CONCENTRIC CONDUCTOR TRANSMISSION SYSTEM

Filed May 23, 1929 2 Sheets-Sheet 1

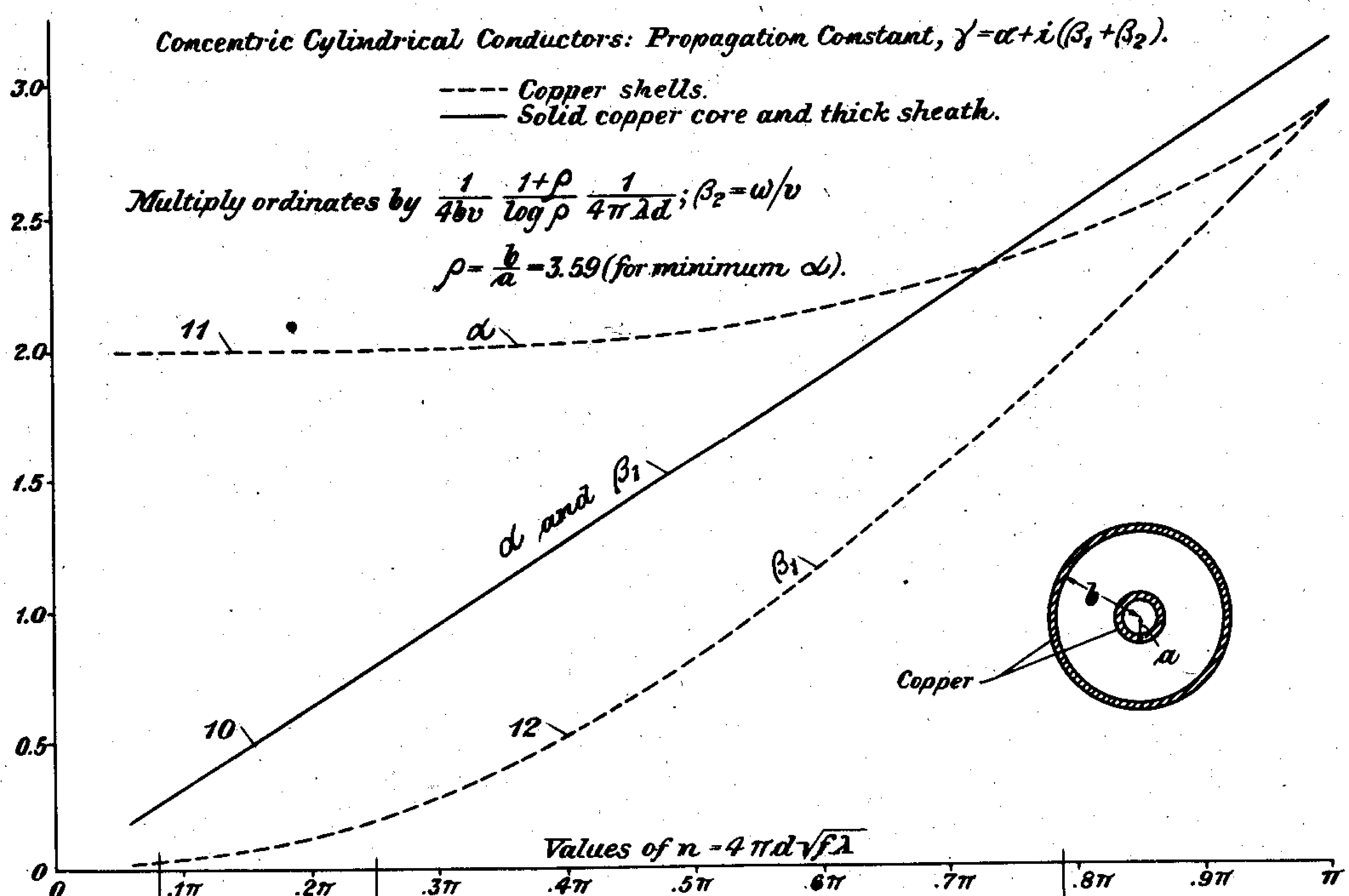

Fig. 1

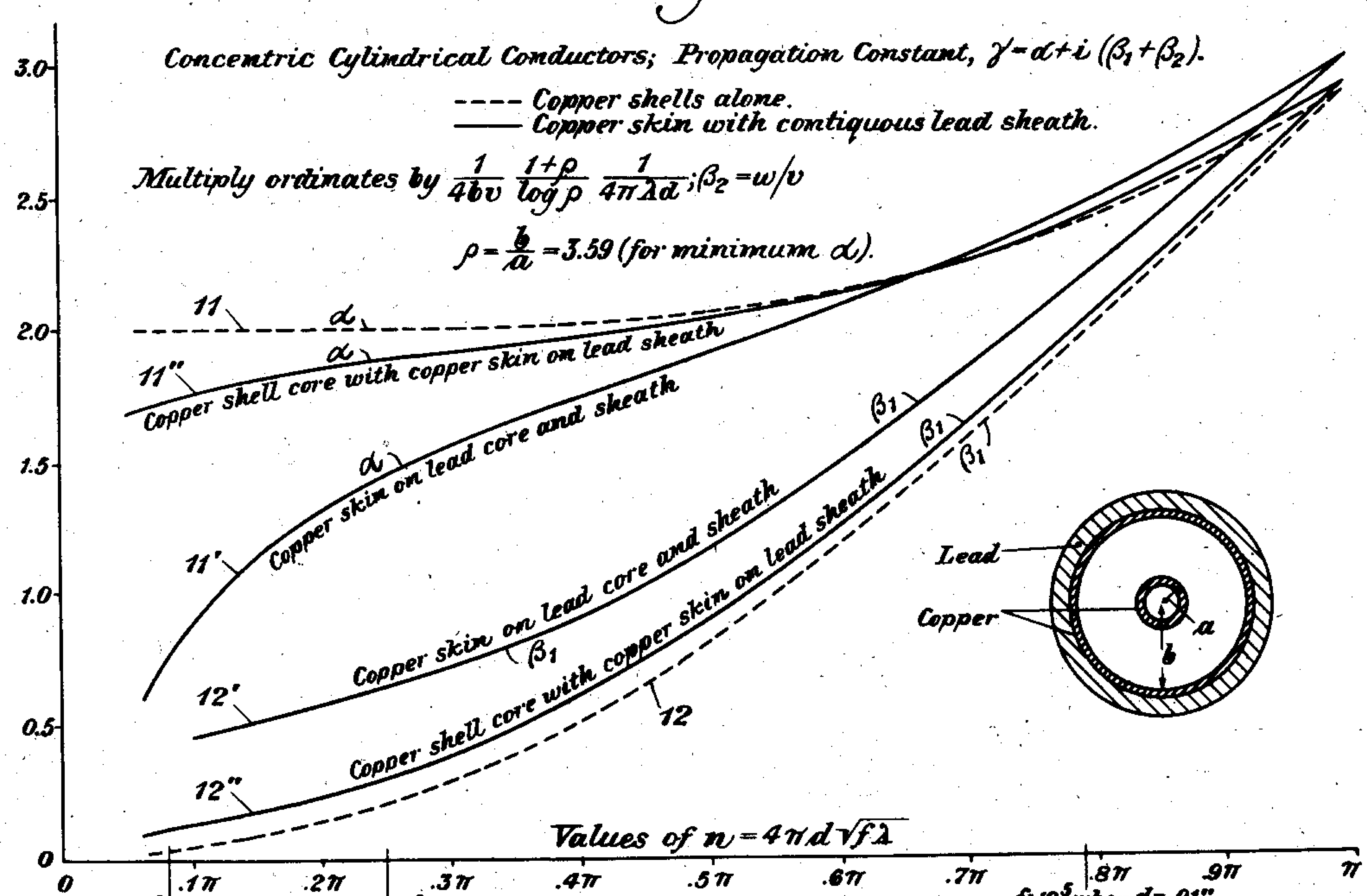

Fig. 2

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CONCENTRIC CONDUCTOR TRANSMISSION SYSTEM

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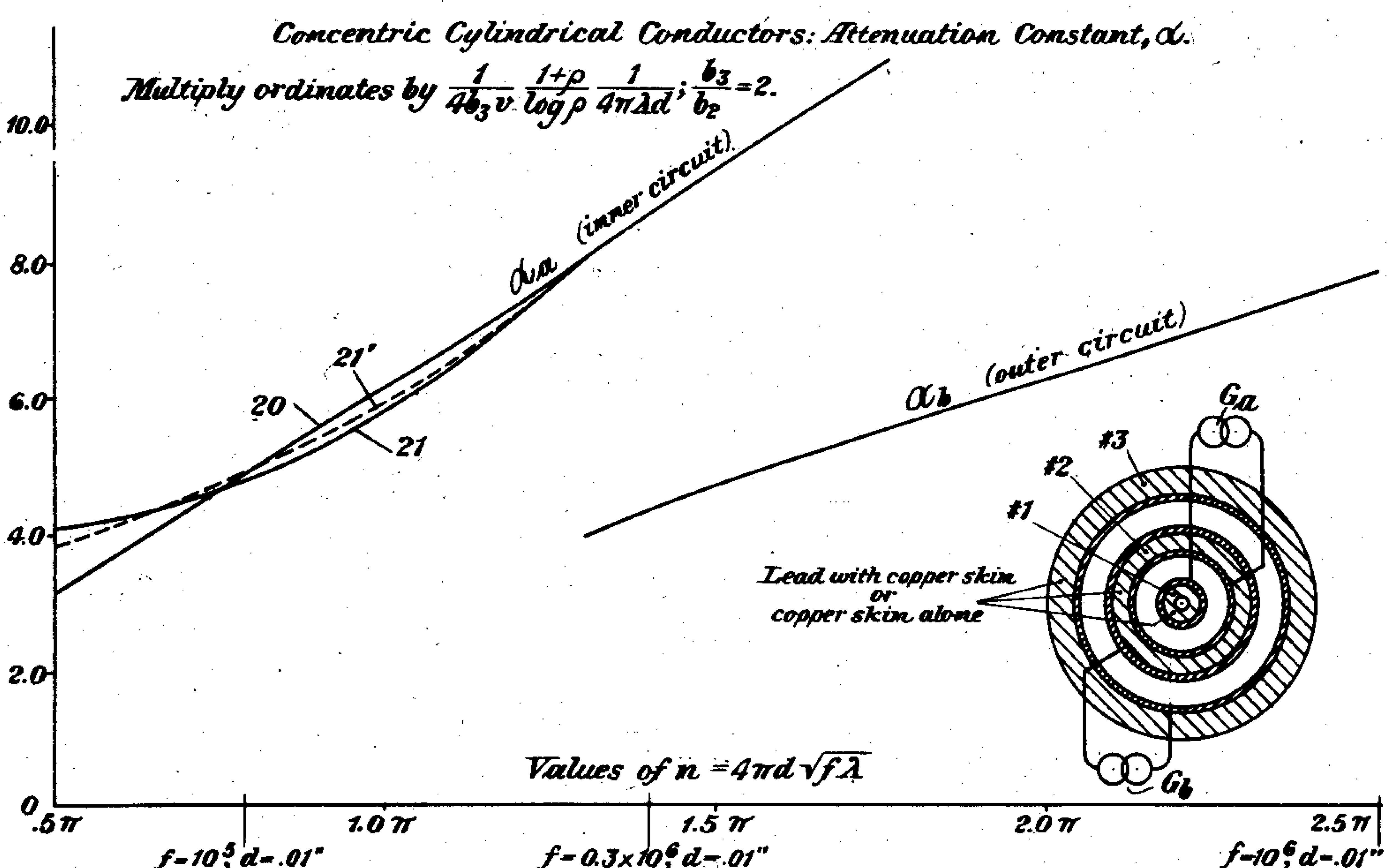

