

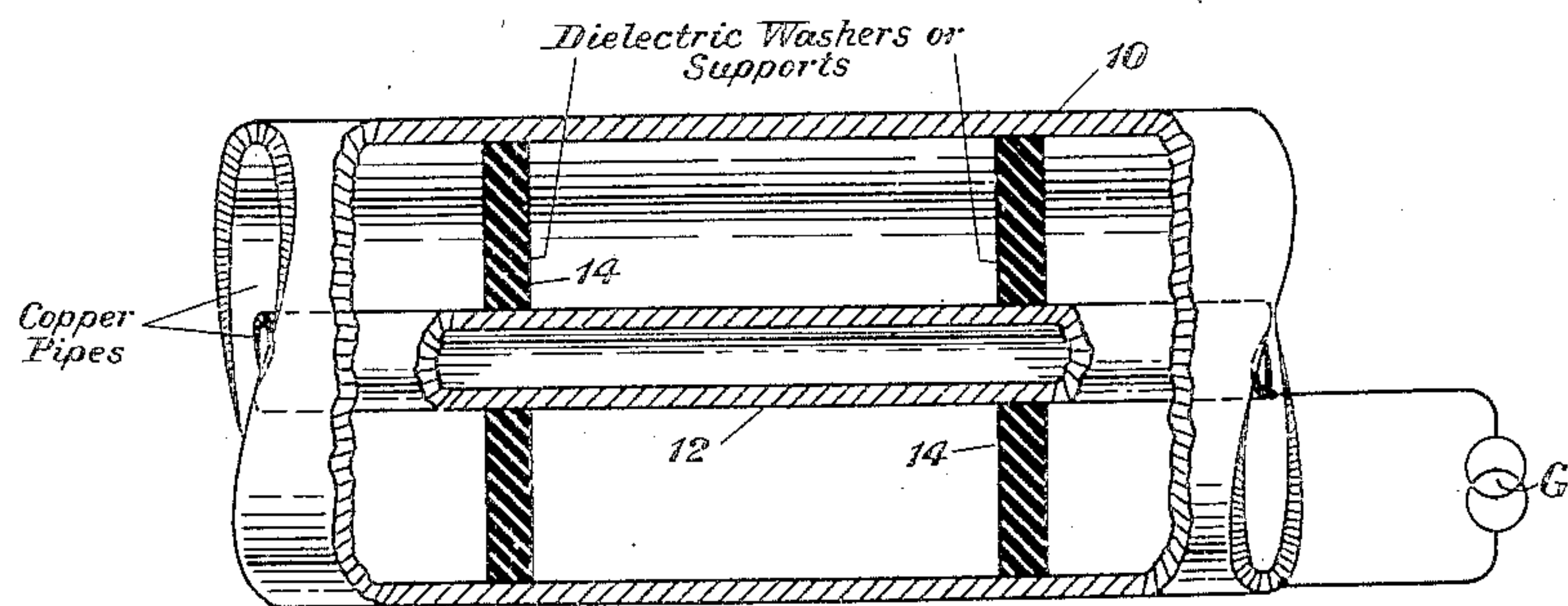
Nov. 1, 1932.

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1,885,195

CONCENTRIC CONDUCTOR TRANSMISSION SYSTEM

Filed May 23, 1929



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CONCENTRIC CONDUCTOR TRANSMISSION SYSTEM

Application filed May 23, 1929. Serial No. 365,519.

This invention relates to a novel form of conductor structure employing concentric cylindrical conductors for the transmission of a wide band of frequencies with relatively low attenuation. The invention principally relates to such proportioning of the relative dimensions of the conductors as to produce minimum attenuation.

If a solid cylindrical conductor or a hollow cylindrical conductor is provided with a return conductor comprising a second hollow cylindrical conductor concentrically arranged with respect to the first conductor, and the two conductors are separated by a dielectric consisting largely of air or other gaseous medium, the transmission line thus formed will have a number of desirable characteristics. Its attenuation at all frequencies will be quite low as compared with the corresponding attenuation of open wire lines and cable circuits such as are now commonly used for telephone transmission. Such a transmission circuit may, therefore, be employed for the transmission of a much wider band of frequencies than has been possible with types of transmission circuits heretofore used. It also has the advantage that it is substantially free from interference from neighboring conductor systems and in itself tends to produce but little interference into adjacent transmission circuits.

