q_s}{2\pi}$$

The output into line L<sub>2</sub> may be obtained by subtracting these groups since the windings of transformers TR<sub>1</sub> and TR<sub>2</sub> oppose each other. This will give the band of frequencies from

$$\frac{q_s - q_k}{2\pi} \text{ to } \frac{q_s - q_1}{2\pi}$$

It will be apparent that these results will be obtained regardless of the phases or magnitudes of the frequency components of the original band extending from

$$\frac{q_1}{2\pi} \text{ to } \frac{q_n}{2\pi}$$

In the treatment just given the use of second order modulators was assumed. That the same result may be obtained using modulators of the third order type will now be demonstrated. Consider again the band of frequencies represented by Equation (1). Referring to Fig. 1 this band of frequencies is introduced to the modulators MD<sub>1</sub> and MD<sub>2</sub> in the same phase, these modulators now being of any desired type of third order device. As before it will be assumed that the band (1) is to be divided into two parts, the

point of division lying between the component frequencies

$$\frac{q_k}{2\pi} \text{ and } \frac{q_m}{2\pi}$$

The carrier frequency oscillator CO generates a frequency

$$\frac{q'_s}{2\pi}$$

such that

$$q_k < 2q'_s < q_m \quad (6)$$

Two components of this frequency differing in phase by 45 electrical degrees are obtained from the phase divider PD. These components may be designated  $\sin q'_s t$  and  $\sin$

$$\left( q'_s t + \frac{\pi}{4} \right)$$

Any odd multiple of 45 degrees [i. e.,

$$(2n-1)\frac{\pi}{4};$$

where  $n$  is any integer] may be used for the phase difference without essentially altering the treatment given. Each of these components is then modulated with the frequency band (1) in modulators MD<sub>1</sub> and MD<sub>2</sub>, respectively. Again only the lower sideband is retained, other undesirable frequencies being suppressed with the low pass filters LP<sub>1</sub> and LP<sub>2</sub>, or by some other suitable means.

As the result of third order modulation of the carrier frequency component  $\sin q'_s t$  with the band (1) in MD<sub>1</sub>, the following frequencies will be produced:

$$\begin{aligned} & +\sin (2q'_s - q_1) t \\ & +\sin (2q'_s - q_2) t \\ & \text{-----} \\ & +\sin (2q'_s - q_k) t \\ & -\sin (q_m - 2q'_s) t \\ & \text{-----} \\ & -\sin (q_n - 2q'_s) t, \end{aligned} \quad (7)$$

while modulation of  $\sin$

$$\left( q'_s t + \frac{\pi}{4} \right)$$

will produce the following frequencies:

$$\begin{aligned} & +\cos (2q'_s - q_1) t \\ & +\cos (2q'_s - q_2) t \\ & \text{-----} \\ & +\cos (2q'_s - q_k) t \\ & +\cos (q_m - 2q'_s) t \\ & \text{-----} \\ & +\cos (q_n - 2q'_s) t \end{aligned} \quad (8)$$

It will be noted now that the sign of the terms in Equation (7) changes at the point where the division is to be accomplished. The procedure from this point on is the same as that considered under second order modulation. The phases of one of these sets of frequencies are shifted by 90 electrical degrees and the two groups are then added or subtracted to eliminate the undesired frequencies. The frequency components resulting from the addition of the frequencies represented by Equation (7) to those of Equation (8) shifted 90 degrees in phase will now extend from

$$\frac{q_m - 2q'_s}{2\pi} \text{ to } \frac{q_n - 2q'_s}{2\pi},$$

while subtraction will give frequencies from

$$\frac{2q'_s - q_1}{2\pi} \text{ to } \frac{2q'_s - q_k}{2\pi}$$

It is, therefore, plain that either second order or



third order modulation will give comparable results and either may be used.

In the disclosure so far the special case in which the relative phase shifts between the carrier components and between the sidebands are 90 electrical degrees (second order modulation) has been discussed. This case is the simplest and most useful one. However, it will be of interest to show briefly the effects of phase shifts other than 90 electrical degrees.

