

July 21, 1931.

R. S. HOYT

1,815,255

IMPEDANCE EQUALIZER FOR ANY SMOOTH LINE

Filed Nov. 20, 1929

2 Sheets-Sheet 1

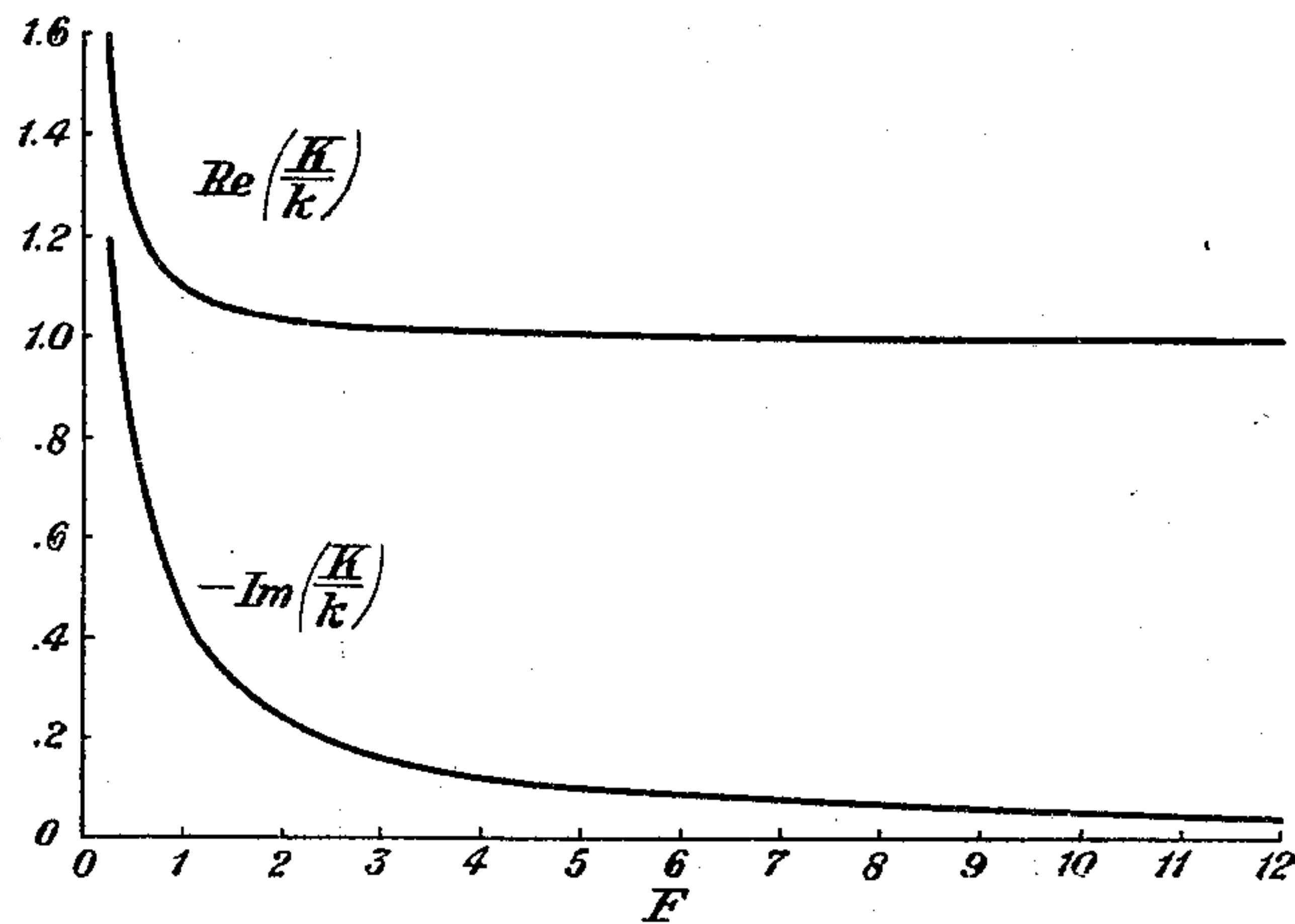


Fig. 1

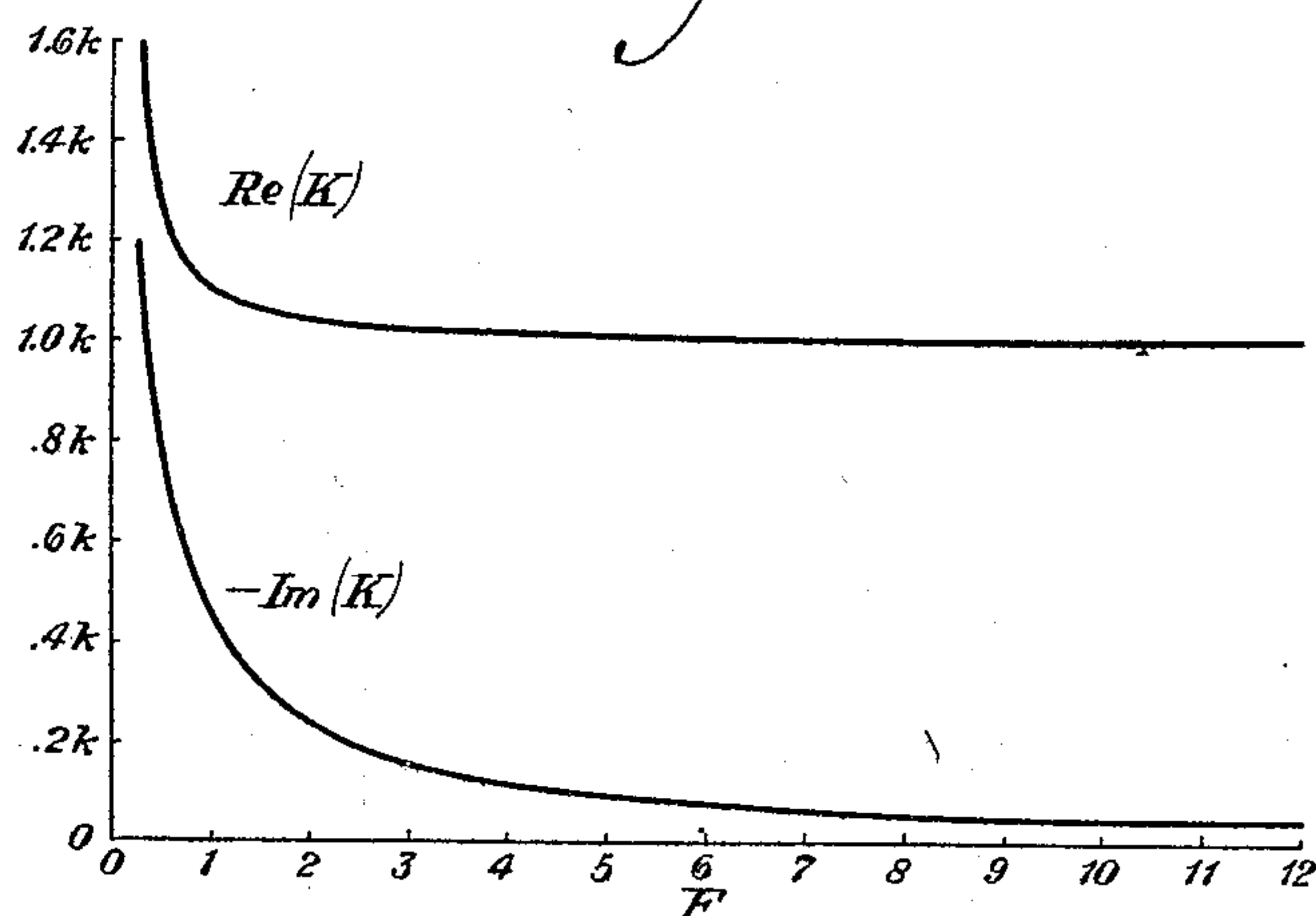


Fig. 2

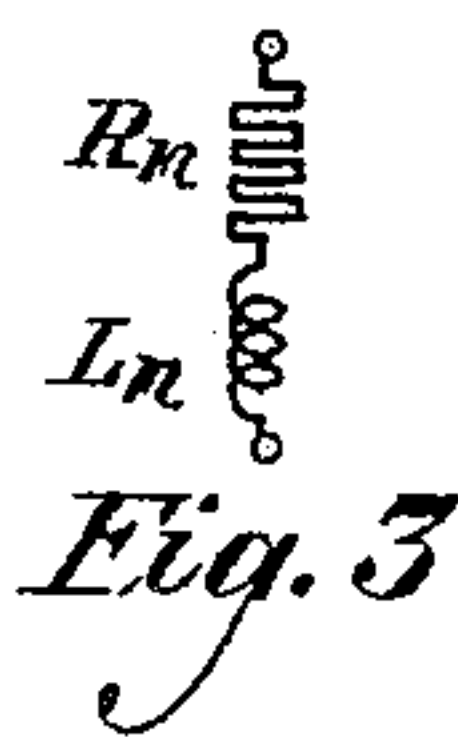


Fig. 3

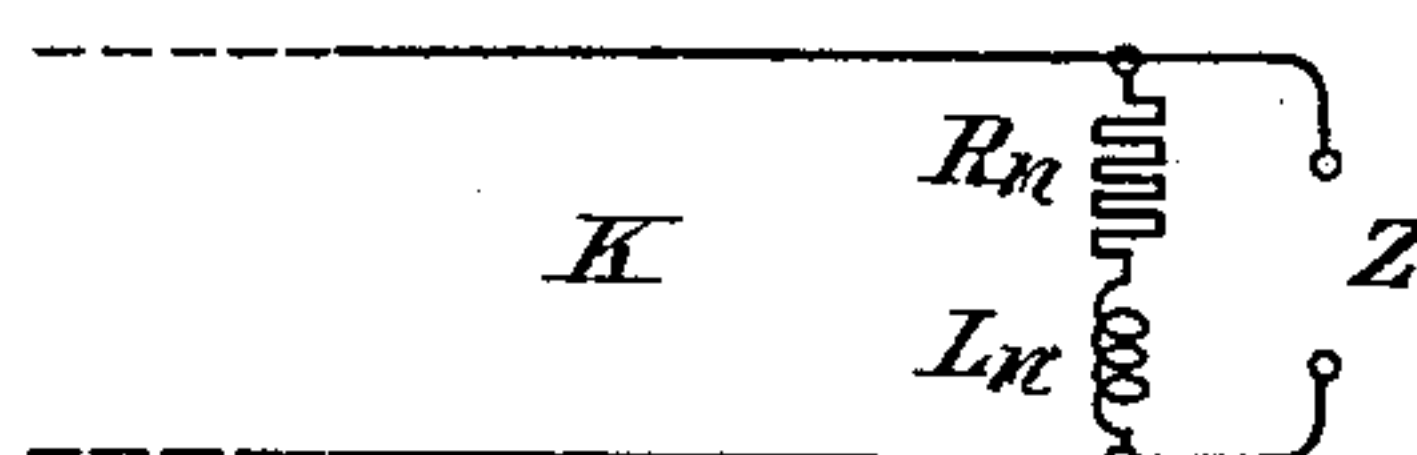


Fig. 4

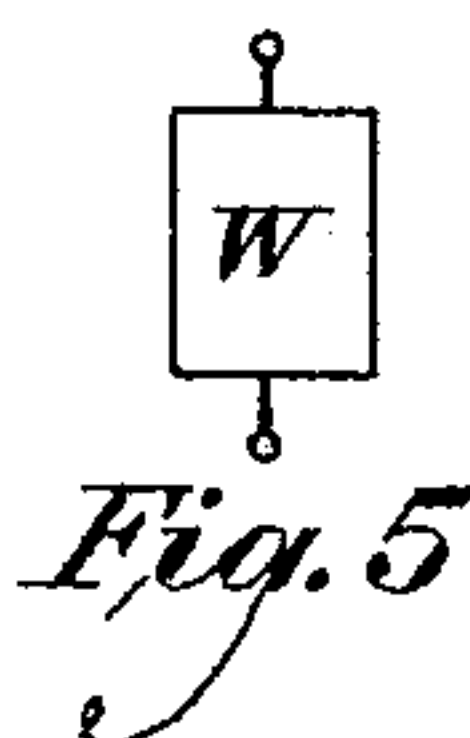


Fig. 5

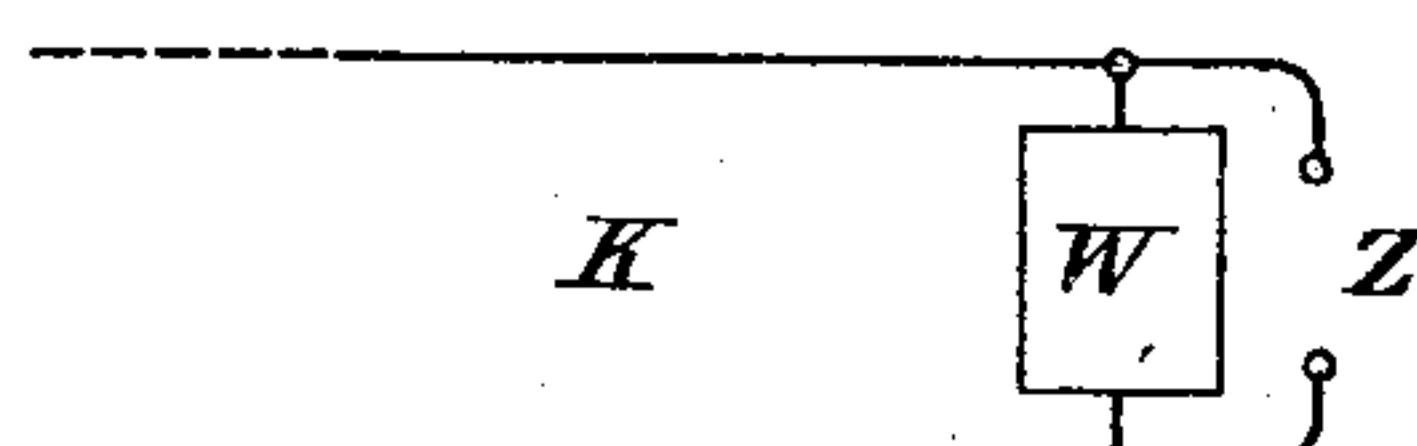


Fig. 6

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2 Sheets-Sheet 2