The present invention is, however, more particularly concerned with the discovery that if a given amount of conducting material is to be used, the conductive material may be most efficiently employed, from the standpoint of attenuation, if a certain ratio exists between the inner diameter of the outer conductor and the outer diameter of the inner conductor.

It is a characteristic of a transmission system of the type herein considered that at high frequencies the current tends to flow at the outer surface of the inner conductor and at the inner surface of the outer conductor.

Therefore, the outer radius of the inner conductor and the inner radius of the outer conductor are of importance from an attenuation standpoint. In accordance with the present invention, it has been found that if a given amount of conducting material is to be employed, the attenuation of the system will be a minimum at all frequencies, when the conductive material is so distributed between the two conductors that the inner radius of the outer conductor bears an optimum ratio to the outer radius of the inner conductor. When the walls of both conductors are of the same thickness, it has been found that this ratio is approximately 4.68. When, however, the walls are of unequal thickness, the optimum ratio decreases when the inner wall is thinner, and increases when the inner wall is thicker than the outer wall.

The invention will now be more fully understood from the following description when read in connection with the accompanying drawing in which the figure is a symbolic representation of a concentric conductor system.

Referring to the figure of the drawing, 10 designates an outer conductor in the form of a hollow cylinder of suitable conducting material. A second cylindrical conductor 12 is mounted concentrically with the outer conductor 10. One of the conductors acts as a return for the other and not as a mere shield, this fact being indicated by the conventional representation of a source of electromotive force G with its terminals connected to the two conductors.

In order that the attenuation may be small at high frequencies, the leakage loss between the conductors should be as small as possible. As the leakage loss is due to the nature of the dielectric interposed between the conductors, the dielectric should be principally of air, since air introduces no leakage loss. Accordingly, the two conductors may

be held in proper concentric relation and out of electrical contact with each other by means of spaced dielectric washers 14. These washers should be separated from each other a suitable distance and should be made as thin as possible consistent with the required mechanical strength. They should also be composed of some dielectric of small loss angle and low dielectric constant, since if these conditions are obtained, the leak-loss (which in the ordinary open wire system comprises a large part of the attenuation) may be made so small as to be practically negligible. For example, hard rubber or preferably pyrex glass or other good insulating material may be used for the insulating washers 14. In this connection it should be noted that as the outer conductor may be made watertight, the insulating washers may be maintained dry and free from dirt or contamination, so that the leakage loss will not increase or change with time but will be maintained at the low value which is characteristic of insulators in the dry and clean condition in which they come from the factory.

Having in mind the foregoing brief description of the type of system to which the invention relates, a mathematical discussion will now be given to show that the condition of minimum attenuation at all frequencies will be attained for a given optimum ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor.

1. Diameter relations for most efficient use of material—Walls of equal thickness

In an application of E. I. Green, Serial No. 365,518, filed May 23, 1929, it is shown that where economy in the use of conducting material is not of importance, the condition of minimum attenuation for a concentric conductor system of the type above described with conductor walls of appreciable thickness, is as follows:

$$\log x = \frac{x+n}{x}$$

where x is the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor, and n is the ratio of the conductivity of the inner conductor to that of the outer. When the conductor walls are made sufficiently thin to secure constant attenuation over a range of frequencies, the condition for minimum attenuation is

$$\log x = \frac{x+mn}{x}$$

where m is the ratio of the thickness of the inner conductor to the thickness of the outer conductor. In the special case where the thickness of both conductor walls is the same, and both conductors are made of the same

material, both m and n become unity so that the above expression reduces to

$$\log x = 1 + 1/x$$

which when solved, gives the value $x=3.59$, so that under the conditions above assumed, minimum attenuation occurs when the inner diameter of the outer conductor is 3.59 times the outer diameter of the inner conductor. In the above discussion and that which follows, the logarithms are natural logarithms.

The above relationship is of considerable importance where the diameter of the outer conductor is fixed by physical limitations and it is then desired to obtain the smallest possible attenuation without regard to the amount of conductive material utilized. The present invention, however, deals with an entirely different situation in which it is desired to obtain the minimum attenuation for a fixed amount of conductor material.