For purposes of simplicity consider only two frequencies in the band of frequencies to be divided,

$$\frac{q_k}{2\pi}$$

and

$$\frac{q_m}{2\pi},$$

one of which is lower than and the other higher than the frequency at which it is desired to divide the band,

$$\frac{q_s}{2\pi}.$$

Let these frequencies have random phase angles, denoted by  $\alpha$  and  $\beta$ , respectively. Then the expression for these frequencies may be written

$$\sin(q_k t + \alpha) + \sin(q_m t + \beta) \quad (9)$$

where

$$q_k < q_s < q_m. \quad (10)$$

Now modulate with these frequencies two component currents of the carrier frequency,

$$\frac{q_s}{2\pi},$$

differing in phase by any angle  $\theta$ , which may be represented as

$$\sin q_s t, \text{ and } \sin(q_s t + \theta) \quad (11)$$

Assuming second order modulators, the lower sideband frequencies resulting from modulation of the first carrier component with (9) will be represented as follows:

$$\cos[(q_s - q_k)t - \alpha] + \cos[(q_m - q_s)t + \beta] \quad (12)$$

and that from modulation of the second component

$$\cos[(q_s - q_k)t - \alpha + \theta] + \cos[(q_m - q_s)t + \beta - \theta]. \quad (13)$$

If now we shift the phases of the components of (13) by some angle  $\phi$ , we obtain

$$\cos[(q_s - q_k)t - \alpha + \theta - \phi] + \cos[(q_m - q_s)t + \beta - \theta - \phi]. \quad (14)$$

Adding the lower sideband (12) to the other shifted by an angle  $\phi$  (14) and combining we get:

$$2 \cos \frac{1}{2}(\theta - \phi) \cos \frac{1}{2}[2(q_s - q_k)t - 2\alpha + \theta - \phi] + 2 \cos \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}[2(q_m - q_s)t + 2\beta - \theta - \phi] \quad (15)$$

The two terms of the above expression represent the two frequencies obtained by modulating the carrier frequency with the original signal frequencies and these frequencies, expressed by the last part of each, are

$$\frac{q_s - q_k}{2\pi} \text{ and } \frac{q_m - q_s}{2\pi}.$$

Of greatest interest, however, are the amplitude coefficients of these two frequencies which are

$$2 \cos \frac{1}{2}(\theta - \phi) \quad (16)$$

and

$$2 \cos \frac{1}{2}(\theta + \phi) \quad (17)$$

In order to eliminate one of the sideband frequencies, one of these amplitude coefficients must be reduced to zero, while the other remains. By inspection it will be obvious that this can readily be accomplished by making the sum or difference of  $\theta$  and  $\phi$  equal 180 electrical degrees.

$$\text{If } (\theta + \phi) = 180^\circ \quad (18)$$

expressions (16) and (17) become

$$2 \cos \frac{1}{2}(\theta - \phi) = 2 \cos(\theta - 90^\circ) = 2 \sin \theta \quad (19)$$

$$2 \cos \frac{1}{2}(\theta + \phi) = 2 \cos 90^\circ = 0 \quad (20)$$

or if

$$(\theta - \phi) = \pm 180^\circ \quad (21)$$

$$2 \cos \frac{1}{2}(\theta - \phi) = 2 \cos \pm 90^\circ = 0 \quad (22)$$

$$2 \cos \frac{1}{2}(\theta + \phi) = 2 \cos(\theta \pm 90^\circ) = \pm 2 \sin \theta \quad (23)$$

By satisfying Equation (18) or (21) it is obvious that either sideband frequency can be deleted, leaving the other. The coefficient of the remaining term,  $2 \sin \theta$ , is of course, a maximum when  $\theta = 90^\circ$ , which is the special case which has been described previously. In this case  $\phi$  is either  $+90^\circ$  or  $-90^\circ$ , depending on which frequency it is desired to eliminate.

The frequencies used in this treatment,

$$\frac{q_k}{2\pi} \text{ and } \frac{q_m}{2\pi} \quad (24)$$

are general expressions,

$$\frac{q_k}{2\pi} \quad (25)$$

being any frequency less than the carrier

$$\frac{q_s}{2\pi} \text{ and } \frac{q_m}{2\pi} \quad (26)$$

any frequency greater than the carrier. Thus the derivation may be applied to a whole band of frequencies, the relations true for

$$\frac{q_k}{2\pi} \quad (27)$$

being true for all the frequencies below the carrier, and those for

$$\frac{q_m}{2\pi} \quad (28)$$

holding for all the frequencies greater than the carrier. Thus, if the angle of phase shift  $\phi$  is exactly the same angle at every frequency elimination of one of these bands of frequencies may be accomplished.

The expression (15) also indicates that the initial phase angles,  $\alpha$  and  $\beta$  in this case, of the frequencies in the band to be divided are immaterial to the operation of the system, since they do not enter into the coefficient expressions (16) and (17).

