

March 10, 1931.

R. S. HOYT

1,795,397

DIRECTIONALLY SELECTIVE RADIO RECEIVING SYSTEM

Filed Dec. 29, 1927

3 Sheets-Sheet 1

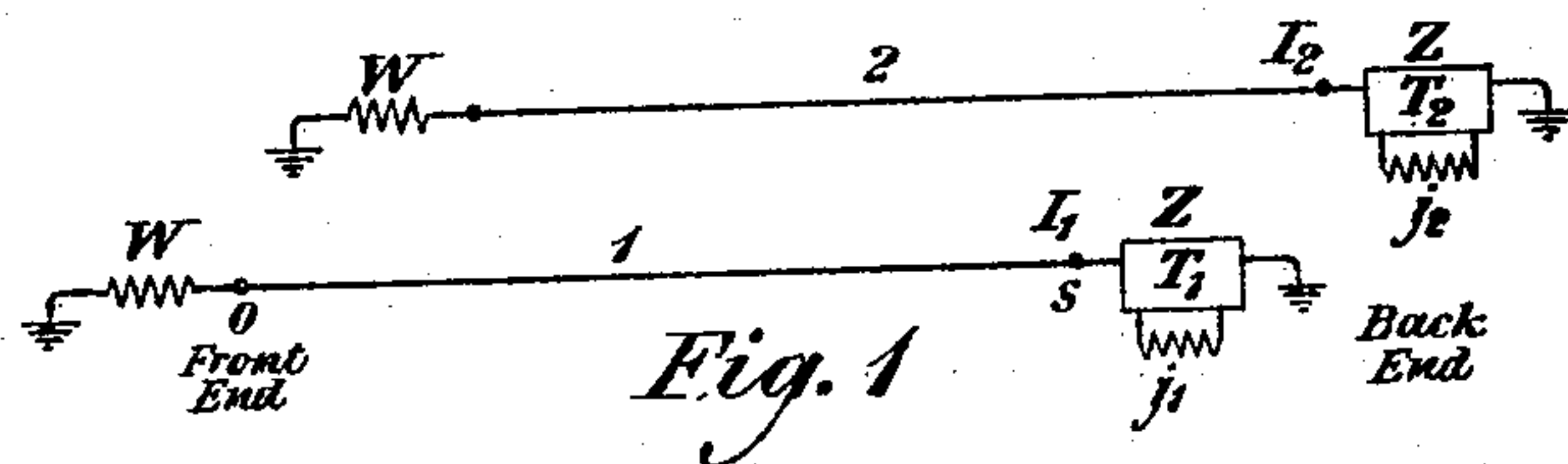


Fig. 1

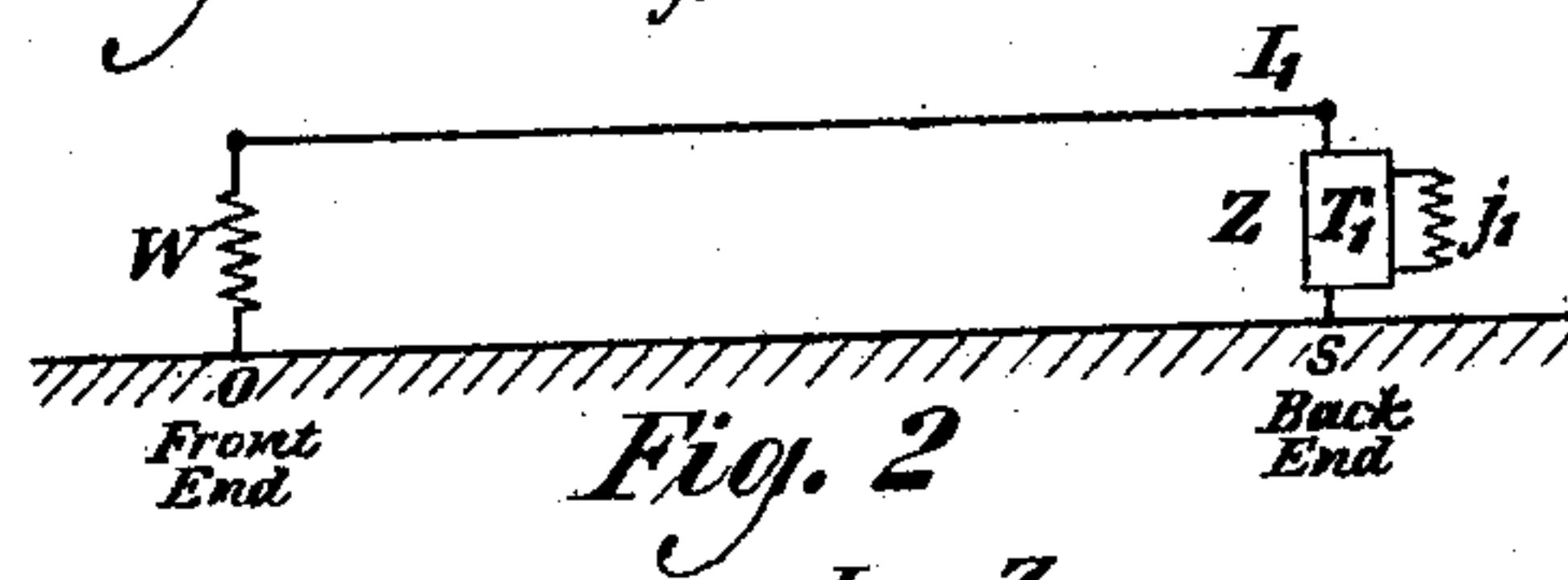


Fig. 2

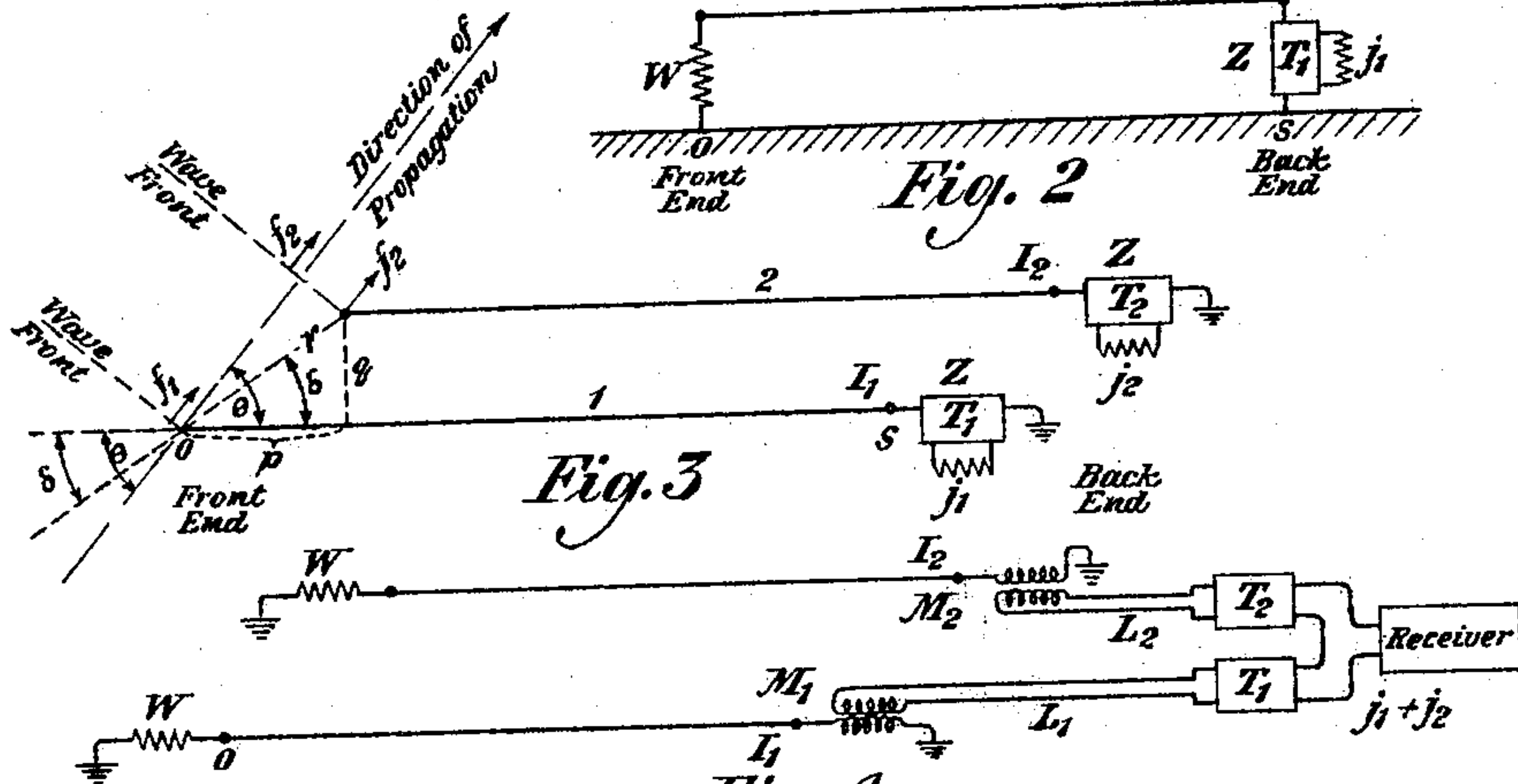


