

Aug. 15, 1933.

J. S. STONE

1,922,115

ANTENNA ARRAY

Filed April 12, 1930

5 Sheets-Sheet 1

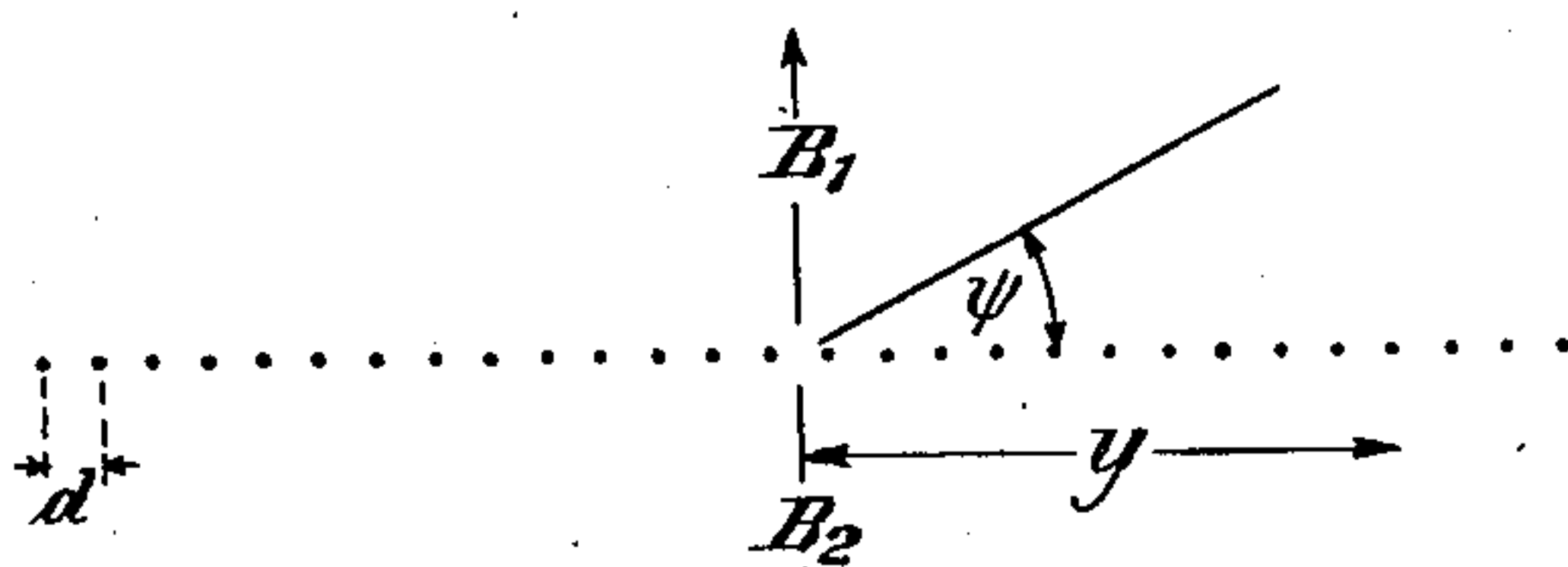


Fig. 1

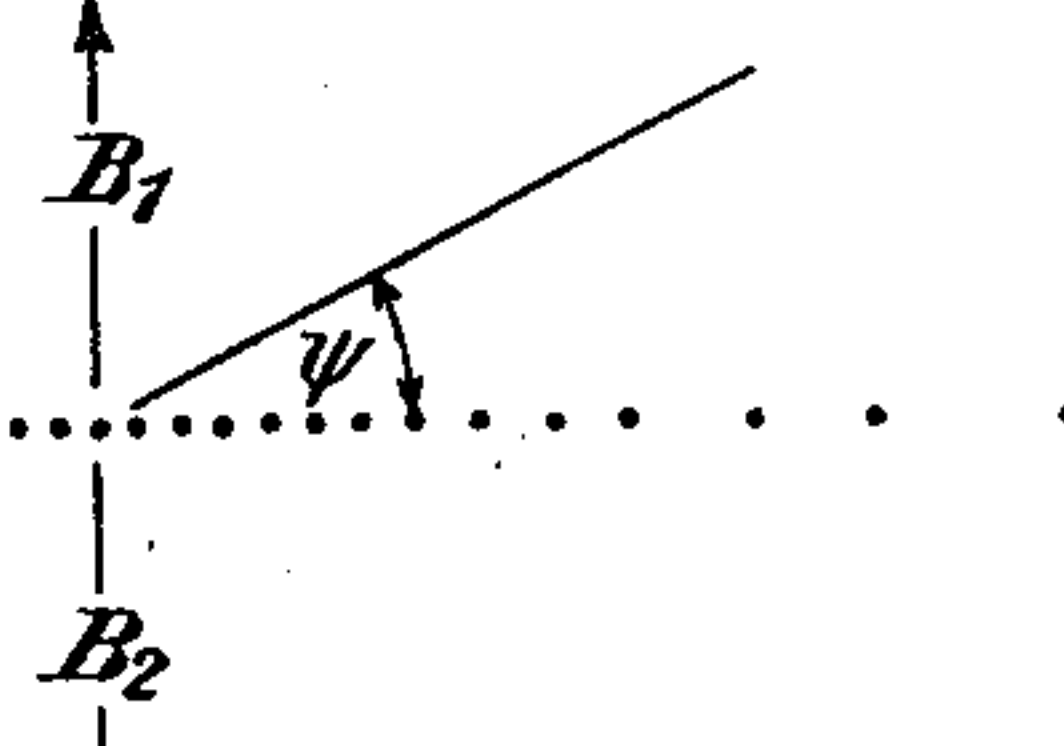


Fig. 2

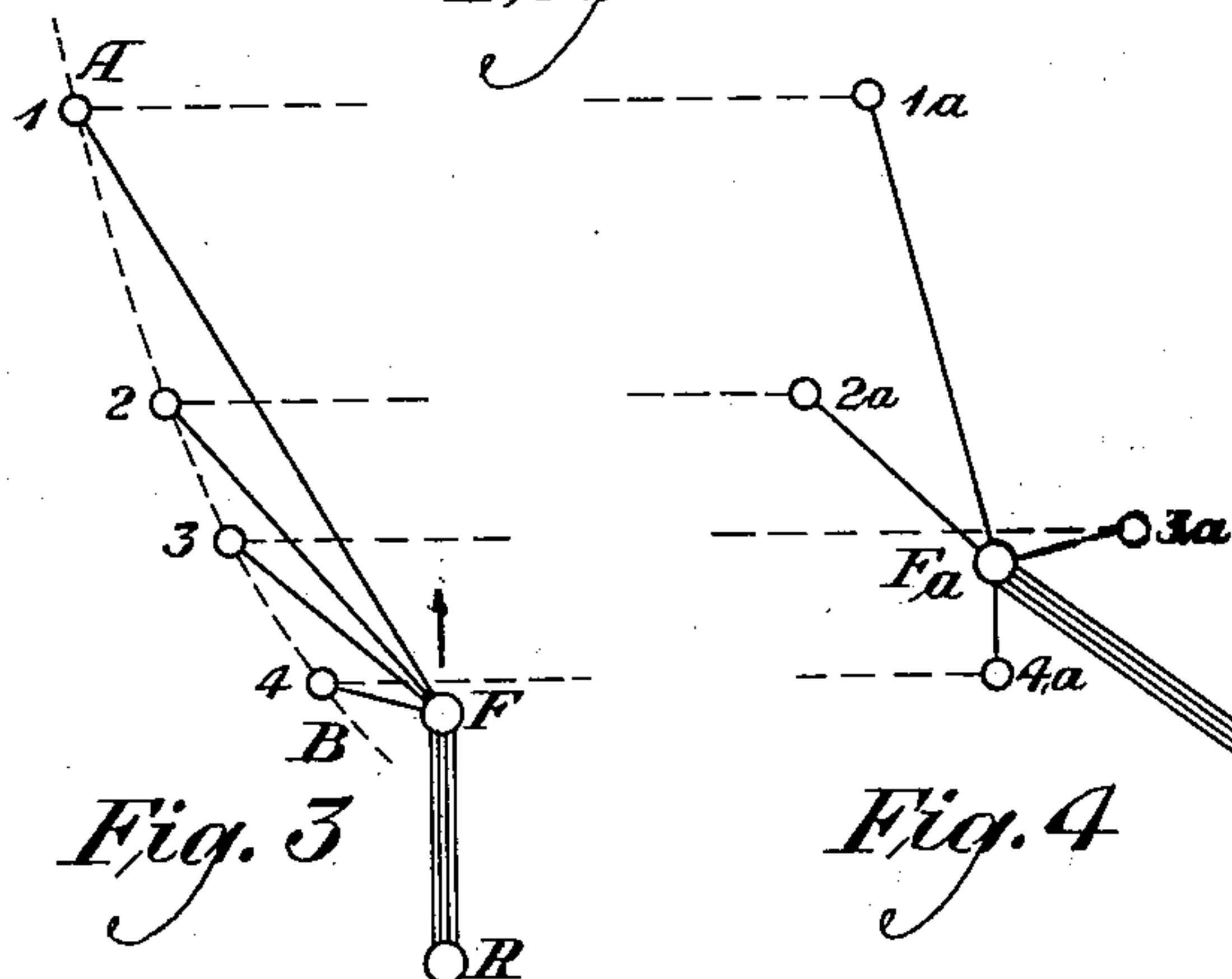


Fig. 3

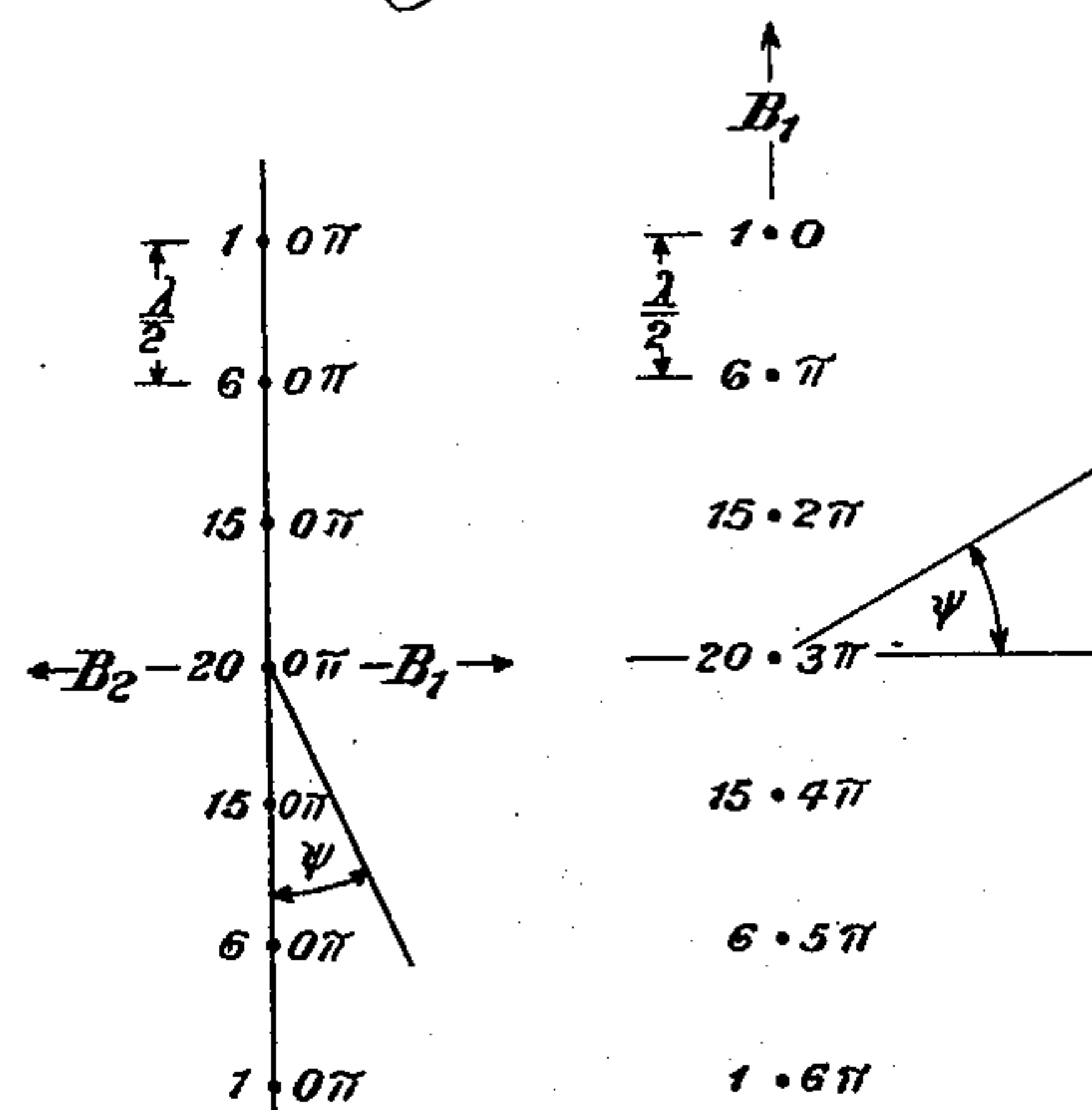
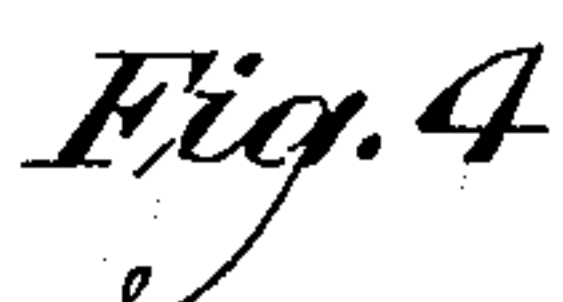