Unless the walls of the coaxial conductors are made extremely thin, the attenuation at high frequencies will be practically independent of the thickness of wall. This is because of the large skin effect, which makes the current flow in a very thin wall on the outside of the inner tube and on the inside of the outer tube. Consequently, the thickness of the conductor walls will ordinarily be determined by mechanical considerations. Under such conditions, the following formulas may be used at high frequencies (above the voice range):

$$\text{Resistance of inner conductor} = R_1 = K_0 \sqrt{f}/b \quad (1)$$

$$\text{Resistance of outer conductor} = R_0 = K_0 \sqrt{f}/c \quad (2)$$

$$\text{Linear inductance} = L = K_1 \log c/b \quad (3)$$

Also

$$\text{Linear capacity} = C = \frac{K_2}{\log c/b} \quad (4)$$

where K_0 , K_1 and K_2 are constants, f is the frequency, c is the inner radius of the outer conductor, and b is the outer radius of the inner conductor. It will be noted that the outer radius of the outer conductor and the inner radius of the inner conductor do not appear in the formulas.

The attenuation at high frequencies is

$$\alpha = \frac{R}{2} \sqrt{C/L} + \frac{G}{2} \sqrt{L/C} \quad (5)$$

where R , L , C and G are the linear resistance, inductance, capacity and leakage conductance, respectively. Let us assume first that air insulation is employed between the two conductors, and $G=0$. We have then

$$\alpha = \frac{R}{2} \sqrt{C/L} = \frac{R_1 + R_0}{2} \sqrt{C/L} \quad (6)$$

Substituting from equations (1) to (4), we obtain

$$\alpha = \frac{K_0 \sqrt{f}}{2} (1/b + 1/c) \sqrt{\frac{K_2}{K_1} \frac{1}{(\log c/b)^2}} \quad (7)$$

Rewriting equation (7), we have

$$\alpha = \frac{K_0}{2} \sqrt{\frac{K_2}{K_1}} \sqrt{f} (1/b + 1/c) \frac{1}{\log c/b} \quad (8)$$

Or, letting

$$K_0/2 \sqrt{K_2/K_1} = K_3$$

$$\alpha = K_3 \sqrt{f} (1/b + 1/c) \frac{1}{\log c/b} \quad (9)$$

The total amount of conductor material employed in the arrangement may be represented by:

$$M = \pi (b^2 - a^2) + \pi (d^2 - c^2) \quad (10)$$

where d is the outer diameter of outer conductor and a is the inner diameter of inner conductor. Now if

$$b - a = d - c = t \quad (11)$$

t being assumed constant, we may write approximately

$$M = 2\pi t (b + c) \quad (12)$$

Or, letting $M/2\pi t = K_{10}$

$$b + c = K_{10} \quad (13)$$

Hence

$$c = \frac{b + c}{b/c + 1} = \frac{K_{10}}{b/c + 1} \quad (14)$$

and

$$b = \frac{b + c}{c/b + 1} = \frac{K_{10}}{c/b + 1} \quad (15)$$

Substituting (14) and (15) in (9), we obtain

$$\alpha = \frac{K_3}{K_{10}} \sqrt{f} (c/b + 1) (b/c + 1) \frac{1}{\log c/b} \quad (16)$$

Or, if $x = c/b$

$$\alpha = \frac{K_3}{K_{10}} \sqrt{f} (x + 1) (1/x + 1) \frac{1}{\log x} \quad (17)$$

Setting $d\alpha/dx = 0$, we find that

$$\log x = \frac{x + 1}{x - 1} \quad (18)$$

the solution for which is

$$x = c/b = 4.68 \quad (19)$$

Consequently, the minimum attenuation for a given amount of conductor material will be obtained when the ratio of the diameters is 4.68.

2. Use of thin walls to secure constant attenuation

If the conductor walls are made sufficiently thin the attenuation of the coaxial arrangement can be made substantially constant at all frequencies below a certain value. In

general, the attenuation will be practically constant for frequencies below about $25/t^2$, where t is the thickness of the conductor wall in centimeters. The maximum frequency for any desired degree of constancy of attenuation and for any assigned value of wall thickness can be determined by plotting a curve of attenuation versus the product $t\sqrt{f}$.