The exposition of the general case has been made assuming the use of second order modulation. Third order modulation could be used, and the derivation would be similar. The angu-



lar phase difference between the carrier components in this case is doubled in the modulation products just as is the frequency of the carrier. The amplitude coefficients, therefore, become

$$2 \cos \frac{1}{2} (2\theta' - \phi) \quad (24)$$

and

$$2 \cos \frac{1}{2} (2\theta' + \phi) \quad (25)$$

and the condition for annulling one of the frequency bands is

$$(2\theta \pm \theta) = \pm 180^\circ \quad (26)$$

It will now be of interest to consider further some of the details of the arrangement discussed in connection with Fig. 1 in which only 90-degree phase shifts are employed. Referring to Fig. 2, a possible embodiment of the invention is shown. A source of signals SS is shown and transformers TR<sub>1</sub> TR<sub>2</sub> are employed to introduce signals to both modulators MD<sub>1</sub> and MD<sub>2</sub> in the same phase. A source of carrier frequency CO is shown. The output of this oscillator is impressed across a resistance R<sub>1</sub> and condenser C<sub>1</sub> in series. The voltage drop across the resistance R<sub>1</sub> is displaced in phase by 90 electrical degrees relative to that across condenser C<sub>1</sub>, the desired relation for second order modulation. The voltage drop across the resistance R<sub>1</sub> is applied to the modulator MD<sub>1</sub> by means of resistance R<sub>4</sub> and that across C<sub>1</sub> is introduced to MD<sub>2</sub> by means of resistance R<sub>5</sub>. The reactance of the condenser C<sub>1</sub> at the carrier frequency is made equal to the resistance R<sub>1</sub>; that is,

$$R_1 = \frac{1}{C_1 \omega_s} \quad (27)$$

where  $\omega_s$  equals  $2\pi$  times the carrier frequency

$$\frac{q_s}{2\pi}$$

The components of the carrier frequency applied to the two modulators are, therefore, equal in magnitude and differ in phase by 90 degrees. The frequency of the carrier oscillator

$$\frac{q_s}{2\pi}$$

is that frequency at which it is desired to divide the incoming band of signals

$$\left( \frac{q_1}{2\pi} \text{ to } \frac{q_n}{2\pi} \right).$$

The modulators are of the balanced type so that the carrier frequency is suppressed in the output. These modulators are not necessarily of the vacuum tube type as shown but may be any type of second order modulator, such as copper oxide units. Only the lower sideband is desired and, therefore, the low pass filters LP<sub>1</sub> and LP<sub>2</sub> are inserted after the modulators MD<sub>1</sub> and MD<sub>2</sub>, respectively. These filters do not need to have sharp cutoff characteristics, because of the separation between the two sidebands, and might be replaced by condensers shunted across the outputs of the modulators. It is, of course, possible to produce only the lower sideband by some method of phasing out the upper sideband such as, for instance, disclosed in my Patents Nos. 1,719,052 and 1,744,044.

In case third order modulation is to be used, a carrier oscillator and phase dividing network similar to those shown in Fig. 3 should be substituted for the oscillator and network (R<sub>1</sub>, C<sub>1</sub>)

shown in Fig. 2. The phase difference between the carrier voltage components should be an odd multiple of 45 electrical degrees, and the carrier frequency should be half that frequency at which it is desired to divide the frequency band

$$\left( \frac{q_1}{2\pi} \text{ to } \frac{q_n}{2\pi} \right).$$

Considering the network shown in Fig. 3, the carrier frequency generated in the oscillator CO' is

$$\frac{q'_s}{2\pi}$$

which is half of the carrier frequency

$$\frac{q_s}{2\pi}$$

used for second order modulation. The output of this oscillator CO' is impressed across a network of capacity C'<sub>1</sub> and resistance R'<sub>1</sub> in series. The voltage across the condenser C'<sub>1</sub> lags that across the resistance R'<sub>1</sub> by 90 electrical degrees. In order to obtain two voltages differing by an odd multiple of 45 degrees the reactance of the capacity C'<sub>1</sub> is made equal to part of the resistance, that is

$$\frac{1}{C'_1 \omega'_s} = .414 R'_1 \quad (28)$$

where  $\omega'_s$  equals  $2\pi$  times the carrier frequency

$$\frac{q'_s}{2\pi}$$

The voltage across this capacity is then added to that across the portion of the resistance .414 R'<sub>1</sub>. Since these voltages are equal and in phase quadrature the resultant voltage E<sub>1</sub> will bear a phase relation to the voltage across the remainder of the resistance which is an odd multiple of 45 degrees, in this case 135 degrees

$$\left( \frac{3\pi}{4} \text{ radians} \right).$$

The magnitude of this component will be proportional to

$$\sqrt{2} \times (.414)$$

or .586, which is the same as that across the remainder of the resistance (.586 R'<sub>1</sub>). The result of modulating the two components of the carrier frequency

$$\frac{q'_s}{2\pi}$$

with the incoming band of frequencies will be equivalent to that obtained when second order modulators are used as described in connection with Fig. 2. The modulator in this case might consist of one or more units of thyrite (a mixture of finely divided particles of carbon and clay). A circuit arrangement for such a modulator, showing thyrite units TU<sub>1</sub> and TU<sub>2</sub> replacing vacuum tubes such as VT<sub>1</sub> and VT<sub>2</sub> of Fig. 2, is shown in Fig. 4.