Fig. 5

Fig. 6

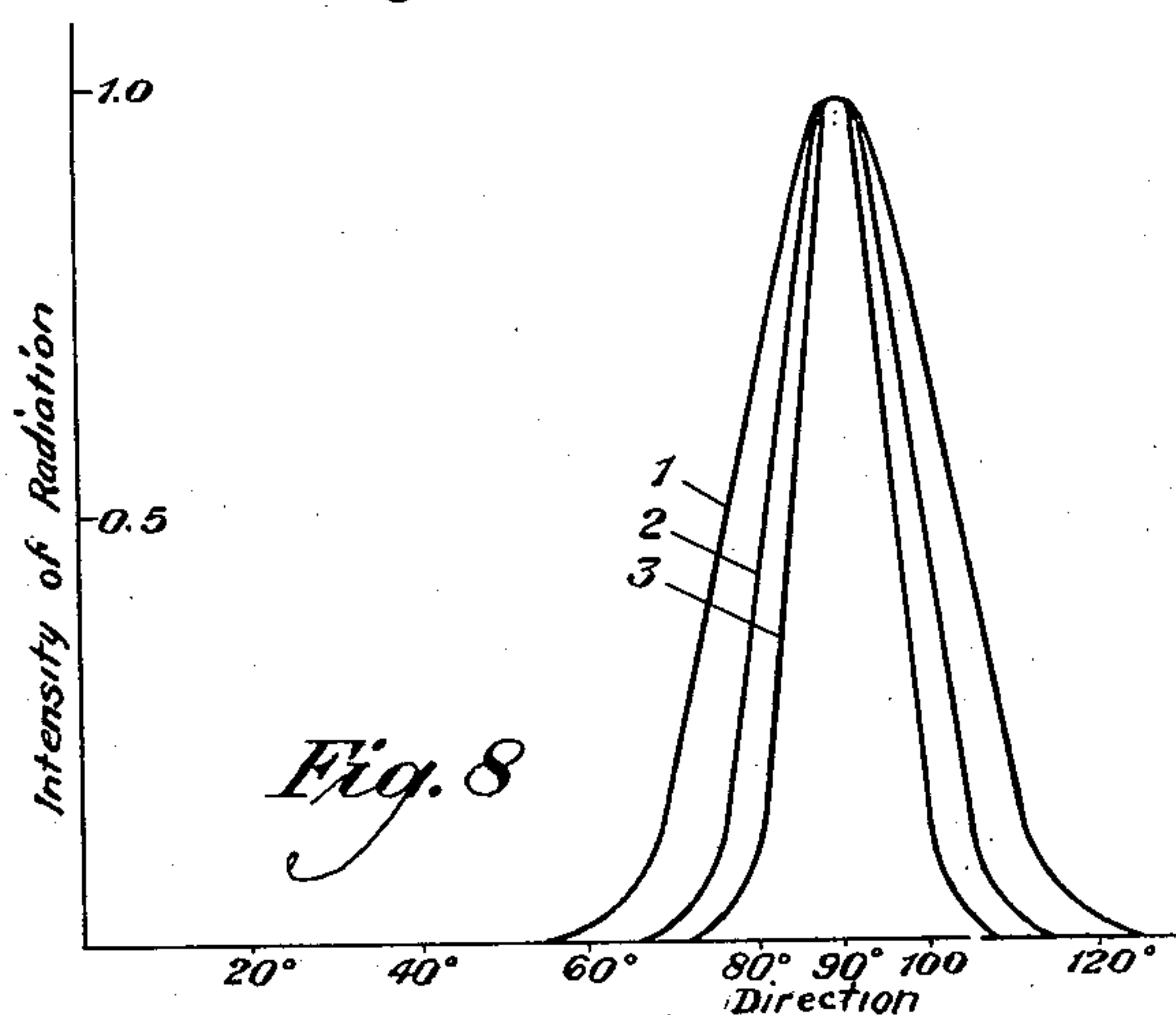


Fig. 8

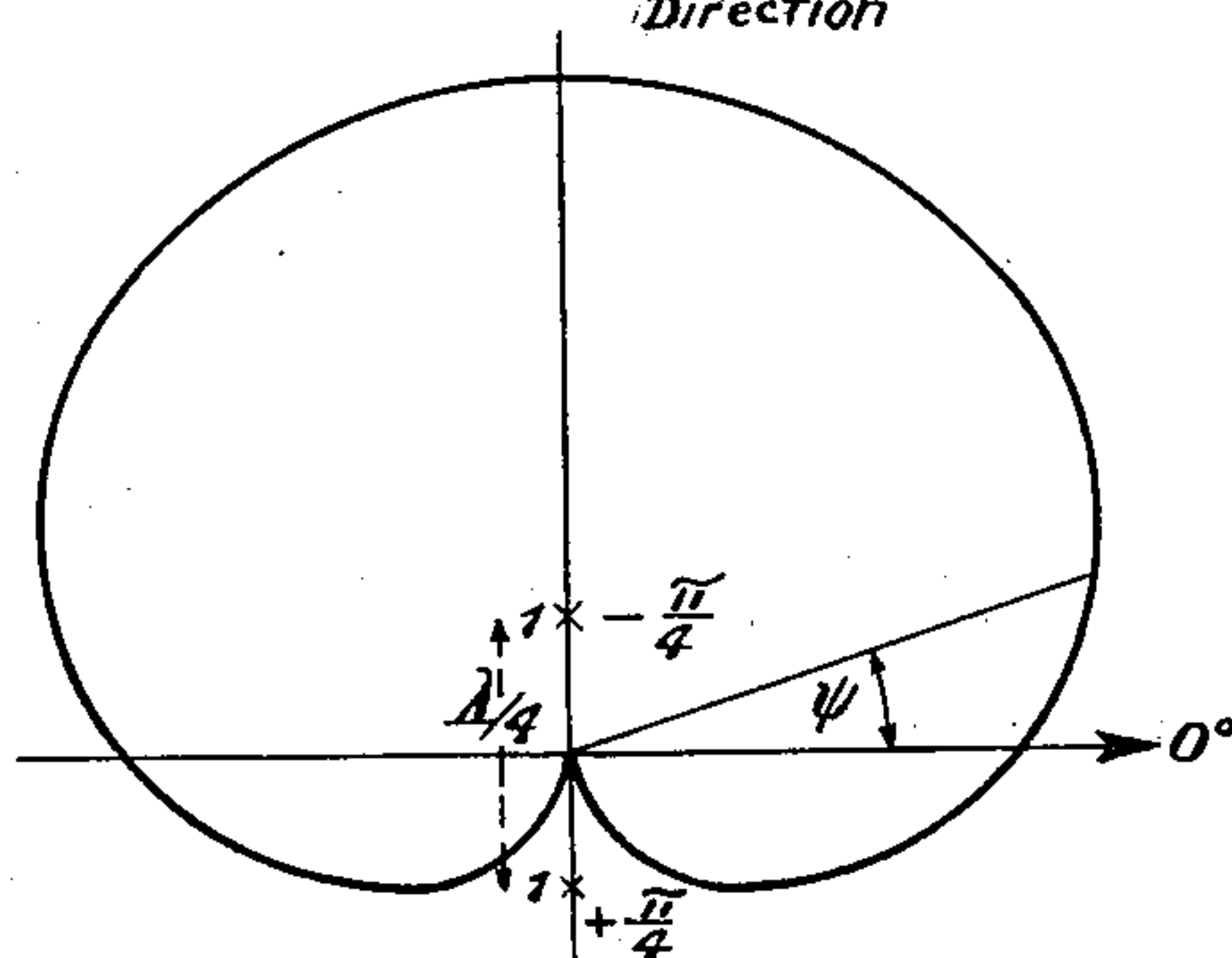


Fig. 9

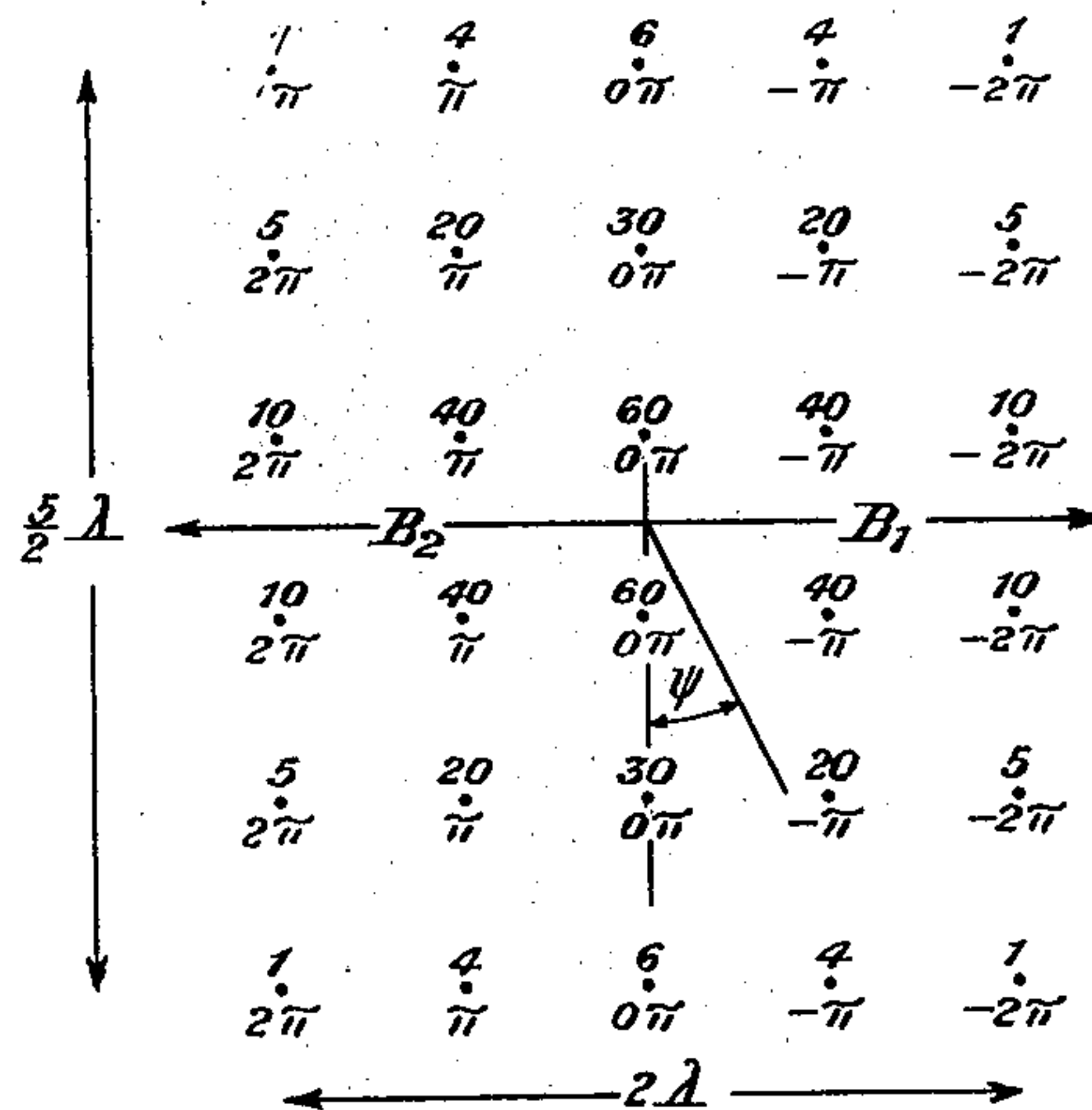


Fig. 7

INVENTOR
John Stone Stone
BY *JEYOR*
ATTORNEY

Aug. 15, 1933.