3. Diameter relations with thin walls

The previous discussion of diameter relations dealt entirely with walls of appreciable thickness. Assuming that the walls of the conductors are made very thin in order to avoid skin effect and secure constant attenuation, the question arises as to the proper diameter relations for minimum attenuation under such conditions. If a equals the inner radius of the inner conductor and d equals the outer radius of the outer conductor, we have the following equations:

$$R_i = K_s / (b^2 - a^2) \quad (20)$$

$$R_o = K_s / (d^2 - c^2) \quad (21)$$

where K_s is a constant. Note that since the walls are so thin that the attenuation and the resistance are uniform over the frequency range considered, the factor f does not appear in equations (20) and (21). L and C are the same as in equations (3) and (4). Consequently, substituting from equations (20), (21), (3) and (4), in equation (6).

$$\alpha = K_s \left(\frac{1}{b^2 - a^2} + \frac{1}{d^2 - c^2} \right) \sqrt{\frac{K_2}{K_1} \frac{1}{\log c/b}} \quad (22)$$

Let us now assume that the thickness of the two conductor walls has the same value, which may be designated t . In other words

$$b - a = d - c = t \quad (23)$$

It will be found that for small values of wall thickness the attenuation is approximately given by the following:

$$\alpha = K_s \sqrt{\frac{K_2}{K_1}} \left(\frac{1}{2bt} + \frac{1}{2ct} \right) \frac{1}{\log c/b} \quad (24)$$

Or, letting $K_s/2 \sqrt{K_2/K_1} = K_9$

$$\alpha = K_9 (1/bt + 1/ct) \frac{1}{\log c/b} \quad (25)$$

Substituting (14) and (15) in equation (25)

$$\alpha = \frac{K_9}{K_{10}} (1/t) (c/b + 1) (b/c + 1) \frac{1}{\log c/b} \quad (26)$$

Or, if $x = c/b$,

$$\alpha = \frac{K_9}{K_{10}} (1/t) (x + 1) (1/x + 1) \frac{1}{\log x} \quad (27)$$

Letting $d\alpha/dx = 0$, we again find

$$\log x = \frac{x + 1}{x - 1} \quad (28)$$

as the condition of minimum attenuation with a fixed amount of conducting material, or, as before

$$x = c/b = 4.68. \quad (29)$$

It is interesting to note that the fixed ratio of c to b gives fixed values of inductance, capacity and characteristic impedance for the concentric arrangement, regardless of the actual dimensions of the conductors. By analogy to formula (55), page 109 of "Calculations of Alternating Current Problems," by L. Cohen, the capacity of the structure herein considered, bearing in mind that the dielectric is equivalent to air, will have the value

$$C = \frac{0.0894}{\log c/b} \times 10^{-8} \text{ farads per mile.}$$

Likewise from formula (56) on page 72 of the Cohen publication, the inductance in abhenries per mile has the value

$$L = 2 \log c/b$$

Reducing this to henries per mile

$$L = \frac{2 \times 1.60935 \times 10^5}{10^9} \log c/b =$$

$$.32187 \times 10^{-3} \log c/b$$

The value of the nominal characteristic impedance corresponding to $c/b = 4.68$ then becomes

$$Z_0 = \sqrt{L/C} = \sqrt{\frac{.32187 \times 10^{-3}}{.0894 \times 10^{-8}}} \times \log c/b$$

$$= 60 \log c/b = 92.6 \text{ ohms.}$$

The corresponding values of inductance and capacity are .497 mh. per mile and .0580 mf. per mile.

4. Diameter relations for most efficient use of material—Walls of unequal thickness

Yet another case for which the most efficient use of material may be desired is that in which the walls of the two conductors are unequal in thickness. Let us assume first that the walls are thick enough for equations (1) and (2) to apply. Under these conditions (see Equation (9))

$$\alpha = K_3 \sqrt{f} (1/b + 1/c) \frac{1}{\log c/b} \quad (30)$$

Let the thickness of the outer conductor be t and that of the inner conductor mt , m and t being assumed constant. The total amount of conductor material is as given by equation (10)