Referring now to Fig. 2, the lower sideband from modulator MD<sub>1</sub> is impressed on the amplifier AM<sub>1</sub> by means of transformer TR<sub>5</sub> and that from MD<sub>2</sub> on amplifier AM<sub>2</sub> by means of transformer TR<sub>6</sub>. Amplifiers AM<sub>1</sub> and AM<sub>2</sub> comprise vacuum tubes VT<sub>5</sub> and VT<sub>6</sub> associated with transformers TR<sub>5</sub> and TR<sub>6</sub> and chokes CH<sub>1</sub> and CH<sub>2</sub>. These elements should be substantially identical and the amplifiers are shown with common battery supplies in order that their gains may be kept equal.



As previously explained, regardless of whether second or third order modulation is employed, the amplified output of one modulator should be shifted in phase 90 degrees with respect to that of the other. A possible method of shifting the phases of the frequencies composing one sideband 90 electrical degrees with respect to the corresponding frequencies of the other sideband is shown in connection with phase shifters PS<sub>1</sub> and PS<sub>2</sub>. The output circuit of amplifier AM<sub>1</sub> comprises a series arrangement of inductance, capacity and resistance, L<sub>2</sub>, C<sub>2</sub> and R<sub>2</sub>. The output circuit of amplifier AM<sub>2</sub> consists of two series circuits of L<sub>3</sub> and C<sub>3</sub> in parallel and a series resistance R<sub>3</sub>. These elements have values

$$L_3 = 2L_2, C_3 = \frac{C_2}{2} \text{ and } R_3 = R_2.$$

It will be readily seen that these two networks PS<sub>1</sub> and PS<sub>2</sub> present equal impedances to the plate circuits of the vacuum tubes VT<sub>5</sub> and VT<sub>6</sub>. The resonant frequency of these networks is adjusted to some frequency within the frequency band occupied by the lower side-band of the modulation. The leads to the transformer TR<sub>7</sub> are connected across a part of the resistance R<sub>2</sub> and hence the voltage V<sub>R</sub> is not shifted in phase with respect to the current. These taps should be so located as to make the voltage V<sub>R</sub> equal to the output voltage V<sub>LC</sub> of the lower network. The voltage V<sub>LC</sub> will be shifted 90 degrees out of phase, however, as will now be explained.

Obviously, the voltage drop across the condenser C<sub>2</sub> of the upper network lags the voltage drop across R<sub>2</sub> by 90 degrees at all frequencies, while the voltage across L<sub>2</sub> leads that across R<sub>2</sub> by 90 degrees. Since the voltage across C<sub>2</sub> is inversely proportional to frequency, and that across L<sub>2</sub> directly proportional to frequency, by reversing the sign of one of these voltages and adding it to the other there can be obtained a voltage which for a considerable range on each side of the resonant frequency has substantially the same magnitude as the voltage drop V<sub>R</sub> across a part of R<sub>2</sub> and differs from it in phase by 90 degrees. This will be clear from the curve V<sub>LC</sub> of Fig. 5 which shows the two reactive voltages combined with one voltage reversed. In the upper network the phases of the reactive voltages are of no importance as the output is taken from the resistance. In the lower network the output is taken from the reactances however, and the reactive voltages are added in reversed relation. This addition of the voltages across the inductance and condenser is obtained by the use of the parallel circuit, as indicated in network PS<sub>2</sub>. The reversal of the one reactive voltage with respect to the other is brought about by reversing the order of the inductances and capacities in the parallel branches, the taps being taken at the junction points of the reactances in the two branches. The voltage V<sub>LC</sub> is thus obtained by superposing the voltage drops across the inductances L<sub>3</sub> upon the voltage drops across the condensers C<sub>3</sub> with their signs reversed. As will be shown below by a concrete example, this voltage V<sub>LC</sub> may be made substantially equal, over a considerable range of frequencies on either side of the resonant frequency of the L<sub>2</sub>-C<sub>2</sub> circuit, to the voltage V<sub>R</sub> which is the drop across a part of the resistance R<sub>2</sub> of the upper network PS<sub>1</sub>.