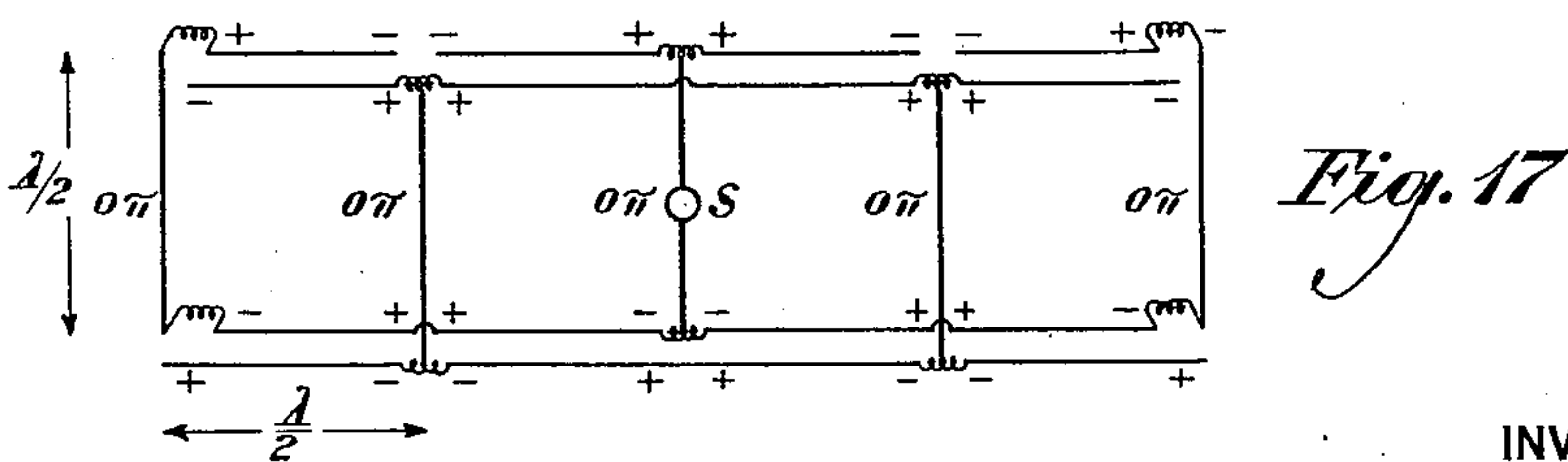
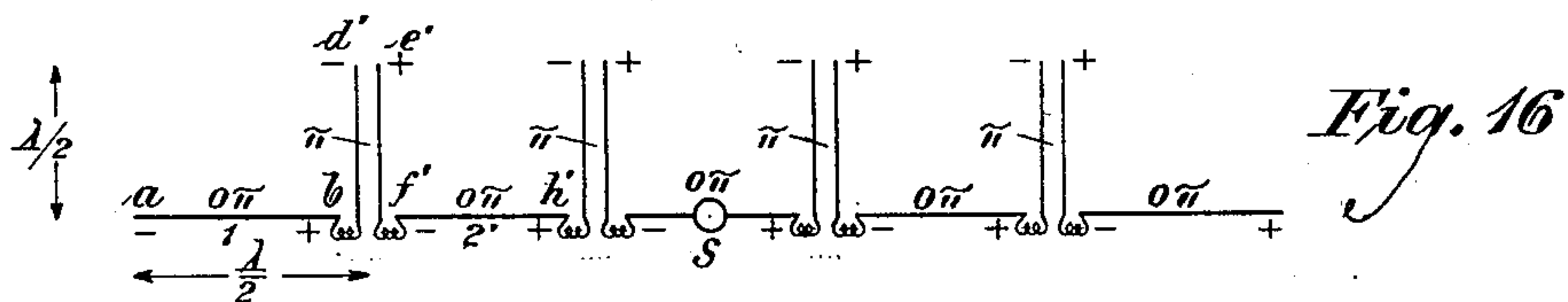
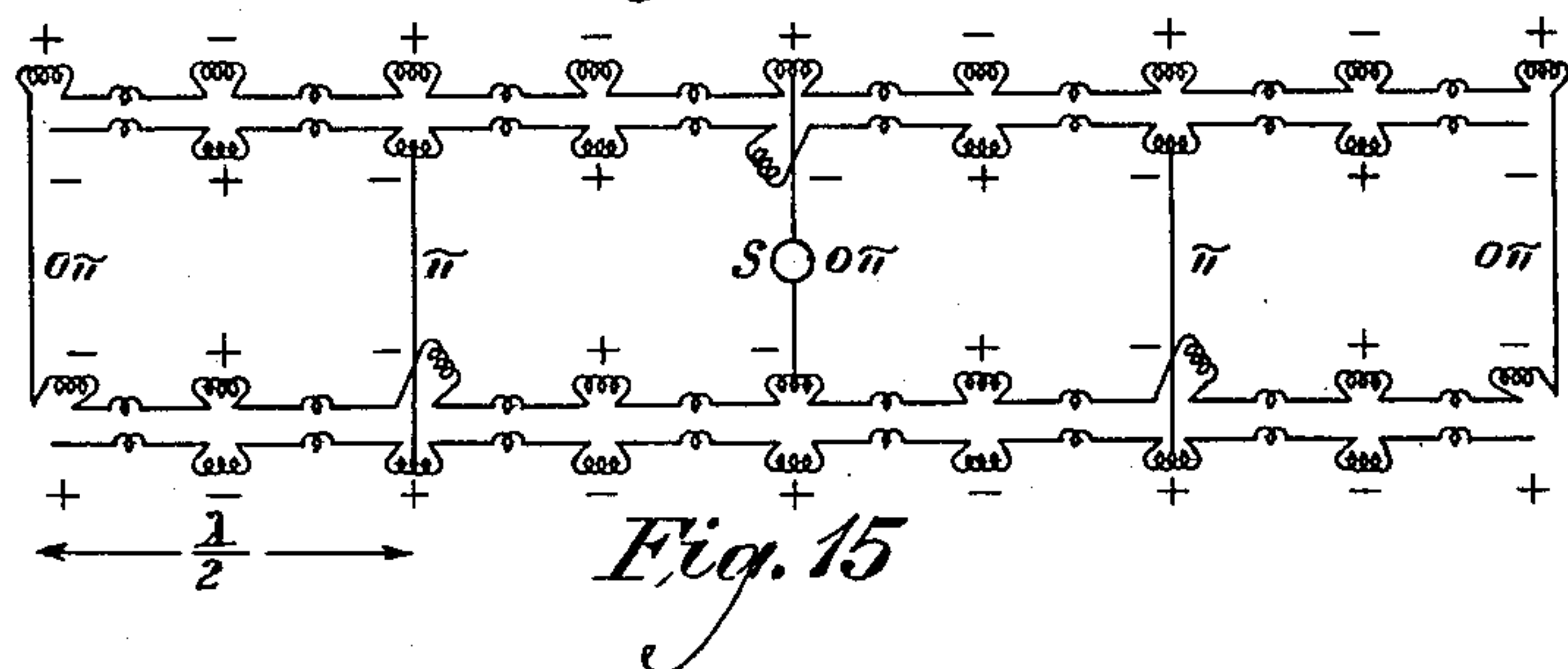
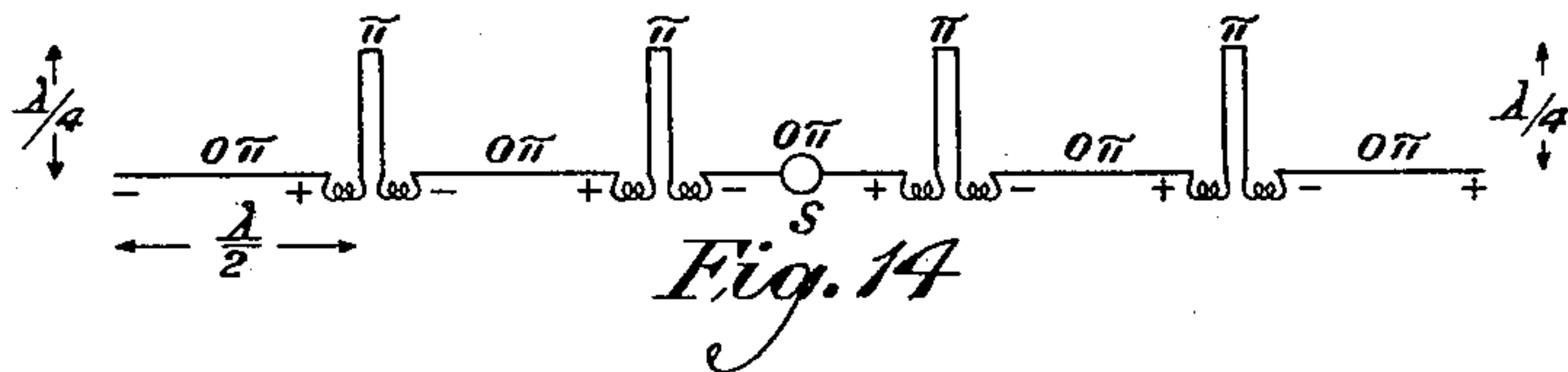
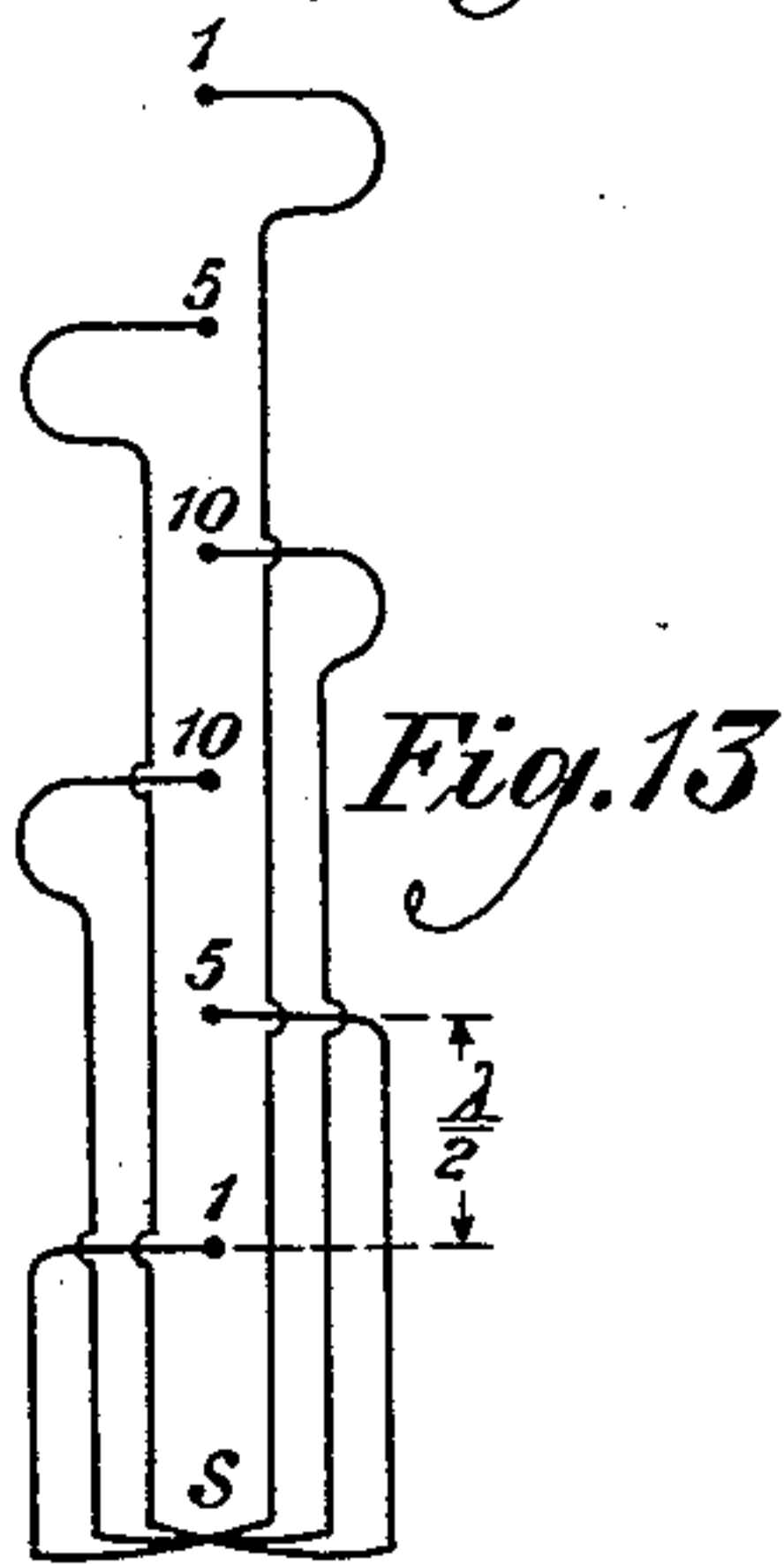
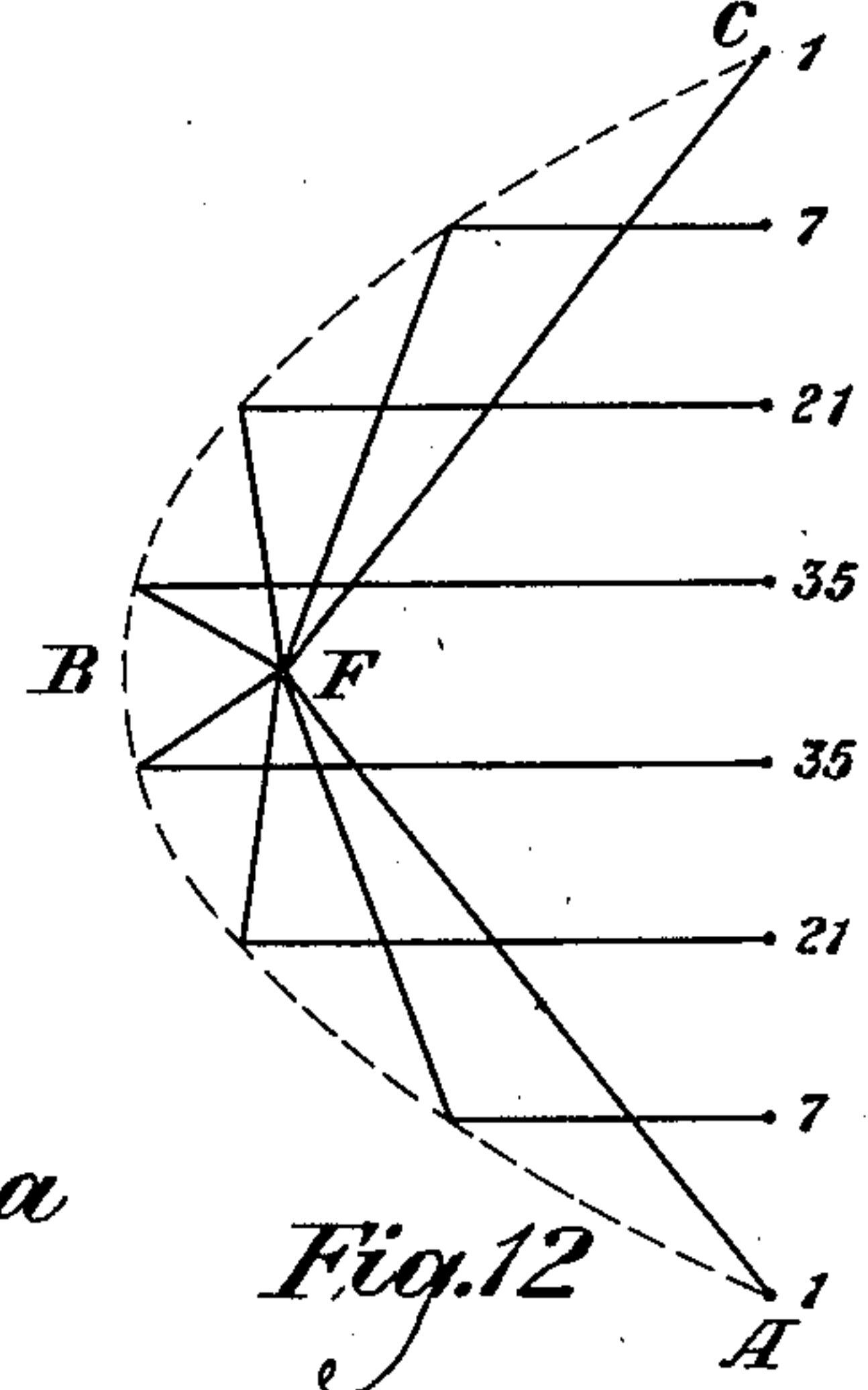
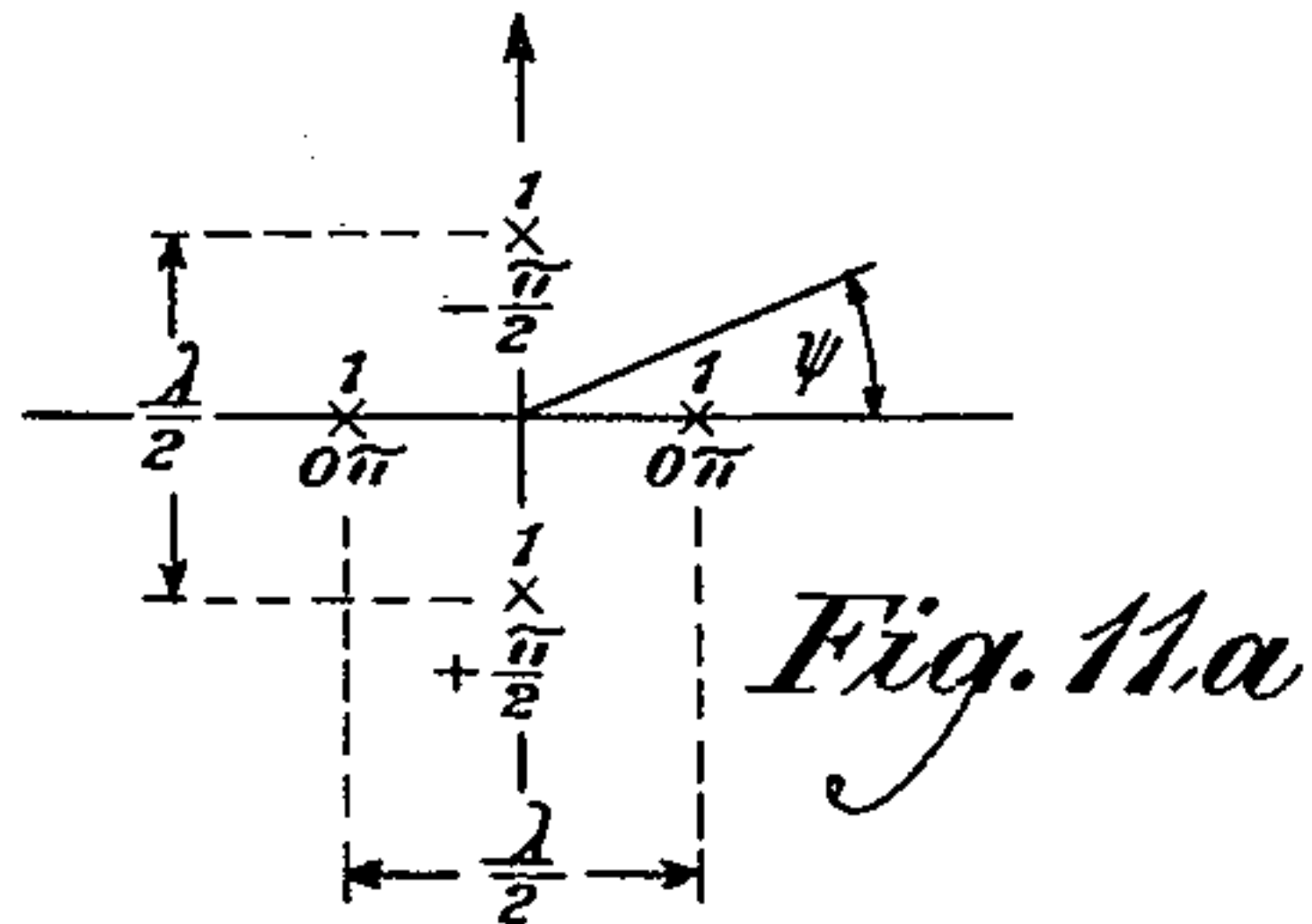
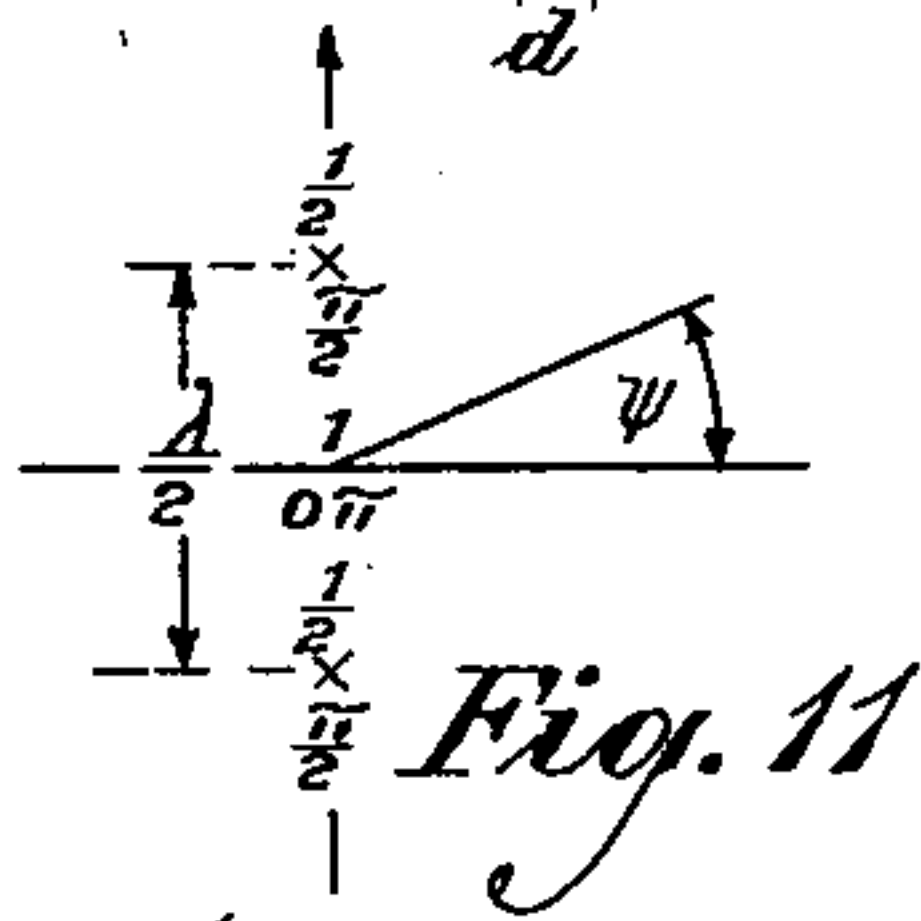
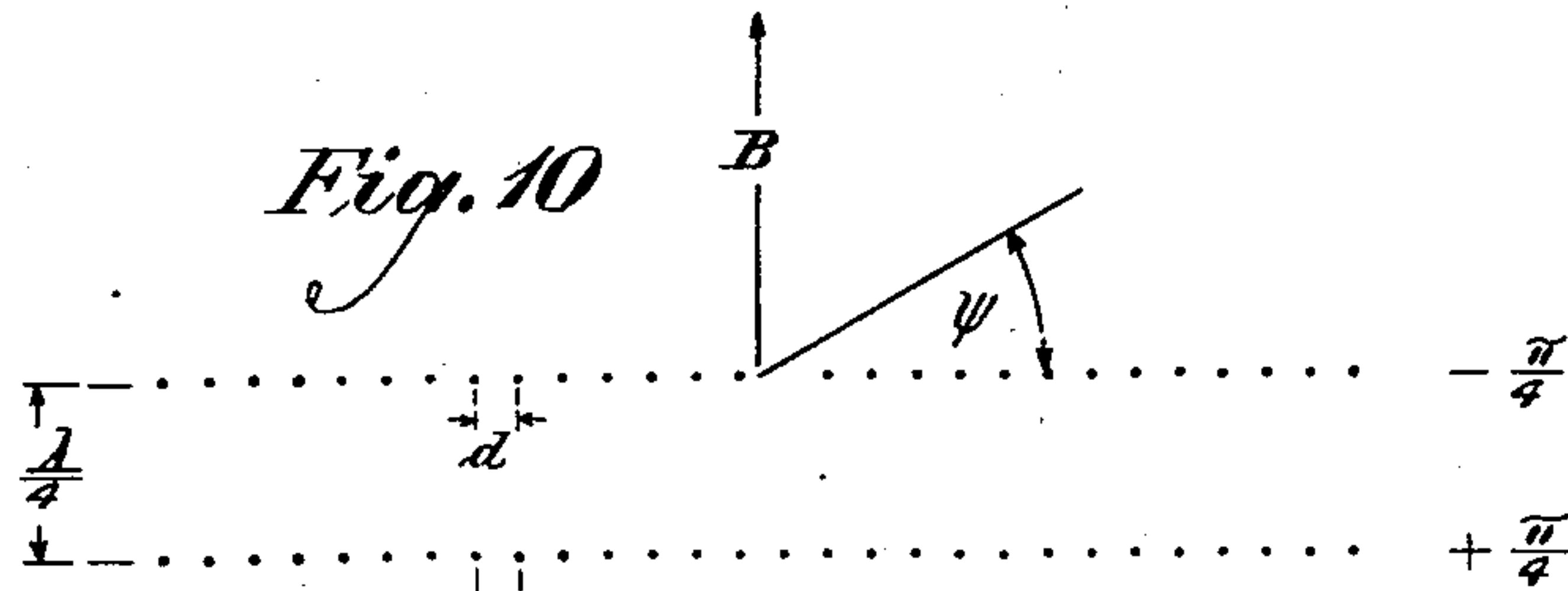
J. S. STONE

1,922,115

ANTENNA ARRAY

Filed April 12, 1930

5 Sheets-Sheet 2



INVENTOR
John Stone Stone
BY
J. E. York
ATTORNEY

Aug. 15, 1933.