Fig. 3

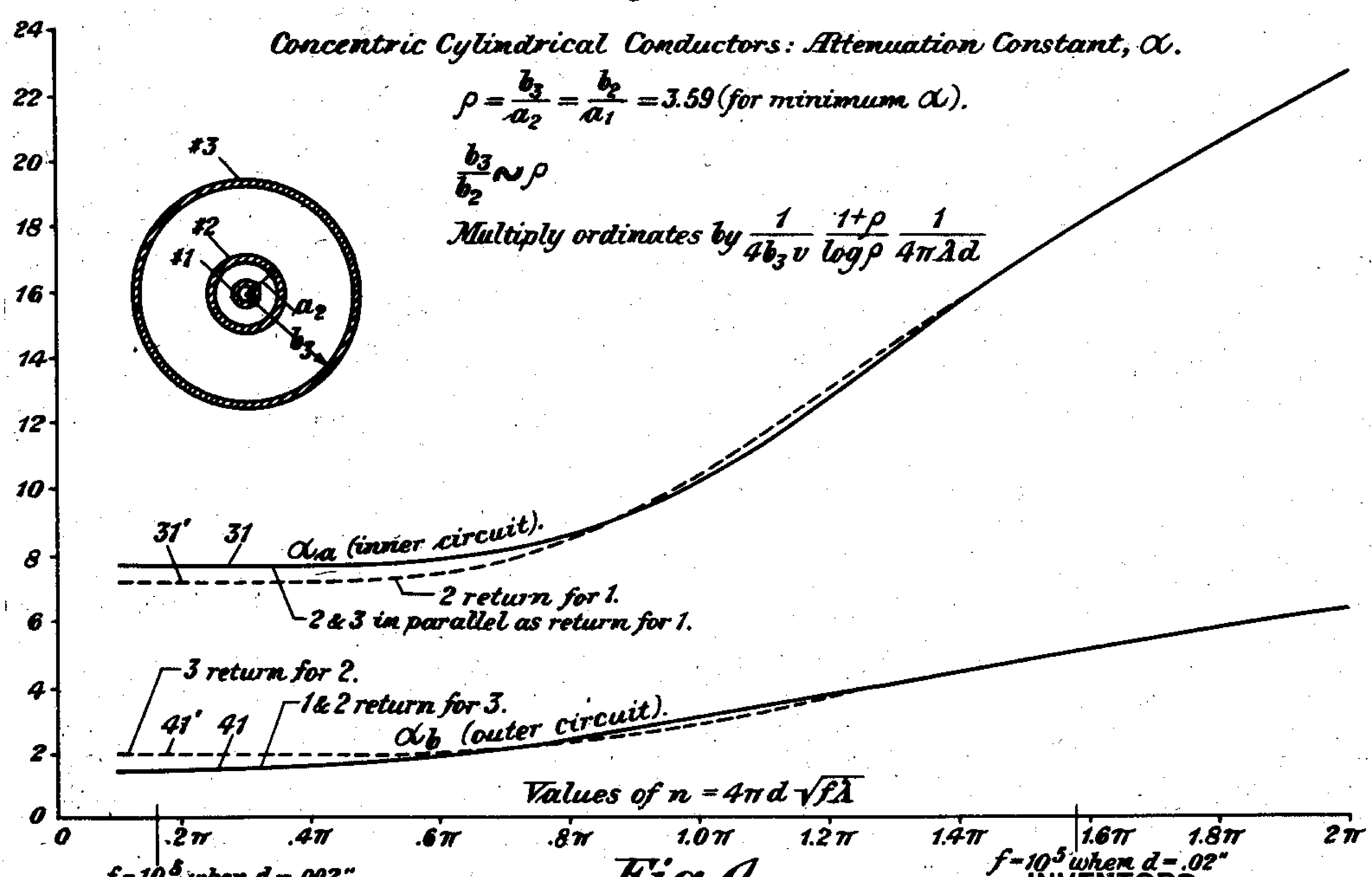

Fig. 4

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Patented Aug. 11, 1931 **1,817,964**

UNITED STATES PATENT OFFICE

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CONCENTRIC CONDUCTOR TRANSMISSION SYSTEM

Application filed May 23, 1929. Serial No. 365,515.