Fig. 3

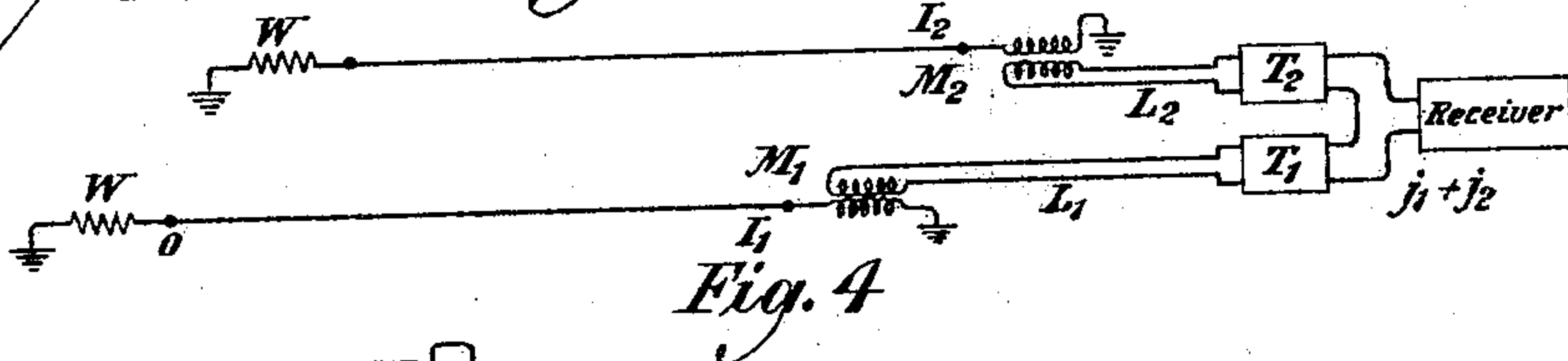


Fig. 4

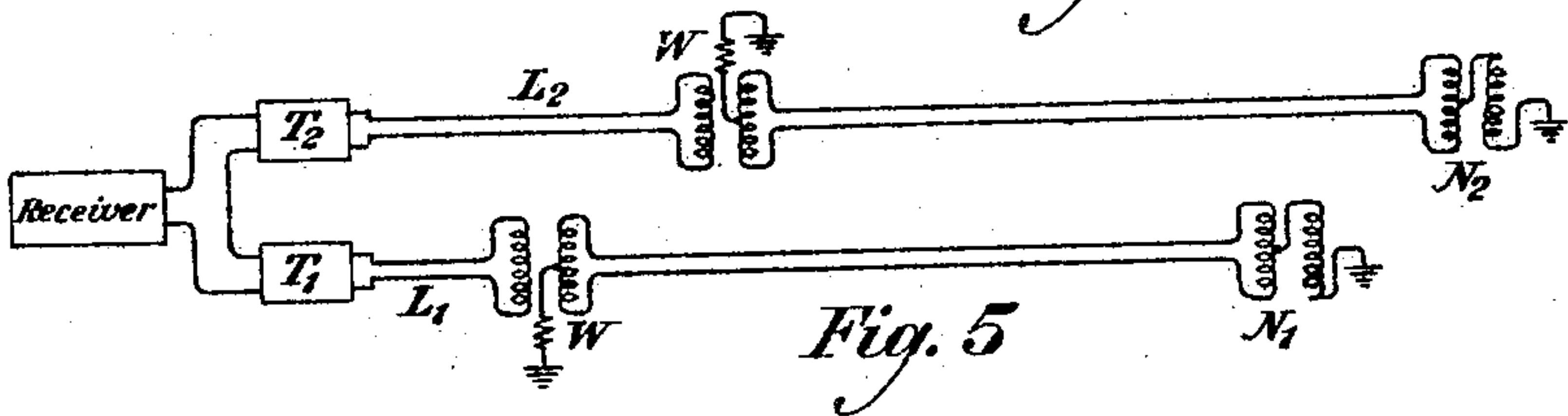


Fig. 5

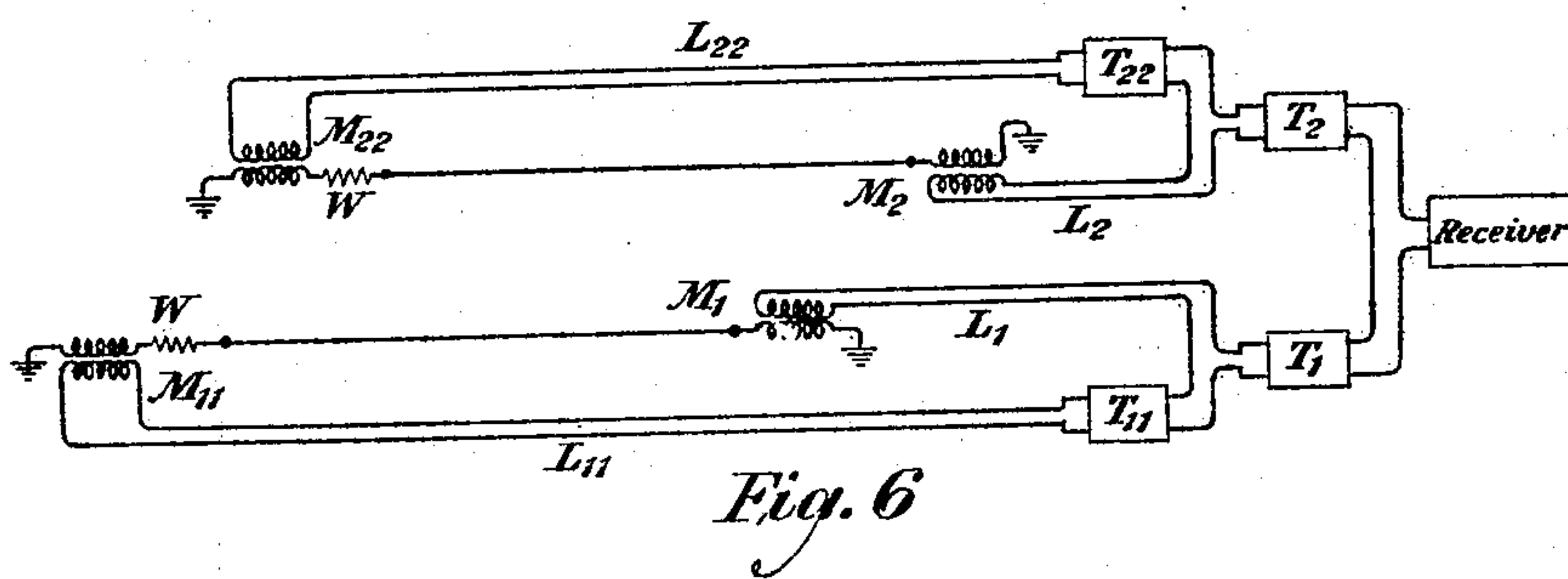