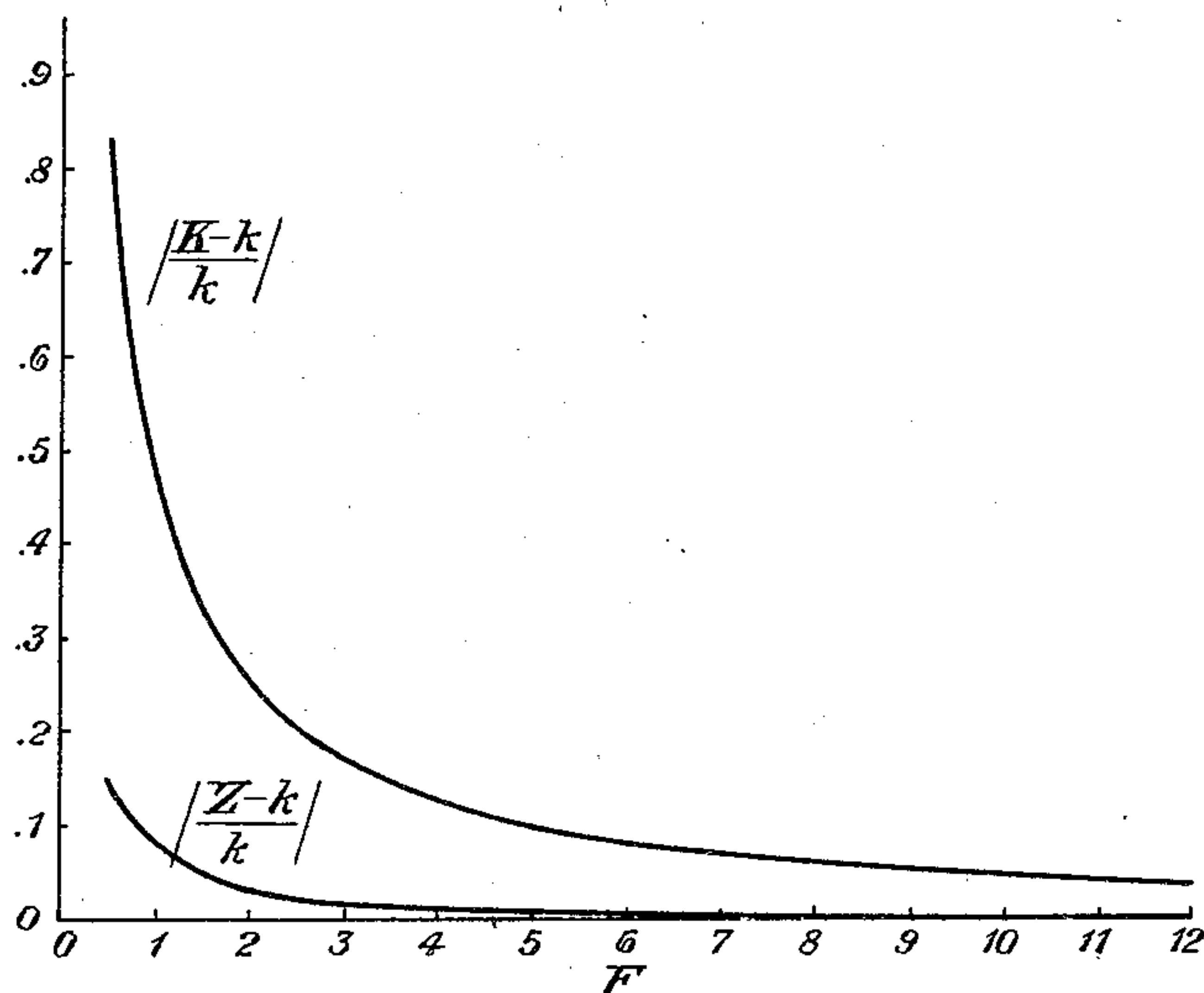


Fig. 7

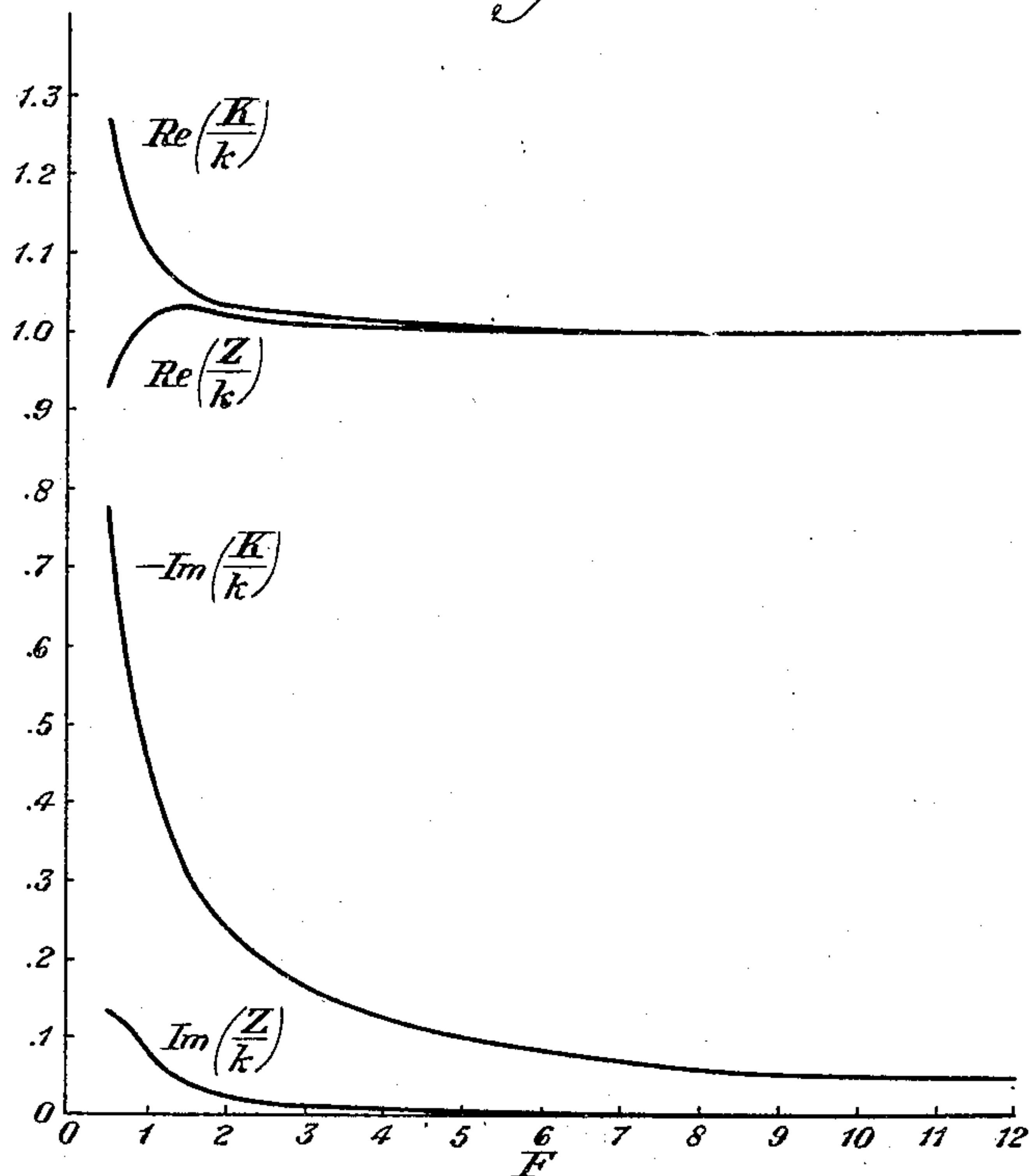


Fig. 8

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IMPEDANCE EQUALIZER FOR ANY SMOOTH LINE

Application filed November 20, 1929. Serial No. 408,577.

It is well-known in communication engineering that the characteristic impedance (iterative impedance) of a smooth line varies considerably with frequency, over the voice frequency-range, particularly toward the lower end of this range, though at high frequencies the impedance approaches a constant resistance.

The term 'smooth line' here includes, as usual, any electrical transmission line whose fundamental parameters (resistance, inductance, leakance, capacitance) are sensibly uniformly distributed along the line; thus among smooth lines are included non-loaded open-wire lines, non-loaded cables, and uniformly loaded cables (aerial, underground, and submarine). As here used, the term 'smooth line' will be understood to include also any line which is effectively smooth in the sense that, over the contemplated frequency-range, its characteristic impedance is approximately the same as though the line parameters were uniformly distributed along the line; thus, for instance, a periodically loaded line is effectively smooth over the frequency-range in which the distance between loads is small compared with the wave-length.

For some purposes the existing rather large departure of the line impedance at the lower frequencies is undesirable, or even very harmful so that it is desirable to have a compensating network to associate with the initial end of the line in order that the resultant impedance shall be approximately a mere resistance over a wide frequency-range; and, more particularly, that this resistance shall be approximately equal to the value of the characteristic impedance at high frequencies. Such a network will here be termed an "impedance-equalizer" or, briefly, an "equalizer".