This invention relates to a novel form of conductor structure employing concentric cylindrical conductors for the transmission of a wide band of frequencies with relatively low attenuation. The invention particularly relates to an arrangement of the conductors such that a fairly uniform attenuation may be obtained for a band of frequencies below a preassigned limit.

If a solid cylindrical conductor or a hollow cylindrical conductor having relatively thick walls is provided, with a return conductor comprising a hollow cylindrical conductor concentrically arranged with respect to the first conductor, and the two conductors are separated by a dielectric consisting largely of air or other gaseous medium, the transmission line thus formed will have a number of desirable characteristics. Its attenuation at all frequencies will be quite low as compared with the corresponding attenuation of open wire lines and cable circuits such as are now commonly used for telephone transmission. Such a transmission circuit may, therefore, be employed for the transmission of a much wider band of frequencies than has been possible with types of transmission circuits heretofore used. It also has the advantage that it is substantially free from interference from neighboring conducting systems and in itself tends to produce but little interference into adjacent transmission circuits.

The present invention is, however, more particularly concerned with the type of conducting system in which the two concentric conductors are either thin copper shells or else comprise two concentric cylindrical structures of relatively low conductivity with a thin skin of good conducting material on the outer surface of the inner conductor and the inner surface of the outer conductor. The last mentioned arrangement is based upon the fact that in a concentric conductor system the current at the higher frequencies tends to crowd to the outer surface of the inner conductor and the inner surface of the outer conductor. Applicants have discovered that if the walls of the concentric conducting cylinders are made sufficiently thin, the attenuation will be substantially uniform over a wide range of frequencies up to an upper limiting frequency which is dependent upon the thickness of the conducting walls. In general, the system may be designed so that if it is desired to obtain uniform attenuation up to a preassigned frequency, the thickness of the conducting walls will bear a definite relation to that frequency. For example, with a conductor having conductivity λ, if the frequency f and the thickness d of the wall in centimeters be approximately related in accordance with the formula

$$f=\frac{1}{64\lambda d^2}$$

the attenuation will be substantially uniform up to the frequency f. Indeed, as will be shown later, the increase in attenuation at the higher frequencies will be quite tolerable if the formula

$$f=\frac{1}{16\lambda d^2}$$

be employed. In other words, if the thickness of the walls d is so chosen that the attenuation will be substantially uniform up to a frequency f, the increase in attenuation from frequency f up to frequency $4f$ will be quite tolerable.

As an alternative arrangement it is proposed, in accordance with the present invention, to obtain a fair degree of equalization to a concentric conductor system by employing three or more concentric cylindrical conductors. For example, a concentric conductor system may comprise an inner, an intermediate and an outer cylindrical conductor, and a frequency band may be split up into two sub-bands, one of which will be, in effect, transmitted over a circuit comprising the inner conductor with the intermediate conductor as a return, the other sub-band being transmitted over the outer conductor with the intermediate conductor as a return. Such a system will have in general two modes of propagation. In accordance with one mode the intermediate and outer conductors act as a return for the inner conductor, while in accordance with the other mode the intermediate and inner conductors act as a return for the outer conductor.

Applicants have found that the attenuation in accordance with these two modes of propagation will be substantially the same as though the system were treated as two independent conducting systems, one of which comprises the inner and intermediate conductors and the other of which comprises the intermediate and outer conductors. At the higher frequencies the return currents, in accordance with the two modes of propagation, tend to concentrate respectively at the two surfaces of the intermediate conductor, and it is unnecessary that these two surfaces of the intermediate conductor be separated by an insulating material. It is, therefore, quite feasible to so connect the conductors at the terminals that one band of frequencies will be transmitted over the inner conductor with the intermediate conductor as a return, and the other band of frequencies (the higher band) will be transmitted over the outer conductor with the intermediate conductor as a return, thus attaining a substantial degree of equalization as between the two bands by reason of the fact that the amount of conducting material employed in the intermediate and outer conductors for the transmission of the higher frequency band will be much greater than that employed for the lower frequency band.

The invention will now be more fully understood from the following description when read in connection with the accompanying drawings, in which Figure 1 shows curves illustrating the transmission characteristics of a conducting system comprising two thin-walled concentric cylindrical conductors; Fig. 2 shows curves illustrating the transmission characteristics of a concentric conducting system comprising two concentric cylindrical members of relatively low conductivity with a thin skin of high conductive material upon the inner surface of the outer cylinder and upon the outer surface of the inner cylinder; Fig. 3 shows the transmission characteristics of a system similar to that of Fig. 2 arranged to attain a substantial degree of equalization of attenuation by the employment of more than two concentric conductors; while Fig. 4 shows the transmission characteristics of a system of multiple transmission conductors each of which comprises a cylindrical conductor with thin walls.

In order to understand the principles of the invention it is desirable to give a general mathematical treatment from which may be derived four general formulæ applying to four general cases, as follows:

System I. A thick copper core and return sheath.

System II. Two thin-walled copper shells.

System III. Two lead conductors with copper skins.

System IV. Three copper shells.

It will be understood, of course, that the mathematical treatment is general and the invention is not limited to the use of copper and lead as relatively good and relatively poor conducting materials, these particular materials being merely chosen for purposes of illustration.

Consider a long cylindrical conductor or core surrounded by another cylindrical conductor (the return conductor) coaxial with the first and separated from it by a layer of dielectric. The transmission of periodic currents over a system of n such coaxial cylinders is analyzed in detail in the paper "Transmission characteristic of the submarine cable" by John R. Carson and J. J. Gilbert, Journal Franklin Institute, December, 1921.

In the present case, denote by E_2 and E_1 the electric forces per unit length at the inner surface (radius$=b$) of the return conductor, and the outer surface (radius$=a$) of the core, respectively, and let I represent the current in the core and γ the propagation constant. Then, in accordance with formula (15) of the paper referred to above,

$$E_2-E_1=-\left(\frac{\gamma^2}{G+i\omega C}-i\omega L\right)I \quad (1)$$

where

$$L=2\mu \log \rho,\ C=\frac{k}{2\log \rho},\ G=\frac{4\pi g}{2\log \rho}$$

Here $\rho=b/a$ and $k=\epsilon/v^2$, ϵ being the dielectric constant in c. g. s. units and v being the velocity of light; μ and k are, respectively, the permeability and specific inductive capacity of the dielectric in c. g. s. electromagnetic units and g, the specific conductivity of the dielectric is assumed, without loss of generality, to be zero. As usual,

$$i=\sqrt{-1}$$

and $\omega=2\pi$ times frequency, f.

Now, writing

$$E_1=Z_1I$$

and

$$E_2=-Z_2I,$$

so that Z_1 and Z_2 represent the internal impedance of the core and return, respectively, and putting G=0, Equation (1) becomes

$$\gamma^2=i\omega C(Z_1+Z_2+i\omega L)$$

$$=(i\omega)^2LC\left(1+\frac{Z_1+Z_2}{i\omega L}\right) \quad (2)$$

or

$$\gamma=i\omega\sqrt{LC}\sqrt{1+\frac{Z_1+Z_2}{i\omega L}}$$

But the term $(Z_1+Z_2)/i\omega L$ is usually small in comparison to unity. In that case,

$$\gamma=i\omega\sqrt{LC}\left(1+\frac{Z_1+Z_2}{2i\omega L}\right)$$

approximately, or, substituting for L and C their values from Equation (1),

$$\gamma=\sqrt{\mu k}\frac{Z_1+Z_2}{4\log\rho}+i\omega\sqrt{\mu k}\quad(3)$$

It now remains to determine Z_1 and Z_2 for the various cases herein disclosed.