J. S. STONE

1,922,115

ANTENNA ARRAY

Filed April 12, 1930

5 Sheets Sheet 3

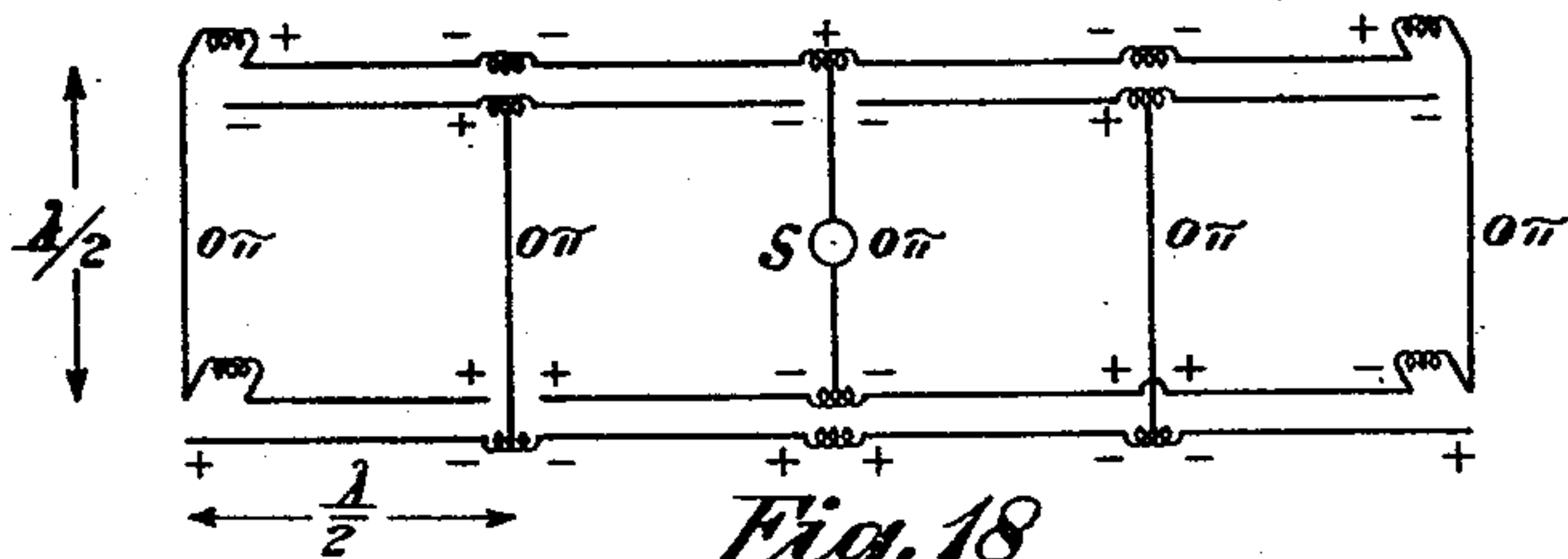


Fig. 18

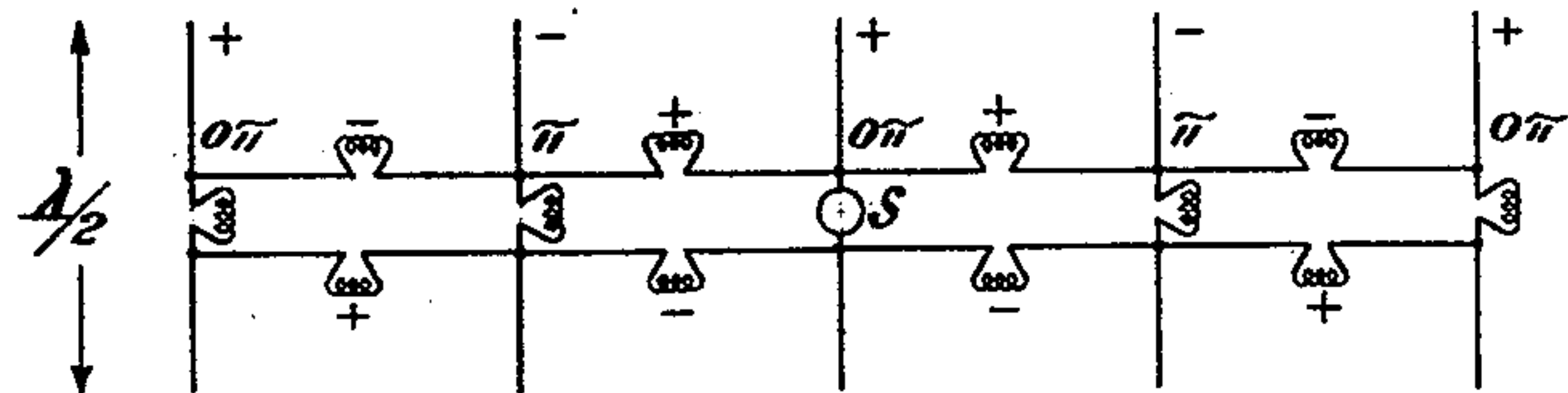


Fig. 19

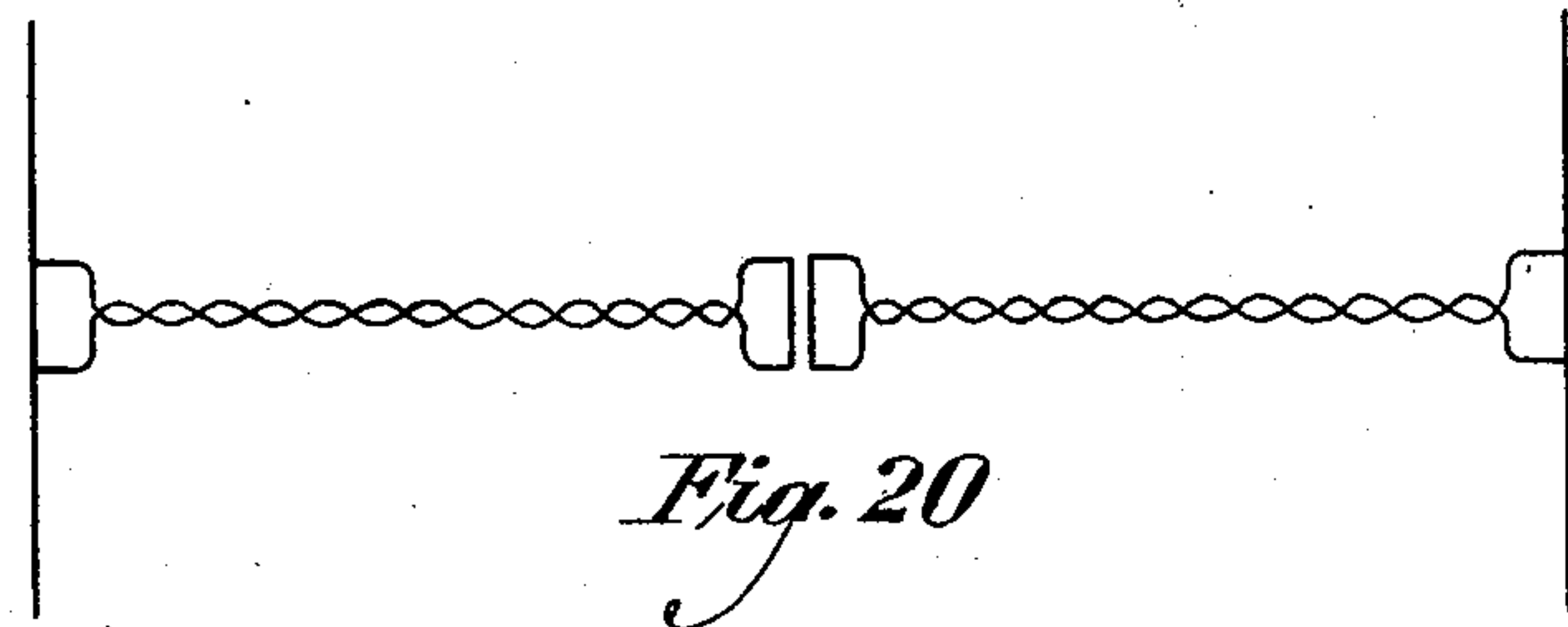


Fig. 20

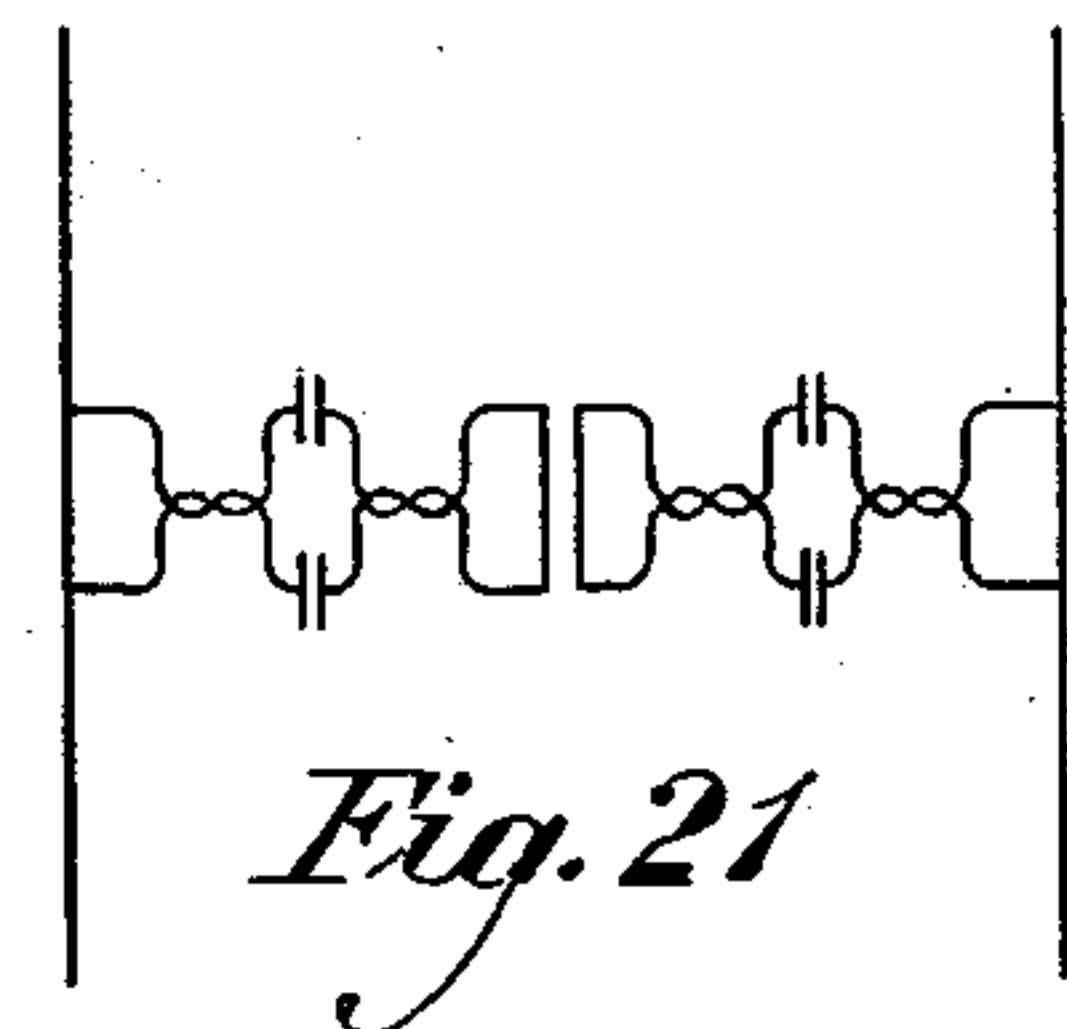


Fig. 21

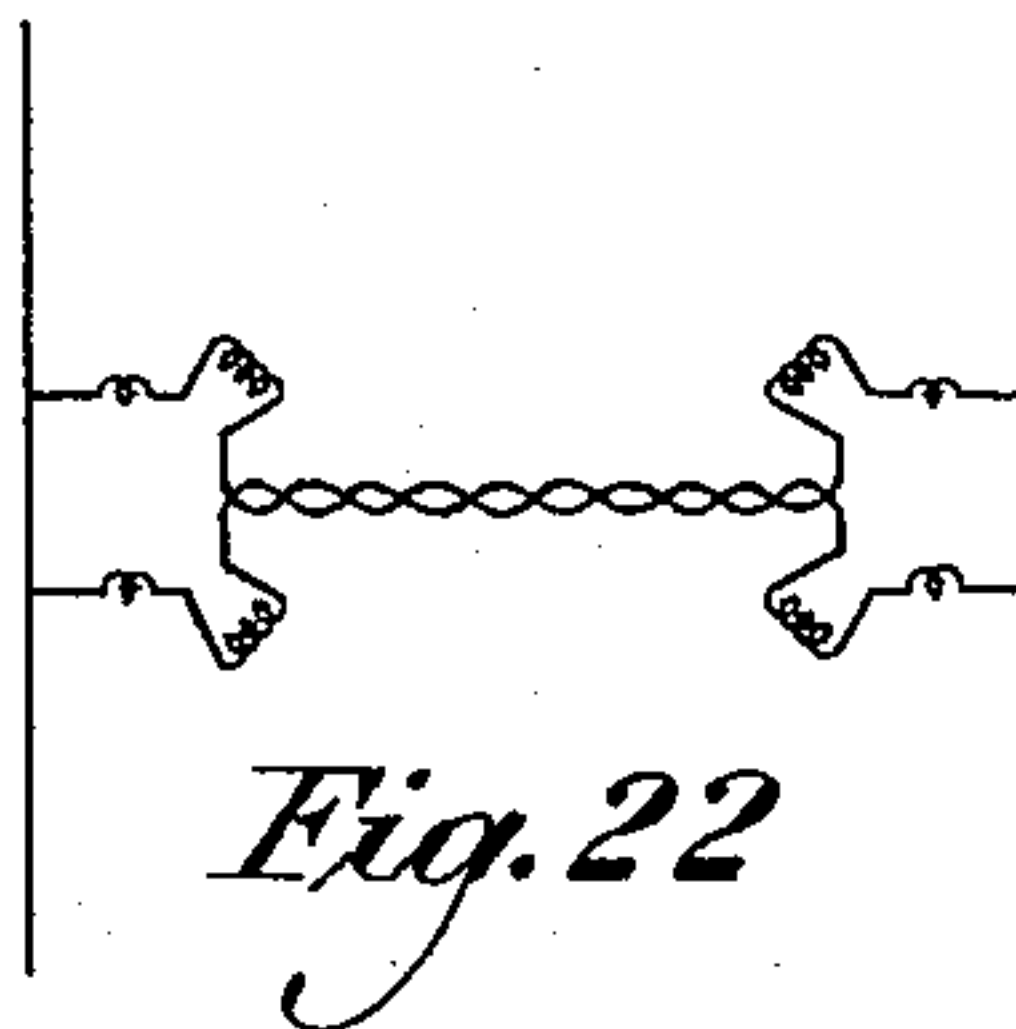


Fig. 22

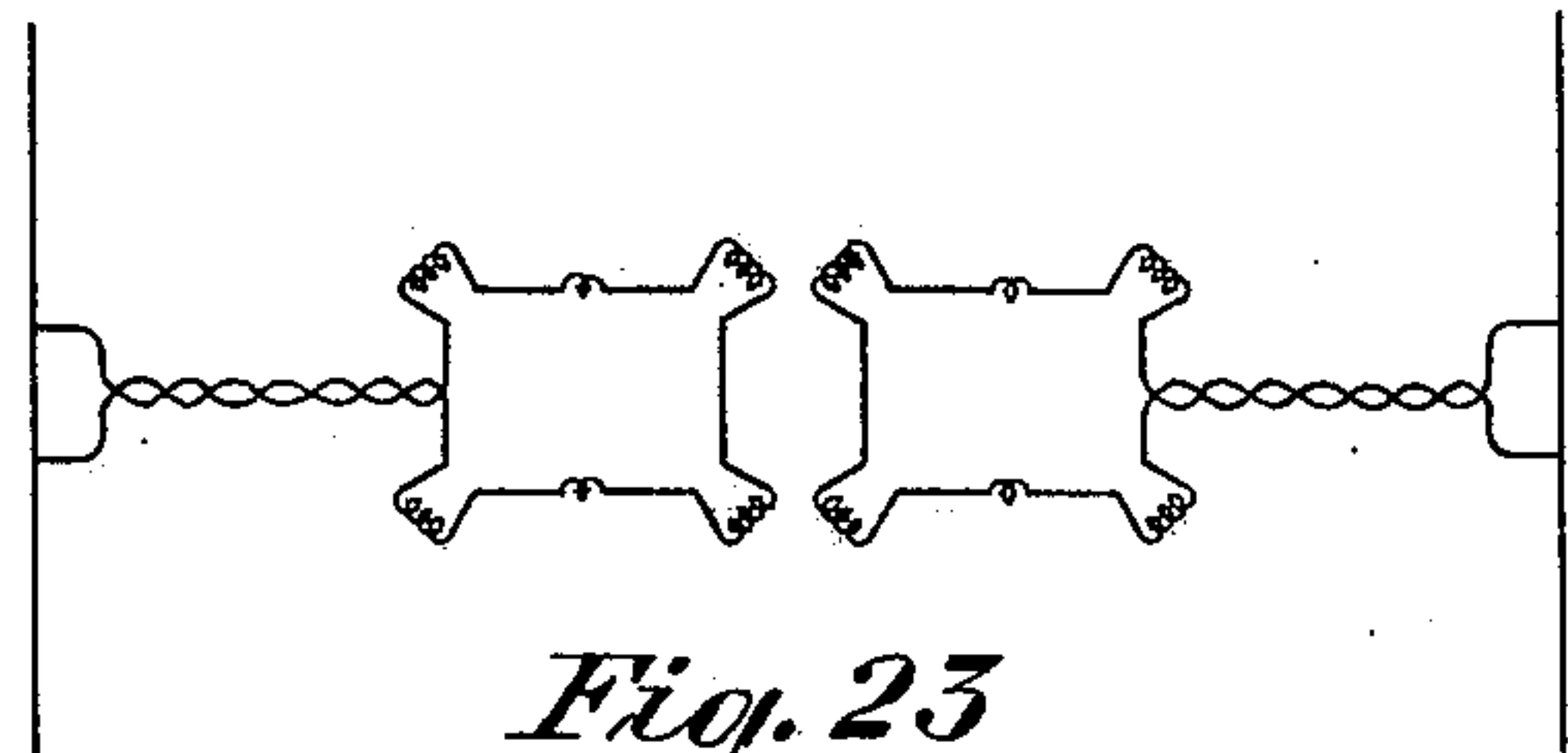


Fig. 23

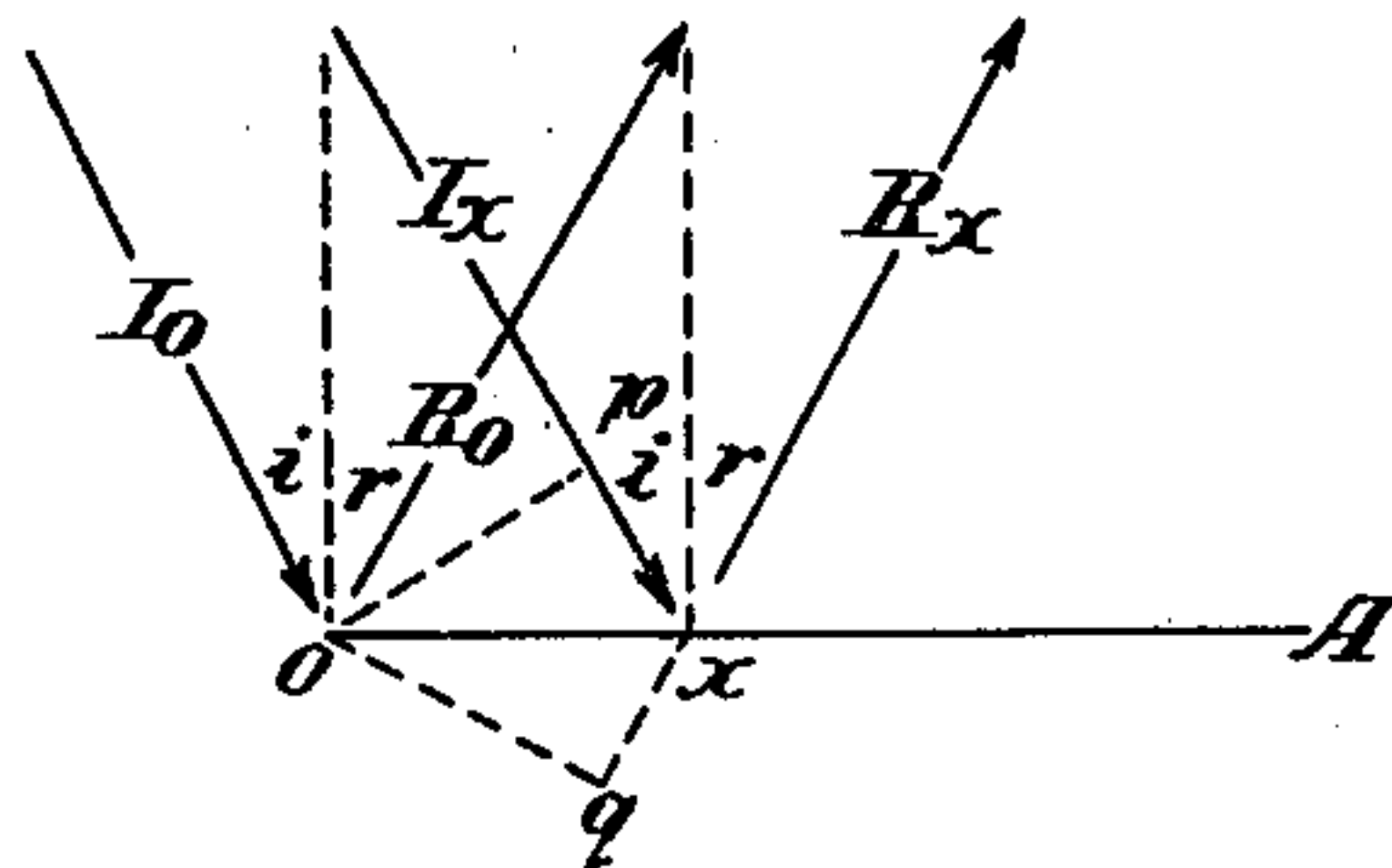


Fig. 24

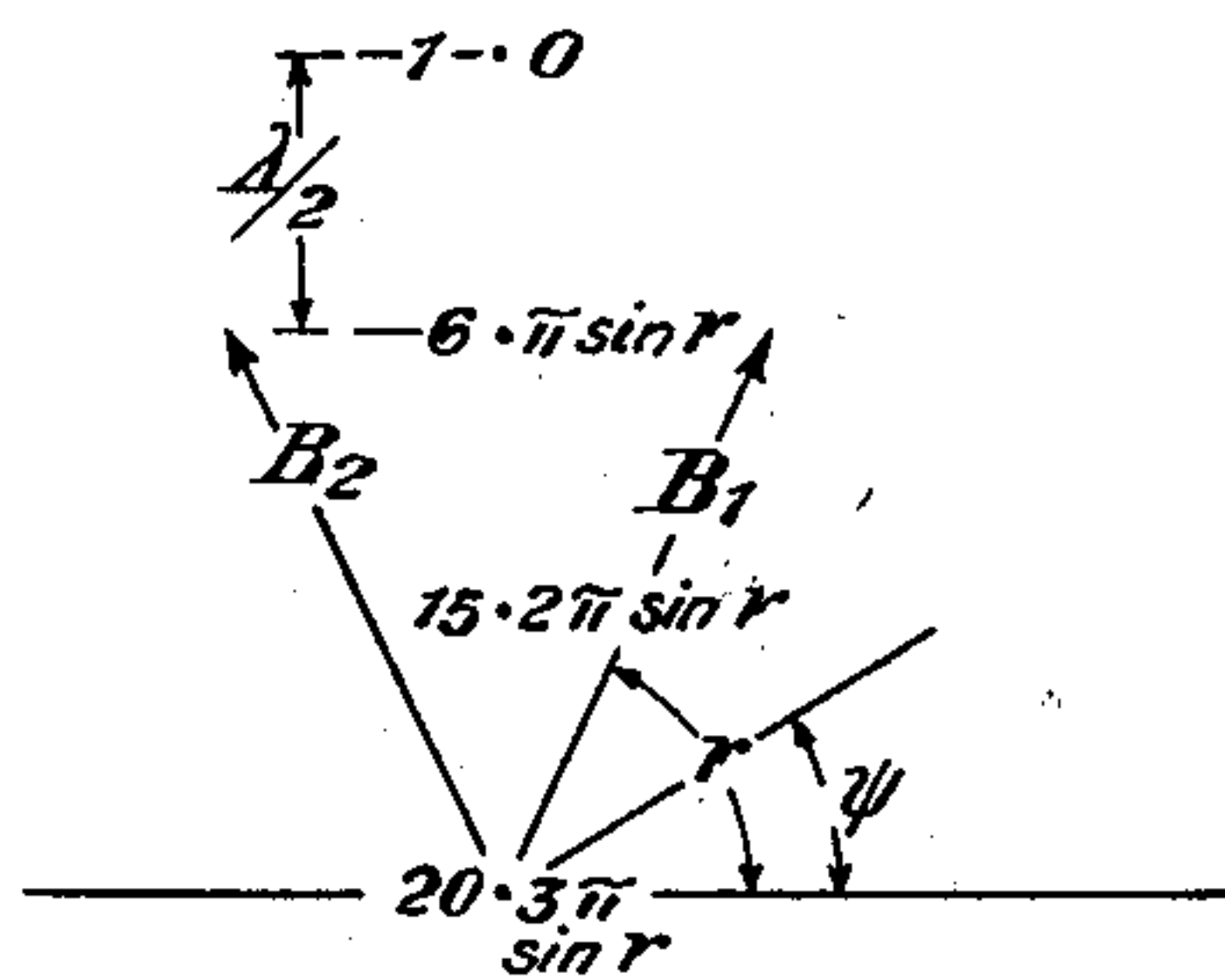


Fig. 25

INVENTOR
John Stone Stone
BY
J. E. York
ATTORNEY

Aug. 15, 1933.