In order to obtain a concrete example of the operation of the phase shifters, PS<sub>1</sub> and PS<sub>2</sub> in

Fig. 2, let it be assumed that the resonant frequency

$$\left( \omega = \frac{1}{\sqrt{L_2 C_2}} \right)$$

is 1000 cycles, that the value of R<sub>2</sub> is 1000 ohms and that L<sub>2</sub>ω and

$$\frac{1}{C_2 \omega}$$

each have a value of 100 ohms at 1000 cycles. Let us assume for the moment that the same voltage E is applied to both the upper and the lower resonant circuits. If reactive voltages are reversed with respect to each other the magnitude of the combined voltages over the range of frequencies from 500 to 2000 cycles will be as shown by the curve V<sub>LC</sub> in Fig. 5. The voltage V<sub>R</sub> in this case is selected as .22 times the voltage across R<sub>2</sub>, because that is about the average value of the voltage V<sub>LC</sub> over the frequency range plotted.

It is evident, therefore, in connection with Fig. 2, that by the use of the two networks PS<sub>1</sub> and PS<sub>2</sub> we can shift one of the applied bands of frequencies exactly 90 degrees in phase relative to the other while producing only a small magnitude distortion over a fairly wide frequency band. The lack of precise equality of magnitude results in a small amount of cross-talk or interference from the unwanted band into the output of the wanted band. It should be noted in connection with the method used to shift the phases of one of the sidebands resulting from the modulation, that the networks PS<sub>1</sub> and PS<sub>2</sub> in Fig. 2 accomplish the desired result only for a limited frequency band. This frequency band is limited by the magnitude relations between the resistive and the combined reactive voltage drops. Too large a difference in the magnitudes of these voltages would impair the performance of the system. From Fig. 5 it will be seen that in the band from about 500 cycles to 2000 cycles the resistive and reactive voltages are substantially equal for the circuit values chosen above. By the use of more complicated networks or other phase shifting devices it would be possible to obtain more precise results with respect to the shifting of the phase of one frequency band relative to the other, and to extend the range of frequencies over which satisfactory results may be obtained.

Returning now to Fig. 2, these voltages V<sub>R</sub> and V<sub>LC</sub> are introduced to the amplifiers AM<sub>3</sub> and AM<sub>4</sub> by means of transformers TR<sub>7</sub> and TR<sub>8</sub>. These amplifiers should have identical gain-frequency characteristics. After being amplified, the side-bands are combined at the outputs of transformers TR<sub>9</sub> and TR<sub>10</sub> by the switch SW. Addition of the side-bands will produce the sideband frequencies corresponding to that part of the original band

$$\frac{q_1}{2\pi} \text{ to } \frac{q_n}{2\pi}$$

which lies above the carrier frequency, that is, the band

$$\frac{q_m - q_s}{2\pi} \text{ to } \frac{q_n - q_s}{2\pi}.$$

By reversing the switch SW, the difference of the two sidebands will give the sideband frequencies corresponding to that part of the original band below the carrier frequency, or

$$\frac{q_s - q_k}{2\pi} \text{ to } \frac{q_s - q_1}{2\pi}.$$



In the systems illustrated in Figs. 2, 3 and 4, the carrier components supplied to the two modulators are shifted relatively either 90 degrees or 45 degrees, and the relative phase shift introduced by the networks  $PS_1$  and  $PS_2$  is also 90 degrees. If phase shifts other than these are to be used in accordance with the principles outlined in connection with Equations (9) to (26) inclusive, some modification in the networks employed will be necessary.

Any desired relative phase shift between the carrier components supplied to the two modulators may be obtained by using a carrier supply circuit of the type shown in Fig. 3. The particular distribution of resistance between the two halves of the circuit shown in Fig. 3 produces a relative phase shift of 45 degrees. By merely changing the ratio of the two parts of the resistance in the two halves of the circuit, relative phase shifts either greater or smaller than 45 degrees will result. For example, by making the resistance in the upper section smaller and smaller, with corresponding increase of that in the lower sections, the phase shift will be increased until in the limiting case, with zero resistance above and full resistance  $R'_1$  below, the phase shift will be 90 degrees. On the other hand, if the upper resistance is increased and the lower resistance is decreased the phase shift may be made to approach zero.

To obtain a relative phase shift other than 90 degrees in the selected output bands of the two modulators, the lower network  $PS_2$  may be replaced by the network shown in Fig. 6, keeping the simple series network  $PS_1$  in the upper branch the same as in Fig. 2. In Fig. 6 it will be observed that a portion ( $aR$ ) of the total series resistance  $R$  is removed from the common branch of the network, thus leaving the remainder  $(1-a)R$  in the common branch, the removed part being in effect divided between the two parallel branches by including a resistance element having a value  $2aR$  in each parallel branch. The three impedance elements in each branch are arranged so their order is reversed in one branch.