System I: Thick copper core and sheath

The conducting system here under consideration may comprise either a solid cylindrical conductor or a hollow cylindrical conductor whose walls are comparatively thick with a return or sheath of suitable conducting material, the sheath being cylindrical and arranged concentrically with respect to the core, and its walls being of sufficient thickness to approximate the effect of a wall of infinite thickness. The two conductors (the core and the sheath) will preferably be separated in such a manner that the intervening dielectric will be equivalent to a gaseous dielectric.

It is well known that the internal impedance of a long cylindrical wire is given by

$$Z_1=2\mu_1 i\omega\frac{J_o(x_1)}{x_1 J'_o(x_1)}\quad(4)$$

where

$x_1=ia\sqrt{ia_1}$,

$a_1=4\pi\lambda_1\mu_1\omega$,

μ_1 = permeability of the conductor,

λ_1 = conductivity of the conductor,

$J_o(x_1)$ = Bessel function of the first kind of zeroth order,

$J'_o(x_1)=\dfrac{dJ_o(x_1)}{dx_1}$

See, for instance, Jahnke and Emde "Funktionentafeln" p. 143; also p. 717 of paper by Carson and Gilbert, above referred to.

Similarly (p. 717, Carson and Gilbert paper), the internal impedance of the outer or return conductor of such electrical thickness that it may be assumed infinite in extent, is given by

$$Z_2=-2\mu_2 i\omega\frac{K_o(y_2)}{y_2 K'_o(y_2)}\quad(5)$$

where

$y_2=ib\sqrt{ia_2}$,

$a_2=4\pi\lambda_2\mu_2\omega$,

μ_2 = permeability of conducting material,

λ_2 = conductivity of conducting material,

$K_o(y_2)$ = Bessel function of second kind of zeroth order,

$K'_o(y_2)=\dfrac{dK_o(y_2)}{dy_2}$

At the very high frequencies considered here, x_1 and y_2 are so large that the limiting values of the Bessel functions for very large arguments may be substituted in Equations (4) and (5), namely:

$$\frac{J_o(z)}{J'_o(z)}=i\text{ and }\frac{K_o(z)}{K'_o(z)}=-i.$$

These give, after simplifying,

$$Z_1=\frac{1}{a}\sqrt{\frac{\mu_1 i\omega}{\pi\lambda_1}},\quad(6)$$

$$Z_2=\frac{1}{b}\sqrt{\frac{\mu_2 i\omega}{\pi\lambda_2}}\quad(7)$$

Now, putting $\mu_1=\mu_2=\mu=1$ (since we are concerned with non-magnetic substances) and $\lambda_1=\lambda_2=\lambda$ and remembering that d=thickness of shell, $\rho=b/a$, $k=\epsilon/v^2$ and $n=4\pi d\sqrt{f\lambda}$, then substituting (6) and (7) in (3) gives Formula I.

$$\gamma=\frac{\sqrt{\epsilon}}{4bv}\frac{1+\rho}{\log\rho}\frac{n(1+i)}{4\pi\lambda d}+i\sqrt{\epsilon}\,\omega/v\quad(\mathrm{I})$$

System II: Two thin copper shells

The conductor arrangement here considered comprises two hollow cylindrical shells with thin walls of copper or other good conducting material, the two shells being arranged concentrically and one acting as a return for the other. In the following mathematical treatment the outer cylindrical shell will be treated as a return for the inner shell, although this is purely conventional. As both the inner and outer radius of each conductor is of importance here, in the following formulæ $a_1=a$ is the outer radius and b_1 the inner radius of the inner shell, while a_2 is the outer radius and $b_2=b$ is the inner radius of the outer shell.

The impedance of a tubular conductor with concentric return outside is given by

$$Z_1=\frac{2\mu_1 i\omega M_o(x_1)}{x_1 M'_o(x_1)}\quad(8)$$

where

$$M_o(x_1)=J_o(x_1)+sK_o(x_1)$$

$$M'_o(x_1)=J'_o(x_1)+sK'_o(x_1)$$

$$s=\frac{J'_o(y_1)}{K'_o(y_1)}$$

$$x_1=ia_1\sqrt{ia_1}$$

$$y_1=ib_1\sqrt{ia_1}$$

(See formula (19), "Wave propagation over parallel tubular conductors" by S. P. Mead, Bell System Technical Journal, April, 1925.)

When the return is inside, the internal impedance is given by interchanging x and y

and reversing the sign, or, for the inner conductor

$$Z_2 = -\frac{2\mu_2 i\omega M_o(y_2)}{y_2 M'_o(y_2)} \quad (9)$$

Equations (8) and (9) may be written,

$$Z_1 = Z^o{}_1 F_1 \quad (10)$$

and

$$Z_2 = Z^o{}_2 F_2 \quad (11)$$

where $Z^o{}_1$ and $Z^o{}_2$ are Z_1 and Z_2 of System I. Now, upon writing the limiting values,

$$\frac{J_o(y)}{J'_o(x)} = i\sqrt{\frac{a}{b}}\,e^{-(a'-b')(1+i)/\sqrt{2}} = -\frac{J'_o(y)}{J_o(x)}$$

$$\frac{K'_o(x)}{K_o(y)} = -i\sqrt{\frac{b}{a}}\,e^{-(a'-b')(1+i)/\sqrt{2}} = -\frac{K_o(x)}{K'_o(y)}$$

$$\frac{J'_o(y)}{J'_o(x)} = \sqrt{\frac{a}{b}}\,e^{-(a'-b')(1+i)/\sqrt{2}} \quad (12)$$

$$\frac{K'_o(x)}{K'_o(y)} = \sqrt{\frac{b}{a}}\,e^{-(a'-b')(1+i)/\sqrt{2}}$$

where $a' = a\sqrt{\alpha}$ and $b' = b\sqrt{\alpha}$ (when $\lambda_1 = \lambda_2$, $\mu_1 = \mu_2 = 1$) F_1 and F_2 become

$$F_1 = F_2 = \frac{1+m}{1-m}, \quad (13)$$

where

$$m = e^{-\sqrt{2}\,(a'-b')(1+i)},$$
$$= e^{-n(1+i)}$$

Thus, Formula II follows immediately from Equations (10) to (13) and (3):

$$\gamma = \frac{\sqrt{\epsilon}}{4bv}\,\frac{1+\rho}{\log\rho}\,\frac{n(1+i)}{4\pi\lambda d}\,\frac{1+m}{1-m} + i\sqrt{\epsilon}\,\omega/v \quad (II)$$

Note that when $m \to 0$, which would be the case when n is large, i. e., when the shell is electrically thick, $F_1 = F_2 \to 1$ and Formula II reduces to Formula I. (From the relation

$$n = 4\pi d\sqrt{f\lambda}$$

it is of course evident that n will be large whenever the frequency f, the conductivity λ, or the thickness d of the shell is large.)

System III: Two lead conductors with copper skins

In the system here considered the effect of two thin-walled shells is obtained by the use of two cylindrical conductors of lead or other material of relatively poor conductivity, arranged concentrically, with a thin skin of copper or other good conducting material upon the inner surface of the outer lead sheath and upon the outer surface of the inner lead core. The copper skins are applied to these two surfaces because at high frequencies the current tends to crowd to the inner surface of the outer conductor and to the outer surface of the inner conductor, due to the so-called skin effect.