Some of the possible uses of "impedance-equalizers" are as follows:

To enable the impedance of any smooth line, over a wide frequency range, to be simu-

lated or to be balanced by a mere constant resistance.

To enable two smooth lines, originally having unequal impedances, to balance each other when a 21-type repeater is worked between them (the equalized lines being rendered equal by the addition of a series resistance to the smaller or by a shunt resistance to the larger, or by the insertion of suitable transformers between the repeater and the equalized lines).

To make the "power factor" of a smooth line, at its terminals, equal to unity over a wide frequency range; that is, to eliminate the wattless component of the current entering the system.

To reduce the building-up time of the entering current, by reducing the transient distortion. (If the impedance of the system were a pure resistance at all frequencies, there would be no transient distortion, and hence the building-up time would be zero; thus the current would attain instantly its steady-state value).

To reduce the discharging time of the system when the source of impressed electromotive force is replaced by an impedance; that is, to reduce the "relaxation time" of the system. (This reduction can be understood from the fact that the removal of an electromotive force is equivalent to the insertion of its negative; the steady-state part of the current produced by this negative e. m. f. exactly annuls the current—supposed to have reached its steady-state value—due to the original e. m. f., and the transient distortion part produced by the negative e. m. f. is reduced by the presence of the impedance equalizer, in accordance with the preceding paragraph.)

The principal object of the present invention is to provide a simple form of shunt-type impedance-equalizer for combination with a smooth line so that the resultant impedance shall be approximately equal to a mere constant resistance over a wide frequency range.

quency-range, such as the voice frequency-range.

Before proceeding further with this specification, the attached figures will be summarized: Figures 1 and 2 show generic graphs of the characteristic impedance of smooth lines, over a wide frequency-range (F being proportional to the frequency). Fig. 3 represents the network of the invention; and Fig. 4 shows it combined with a smooth line. Figs. 5 and 6 are generalizations of Figs. 3 and 4, respectively, W denoting any impedance; these two figures, 5 and 6, are introduced for the purpose of attaining greater generality and simplicity in the exposition. Figs. 7 and 8 present graphs showing the precision of equalization attainable by means of the network of Fig. 3 when combined with a smooth line as in Fig. 4.

Before setting forth the theory, design-methods, and design-formulae of this invention, it will be desirable to review briefly but in a precise manner the nature of the dependence of the characteristic impedance of a smooth line on the frequency. By the characteristic impedance of any transmission line is here meant, as usual, the line impedance when the line is so long that its impedance is sensibly independent of the distant terminating impedance.

The well known general formula for the characteristic impedance K of any smooth line is

$$K = \sqrt{(R + i\omega L)/(G + i\omega C)} \quad (1)$$

R, L, G, C denoting the line parameters per unit length, namely the resistance, inductance, leakance, capacitance; ω denoting 2π times the frequency f ; and $i = \sqrt{-1}$.

The foregoing formula (1) for K has been very thoroughly studied in my article "Impedance of smooth lines and design of simulating networks" published in the Bell System Technical Journal of April, 1923; also in my U. S. Patent 1,713,603, issued May 21, 1929. As there stated, the effects produced on the impedance K by normal values of the leakance G are slight except at very low frequencies (below the voice range). Hence, for the present expository discussion of the nature of the dependence of the impedance K on the frequency, the leakance may be neglected. This results in a great gain in simplicity of exposition; for, with G set equal to zero, Equation (1) can be written

$$K = k\sqrt{1 - i/F} \quad (2)$$

where

$$k = \sqrt{L/C} \quad (3)$$

and

$$F = \omega L/R \quad (4)$$

Thus when the line parameters (R, L, C) are independent of the frequency, k is a mere constant and F is directly proportional to the fre-

quency; k , called the nominal impedance, is the limiting value of K at very high frequency, as is seen from (2) by setting $F = \infty$.

Usually the line parameters (R, L, C) are very nearly independent of the frequency over at least the voice-frequency range.

Since Equation (2) can be written

$$K/k = \sqrt{1 - i/F} \quad (5)$$

it is seen that the nature of the dependence of the characteristic impedance of all non-leaky smooth lines on the frequency can be represented by a single graph, namely a graph of

$$\sqrt{1 - i/F}$$

as function of F . In Fig. 1, which presents such a graph, the upper and lower curves depict respectively the real (Re) part and the negative of the imaginary (Im) part of

$$K/k = \sqrt{1 - i/F}$$

Hence, in any specific case, these curves, after their ordinates are multiplied by the specific value of k , depict respectively the resistance component and the negative of the reactance component of K , as functions of F , as in Fig. 2. The considerable departure of K from its limiting value k , particularly at the lower values of F , is clearly and simply shown by Fig. 2.