By comparing this network with the network  $PS_2$  of Fig. 2 in which the reactance elements introduce a phase shift of 90 degrees between the output voltage  $V_{LC}$  and the voltage across the resistance  $R_3$ , it will be evident that the effect of the resistance elements in the two parallel branches in Fig. 6 is to produce a phase shift of less than 90 degrees. This will be clear when we consider that in network  $PS_2$  the drop across the inductance  $2L_2$  in one of the parallel branches is  $90^\circ$  out of phase with the drop through the series resistance  $R_3$ , while in Fig. 6 the drop through the resistance  $2aR$  in series with inductance  $2L$  in one branch will be out of phase with the drop through resistance  $(1-a)R$  by an amount determined by the vectorial sum of the drops through resistance  $2aR$  and inductance  $2L$ .

From what has already been stated with reference to the action of the networks  $PS_1$  and  $PS_2$  of Fig. 2, in which two separate voltages applied to the networks may be relatively shifted in phase 90 degrees, it will be evident that by an analogous arrangement an applied voltage may be translated into two separate voltages in phase quadrature. Such an arrangement is shown in Fig. 7. Here the applied voltage  $V_0$  is applied to a network somewhat like network  $PS_2$  of Fig. 2, said network having a resistance  $R$  in series with two parallel branches each having

inductance  $L$  and capacity  $C$  in series, these reactance elements being reversed in the two branches. By taking two sets of taps off as shown, a voltage  $V_R$  may be taken from the lower set of taps which is in phase quadrature with respect to a voltage  $V_{LC}$  taken off the upper set of taps. By properly setting the taps these two voltages may be made substantially equal in magnitude over a considerable range of frequencies.

Having obtained by any of the methods previously described, separate frequency bands representing the two desired portions of the original band of frequencies, modulated to the carrier frequency, it is possible to translate either band to any desired position in the frequency spectrum, including, of course, that originally occupied by said band. A single sideband modulator of any of several well known types may be used for this purpose, such as, for instance, those disclosed in my patents referred to above.

By additional steps of division in the manner already described, the original frequency band may be divided into any desired number of separate bands of frequencies. In this way my invention may be made to perform the functions of one or more band filters, as well as those of high and low pass filters.

As compared with selection by the use of filters, where the dead space or separations between adjacent selected bands becomes greater as we go up in the frequency spectrum, the method of selection herein disclosed has the advantage that the separation between bands is the same at higher frequencies as at low frequencies. Hence even if the design is such that the cutoff is not as sharp as a band filter at low frequencies, it will give sharper selection at high frequencies.

It is scarcely necessary to discuss the applications of my invention. It has been disclosed as a method and means for performing functions similar to those of high and low pass filters. With additional steps the functions of band selection and band elimination filters may also be performed. It is obvious that a device capable of performing the above functions will have manifold uses. One possible application, will however, be mentioned by way of illustration.

A possible application of this method of dividing a band of frequencies is in the reception of a single sideband of a double sideband transmission. The arrangement of apparatus shown in Fig. 2 may be easily adapted for radio reception. The source of signals  $SS$  will for this purpose comprise a radio antenna with or without a radio frequency amplifier and tuned circuits for discriminating in favor of the desired radio transmission. Both sidebands will be transmitted to the modulators  $MD_1$  and  $MD_2$ . The carrier frequency oscillator  $CO$  will generate a frequency equal to the carrier frequency of the radio transmission, or it might even be derived from the radio carrier. The operation of the apparatus has already been described and it will be seen that by either adding or subtracting the two demodulated waves, at the output, the resulting audible frequencies obtained will correspond to either the upper or the lower sideband of the original transmission.

While this invention has been disclosed as embodied in certain particular forms, it is capable of embodiment in other and different forms without departing from the spirit and scope of the appended claims.

What is claimed is:

1. The method of dividing a band of frequencies, which consists in producing differently



phased components of the same carrier frequency, modulating one of said components with a part of the energy of said band, modulating the other component with an identically phased part of the energy of said band, and utilizing the different phase relations between frequencies resulting from the two modulations as a basis for obtaining two separate bands of frequencies which correspond, respectively, to different sub-bands of said original frequency band.

2. The method of dividing a band of frequencies, which consists in separately modulating differently phased components of the same carrier frequency with said band, after such modulation shifting the phase of certain component frequencies resulting from one modulation with respect to corresponding component frequencies of the other modulation, and combining said phase shifted components with said other components so as to obtain frequency bands corresponding to certain portions only of the original band.

3. The method of dividing a band of frequencies, which consists in separately modulating differently phased components of the same carrier frequency with said band, after such modulation shifting the phase of certain component frequencies resulting from one modulation with respect to corresponding component frequencies of the other modulation, and combining said phase-shifted components with said other components so as to obtain a frequency band corresponding to a portion only of the original band and suppress another portion of the original band.