Consider the outer bi-metallic conductor comprising a lead sheath with a copper skin on its inner surface. The internal impedance Z_2 of the outer bi-metallic conductor is defined as before by the relation

$$E'_2 = -Z_2 I, \quad (14)$$

I being the current in the core or inner bi-metallic conductor and E'_2 the electric force at the inner surface of the outer conductor, i. e., at the inner surface of the copper skin. Referring now by subscript "2" to the copper skin and by subscript 3 to the lead sheath, as shown in the paper by Carson and Gilbert (page 716 in particular), E'_2 may be written as a linear function of I, the current in the inner conductor, and I_2 the current in the copper skin of the outer conductor, or

$$E'_2 = v'_2(I+I_2) - u'_2 I \quad (15)$$

where u'_2 and v'_2 are complicated expressions in Bessel functions involving the constants and dimensions of the copper skin. The ratio I_2/I is required. Expressions analogous to (14) for E_2, the electric force at the outer surface of the copper skin and for E'_3 at the inner surface of the lead sheath, are

$$E_2 = v_2\,(I+I_2) - u^2\,I,$$
$$E'_3 = v'_3(I+I_2+I_3) - u'_3(I+I_2) \quad (16)$$

where I_3 is the current in the lead sheath. But $I+I_2+I_3=0$, as there is no current outside of the lead sheath and, by the law of the continuity of the tangential electric force at the surface of separation of two conductors,

$$E_2 = E'_3.$$

Thus

$$\frac{I_2}{I} = \frac{u_2 - v_2 - u'_3}{u'_3 + v_2}, \quad (17)$$

and, from (14), (15) and (17),

$$Z_2 = u'_2 - \frac{u_2 v'_2}{v_2 + u'_3} \quad (18)$$

For the very high frequencies which we are considering,

$$u'_2 = \frac{1}{b_2}\,\frac{n}{4\pi\lambda d}\,\frac{1+m}{1-m}\,(1+i)$$

$$u_2 = v'_2 = \frac{1}{\sqrt{a_2 b_2}}\,\frac{n}{4\pi\lambda d}\,\frac{2m^{\frac{1}{2}}}{1-m}\,(1+i) \quad (19)$$

$$v_2 = \frac{b_2}{a_2}\,u'_2$$

$$u'_3 = \frac{1}{b_3}\sqrt{\frac{u_o f}{\lambda_o}}\,(1+i)$$

where a_2 and b_2 are the outer and inner radii of the copper skin, while a_3 and b_3 are the

the corresponding dimensions of the sheath. Here, as before,

$n=4\pi d\sqrt{f\lambda}$
λ = conductivity of copper skin
d = thickness of copper skin
$m=e^{-n(1+i)}$
μ_o, λ_o = constants of outer sheath (permeability and conductivity)

It is assumed here that $d_3=a_3-b_3$ is so large that

$$e^{-4\pi d_3\sqrt{\mu_o f\lambda_o}}=0.$$

Note that when

$$m^{\frac{1}{2}}=e^{-2\pi d\sqrt{f\lambda}}\rightarrow 0,$$

Equation (17) gives

$$\frac{I_2}{I}\rightarrow -1$$

or the current is confined to the copper skin and

$$Z_2\rightarrow u'_2.$$

The conductor is then equivalent to a copper shell alone. In fact, $Z_2=u'_2$ is identical with Formulas (9) and (11).

Turning now to the inner conductor or core, and referring to the copper skin by subscript "1" and to the material within by subscript "o", and having as before $I=I_o+I_1$, similarly,

$$\frac{I_1}{I}=\frac{-v'_1+u'_1+v_o}{u'_1+v_o}\quad(20)$$

and

$$Z_1=v_1-u_1\frac{v'_1}{v_o+u'_1}\quad(21)$$

where, assuming $a_1-b_1=a_2-b_2=d$,

$$u'_1=\frac{a_2}{b_1}v_2$$

$$u_1=v'_1=\sqrt{\frac{b_2}{a_1}\cdot\frac{a_2}{b_1}}u_2$$

$$v_1=\frac{a_2}{a_1}v_2=\frac{b_2}{a_1}u'_2$$

$$v_o=\frac{b_2}{a_o}u'_2=\frac{a_2}{b_1}u'_2$$

When n is so large, i. e., the frequency so high, that $m\rightarrow 0$, the current is confined to the copper skin and $Z_1=v_1$, which is identical with Equation (8). But $b_2/a_1=\rho$ and, since d is very small with respect to b_2 and a_1, with negligible error, we have also,

$$\frac{a_2}{b_1}=\rho.$$

Thus, from (21) and (18),

$$Z_1=\rho\, Z_2,\quad(22)$$

and, we may write

$$Z_2=u'_2F\quad(23)$$

where

$$F=1-\sqrt{\frac{\lambda_o}{\mu_o\lambda}}\,\frac{4m/(1-m^2)}{1+\sqrt{\frac{\lambda_o}{\mu_o\lambda}}\,\frac{1+m}{1-m}}\quad(24)$$

and, as in (19),

$$u'_2=\frac{1}{b}\,\frac{n(1+i)}{4\pi\lambda d}\,\frac{1+m}{1-m}.$$

Thus, on substituting Z_1 and Z_2 from Equations (22) and (23) in Formula (3), Formula III follows immediately; namely,

$$\gamma=\frac{\sqrt{\epsilon}}{4bv}\,\frac{1+\rho}{\log\rho}\,\frac{n(1+i)}{4\pi\lambda d}\,\frac{1+m}{1-m}F+i\sqrt{\epsilon}\;\omega/v\quad\text{(III)}$$

If, however, the bi-metallic core is replaced by a copper shell alone, Z_1 is given by

$$Z_1=v_1=\rho u'_2$$

so that

$$Z_1+Z_2=(F+\rho)u'_2$$

and F in III should be replaced by $(F+\rho)/(1+\rho)$.

It is interesting to note that when $m\rightarrow 0$, Formula III becomes Formula I, indicating that when the frequency is so high that the current is confined entirely to the copper, the thinness of the copper itself is no longer significant, that is, the copper skin itself is electrically thick. When $\lambda o=0$, Formula III becomes Formula II.

System IV: Three copper shells

The system here considered involves three concentric cylindrical conductors, an inner conductor, an intermediate conductor and an outer conductor, separated by two dielectric spaces which should be practically equivalent to air, or other gaseous dielectric. The three conductors, as will be shown later, may be treated as equivalent to two conducting systems, one involving the outer conductor as a return for the intermediate conductor and the other involving the intermediate conductor as a return for the inner conductor. From this standpoint the intermediate conductor may be formed of an inner and outer skin of good conducting material upon a shell of some low conducting material such as lead, or the two skins may be separated by dielectric. (In the latter case, the two intermediate skins could be used as a third circuit upon which an intermediate band of frequencies might be impressed.) If desired, the outer conductor may comprise a skin upon the inner surface of a lead sheath and the inner conductor may comprise a conducting skin upon a core of lead. At high frequencies, however, as will more clearly appear later, the inner and outer skins of the intermediate conductor may not be separated but may merge together without in any way interfering with the normal transmission characteristics of the system.

In this system of three conductors separated from one another by two dielectric spaces, an equation analogous to (1) holds for each dielectric space. Referring to the conductors, beginning with the innermost, by subscripts "1", "2" and "3" respectively, and distinguishing the potentials at the inner surfaces from those at outer surfaces by primes,

$$E'_2-E_1=-\left(\frac{\gamma^2}{i\omega C_{12}}-i\omega L_{12}\right)I_1 \quad (25)$$

and

$$E'_3-E_2=-\left(\frac{\gamma^2}{i\omega C_{23}}-i\omega L_{23}\right)(I_1+I_2) \quad (26)$$

Here, if b_1, b_2 and b_3 designate the inner radii of conductors 1, 2 and 3, and if a_1, a_2 and a_3 designate the outer radii, then by putting $b_3/a_2=b_2/a_1=\rho$,

$$C_{12}=C_{23}=\frac{k}{2\log\rho}$$

and

$$L_{12}=L_{23}=2\log\rho.$$

Introducing in (25) and (26) the linear expressions in terms of I_1 and I_2 for E_1, E_2, E'_2 and E'_3, gives two equations to be solved simultaneously for I_2/I_1 and λ^2. Eliminating I_2/I_1 gives a quadratic in γ^2, the two solutions γ_a and γ_b corresponding to the two possible modes of propagation in the system. The mode of propagation according to γ_a involves the conception of the intermediate and outer conductors acting in parallel as a return for the inner conductor, while the mode of propagation in accordance with γ_b involves the idea of the inner and intermediate conductors acting in parallel as a return for the outermost conductor.