J. S. STONE

1,922,115

ANTENNA ARRAY

Filed April 12, 1930

5 Sheets-Sheet 4

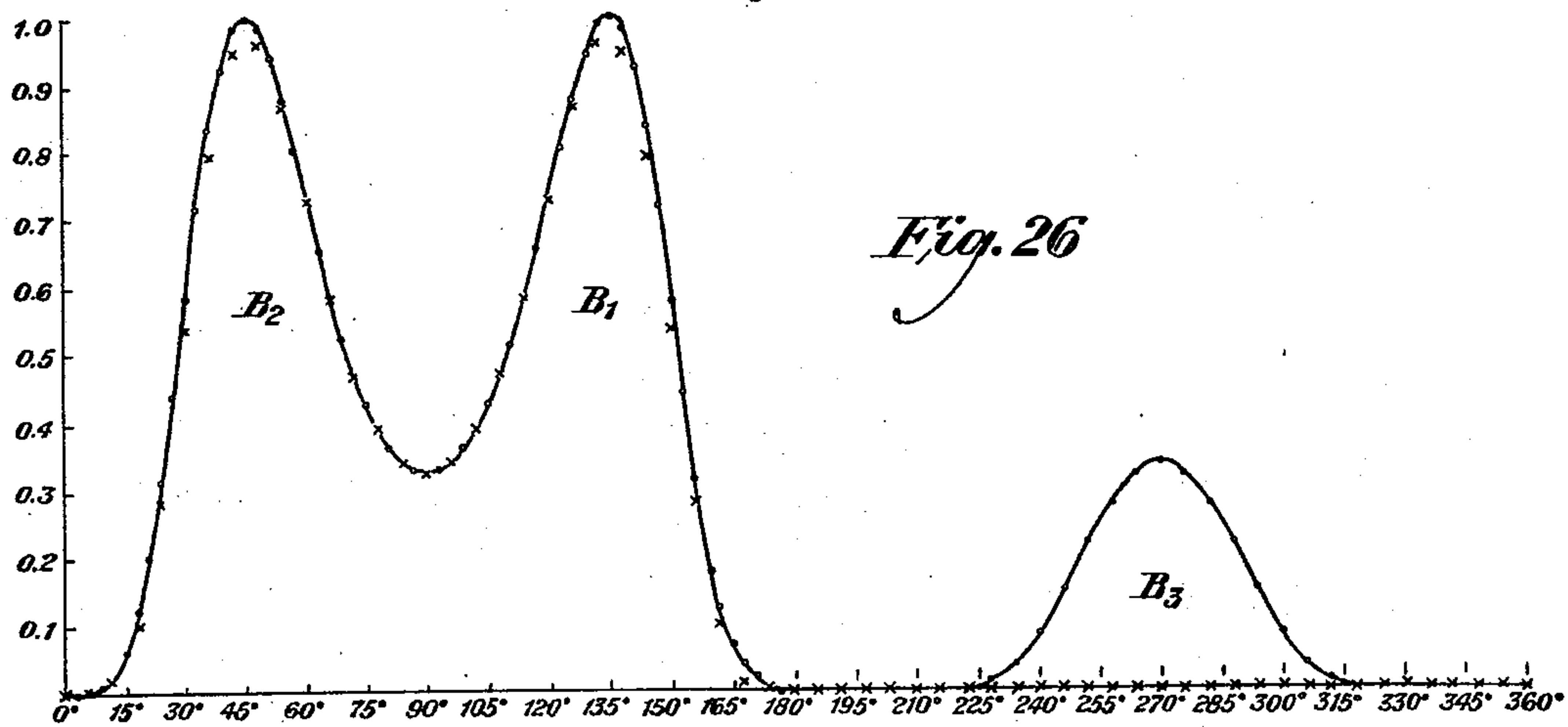


Fig. 26

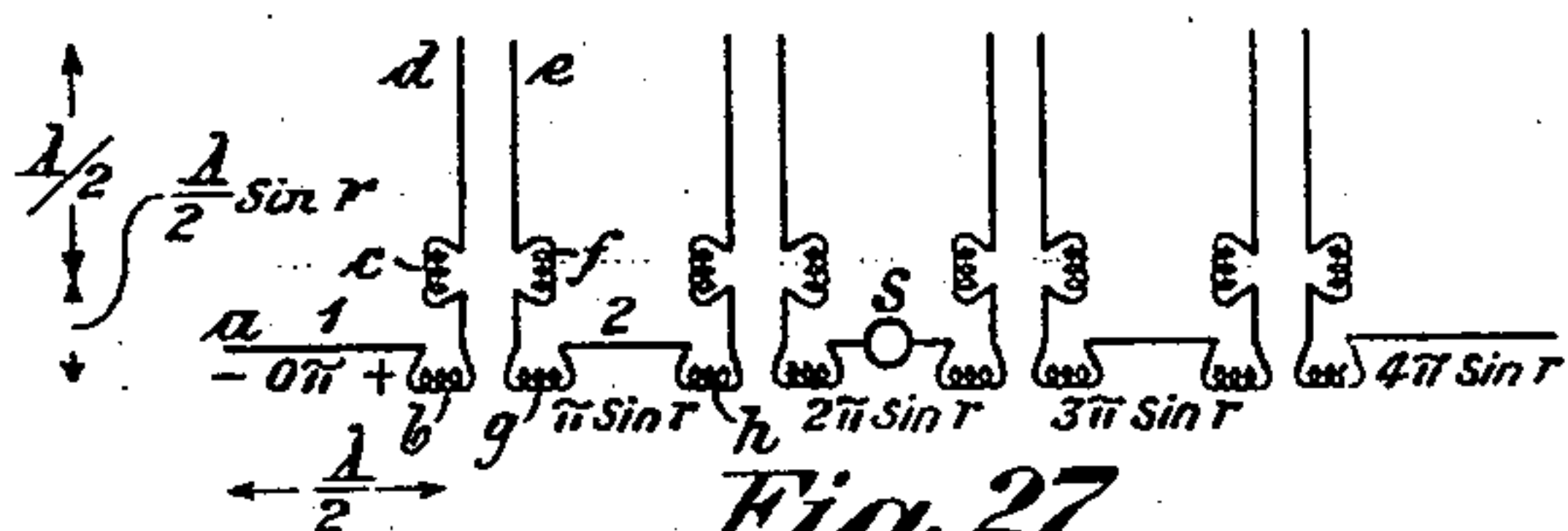


Fig. 27

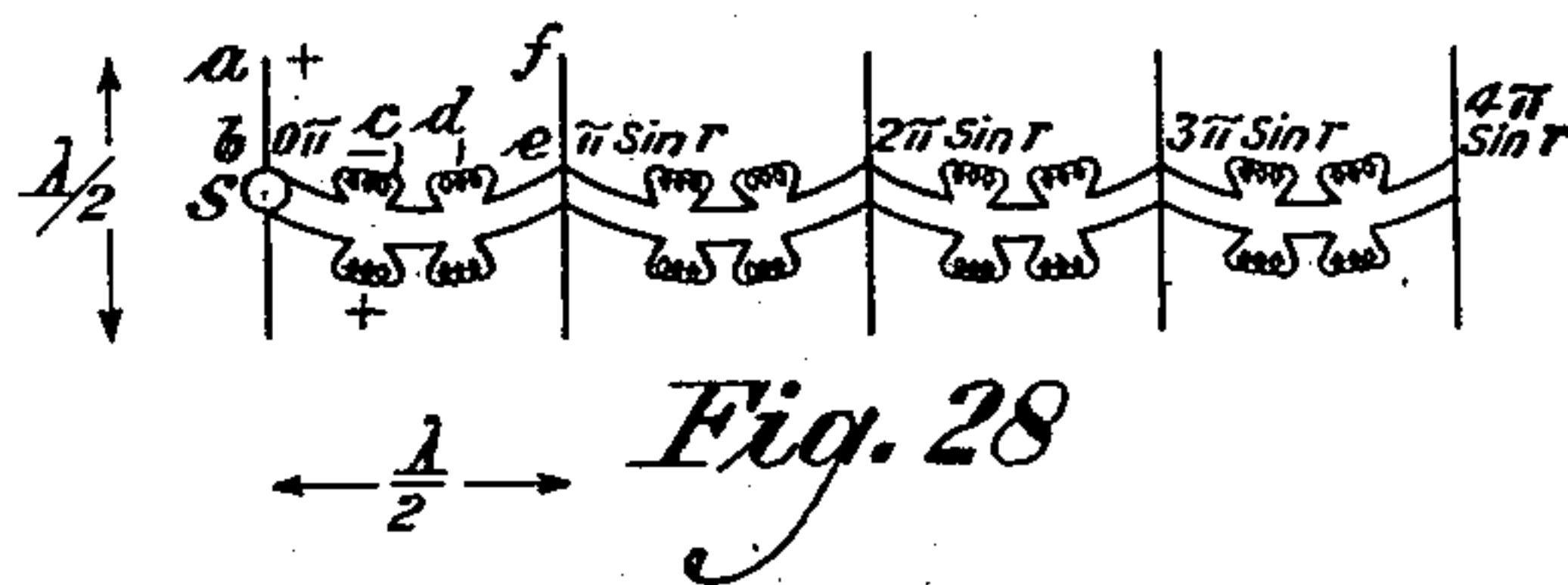


Fig. 28

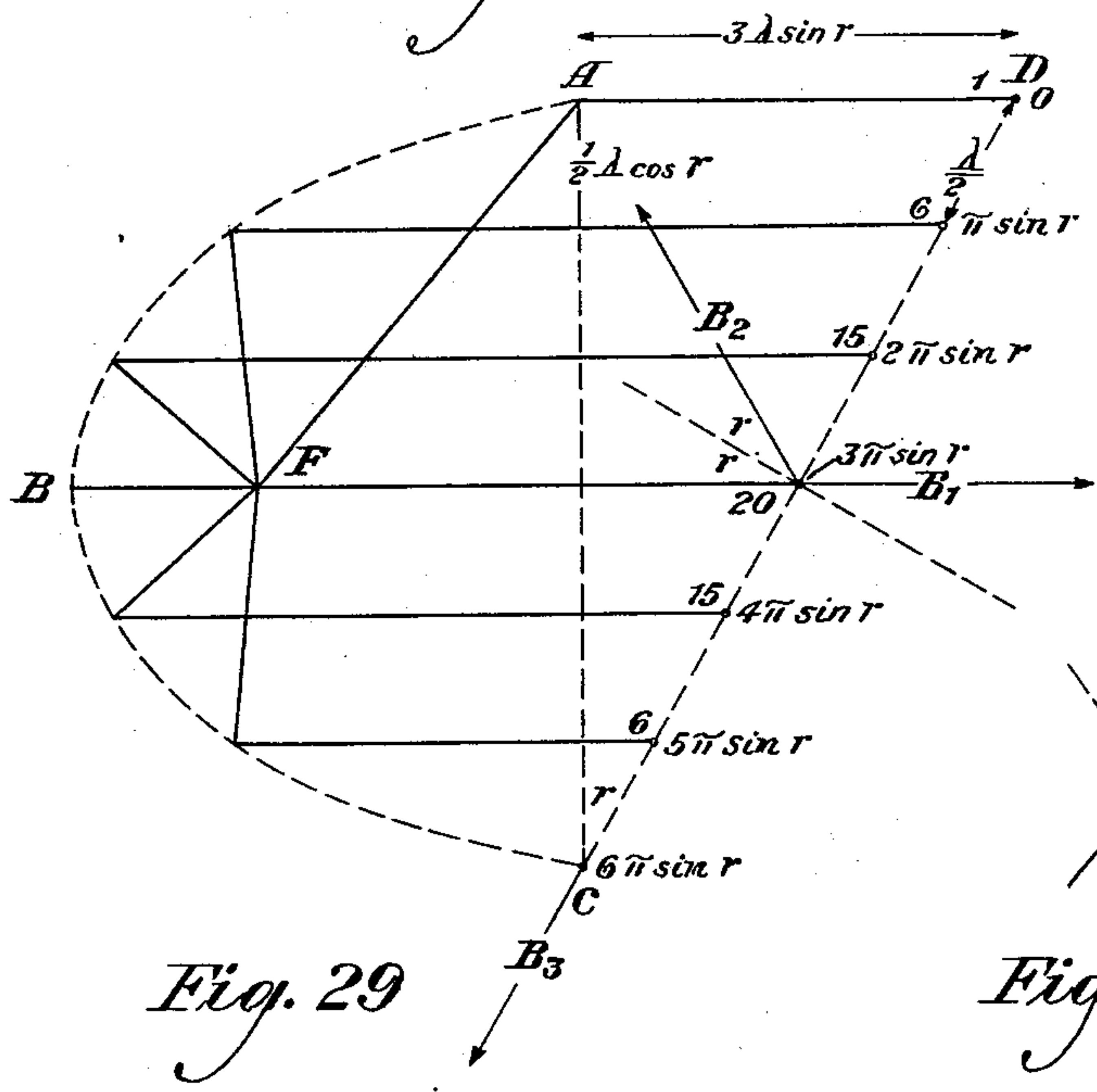


Fig. 29

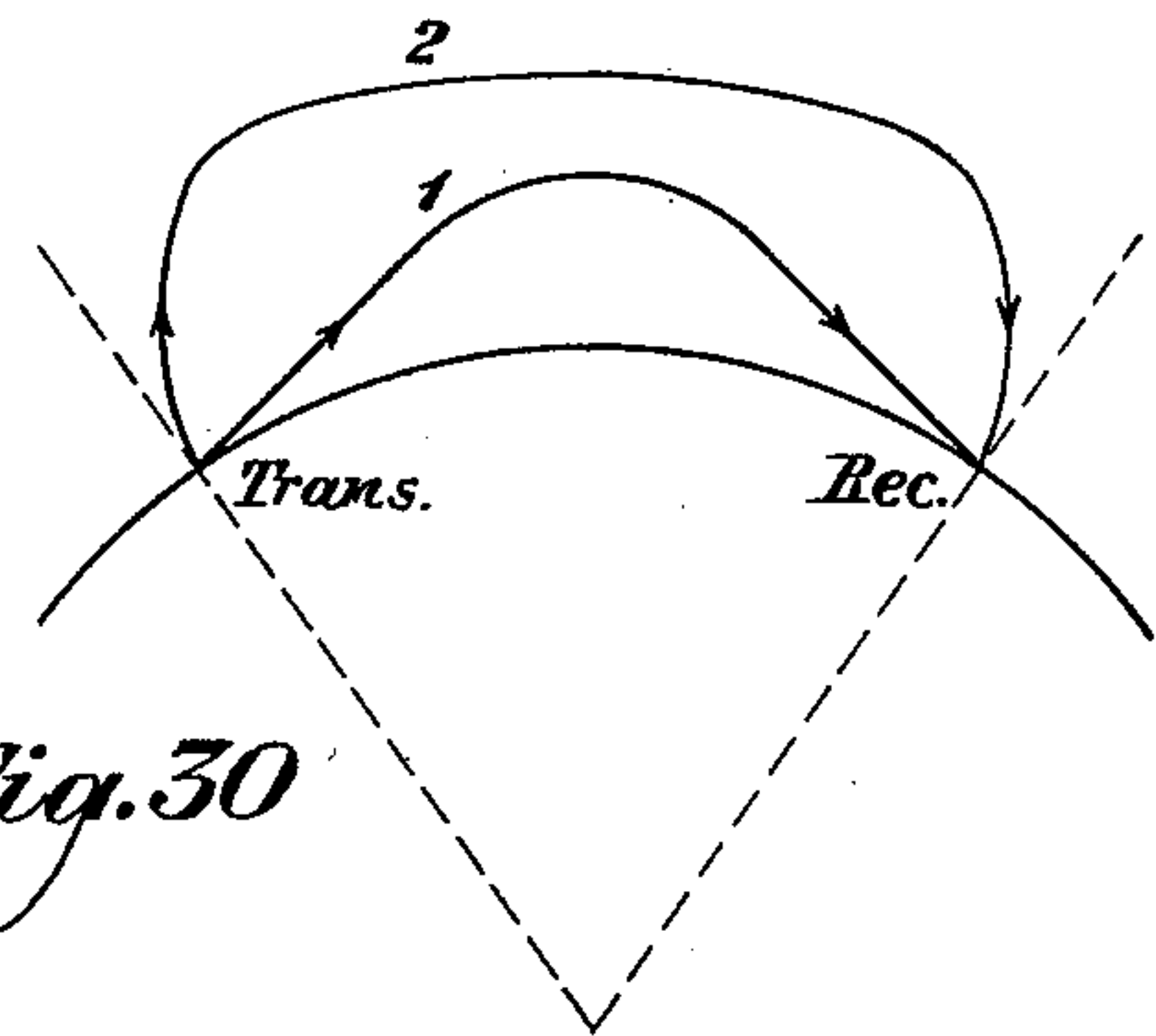