Fig. 3 represents the equalizing network of my invention, consisting of a resistance R_n in series with an inductance L_n , so that the impedance W of this network at any frequency $f = \omega/2\pi$ is

$$W = R_n + i\omega L_n \quad (6)$$

Fig. 4 represents the equalizing network of Fig. 3 bridged across the initial end of a smooth line of impedance K ; the resultant impedance of the combination is denoted by Z . Like the other impedances, Z is of course a function of the frequency $f = \omega/2\pi$, and can be expressed in terms of F by means of the substitution $\omega = RF/L$ obtained from (4).

The formula for the resultant impedance Z of Fig. 4, will now be studied for the purpose of demonstrating the impedance equalizing properly possessed by the network (Fig. 3) of my invention, and also for the purpose of deriving general design formulae for its proportioning in any specific case.

For this purpose it is convenient to start with the generic case represented by Figs. 5 and 6, in which W denotes any impedance. In terms of K and W the resultant impedance Z of the system of Fig. 6 is

$$Z = \frac{KW}{K + W} \quad (7)$$

If the desired or prescribed value of Z is denoted by B , then the precision of equalization attained will evidently be shown by a graph of the ratio Z/B as function of F ; and

also by a graph of the fractional departure $(Z-B)/B$, where the two vertical bars enclosing an expression denote that the absolute value of the expression is to be taken. From (7), and the equation

$$\lambda = B/k \quad (8)$$

defining λ , we get

$$\frac{Z}{B} = \frac{(K/k)(W/k)}{\lambda[(K/k) + (W/k)]} \quad (9)$$

from which $(Z-B)/B$ is obtainable by merely subtracting unity, since

$$\frac{Z-B}{B} = \frac{Z}{B} - 1 \quad (10)$$

In order that the equalization shall be exact, Z/B must be equal to unity, whence the requisite value of W/B for producing exact equalization is

$$\frac{W}{B} = \frac{K/k}{(K/k) - \lambda} \quad (11)$$

which thus constitutes the general design formula for expressing the requisite value of W/B in terms of K/k and λ , as function of F .

In particular, when Z is prescribed to be equal to k , so that $B=k$ and $\lambda=1$, and when the line leakance G is negligible so that K/k has the value expressed by Equation (5), then the requisite value of W/k is

$$\frac{W}{k} = \frac{\sqrt{1-i/F}}{\sqrt{1-i/F}-1} \quad (12)$$

or, on multiplying numerator and denominator by

$$iF(\sqrt{1-i/F}+1).$$

$$W/k = 1 + iF + iF\sqrt{1-i/F} \quad (13)$$

For the particular case here contemplated, each of these equivalent equations, (12) and (13), constitutes an exact design-formula for W/k ; but (13) will usually be the more convenient.

It it were possible to devise a network whose impedance W would vary with F in exact accordance with Equation (11), that network, when associated with the line as in Fig. 6, would make the resultant impedance Z exactly equal to B at all frequencies. It will now be shown how closely this can be attained by the network of Fig. 3 of my invention.

To arrive at first approximation design-formulae for the network of Fig. 3, it may be noted that, for all values of F considerably greater than unity, the design-formula (13) reduces approximately to

$$W/k = 1 + i2F \quad (14)$$

which, by (4), can be written

$$W = k + i\omega(2kL/R) \quad (15)$$

This, when compared with (6), shows that

$$R_n = k \quad (16)$$

$$L_n = 2kL/R \quad (17)$$

These two equations, (16) and (17), therefore constitute the design-formulae for so proportioning the network of Fig. 3 as to secure good precision of equalization for at least all values of F considerably greater than unity. When the network of Fig. 3 is proportioned in accordance with formulae (16) and (17), the exact formula for Z/k is, by (9),

$$\frac{Z}{k} = \frac{(1+i2F)(K/k)}{1+i2F+(K/k)} \quad (18)$$

For the case where the line leakance G is negligible, so that K/k is given by (5), the precision of equalization attained by means of the network of Fig. 3, when proportioned in accordance with equations (16) and (17), is depicted by the graphs in Figs. 7 and 8. Each of these figures shows that the equalization attained is tolerably good down to values of F as low even as $F=1$ and rapidly improves with increasing F . These facts are shown most simply by Fig. 7, which gives a graph of the fractional departure $|(K-k)/k|$ of the line before equalization, and a graph of the fractional departure $|(Z-k)/k|$ after equalization. Fig. 8 gives graphs of K/k and Z/k by giving graphs of their real (Re) parts and of their imaginary (Im) parts—except that, for convenience, the negative of the imaginary part of K/k is shown.