Solving (25) and (26) for $x^2=\gamma^2+\omega^2LC$ instead of directly for γ^2 gives,

$$x=\frac{i\omega}{2}\Big\{C_{23}(v_2+u'_3)+C_{12}(u'_2+v_1)\pm$$
$$\sqrt{[C_{23}(v_2+u'_3)-C_{12}(v_1+u'_2)]^2+4u_2v'_2C_{12}C_{23}}\Big\} \quad (27)$$

where, as above, using the general subscript "i" in place of 1, 2 or 3 as the case may be

$$u'_i=\frac{1}{b_i}\,\frac{n(1+i)}{4\pi\lambda d}\,\frac{1+m}{1-m}=\frac{a_i}{b_i}v_i$$

$$u_i=\frac{1}{\sqrt{a_ib_i}}\,\frac{n(1+i)}{4\pi\lambda d}\,\frac{2m^{1/2}}{1-m}=v'_i$$

Then

$$\gamma^2=-\frac{\epsilon\omega^2}{v^2}\left(1-\frac{x^2}{\epsilon\omega^2/v^2}\right)$$

and, to a good approximation,

$$\gamma=\frac{i\sqrt{\epsilon}\omega}{v}\left(1-\frac{1}{2}\,\frac{x^2}{\epsilon\omega^2/v^2}\right) \quad (28)$$

Then, from Equations (27) and (28), Formula IV follows immediately; namely,

$$\left.\begin{matrix}\gamma_a\\ \gamma_b\end{matrix}\right\}=\frac{\sqrt{\epsilon}}{4b_3v}\,\frac{1+\rho}{\log\rho}\,\frac{n(1+i)}{4\pi\lambda d}\,\frac{1+m}{1-m}G+i\sqrt{\epsilon}\,\omega/v \quad (IV)$$

where

$$G=\frac{1}{2}\left\{1+\frac{b_3}{b_2}\mp\sqrt{\left(1-\frac{b_3}{b_2}\right)^2+\frac{16m}{(1+m)^2}\,\frac{b_3}{b_2}\,\frac{\rho}{(1+\rho)^2}}\right\}$$

Note that when the frequency is high so that $m\rightarrow 0$, G approaches either of two values,

$$\frac{b_3}{b_2}$$

or 1. Therefore

$$\gamma_a=\frac{\sqrt{\epsilon}}{4b_2v}\,\frac{1+\rho}{\log\rho}\,\frac{n(1+i)}{4\pi\lambda d}\,\frac{1+m}{1-m}+i\sqrt{\epsilon}\;\omega/v \quad (IVa)$$

and

$$\gamma_b=\frac{\sqrt{\epsilon}}{4b_3v}\,\frac{1+\rho}{\log\rho}\,\frac{n(1+i)}{4\pi\lambda d}\,\frac{1+m}{1-m}+i\sqrt{\epsilon}\;\omega/v \quad (IVb)$$

In this circumstance, the system may be treated theoretically as two independent pairs of conductors although the intermediate shell is common to the two circuits. In other words, since IV*a* is the equation for conductor No. 2 acting as a return for No. 1, while IV*b* is the equation for No. 3 acting as a return for No. 1, the two systems are independent.

Transmission characteristics

We have now developed general Formulæ I, II, III and IV for four different cases of concentric conductor systems. We will now show how from these equations the attenuation and phase distortion may be plotted with respect to a function of the frequency to produce curves which are perfectly general for a conductor system of each type, regardless of its dimensions.

Considering first System I, it is well known that γ, the propagation constant per unit length of any conducting system, may be expressed

$$\gamma=\alpha+i\beta \quad (29)$$

where α is the attenuation constant per unit length and β the phase constant per unit length. For convenience, the phase constant may be separated into two components, one of which is proportional to frequency and the other of which is a much smaller component representing the phase distortion.

Thus,

$$\beta=\beta_1+\beta_2 \quad (30)$$

where β_1 is the phase distortion and β_2 is the component proportional to frequency which may be expressed

$$\beta_2=\sqrt{\epsilon}\frac{\omega}{v} \quad (31)$$

Rewriting (29) we have

$$\gamma=\alpha+i(\beta_1+\beta_2) \quad (32)$$

From Equation (32) and Equation (I), assuming that $\epsilon=1$, as will be the case where the dielectric is air or an equivalent material, we have

$$\gamma=\left(\frac{1}{4bv}\cdot\frac{1+\rho}{\log\rho}\cdot\frac{1}{4\pi\lambda d}\right)(n+in)+i\frac{\omega}{v} \quad (33)$$
$$=\alpha+i\beta_1+i\beta_2$$

Since from Equation (31) $i\beta_2=i\frac{\omega}{v}$, it is evident from Equation (33) that we may write

$$\alpha+i\beta_1=\left(\frac{1}{4bv}\cdot\frac{1+\rho}{\log\rho}\cdot\frac{1}{4\pi\lambda d}\right)(n+in) \quad (34)$$

Equating the real parts and the imaginary parts of Equation (34) we have

$$\alpha=\left(\frac{1}{4bv}\cdot\frac{1+\rho}{\log\rho}\cdot\frac{1}{4\pi\lambda d}\right)n \quad (35)$$

and

$$i\beta_1=i\left(\frac{1}{4bv}\cdot\frac{1+\rho}{\log\rho}\cdot\frac{1}{4\pi\lambda d}\right)n \quad (36)$$

From Equations (35) and (36) it is clear that α and β_1 may be plotted as the same straight line, as shown for example by the solid line 10 of Fig. 1. The curve of Fig. 1 may be taken as representing the values proportional to either α or β_1 plotted as ordinates against the various values of

$$n=4\pi d\sqrt{f\lambda}$$

plotted as abscissæ. Since n is a function of frequency, the values of α and β_1 may be obtained from the curve 10 for any frequency, and the curve is perfectly general for any conducting system of type I. In determining the actual values of α and β_1 in attenuation units and radians respectively, at any given frequency from the curve 10 of Fig. 1, it is necessary to multiply the corresponding ordinate by the factor

$$\left(\frac{1}{4bv}\cdot\frac{1+\rho}{\log\rho}\cdot\frac{1}{4\pi\lambda d}\right)$$

which is common to both α and β_1, as shown by Equations (35) and (36). The actual ordinates used in plotting the curve 10 were obtained by first dividing out this factor. The values of α and β_1 will be in c. g. s. units and, therefore, to obtain the attenuation and phase distortion per mile it is necessary to multiply the ascertained values of α and β_1 by $.161\times10^6$, the number of centimeters in a mile.

It is interesting to note that the factor $(1+\rho)/\log\rho$ has a minimum when $\rho=3.59$. Consequently, both the attenuation and phase distortion will be a minimum when the ratio $b/a=3.59$. Assuming this optimum ratio and 2⅜ inches for the inside diameter of the sheath or outer conductor, when $f=10^6$ we have $\alpha=0.56$ TU per mile, $\beta_1=0.07$ and $\beta_2=33.7$ radians per mile. Hence, the phase distortion is practically negligible and the attenuation is comparatively small for a frequency of 10^6 cycles. This value of attenuation, it will be seen, gives a current ratio of 1/10th in about 35 miles.