4. The method of dividing a band of frequencies, which consists in producing two carrier currents of the same frequency and having a predetermined phase relation, said carrier frequency bearing a predetermined relation to the frequency at which it is desired to divide said band of frequencies, separately modulating each of said carrier currents with said band of frequencies, retaining only the lower sideband of each modulation, shifting the phases of the frequencies of one of said lower sidebands uniformly by a predetermined amount with respect to those of the other of said bands, and combining the two resultant bands so that frequencies corresponding to one portion of the original band are obtained.

5. The method of dividing a band of frequencies, which consists in producing two carrier currents of the same frequency and having a predetermined phase relation, said carrier frequency bearing a predetermined relation to the frequency at which it is desired to divide said band of frequencies, separately modulating each of said carrier currents with said band of frequencies, retaining only the lower sideband of each modulation, shifting the phases of the frequencies of one of said lower sidebands uniformly by a predetermined amount with respect to those of the other of said bands, and combining the two resultant bands so that frequencies corresponding to one portion of the original band are obtained in one circuit and frequencies corresponding to another portion of the original band are obtained in another circuit.

6. The method of segregating a portion of a band of frequencies, which consists in producing two carrier currents of the same frequency and having a predetermined phase relation, said carrier frequency bearing a predetermined relation to the boundary of the band to be segregated, separately modulating each of said carrier frequency currents with the original band of frequencies, retaining only the lower sideband of

each modulation, shifting the phases of the frequencies of one of said lower sidebands uniformly by a predetermined amount with respect to those of the other of said sidebands, and combining the resultant bands so that a frequency band corresponding to the desired portion of the original band is obtained and components corresponding to the unwanted portion of the original band are substantially annulled.

7. The method of separating out a component band of frequencies from a larger band, which consists in dividing said larger band by the method of claim 6, and dividing again by the same method the band resulting from said first division.

8. The method of segregating a portion of a frequency band, consisting in generating a carrier frequency at which it is desired to divide said band, obtaining two components of said frequency equal in magnitude but differing in phase by some angle, modulating each of said carrier frequency currents separately with said frequency band by second order modulation, retaining only the lower sidebands resulting from said modulations, shifting the phases of the frequencies of one of said sidebands by some second angle relative to those of the other of said sidebands, combining said sidebands, and choosing said first mentioned and said second mentioned angles of phase shift so that their sum is equal to 180 electrical degrees, thus eliminating by said combination the frequencies on one side of the carrier frequency.

9. The method of segregating a portion of a frequency band, consisting in generating a carrier frequency at which it is desired to divide said band, obtaining two components of said frequency equal in magnitude but differing in phase by some angle, modulating each of said carrier frequency currents separately with said frequency band by second order modulation, retaining only the lower sidebands resulting from said modulations, producing a phase shift at all frequencies of one of said sidebands of some second angle relative to those of the other sideband, combining said sidebands, and choosing said first mentioned and said second mentioned angles of phase shift so that their difference is equal to 180 electrical degrees, thus eliminating by said combination the frequencies on one side of the carrier frequency.

10. The method of dividing a band of frequencies, which consists in producing two carrier currents of the same frequency having a quadrature phase relation, said carrier frequency lying between adjacent component frequencies of said band of frequencies, separately modulating by second order modulation each of said carrier currents with said band of frequencies, retaining only the lower sidebands resulting from said modulations, shifting the phases of the currents of one of said lower sidebands 90 electrical degrees with respect to the corresponding currents of the other of said lower sidebands, and combining said phase-shifted sideband with said other sideband in two circuits so that the sideband frequencies corresponding to one portion of the original band appear in one of said circuits, and those corresponding to the other portion appear in the other of said circuits.

11. The method of segregating a portion of a band of frequencies, consisting in generating a carrier frequency corresponding to one boundary of said portion, obtaining two equal current components of said carrier frequency in phase quadrature, modulating by second order modulation each of said components separately with said



band of frequencies, retaining only the lower sidebands resulting from said modulations, producing a phase shift of 90 electrical degrees in one of said lower sidebands, and combining said phase-shifter sideband with the other of said sidebands in such a way that the frequency band corresponding to that portion of the original band to be segregated is retained, the undesired frequency band being annulled.

12. The method of separating out a component band of frequencies from a larger band, consisting in segregating a portion of said larger band by the method of claim 11, and by the same method segregating a portion of the resulting band.

13. The method of dividing a band of frequencies, consisting in producing two carrier currents of the same frequency having a phase difference equal to 45 electrical degrees multiplied by an odd number, said carrier frequency having a value of one half the frequency at which it is desired to divide said band of frequencies, separately modulating by third order modulation each of said carrier currents with said band of frequencies, retaining only the lower sidebands of said modulations, shifting the phases of the currents of one of said lower sidebands 90 electrical degrees relative to the corresponding currents of the other of said lower sidebands, and combining said phase-shifted sideband with said other sideband so that the sideband frequencies corresponding to one portion of the original band are obtained.