Fig. 30

INVENTOR
John Stone Stone
BY
J. E. York
ATTORNEY

Aug. 15, 1933.

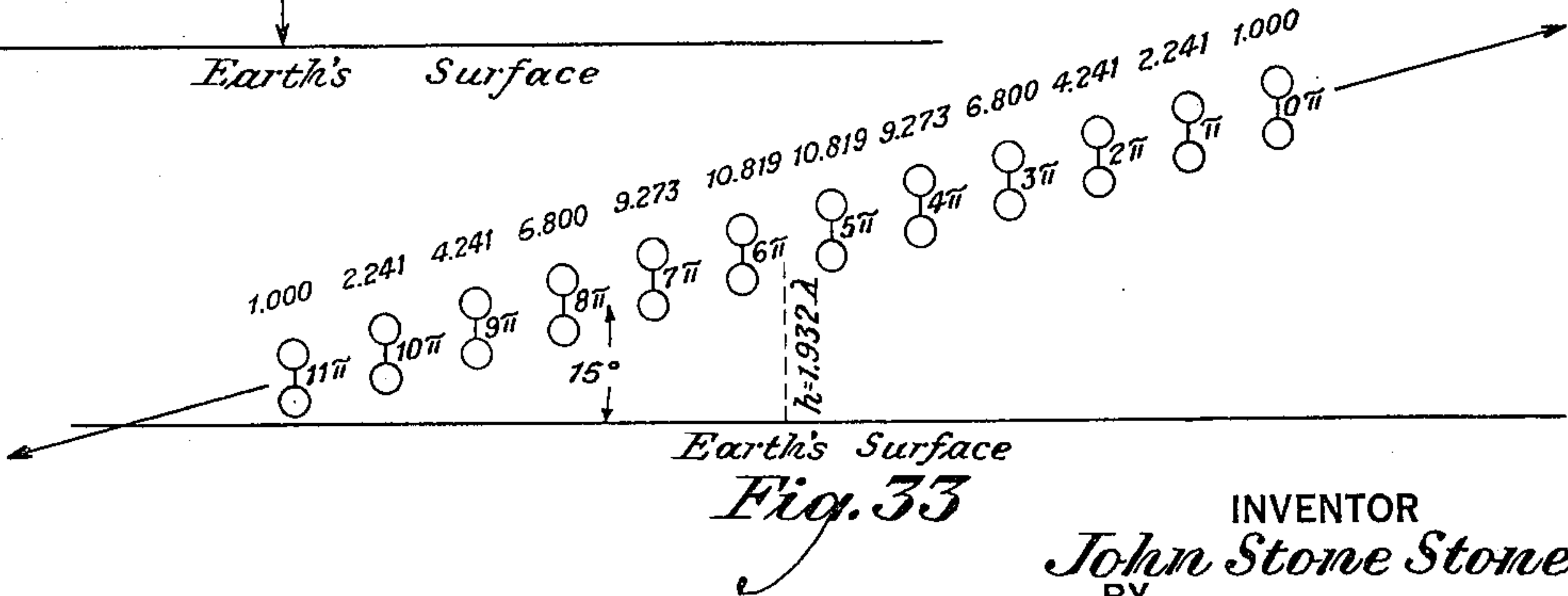
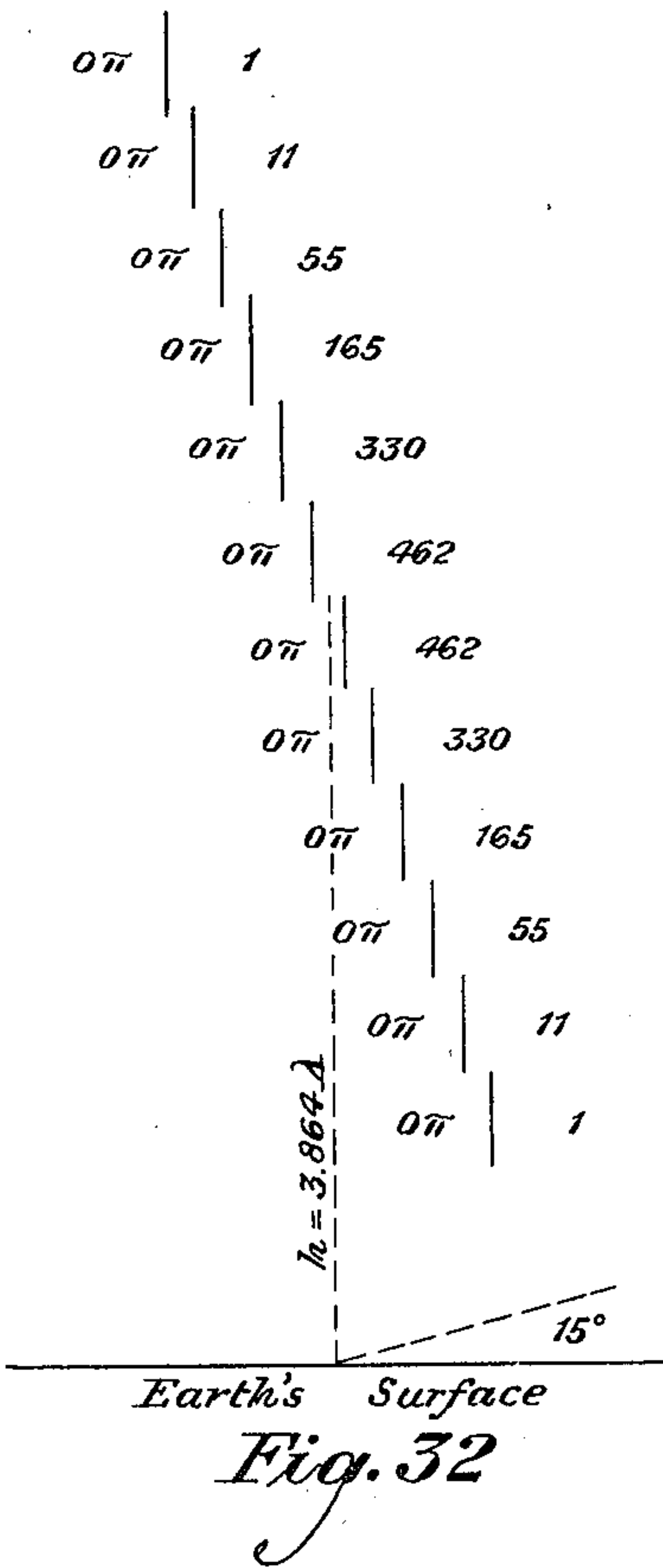
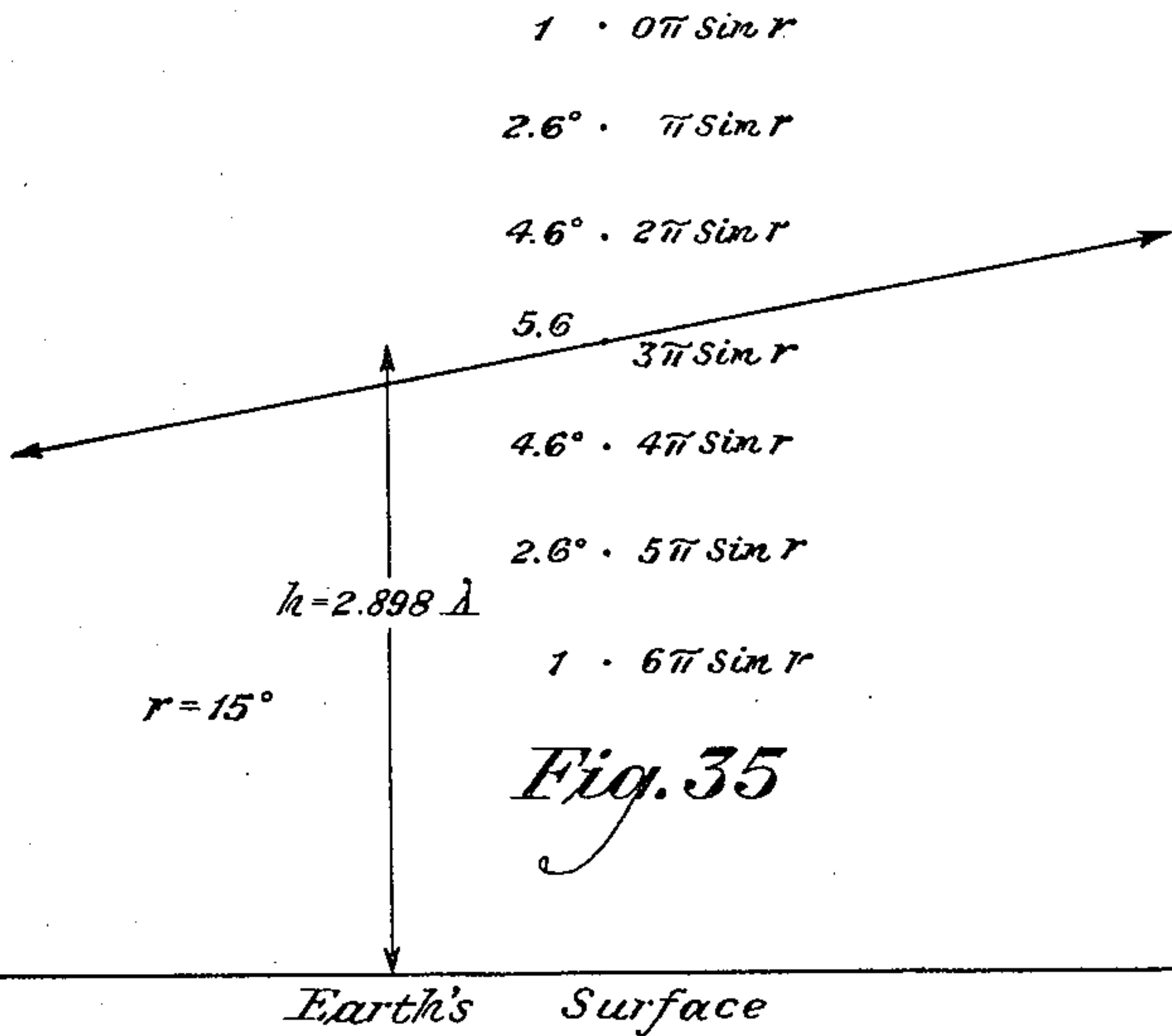
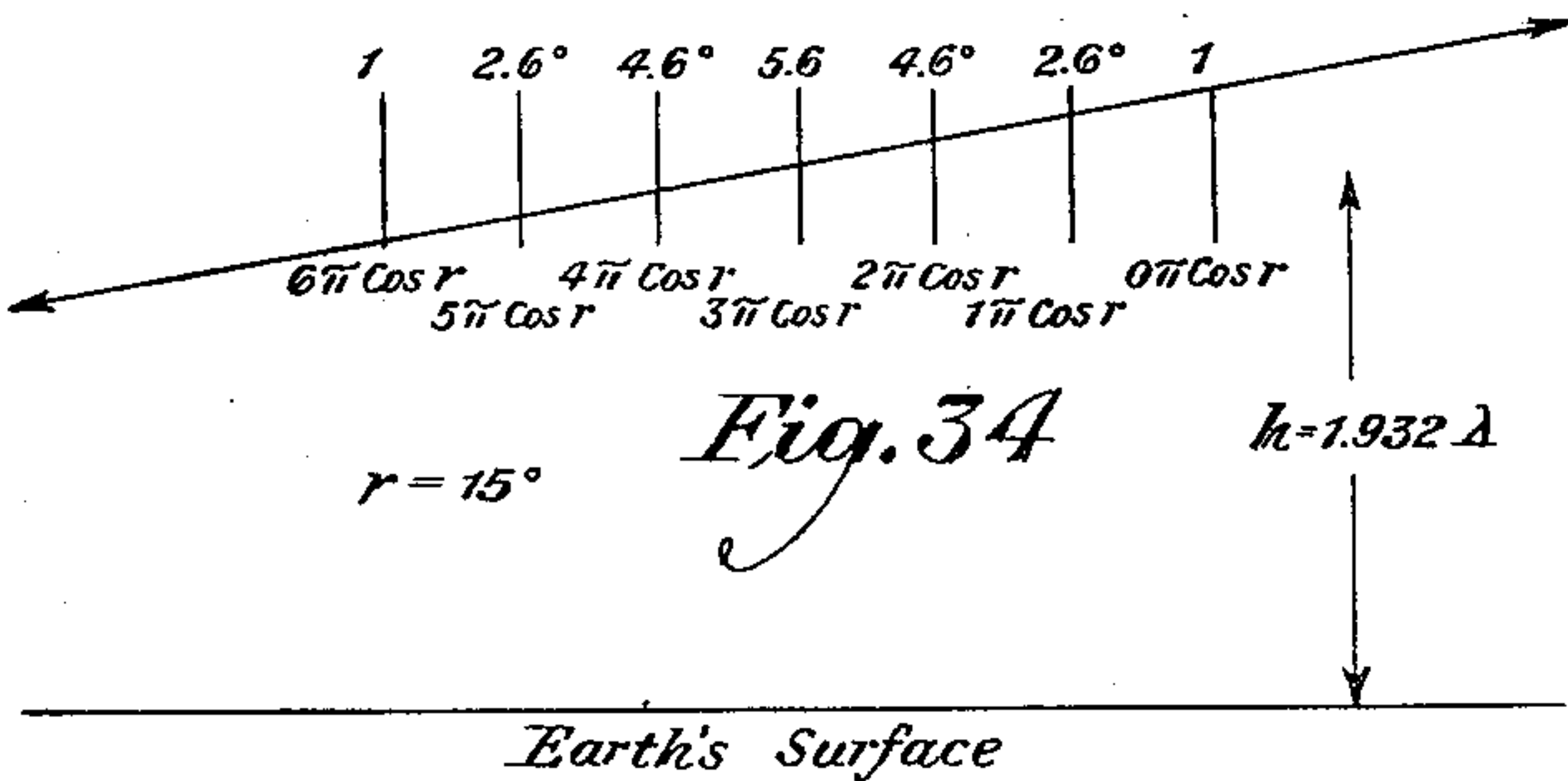
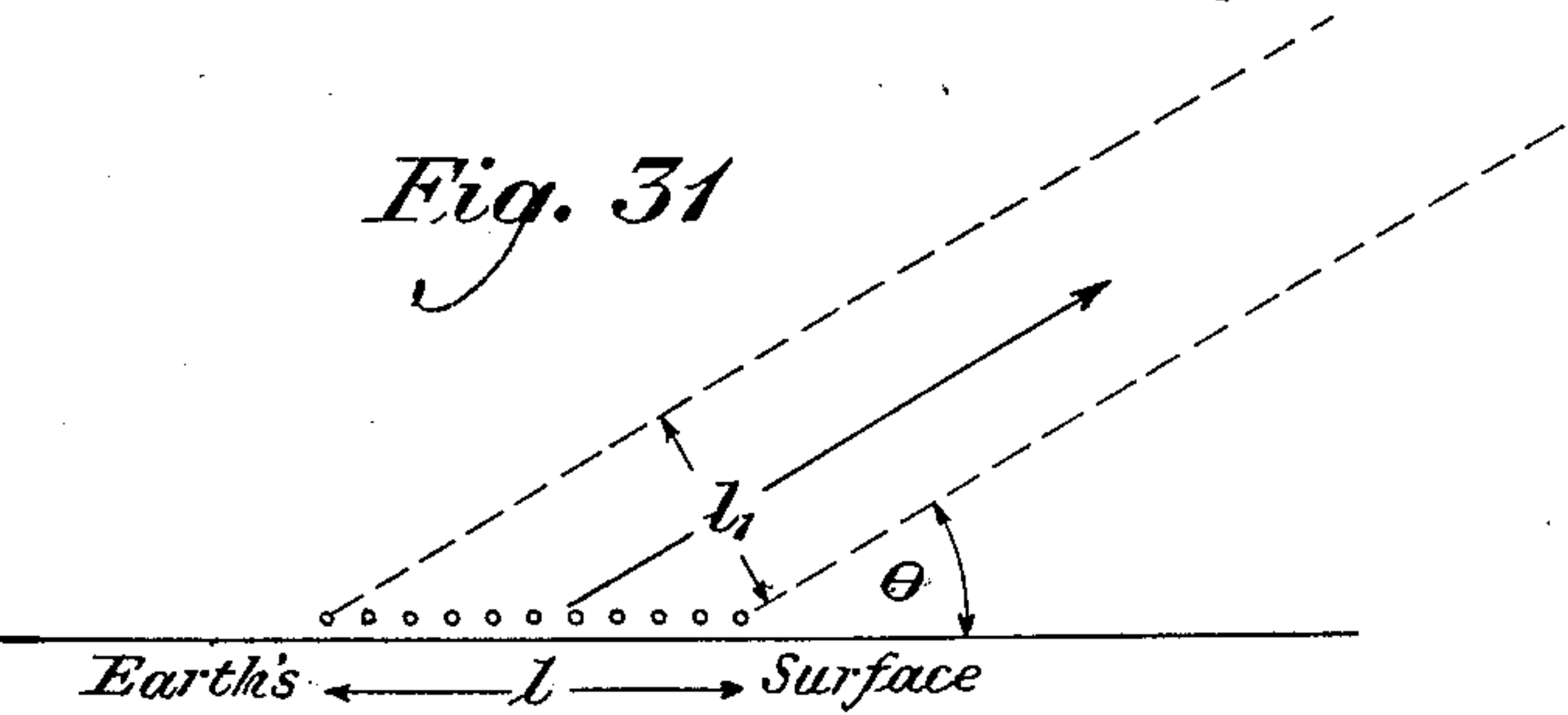
J. S. STONE