Coming now to System II, it is evident that an equation in terms of α and β_1 may be obtained from Equation (II) in a manner analogous to that by which Equation (34) was obtained from Equation (I). This equation is as follows:

$$\frac{\sqrt{\epsilon}}{4bv}\frac{1+\rho}{\log\rho}\frac{n(1+i)}{4\pi\ \lambda d}\frac{1+m}{1-m}=\alpha+i\beta_1 \quad (37)$$

Equating the real part of the left-hand term of Equation (37) to α and the imaginary part to $i\beta_1$, it will be found that

$$\left(\frac{1}{4bv}\cdot\frac{1+\rho}{\log\rho}\cdot\frac{1}{4\pi\ \lambda d}\right)$$

will be a common factor to both α and β_1. Dividing both α and β_1 by this factor and plotting curves corresponding to α and β_1 we obtain the dotted curves 11 and 12 of Fig. 1, these curves representing the case of two thin shells. As in the case of curve 10, the values of α and β_1 as given by the dotted line curves must be multiplied by the above factor to give the true values of the attenuation in TU and the phase distortion in radians.

It will be noted that the dotted curve 11 (which shows how the attenuation α varies with respect to frequency) departs but little from constant attenuation up to value of $n=.5\pi$, so that we have as a characteristic of the thin shell type of circuit substantial attenuation equalization up to the frequency corresponding to the value $n=.5\pi$. Remembering that

$$n=4\pi d\sqrt{f\lambda},$$

and setting

$$.5\pi=4\pi d\sqrt{f\times6.06\times10^{-4}},$$

where for copper $\lambda=6.06\times10^{-4}$, we have

$$f=\frac{25}{d^2}$$

approximately as the frequency up to which substantial equalization is obtained. This relation is the criterion for determining the thickness of copper conductor necessary to equalize up to any given frequency.

Again noting the shape of the curve 11, it will be found that even up to a value of $n=\pi$ the increase in attenuation is quite tolerable. On the assumption that we will tolerate the amount of increase in attenuation indicated at π, we have

$$n=\pi=4\pi d\sqrt{f\lambda},$$

from which

$$f=\frac{100}{d^2}$$

approximately becomes the criterion for determining the thickness of copper conductor ($\lambda=6.06\times10^{-4}$) necessary to equalize within the assumed limits of variation up to a given frequency. For example, for the case where $f=10^6$, we have

$$d=\sqrt{\frac{100}{f}}=\frac{1}{100}$$

centimeters approximately.

If in Fig. 1 the curves 11 and 12 were plotted for higher values of n, it would be found that at a value of about $n=1.5\pi$ the curves 11 and 12 reached their common asymptote, which is the straight line 10. This means that, due to the skin effect, the current at these high frequencies is now confined to the surfaces of the thin shell of System II, so that the thin-walled conductors are now equivalent electrically to the thick-walled conductors considered under case I.

System III, as has already been stated, involves the use of a solid core of lead or other low conducting material, or at least a core comprising a hollow cylinder with thick walls, and a return conductor or sheath likewise comprising a thick-walled hollow cylinder of lead or other material of low conductivity, with a thin skin of copper or other good conducting material upon the inner surface of the outer lead sheath and upon the outer surface of the lead core. From Equation (III) it will be evident, by analogy to the treatment already applied to Equations (I) and (II), that the factor

$$\left(\frac{1}{4bv}\cdot\frac{1+\rho}{\log\rho}\cdot\frac{1}{4\pi\lambda d}\right)$$

is again common to both the attenuation α and the phase distortion β_1, as obtained from Equation (III). With this factor out, the corresponding values proportional to α and β_1 may be plotted as shown by the curves 11′ and 12′ in Fig. 2. Comparing these curves with the curves 11 and 12 for the case of thin shells without the lead sheath and core, it will be seen that there is some increase in the phase distortion due to the lead, and that the equalization of attenuation at different frequencies is not so good as in the case of the thin copper shells.

If, however, the lead core with outer skin of copper be replaced by a copper shell, and the copper skin on the outer lead sheath be retained for the outer or return conductor, the equalization of attenuation is very good, as is indicated by the curve 11″ of Fig. 2, and the phase distortion is decreased to values approximating those of system II, as is indicated by the curve 12″ of Fig. 2. The curves 11″ and 12″ are obtained by substituting for F in Equation (III) the factor

$$\frac{F+\rho}{1+\rho}$$

as was previously indicated in the general mathematical treatment relating to System III (see particularly the comments immediately following Equation III).

In the case where the lead is employed in connection with both the inner and outer conductors, as well as in the case where it is only employed in the outer conductor, the values of α and β_1 approach their values for thin copper shells alone when n is in the neighborhood of π, as will be clear from the curves of Fig. 2. At this point, however, the current is practically confined to the copper, so that α and β_1 rapidly approach their values for thick copper conductors. If the curves were plotted for higher values of n, it would be found that they all reach their common asymptote, a straight line corresponding to 10 of Fig. 1, at about 1.5π.

At frequencies as high as those we are here considering (from the neighborhood of 100,000 cycles to 1,000,000 cycles or more), a lead sheath of the ordinary thickness, which is about ⅛th of an inch, will serve as a shield from interference. For a frequency of 10^5 cycles and a thickness of ⅛th inch,

$$n=4\pi\times.125\times2.54\sqrt{.444\times10^{-4}f}=8.4.$$

(In the foregoing expression 2.54 is the number of centimeters in an inch and $.444\times10^{-4}$ is the conductivity of lead.) The ratio of the electric force E at the outer surface of the lead to the electric force E′ at the inner surface, which is roughly a measure of the protection against interference, is proportional to $e^{-n/2}$, which is less than 2 per cent. in this case, and rapidly decreases for higher frequencies.

When for mechanical or other reasons it is necessary to make the copper skin somewhat thicker than is required for good equalization, as described in connection with systems II and III, or when it is desired to extend the frequency range over which equalization is obtained so that the corresponding values of α no longer lie on the equalized portion of the curve, the attenuation may be substantially equalized by employing a multiple concentric conductor system such as outlined under the heading System IV. As has been previously pointed out, the intermediate conductor may be lead or some flexible insulating material with a skin of high conductivity on both sides, or it may be composed entirely of a good conducting material, such as copper. Then, by dividing the frequency range between the two circuits thus formed (the inner conductor with the intermediate conductor as a return and the intermediate conductor with the outer conductor as a return) considerable attenuation equalization is obtainable. This is due to the fact that the attenuation is inversely proportional to the inner radius of the outer of the two conductors (see factor b in Equations (I), (II) and (III) and factors b_2 and b_3 in Equations (IVa) and (IVb)). If, for example, the lower frequency band be impressed upon a circuit having a given radius b for the inner surface of its outer conductor, the attenuation of the higher band will be decreased if such higher band is impressed upon the circuit having the larger b. This is illustrated in Fig. 3. The three-conductor system is in this figure shown as comprising three cylindrical members, No. 1, No. 2 and No. 3. The lower frequency band is symbolically represented at G_a as being applied between conductor No. 1 and conductor No. 2, while the higher frequency band is symbolically represented at G_b as being applied between conductor No. 2 and conductor No. 3.

Treating the two transmission systems as independent systems and employing Formula (III), the attenuation for the inner circuit is represented by the group of curves designated α_a and the attenuation for the outer circuit is indicated by the curve designated α_b. These curves are plotted on the assumption that b_3, the radius of the inner surface of shell No. 3, is twice b_2, the radius of the inner surface of shell No. 2. Assuming the copper skin to be 0.01 inch in thickness, when $f=10^5$, $n=0.79\pi$, and when $f=10^6$, $n=2.5\pi$.

In the group of curves designated α_a in Fig. 3, the straight line curve 20 represents the attenuation for the case when the inner and intermediate conductors comprise two copper cylinders with thick walls. The solid line curve 21 represents the case where the conductors No. 1 and No. 2 are thin-walled copper shells, while the dotted line curve 21′ represents the case for a system in which conductors No. 1 and No. 2 are lead conductors with copper skins on their contiguous surfaces. The proximity of curves 21 and 21′ when $n>0.5\pi$ indicates that the current in that region is almost entirely confined to the copper so that the presence of the lead makes practically no difference and the convergence of the two curves with the straight line at about 1.3π indicates skin conduction. The curve α_b is (over the range plotted) a straight line curve indicating that at the high frequencies involved the transmission over conductors No. 2 and No. 3 is entirely confined to the copper skin. Obviously, therefore, some other material of low conductivity might be substituted for the lead between the two copper skins of conductor No. 2, or the entire conductor might be solid copper. In the latter case it would, however, be necessary to use selective apparatus at the terminal to separate the two frequency ranges involved.