14. The method of dividing a band of frequencies, consisting in producing two carrier currents of the same frequency having a phase difference equal to 45 electrical degrees multiplied by an odd number, said carrier frequency having a value of one half the frequency at which it is desired to divide said band of frequencies, separately modulating by third order modulation each of said carrier currents with said band of frequencies, retaining only the lower sidebands of said modulations, shifting the phase of the currents of one of said lower sidebands 90 electrical degrees relative to the corresponding currents of the other of said lower sidebands, and combining said phase-shifted sideband with said other sideband so that the sideband frequencies corresponding to one portion of the original band are obtained in one circuit, and those corresponding to the other portion are obtained in another circuit.

15. In a system for dividing a band of frequencies, a source of signals, a pair of modulators of the second order type, means for introducing signals from said source into said modulators in the same phase, a source of carrier frequency, said carrier frequency lying between adjacent component frequencies of said band of frequencies, means for introducing into said modulators currents of said carrier frequency equal in magnitude but in phase quadrature, means for shifting the phases of the frequencies of the lower sideband from one of said modulators 90 electrical degrees relative to those of the lower sideband from the other of said modulators, means for combining said phase-shifted sideband with said other means so that the resultant sideband frequencies correspond to the frequencies in the original band of signals on one side of said carrier frequency.

16. In a system for dividing a band of frequencies, a source of signals, a pair of modula-

tors of the second order type, means for introducing signals from said source into said modulators in the same phase, a source of carrier frequency, said carrier frequency lying between adjacent component frequencies of said band of frequencies, means for introducing into said modulators currents of said carrier frequency equal in magnitude but in phase quadrature, means for shifting the phases of the frequencies of the lower sideband from one of said modulators 90 electrical degrees relative to those of the lower sideband from the other of said modulators, means for combining said phase-shifted sideband with said other sideband so that the resultant sideband frequencies correspond to the frequencies in the original band of signals on one side of said carrier frequency, means for combining said phase-shifted sideband with the other sideband so that the resultant sideband frequencies correspond to those frequencies in the original band on the other side of said carrier frequency.

17. In a system for dividing a band of frequencies, a source of signals, a pair of third order modulators, means for introducing signals from said source into said modulators in the same phase, an oscillator generating a carrier frequency having a value of half the frequency at which it is desired to divide said band of frequencies, means for introducing into said modulators currents of said carrier frequency equal in magnitude but differing in phase by 45 electrical degrees multiplied by an odd number, means for shifting the phases of the frequencies of the lower sideband from one of said modulators 90 electrical degrees relative to those of the lower sideband from the other of said modulators, means for combining said phase-shifted sideband with said other sideband so that the resultant sideband frequencies correspond to the frequencies in the original band of signals on one side of said carrier frequency.

18. In a system for dividing a band of frequencies, a source of signals, a pair of third order modulators, means for introducing signals from said source into said modulators in the same phase, an oscillator generating a carrier frequency having a value of half the frequency at which it is desired to divide said band of frequencies, means for introducing into said modulators currents of said carrier frequency equal in magnitude but differing in phase by 45 electrical degrees multiplied by an odd number, means for shifting the phases of the frequencies of the lower sideband from one of said modulators 90 electrical degrees relative to those of the lower sideband from the other of said modulators, means for combining said phase-shifted sideband with said other sideband so that the resultant sideband frequencies correspond to the frequencies in the original band of signals on one side of said carrier frequency, and means for combining said phase-shifted sideband with the other sideband so that the resultant sideband frequencies correspond to those frequencies in the original band on the other side of said carrier.

19. In combination, a source of frequencies extending over a range, means for deriving from said source two components one of said components having a phase shift at each frequency of approximately 90 electrical degrees relative to the corresponding frequency of the other of said components, the relative current magnitude



at each frequency in said two components being substantially constant, said means consisting in an impedance network comprising two identical series circuits of inductance and capacitance 5 connected in parallel, one reversed with respect to the other, and a resistance in series with said parallel combination, one of said components being the voltage between the midpoints of said two series circuits, and the other the voltage 10 across part of said resistance.

20. The method of receiving a radio transmission comprising two sidebands and a carrier frequency, consisting in producing locally a carrier frequency substantially equal to that of said

radio transmission, obtaining from said local carrier frequency two currents having a quadrature phase relation, demodulating in second order demodulators said radio transmission with each of said locally produced carrier currents, 5 producing a 90-degree phase shift in one of the demodulated frequency bands after demodulation, and combining the demodulated frequency bands after such shift in phase in such a way that only the frequencies resulting from the de- 10 modulation of one of said side-bands are retained, the frequencies resulting from the other sideband being annulled.

ESTILL I. GREEN.