From the nature of their derivation, the design-formulae (16) and (17) favor the higher frequencies. In order to render these formulae elastic, let them now be written in the more general forms.

$$R_n = pk \quad (19)$$

$$L_n = 2qkL/R \quad (20)$$

whence

$$W = k(p + i2qF) \quad (21)$$

where p and q are constants, each usually in the neighborhood of unity, which can be so evaluated as to favor the frequencies in the neighborhood of any specified frequency; such evaluation may be accomplished by plotting a series of curves of which each curve corresponds to a certain pair of values of p and q . For any fixed values of p and q , the precision of the equalization attained by the network of Fig. 3 when designed in accordance with formulae (19) and (20) can be investigated by means of the formula

$$\frac{Z}{k} = \frac{(p + i2qF)(K/k)}{(p + i2qF) + (K/k)} \quad (22)$$

obtained from Equations (9) and (21) with $B=k$ so that $\lambda=1$. In the limiting case where $p=q=1$, Equation (22) reduces of course to (18).

It being known from the foregoing analysis that the network (Fig. 3) of my invention is capable of being so proportioned as to possess equalizing properties, the perfectly general and exact equation (11) may be utilized as the basis of the following alternative design-method: With the prescribed value of B (supposed to be real), and at the particular frequency, say f' , where it is decided that the equalization shall be exact, calculate the requisite value of W by means of Equation (11), and let its real and imaginary parts be denoted by U' and V' , so that

$$W = U' + iV' \quad (23)$$

Then comparison of (23) with (6) shows that the requisite values of R_n and L_n , in order to produce exact equalization at $f=f'$, are

$$R_n = U' \quad (24)$$

$$L_n = V' / \omega' \quad (25)$$

When these values of R_n and L_n are assigned to the network of Fig. 3, the impedance Z of the system of Fig. 4 at the frequency f' will be exactly equal to B. The precision of equalization over any contemplated frequency-range will of course have to be investigated, by means of Formula (9).

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network associated with the line and having such impedance with respect to that of the line that the impedance of the combination will be substantially a pure resistance.
2. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network connected in shunt across the line and having such impedance with respect to that of the line that the impedance of the combination will be substantially a pure resistance.
3. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network including a series combination of a resistance and an inductance, the resistance being substantially equal to the nominal impedance of the line and the inductance being of such value that the impedance of the line and associated network will be substantially a pure resistance.
4. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range, comprising a network including a series combination of a resistance and an inductance, the resistance being substantially equal to k , the nominal impedance of the line, and the inductance being substantially equal to $2kL/R$ when L is the line inductance and R is the resistance of the line.
5. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network associated with the line and having such impedance with respect to that of the line that the impedance of the combination will be substantially a pure resistance, said network including a series combination of resistance and inductance.
6. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network connected in shunt across the line and having such impedance with respect to that of the line that the impedance of the combination will be substantially a pure resistance, said network including a series combination of resistance and inductance.
7. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network connected in shunt across the line and including a series combination of a resistance and an inductance, the resistance being substantially equal to the nominal impedance of the line and the inductance being of such value that the impedance of the line and associated network will be substantially a pure resistance.
8. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range, comprising a network connected in shunt across the line and including a series combination of a resistance and an inductance, the resistance being substantially equal to k , the nominal impedance of the line, and the inductance being substantially equal to $2kL/R$ when L is the line inductance and R is the resistance of the line.
9. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network associated with the line and having such impedance with respect to that of the line that the impedance of the combination will be substantially a pure and constant resistance.
10. In a smooth line whose characteris-

tic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network connected in shunt
5 across the line and having such impedance with respect to that of the line that the impedance of the combination will be substantially a pure and constant resistance.

11. In a smooth line whose characteristic
10 impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network including a series combination of a resistance and an induc-
15 tance, the resistance being substantially equal to the nominal impedance of the line and the inductance being of such value that the impedance of the line and associated network will be substantially a pure and constant resistance.
20

12. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of said smooth line over a wide frequency range
25 comprising a network associated with the line and having such impedance with respect to that of the line that the impedance of the combination will be substantially a pure and constant resistance, said network including a series combination of resistance and inductance.
30

13. In a smooth line whose characteristic impedance includes reactance, means to equalize the characteristic impedance of
35 said smooth line over a wide frequency range comprising a network connected in shunt across the line and having such impedance with respect to that of the line that the impedance of the combination will be substantially a pure and constant resistance, said
40 network including a series combination of resistance and inductance.

14. In a smooth line whose characteristic impedance includes reactance, means to
45 equalize the characteristic impedance of said smooth line over a wide frequency range comprising a network connected in shunt across the line and including a series combination of a resistance and an inductance, the re-
50 sistance being substantially equal to the nominal impedance of the line and the inductance being of such value that the impedance of the line and associated network will be substantially a pure and constant resistance.

55 In testimony whereof, I have signed my name to this specification this 18th day of November, 1929.

RAY S. HOYT.