1,922,115

ANTENNA ARRAY

Filed April 12, 1930

5 Sheets-Sheet 5



INVENTOR
John Stone Stone
BY
g v h o k
ATTORNEY

UNITED STATES PATENT OFFICE

1,922,115

ANTENNA ARRAY

John Stone Stone, San Diego, Calif., assignor to
American Telephone and Telegraph Company,
a Corporation of New York

Application April 12, 1930. Serial No. 443,830

9 Claims. (Cl. 250—33)

The principal object of my invention is to provide a new and improved antenna array for the transmission or reception of radio communication waves. Another object is to provide a method for directionally selective transmission or reception of such waves. Another object is to provide for directional selectivity with suppression of beams of transmission or receptivity in particular undesired directions. Other objects have to do with securing the advantages of directional selectivity by means of antenna arrays of limited extent; distributing the power to the antennas of an array (or receiving the power therefrom) so as to coordinate their operation; and directing a beam of radiation at any suitable desired angle relatively to the axis of the array. All these objects and other objects of my invention will become apparent on consideration of a limited number of examples of practice of the invention which are presented in the following specification. It will be understood that this disclosure relates to these particular examples of the invention and that the scope of the invention is intended to be indicated in the appended claims.

Referring to the drawings, Figure 1 is a plan diagram illustrating an array tapered to form what I call a "parabolic image array"; Fig. 2 is a plan diagram of an array similarly tapered but in a manner differing in detail from Fig. 1; Fig. 3 is a plan diagram for a parabolic array with its power supply lines; Fig. 4 is a plan diagram of an array equivalent in its effect at a distance to the array of Fig. 3; Fig. 5 is a plan diagram of what I call a "binomial law array" with a maximum of intensity in a direction transverse to the extent of the array; Fig. 6 is a diagram of a similar array but with the maximum longitudinal to the extent of the array; Fig. 7 is a plan diagram of a two-dimensional or rectangular binomial law array; Fig. 8 is a curve diagram giving a comparison of results from curtailed and non-curtailed tapered arrays; Fig. 9 is a cardioid diagram for a certain antenna couplet employed in some embodiments of my invention; Fig. 10 is a diagram showing an array obtained by substitution of the couplet of Fig. 9 for each of the units of Fig. 1; Figs. 11 and 11a are plan diagrams for a triplet and a quadruplet, respectively, of properties similar to the couplet of Fig. 9; Fig. 12 is a plan diagram showing a system of power lines to energize a binomial law array for a transverse beam of radiation; Fig. 13 is a similar diagram

for a longitudinal beam; Figs. 14 and 15 are diagrams illustrating power distribution to the antennas of an array in sequence in a network; Figs. 16, 17, 18 and 19 are further diagrams similar to Figs. 14 and 15; Figs. 20, 21, 22 and 23 are diagrams showing conductive connections between antennas to neutralize their interaction through space; Fig. 24 is a diagram for phase relations for an oblique beam of transmission by analogy to reflection of light from a plane mirror; Fig. 25 is a diagram of a binomial law array with the antennas energized in proper phase relation to give an oblique beam; Fig. 26 is a curve diagram showing the intensity of radiation for Fig. 25 as it stands and also after one beam has been suppressed by substitution of the couplet of Figs. 9; Figs. 27 and 28 are diagrams for network power distribution in an array so as to give an oblique beam of radiation therefrom; Fig. 29 is a diagram showing power supply conductors adapted to give an oblique beam of radiation from an array; Fig. 30 is a diagrammatic elevation showing the course of wave trains in relation to the earth's surface and the "Heaviside layer"; Fig. 31 is a diagram illustrating an array disposed and adapted for sending a beam in an upwardly inclined direction relatively to the earth's surface; Fig. 32 is a diagrammatic elevation of an array with its axis of extent tilted in order to secure a tilted beam of radiation transverse thereto; Fig. 33 is a similar diagram except that in this case the beam is longitudinal to the extent of the array; Fig. 34 is a diagrammatic elevation of a horizontal array with which an upwardly inclined beam is secured by establishing suitable phase relations among the antennas; and Fig. 35 is a similar diagram except that the extent of the array is vertical instead of horizontal.

For convenience and definiteness of exposition, I shall confine the following description to antennas used for transmitting. It is well understood that to a great extent the principles of directional selectivity for transmission are applicable similarly for reception and this will be readily appreciated in the present connection.

Referring to Fig. 1, this shows a straight-line array of equally spaced antennas separated by the interval d , where $d \approx \lambda/4$. These antennas are intended to be operated all in the same phase and at intensities given by the law

$$I = 4a/(4a^2 + y^2) \quad (1)$$

where a is a constant and y is the distance from

the center of the array to the particular antenna for which the formula is employed. In the present example I assume that the number of antennas is an even number. It will readily be seen how to extend the principles here presented to an odd number. Let this even number be n . Then the intensity of the radiation in any direction given by the angle ψ is

$$I = 2 \sum_{m=1}^{m=2n-1} 4a \cos\left(\frac{m\pi d}{2} \cos\psi\right) / \left(4a^2 + \frac{m^2 d^2}{4}\right) \quad (2)$$

There are two equal maxima of transmission in the directions B_1 and B_2 .

Instead of spacing the antennas equally as in Fig. 1, they may be spaced unequally as in Fig. 2, making the density of their spacing according to the proportion given by Formula (1), and energizing the individual antennas alike. In the case of Fig. 2, Formula (2) is not applicable for it is based on a uniform interval d . The corresponding formula for Fig. 2 is

$$I = 2 \sum_{m=1}^{m=2n-1} \cos\left(\frac{4\pi}{2} a \tan \frac{m\theta_1}{4} \cos\psi\right) \quad (3)$$

where $\theta_1 = 4 \tan^{-1} \frac{d}{4a}$. In this case d stands for the minimum distance between consecutive antennas in the array.

In a modification of the parabolic image array, the individual antennas may be distributed otherwise than in a straight line, in which case they will not, in general, be energized all in the same phase. In Fig. 3, the antennas 1, 2, 3 and 4 lie along a dotted parabola AB with focus at F and they are energized by conductors that go from central station R to F and thence along straight lines, respectively, to the antennas. It is assumed that the speed of transmission in the conductors such as F1, F2, etc., is the same as in the ether. The maximum of transmission will be in the direction of the arrow or in the direction opposite thereto.

The antennas do not need to be located on a parabola but may be scattered irregularly as shown at $1a, 2a$, etc., in Fig. 4. Here the electrical lengths of the respective conductors may properly be made equal to the corresponding values in Fig. 3. Thus, $Fa1a = F1$, $Fa2a = F2$, etc.

The foregoing discussion relates to linear tapered parabolic image arrays. But a linear array may be tapered with similar effect according to a law which I call "the binomial law" because of its analogy to the coefficients in the expansion of a binomial to a power given by a positive integer. Thus, for example, in the plan Fig. 5, the dots represent antennas energized in the same phase and spaced $\lambda/2$, where λ is the wave length. In the array shown in Fig. 5 there are 7 antennas. The binomial expansion to the sixth power gives seven terms and the coefficients of these seven terms are 1, 6, 15, 20, 15, 6 and 1 in order; accordingly, these numbers represent the relative power magnitudes on the respective antennas of the array of Fig. 5. The symbol 0π opposite each antenna is meant to indicate, consistently with a notation that will be employed later, that the antennas are energized in the same phase.

For the array of Fig. 5 the intensity of radiation is according to the formula

$$I = 2^6 \cos^6\left(\frac{\pi}{2} \cos\psi\right) \quad (4)$$

It will readily be seen from Fig. 5 and the foregoing disclosure in connection therewith how to construct and energize a linear array according to the same principle for any number of antennas in the array.

In the linear antenna arrays that have been discussed thus far, the direction of maximum transmission is transverse to the extent of the array. I will now show how a binomial law array can be built up in which there will be two opposite maxima of transmission intensity along the axis of the array. Referring to Fig. 6, this shows a linear array of seven antennas spaced by the distance $\lambda/2$ and energized in intensity according to the binomial law. But in Fig. 6 they are not in the same phase but are alternately opposite in phase as indicated by the symbols $0, \pi, 2\pi, 3\pi$, etc. Here the maxima of transmission are in the direction of the arrows B_1 and B_2 and the intensity in any direction is given by the formula

$$R = 2^6 \sin^6\left(\frac{\pi}{2} \sin\psi\right) \quad (5)$$

Rectangular arrays can be built up from the linear arrays that have been disclosed above. In general, the method of procedure is to start with a linear tapered array and replace each of its units by a linear tapered array lying in the transverse direction and varying the total energy in each component linear array according to the law for the initial linear array whose units have been replaced.

A tapered rectangular array of a given number of units produces a beam which is less narrow than that from a tapered linear array of the same number of units whose axis of extension is perpendicular to the beam; on the other hand, such a tapered rectangular array gives a beam which is more narrow than that from a tapered linear array of an equal number of units whose axis of extension is parallel to the beam.

Tapered rectangular arrays give radiation which is restricted in two principal planes perpendicular to each other, instead of one principal plane as for tapered linear arrays. This double restriction of the radiation in the case of tapered rectangular arrays gives an economy of power in transmission and is of advantage in the case of receivers through the diminution of the effect of strays or static. The rectangular arrays have the advantage that they permit the use of a large number of antennas in a relatively restricted space.

A tapered rectangular array is shown in Fig. 7. This array may be conceived as being built up according to the following plan; first a linear array with the units in the same phase energized according to the binomial law is set up in a direction transverse to that of desired maximum transmission which is indicated by the arrows B_1 and B_2 . Then each unit of this linear array is replaced by a linear array extending transversely thereto and of the type in which the units are not in the same phase, and these last mentioned linear arrays, each as a whole, are energized according to the binomial law. This gives the result shown in Fig. 7 with the data indicated by the legends thereon. In gen-

eral for such a rectangular array the intensity or radiation in any direction in the equatorial plane of the oscillators will be given by the formula

$$I = 2^{N_1+N_2-2} \cos^{N_1-1} \left(\frac{\pi}{2} \cos \psi \right) \sin^{N_2-1} \left(\frac{\pi}{2} \sin \psi \right) \quad (6)$$

From the foregoing description showing how linear arrays can be compounded into rectangular arrays, it will readily be seen that one can set up a linear array and replace each unit by a rectangular array and thus get a three-dimensional array.

In any of the arrays heretofore considered, linear, rectangular or three-dimensional, it will be seen that the great bulk of the power of the array lies in the central units. For example, in Fig. 7, the antennas nearest the center of the array each carry 60 units of energy and those at the corners each carry only 1 unit. As the result of a study that I have made, I find that some of the marginally located low power units can be suppressed with only a negligible impairment of the directional selectivity, but with decided reduction of the physical extent of the antenna array. For example, in a simple tapered linear array of 11 oscillators, the two end oscillators may be omitted. This omission will introduce a fringe of radiation in a direction transverse to the maximum and amounting to about 0.2 per cent. of the intensity of the main beam, but with this slight impairment of directional selectivity goes a reduction of 18 per cent. in the physical extent of the array. Thus it will be seen that tapered arrays can be very much simplified if and when we can tolerate the presence of fringes whose intensities, relatively to that of the main beam of the array, are rather small. The more extensive the array, the greater the simplification which can be effected. Again, consider a tapered linear array of 21 oscillators. This can be cut down to 11 oscillators—that is, it can be cut down in physical extent nearly 50 per cent.—and yet the resulting fringes that will appear in directions transverse to the main beam will be less than 1.18 per cent. of that beam in their intensity.

Putting the proposition in slightly different form, if we limit ourselves to a certain number of unit antennas, we can get far better directional selectivity in a curtailed tapered array than in one not curtailed, provided we are willing to endure the presence of small transverse fringes. In Fig. 8, the intensity of radiation is plotted as a function of direction for two arrays of 12 units each. The least selective diagram (curve No. 1) is for a 12-unit complete tapered array. The more selective diagram (curve No. 2) is for a 26-unit array which by curtailment, has been brought down to 12 units. In the latter case there will be fringes of less than 1.463 per cent. Curve No. 3 is for a 51-unit tapered array not curtailed.