$$M = \pi(b^2 - a^2) + \pi(d^2 - c^2) \quad (31)$$

which may be reduced to

$$M = \pi t [2(mb + c) + t(1 - m^2)] \quad (32)$$

In other words, since m and t are constant and M is assumed fixed

$$mb + c = \text{const.} = K_{11} \quad (33)$$

Noting that

$$c = \frac{mb + c}{mb/c + 1} = \frac{K_{11}}{mb/c + 1} \quad (34)$$

and rewriting (30) as follows:

$$\alpha = K_3 \sqrt{f} \frac{c/b + 1}{c} \frac{1}{\log c/b} \quad (35)$$

then by substituting (34) in equation (35) we have

$$\alpha = \frac{K_3}{K_{11}} \sqrt{f} (c/b + 1) (mb/c + 1) \frac{1}{\log c/b} \quad (36)$$

Or, as before, letting $x = c/b$

$$\alpha = \frac{K_3}{K_{11}} \sqrt{f} (x + 1) (m/x + 1) \frac{1}{\log x} \quad (37)$$

Setting $d\alpha/dx = 0$, we find that the condition for minimum attenuation is

$$\log x = \frac{(x + m)(x + 1)}{x^2 - m} \quad (38)$$

The values of x which satisfy this relation are given in the following table for various values of m

$m \left(\frac{b-a}{d-c} \right)$	$x \left(\frac{c}{b} \right)$
.25	3.90
.33	3.90
.5	4.18
1.	4.68
2.	5.56
3.	6.33
4.	7.03

The solution of the corresponding case in which the walls are thin and of unequal thickness, and it is desired to use the material most efficiently, is also of some interest. The method employed in this case is similar to that already utilized. Assuming, as before, that

$$b - a = mt \text{ and } d - c = t \quad (39)$$

where m and t are assumed constant, it is found that α is approximately given as follows:

$$\alpha = K_{12} (c/mb + 1) (mb/c + 1) \frac{1}{\log c/b} \quad (40)$$

Differentiation shows that the condition for minimum attenuation with a constant amount of conductor material is

$$\log x = \frac{x + m}{x - m} \quad (41)$$

where, as before $x = c/b$. The values of x

which satisfy this relation are given in the following table for various values of m .

$m \left(= \frac{b-a}{d-c} \right)$	$x \left(= \frac{c}{b} \right)$
.25	3.22
.33	3.38
.50	3.71
1.	4.68
2.	6.55
3.	8.35
4.	10.15

diameter of the inner conductor, and m being the ratio of thickness of wall of the inner conductor to that of the outer.

5. A concentric conductor system comprising two concentrically arranged cylindrical conductors, the inner diameter of the outer conductor and the outer diameter of the inner conductor bearing such ratio to each other that with a predetermined amount of conductive material per unit of length apportioned to the two conductors, the characteristic impedance will be approximately 93 ohms at frequencies above the voice range.

In testimony whereof, I have signed my name to this specification this 21st day of May, 1929.

ESTILL I. GREEN.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. A concentric conductor system comprising two concentrically arranged cylindrical conductors, said conductors having a predetermined amount of conductive material per unit of length apportioned thereto, the inner diameter of the outer conductor and the outer diameter of the inner conductor bearing such ratio to each other that for the predetermined apportionment of conductive material to the two conductors the attenuation of the system will be a minimum at high frequencies.

2. A concentric conductor system adapted for high frequency transmission comprising two concentrically arranged cylindrical conductors, the dimensions of the conductors being such that with a predetermined amount of conductive material per unit of length apportioned to the two conductors the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor will be approximately 4.7.

3. A concentric conductor system comprising two concentrically arranged cylindrical conductors, the dimensions of the conductors being such that with a predetermined amount of conductive material per unit of length apportioned to the two conductors the relation

$$\log x = \frac{x+1}{x-1}$$

will approximately hold for high frequency transmission, x being the ratio of the inner diameter of the outer conductor to the outer diameter of the inner conductor.

4. A concentric conductor system comprising two concentrically arranged cylindrical conductors, the dimensions of the conductors being such that with a predetermined amount of conductive material per unit of length apportioned to the two conductors the relation

$$\log x = \frac{(x+m)(x+1)}{x^2-m}$$

will approximately hold for high frequency transmission, x being the ratio of the inner diameter of the outer conductor to the outer

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100

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