The curves 20, 21 and 21′ are not plotted in Fig. 3 for values of n lower than $.5\pi$. The curve 20, however, continues as a straight line at the lower frequencies just as did the curve 10 of Fig. 1. The character of curves 21 and 21′ for values of n lower than $.5\pi$ may be obtained from curves 11 and 11′ of Fig. 2 by multiplying the ordinates plotted in the latter figure by 2. This is for the reason that in the case of the conducting system plotted in Fig. 3 it was assumed that the inner diameter b_3 of conductor No. 3 was the same as the inner diameter b of the outer conductor of Fig. 2, while in Fig. 3 we are further assuming $b_3/b_2=2$.

Returning again to Fig. 3, it will be noted that α_a, as indicated by curve 21, increases relatively from 4 at $n=.5\pi$ to 8 at $n=1.3\pi$, the latter corresponding to a frequency of about 2.5×10^5 cycles per second. Now, if this latter frequency were impressed between the two outer conductors instead of between the two inner conductors α, the attenuation would be reduced by the factor b_2/b_3 which, in the case assumed in the drawing, is ½. Consequently, if all frequencies from 2.5×10^5 to 10^6 are impressed on the outer circuit, the attenuation α in this band will also increase in relative value approximately from 4 to 8. On the other hand, if the outer circuit comprising conductors No. 2 and No. 3 had been used for the whole frequency range, the increase in attenuation would be from about 2 to 8.

The range of variation in attenuation may be reduced by increasing the number of intermediate conductors. The number is, of course, limited by the overall size of the system, and the absolute value of the attenuation will naturally increase as the ratio of the diameters of the outer and innermost circuits increases.

In the foregoing discussion of Fig. 3, it has been assumed that the two conducting systems (conductor No. 1 with conductor No. 2 as a return and conductor No. 2 with conductor No. 3 as a return) may, without substantial error, be treated as two independent conducting systems. Consequently, the curves α_a and α_b in Fig. 3 were plotted by using Formula (III). How little error is involved in this assumption will be clear from a consideration of Fig. 4. Here attenuation curves are plotted for a system involving conductors No. 1, No. 2 and No. 3, each comprising a thin-walled copper shell. The radii for the several shells were so chosen that the value of the ratio ρ for minimum attenuation is obtained. In other words, b_3/a_2 and b_2/a_1 are both made equal to 3.59. As it happens these particular values are not well chosen from the standpoint of equalization of attenuation, and better equalization would have been obtained if the radius of conductor No. 2 had been made larger. However, this does not affect the point which is now being made, namely, that no substantial error is involved in the assumption that the three-conductor system may be treated as two two-conductor systems.

By the use of Equation (IVa) in a manner analogous to that already described in connection with Equation (I) the full line curve 31 is obtained. This curve represents the attenuation when the system is employed with conductors No. 2 and No. 3 in parallel as a return for conductor No. 1. The dotted line curve 31′ of Fig. 4 is plotted in accordance with Formula (II) for the case where conductor No. 2 acts as a return for conductor No. 1. The close approximation of the two curves is evident from the drawing.

In a similar manner the curve 41 of Fig. 4 is plotted from Equation (IVb) and represents the attenuation when the system of conductors is used with conductors No. 1 and No. 2 in parallel as the return for No. 3. Curve 41′ represents the attentuation for conductor No. 3 as a return for conductor No. 2 plotted from Formula (II). As before, the two curves closely approximate each other, and in both cases when n is in the neighborhood of π or larger, the two sets of curves become practically identical. It is, therefore, evident that we are justified in treating any two adjacent conductors, when used as a transmission system, as though the third conductor were not present.

As previously stated, by making the radius of conductor No. 2 larger than was chosen for the particular curves plotted in Fig. 4, the attenuation α_a would be lower and the attenuation α_b would be higher, so that a fair degree of equalization of attenuation would be obtained by transmitting the lower band of frequencies over one pair of conductors and the higher band of frequencies over the other pair, as described in connection with Fig. 3.

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. In a conducting system for the communication of intelligence, a pair of conductors in the form of concentrically arranged cylindrical shells of conducting material, the thickness of the walls being proportioned with respect to a preassigned frequency, so that a band of frequencies from zero up to said preassigned frequency will be transmitted with substantially uniform attenuation and negligible phase distortion.

2. In a conducting system for the communication of intelligence, a pair of conductors in the form of concentrically arranged cylindrical shells of conducting material, the thickness of the walls being proportioned with respect to a preassigned frequency substantially in accordance with the relation

$$f=\frac{1}{64\lambda d^2},$$

where f is the preassigned frequency and d is the thickness of the wall in centimeters, whereby all frequencies from zero up to f will be transmitted with substantially uniform attenuation and negligible phase distortion.

3. In a conducting system for the communication of intelligence, a pair of conductors in the form of concentrically arranged cylindrical shells of conducting material, the thickness of the walls being proportioned with respect to a preassigned frequency substantially in accordance with the relation

$$f=\frac{1}{16\lambda d^2},$$

where f is the preassigned frequency and d is the thickness of the wall in centimeters, whereby all frequencies from zero up to $f/4$ will be transmitted with substantially uniform attenuation and the increase in attenuation of frequencies from $f/4$ to f will be less than fifty per cent.

4. In a conducting system for the communication of intelligence, a pair of conductors in the form of concentrically arranged cylinders, the outer cylinder comprising a skin of highly conductive material on the inner surface of a sheath of less conductive material, the thickness of said skin being proportioned with respect to a preassigned frequency, so that a band of frequencies from zero up to said preassigned frequency will be transmitted with substantially uniform attenuation and negligible phase distortion.

5. In a conducting system for the communication of intelligence, a pair of conductors in the form of concentrically arranged cylinders, the outer cylinder comprising a skin of highly conductive material on the inner surface of a sheath of less conductive material, the thickness of said skin being proportioned with respect to a preassigned frequency substantially in accordance with the relation

$$f=\frac{1}{64\lambda d^2},$$

where f is the preassigned frequency and d

is the thickness of the wall in centimeters, whereby all frequencies from zero up to f will be transmitted with substantially uniform attenuation and negligible phase distortion.

6. In a conducting system for the communication of intelligence, a pair of conductors in the form of concentrically arranged cylinders, the outer cylinder comprising a skin of highly conductive material on the inner surface of a sheath of less conductive material, the thickness of said skin being proportioned with respect to a preassigned frequency substantially in accordance with the relation

$$f=\frac{1}{16\lambda d^2}$$

where f is the preassigned frequency and d is the thickness of the wall in centimeters, whereby all frequencies from zero up to $f/4$ will be transmitted with substantially uniform attenuation and the increase in attenuation of frequencies from $f/4$ to f will be less than fifty per cent.

7. In a conducting system for the communication of intelligence, at least three cylindrical conductors concentrically arranged, the inner and intermediate conductors being connected to form one transmission circuit, and the intermediate and third conductors forming another and independent transmission circuit.

8. In a conducting system for the communication of intelligence, at least three cylindrical conductors concentrically arranged, the inner and intermediate conductors being connected to transmit one range of frequencies, and the intermediate and third conductors being connected to transmit another range of frequencies.

9. In a conducting system for the communication of intelligence, at least three cylindrical conductors concentrically arranged, the inner and intermediate conductors being connected to transmit a lower range of frequencies, and the intermediate and third conductors being connected to transmit a higher range of frequencies.

In testimony whereof, we have signed our names to this specification this 16th day of May, 1929.

JOHN R. CARSON.
SALLIE P. MEAD.