One advantage of the curtailed tapered arrays is the reduction in the disparity of intensity along the antennas. Thus in a complete array of 12 units, the binomial coefficients range from 1 up to 462. But in an array on the basis of 26 units curtailed to 12, the binomial coefficients range from 480,700 up to 5,200,300, a ratio of about 1 to 18.

In the arrays that have been considered thus far there have been two principal maxima of radiation in opposite directions. In general, a beam of radiation in a particular direction can be suppressed by substituting for each unit of

the array a couplet, or other small group, as will now be pointed out more particularly.

Consider the couplet or pair of oscillators each marked 1 in Fig. 9. If these are excited in equal intensity and in the phase relation and wave length relation indicated in the drawings, they will give a polar diagram of radiation intensity according to the cardioid curve of the drawings, and it will be noticed that in one particular direction radiation is nil.

Now in any array such, for example, as the array of Fig. 1, if it is desired to suppress a beam of radiation in a particular direction, such as the beam B_2 of Fig. 1, then for each unit of Fig. 1 substitute the couplet of Fig. 9 with its direction of null radiation in the direction of desired suppression. This gives Fig. 10 instead of Fig. 1. The radiation as a function of direction in the equatorial plane of the oscillators is given by the formula.

$$I = 4 \sum_{m=1}^{m=2n-1} \frac{16a}{16a^2 - m^2d^2} \cos \left(\frac{m\pi d}{\lambda} \cos \psi \right) \cos \frac{\pi}{4} (\sin \psi - 1) \quad (7)$$

The foregoing formula is obtained by multiplying the formula for Fig. 1 by that for Fig. 9, and this is the general method.

Instead of the couplet of Fig. 9 being substituted for each unit oscillator of an array, the triplet of Fig. 11 may be substituted or the quadruplet of Fig. 11a. Each of these aggregations gives an effect in substitution similar to the couplet of Fig. 9. With the legends on Fig. 11 and 11a, further description of their structure will not be necessary. It should be noted that by the substitution in an array of antennas of a couplet or triplet or quadruplet to suppress a beam in a particular direction, not only is this suppression accomplished but the remaining beam is narrowed somewhat.

The advantage of this substitution lies not alone in economizing the radiated energy but in reducing the radiation in directions in which it might be regarded as a serious interference with receiving stations lying in those directions.

I have already touched upon the question of distributing the power to the respective antennas of an array; this was in connection with Figs. 3 and 4 in which the power conductors between the central receiving station R and the various antennas were each given the proper electrical length. Another way to distribute the power with certain advantages will now be disclosed. Suppose, for example, that we have the tapered linear array whose eight antenna units are indicated by the numerals 1, 7, 21, 35, 35, 21, 7 and 1 in Fig. 12. These antennas may be considered to lie across the axis of a parabola whose focus is to be F. Accordingly, power lines connecting F to these antennas will have the proper lengths if they are drawn as shown in Fig. 12 and have equal phase speeds; this will readily be appreciated in accordance with the geometrical properties of the parabola. Of course, this is only one way, but a very symmetrical way, of representing the power circuits. It may be departed from, provided that the lengths and phase speeds of the several separate supply circuits are maintained the same as here represented. The foregoing disclosure in connection with Fig. 12 is for a case in which the beam of radiation is at a right

angle to the direction of extent of the linear array. For a case in which the beam of radiation is along the direction of extent of the linear array, the distribution of power between the central station S and the respective antennas may be as shown in Fig. 13. Each line from S to an antenna unit represents a two-wire circuit. The difference between the lengths of the circuits leading to any two consecutive antenna units is equal to the distance between them; this is based on the assumption that the phase speed in the supply circuits does not differ materially from the velocity of transmission in the ether.

The power and phase distribution to the antenna units may be effected without the necessity of having a separate lead extending from the source to each of the oscillators or antennas of the array. This may be effected by making the antennas part of a network so constructed and connected that each antenna oscillates resonantly to the centrally impressed force and at the intensity and phase proper to its position in the array. In such a network the conductors whose functions are merely to distribute power should be non-radiative; this condition will be secured by disposing them in pairs, the members of each pair being close together compared to a quarter-wave-length and so that the potentials and currents in the two conductors of such a pair shall be substantially equal in intensity and opposite in phase, or else, by arranging that the currents and potentials in the distributing conductors shall be of such small magnitude that they do not give a disturbing radiation.

In the networks according to the present plan, the radiative linear oscillators are, of course, in resonance with the forces impressed on the system, and certain other parts of the network are likewise in resonance by virtue of their lengths. The resonant sections are marked off by the introduction of inductance coils at the points of current nodes and potential loops. For example, in Fig. 14 the linear oscillators are arranged end to end and energized from a central source S. The connecting conductors between adjacent ends are of such length that the phase relations are established as indicated in the drawings. There is a current node and a potential loop at each of the inductances. The extended linear oscillators are radiative but the loop circuits between them are practically non-radiative in accordance with the principles set forth above.

It may be desirable to decrease or increase the wave speed in the power supply conductors. A reduction of this speed may be accomplished by the use of small serially connected inductances and an increase of the speed may be accomplished by the use of small serially connected capacities. The use of serially connected inductances as here suggested is shown in Fig. 15.

Other examples of power distribution systems for antennas according to the foregoing principles are shown in Figs. 16, 17 and 18. The connecting conductors in Fig. 16 are open-ended and therefore, of double the length of the closed-ended connecting conductors of Fig. 14. In each of Figs. 14 and 16 to 18 the axis of the beam of radiation is perpendicular to the axis of extension of the array. When the axis of the beam of radiation is coincident with the axis of extension of the array, a different system of distribution will be employed which is illus-

trated in Fig. 19, and in Fig. 15 mentioned heretofore.

In the system shown in Figs. 14 and 16, likewise in Fig. 27, the supply branches are non-resonant; therefore the currents and potentials in them are too small to produce effective radiation. On the other hand, in the systems shown in Figs. 17, 18 and 19 the supply branches are resonant but are disposed in pairs of adjacent conductors so that the currents and potentials are approximately equal in intensity and opposite in phase, and therefore they give rise to no considerable radiation.

To this point the matter of interaction among the oscillators of an array has been ignored. Clearly, however, each oscillator of an array will be under the influence of the field force of all the other oscillators of the array and will, therefore, be subject to electromotive forces other than those conveyed to it by the power and phase distributing circuits. Unless we annul these interactions the adjustment of the power and phase determining arrangements as previously described can only be regarded as a first approximate adjustment, and the final adjustment must in every instance be empirical. This final adjustment will be in the nature of a correction of the theoretical adjustment of power and phase previously described.

The interaction of the oscillators of an array for any given intensity and phase distribution is, of course, capable of predetermination. But the result will generally be complex and a better procedure is simply to measure the interaction, which is easily done, and make correction accordingly. The correction will in all instances consist in a slight lengthening or shortening of the electrical length of a segment or segments of the power and phase distributing circuits. It is to be remembered that the electrical length of a conductor may be increased by the addition of a small inductance at its electrical center and decreased by the addition of a large serial capacity at this center.

Another direct method for meeting the problem presented by the natural interaction of the oscillators of an array is to neutralize the electromotive forces due to this natural interaction by the introduction of couplings between them. For example, suppose the two oscillators of Fig. 20 are sufficiently far apart to insure that the time required for radiation to pass from one to the other shall be substantially their distance divided by the velocity of transmission in the ether; then the coupling illustrated in Fig. 20 will be of the type proper to suppress the effect of the direct interaction between them. It is here assumed that the wave speed in the two-wire circuit connecting the oscillators is substantially that of radiation through the ether.

When the oscillators are nearer together than in the case of Fig. 20, it may be necessary to enhance the phase speed in the coupling circuit by the use of condensers, as shown in Fig. 21. On the other hand, Figs. 22 and 23 show how, by the use of inductances, the effective length of the coupling circuit may be shortened.

We have considered linear arrays and other arrays made up of linear arrays in all of which the radiation is either transverse to the linear extent of the array or longitudinal thereto. It will now be shown how to secure a beam of maximum radiation in a direction oblique to the extent of a linear array. A theory of ob-

lique beams may be developed readily by consideration of the optical analog. In Fig. 24 suppose that OA is a plane mirror viewed edgewise and that incident light represented by the rays I_0 and I_x is reflected as represented by the arrays R_0 and R_x . According to the electromagnetic theory of light, the incident light sets up an electrodynamic condition in the elemental particles in the surface of the mirror, and these elements acting as oscillators, generate and emit the reflected light. Therefore, without thinking of the incident light, we can think of the reflected light as due to elementary oscillators lying in the surface of the mirror and operating in such phase relation that they emit a beam of light in the direction R_0-R_x . Knowing the angle of reflection and the dimensions, the phase relation between the points 0 and x can easily be deduced.

Without further pursuing the illustrative case of light from a mirror, let us turn to the case of a linear antenna array which may be taken to be analogous to the element OA of the light-reflecting mirror shown in Fig. 24. In Fig. 25 we have a linear array of antennas energized according to the binomial taper law and it is desired that they shall emit a beam of maximum radiation in the direction of the arrow B_1 whose angle with the normal is r . Accordingly, the phase relation in the antenna units will be given in angular measure by the expression $0, \pi \sin r, 2\pi \sin r, 3\pi \sin r$, etc.

In this case the formula for the intensity of the radiation in any direction in the equatorial plane of the oscillators according to the value of the angle ψ will be

$$I = 2^{N-1} \cos^{N-1} \left(\frac{\pi}{2} \sin \psi - \frac{\pi}{2} \sin r \right) \quad (8)$$

From this formula it will be seen that there are two other principal maximum directions as shown in Fig. 25 by the arrows B_2 and B_3 . Radiation in the direction of the arrow B_3 can readily be annulled by substituting the couplet of Fig. 9 for each unit of Fig. 25 as heretofore explained in connection with Fig. 9. Or, the triplet or Fig. 11 or the quadruplet of Fig. 11a may be substituted for each unit of Fig. 25.

By varying or adjusting the phase difference between consecutive unit antennas of the array of Fig. 25, and desired value can be given to the angle r which measures or indicates the direction of radiation in the beam B_1 . Fig. 26 is a diagram showing the intensity of radiation in all horizontal directions for the array of Fig. 25, assuming that the angle r has the value 45 degrees. When the couplet of Fig. 9 is substituted for each of the unit antennas of Fig. 25, the curve is changed from its course through the little circles to the course indicated by the little crosses. It will be seen that the change is very slight for the beams in the directions B_1 and B_2 , but that the beam B_3 is almost completely annulled.

This method of getting a beam of maximum radiation at a desired angle by adjusting the phase relation among the component unit antennas may be practiced in a wide variety of situations. Supplementing the foregoing discussion in connection with Figs. 14 to 19, Figs. 27 and 28 illustrate a case of power distribution for an array of five linear oscillators so as to get the beams of radiation with their axes oblique to the corresponding axes of the arrays. This is accomplished in Fig. 27 by making the

connecting circuits longer (by the segments such as

$$bc = \frac{\lambda}{2}$$

$\sin r$) than in Fig. 16. Likewise in Fig. 28 the connecting circuits are made longer by separating the intermediate inductance coils each into two parts and interposing a length cd of conductor pair between them; in this respect Fig. 28 may be contrasted with Fig. 19.

Corresponding to the discussion of dimensions for the connecting circuits as presented in connection with Fig. 12, if it is desired to make the radiation oblique, we may proceed as in Fig. 29 in which I construct a parabola whose axis is the direction line B_1 through the central antenna of the array. From the focus F of this parabola the lines are drawn as shown and they will then have the proper lengths as power conductors from F as central station to give the maximum of radiation in the desired direction. Having obtained these lengths from Fig. 29, we may employ them in a system which departs geometrically from Fig. 29.

As already pointed out and as is readily apparent, with a fixed array of antennas the angle of the beam of maximum radiation can be adjusted by adjusting the phase difference between consecutive antennas. I will point out a case in which this may be especially useful. According to common opinion "fading" is due to interference at the receiving station of waves that have been reflected or refracted differently in relation to the "Heaviside" layer. For example, the transmission may be over two courses as shown in Fig. 30 and the waves over the path designated 1 in that figure may be interfered with and annulled at the receiver by those over the path designated 2. Changing conditions in the Heaviside layer, such as variations in the height of its lower bounding surface, cause the zone of interference on the earth's surface to vary, and this gives the well-known intermittent character of fading.

In Fig. 31 a horizontal linear array is shown with adjustment of the phase relations according to the principles heretofore discussed, so that a maximum beam of radiation will be given in the direction indicated. Comparing Fig. 31 with Fig. 30, it will be seen that the radiation from the transmitting station will be almost entirely suppressed along the path 2 of Fig. 30, and thus, in great measure, fading at the receiving station will be obviated. Due to changing conditions in the Heaviside layer, the angle of elevation of the maximum beam in Fig. 31 may require adjustment from time to time. The operator at the transmitting station will vary this angle until the operator at the receiving station reports to him that the optimum has been secured. Or, if Fig. 31 represents a receiving station, the operator there will vary the angle until he gets the optimum.

Another case in which an adjusted inclination of the axis of the beam to the extent of the antenna array may be desirable is when wave lengths are used of the same order of magnitude as the ocean waves or swells. It is then advantageous to direct the radio waves at an angle of about 15 degrees, or a little less, to the earth's surface. Evidently this can be done in the manner explained above for Fig. 31. Another way in which an upward tilt of 15 degrees can be secured is shown in Fig. 32. Here we

have a tapered linear array of 12 oscillators whose axis of extension is inclined 15 degrees from the normal to the earth's surface. Of course we must reckon with the reflection from the earth's surface which will produce an interference pattern. Therefore the height of the midpoint of the array in relation to the wave length must be such that in the desired direction there will be reenforcement of the direct waves by the reflected waves. It can readily be shown that in Fig. 32 if the height of the central point of the array is made 3.864 times the wave length, this insures that the direct beam from the array and the reflected beam from the earth's surface shall reinforce each other at the angle of 15 degrees.

Again, in the case of having the beam of maximum transmission along the axis of the array, the construction may be shown as in Fig. 33, which represents a curtailed tapered array. This is on the basis of 26 antennas cut down to 12. The binomial coefficients for 26 terms run 1, 25, 300, 2300, 12650, 53130, 177100, 480700, 1,081,575, etc. After curtailment the terminal numbers become 480700. Dividing the remaining numbers through by 480700 gives the coefficients in Fig. 33, 1000, 2241, 4241, etc. The height, $h=1932 \lambda$, has been adjusted to get reenforcement in the interference pattern along the direction at angle 15°.

Fig. 34 shows how the beams at angle 15° may be obtained with a linear array extending horizontally. In this case the array is on the basis of 11 antennas curtailed to 7. Fig. 35 shows the same preferred direction with a vertically extending array tapered and curtailed as in Fig. 34. In both Figs. 34 and 35 the value of h has been chosen to get reenforcement along the preferred direction between the direct and reflected waves.

I claim:

1. A linear antenna array consisting of a certain number of antennas and means to energize them in intensities along the array graded according to the intermediate terms of the binomial expansion with a number of terms greater than the number of the antennas.

2. A certain number of antennas in linear array and means to energize them in graded intensity at values corresponding to the intermediate coefficients of the binomial expansion for a number of terms greater than the number of the antennas.

3. The method of radiating a narrow beam of energy from a linear array of a limited number of antennas which consists in energizing the antennas in graded intensities corresponding to the intermediate coefficients of the binomial expansion for a number of terms greater than the number of the antennas.

4. The method of radiating a narrow beam of energy from a linear array of a limited number of antennas with moderate disparity of intensity of energy from the different antennas which consists in energizing the antennas in

graded intensities corresponding to the intermediate coefficients of the binomial expansion for a number of terms greater than the number of the antennas.

5. In an antenna array of a certain number of dimensions, means for enhancing the directional selectivity and reducing the disparity of intensity, which consists in energizing the antennas along one dimension of the array in accordance with the intermediate coefficients of the binomial expansion for a number of terms greater than the number of antennas along a row in that dimension.

6. The method of producing a beam of radiation from an antenna array in a desired direction oblique to the direction of extent of the array which consists in energizing the antennas in said direction of extent according to the coefficients of a binomial expansion and with the proper phase difference between consecutive antennas to adjust the beam to the desired direction.

7. An antenna array adapted for a beam of radiation in a direction oblique to the direction of its extent and means to energize its antennas in intensities graded according to the coefficients of a binomial expansion and at a proper constant phase difference differing from zero and 180 degrees to give the beam its proper direction.

8. An antenna array consisting of a series of directional antenna units lying along an axis inclined to the horizontal and at a distance above the earth's surface to make the direct and reflected radiation therefrom combine with maximum effect in a desired inclined direction, each such unit consisting of a small number of antennas energized to have a null direction of transmission and all said units being oriented alike in respect to this null direction, a central power supply, conductor pairs therefrom to said antennas, the conductors of said pairs being combined to be non-radiative, phase adjusters in said conductors to give a desired directional selectivity, and means to apportion energy to said units according to the intermediate coefficients of the binomial expansion for an exponent somewhat higher than the number of units in said series.

9. An antenna array consisting of a series of directional antenna units lying along an axis each such unit consisting of a small number of antennas energized to have a null direction of transmission and also said units being oriented alike in respect to this null direction, a central power supply, conductor pairs therefrom to said antennas, phase adjusters in said conductors to give a proper phase relation for a desired directional selectivity, and means respective to said conductors to apportion energy to said units according to the intermediate coefficients of the binomial expansion having an exponent that is somewhat higher than the number of units in said series.

JOHN STONE STONE.