Fig. 6

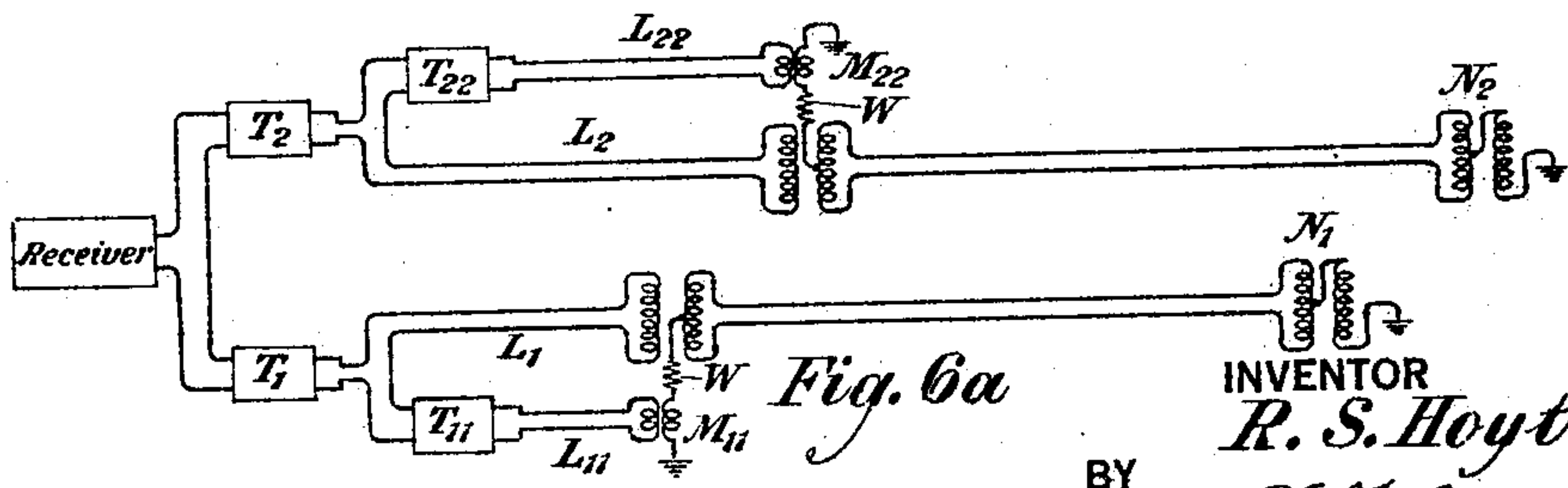


Fig. 6a

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Fig. 7

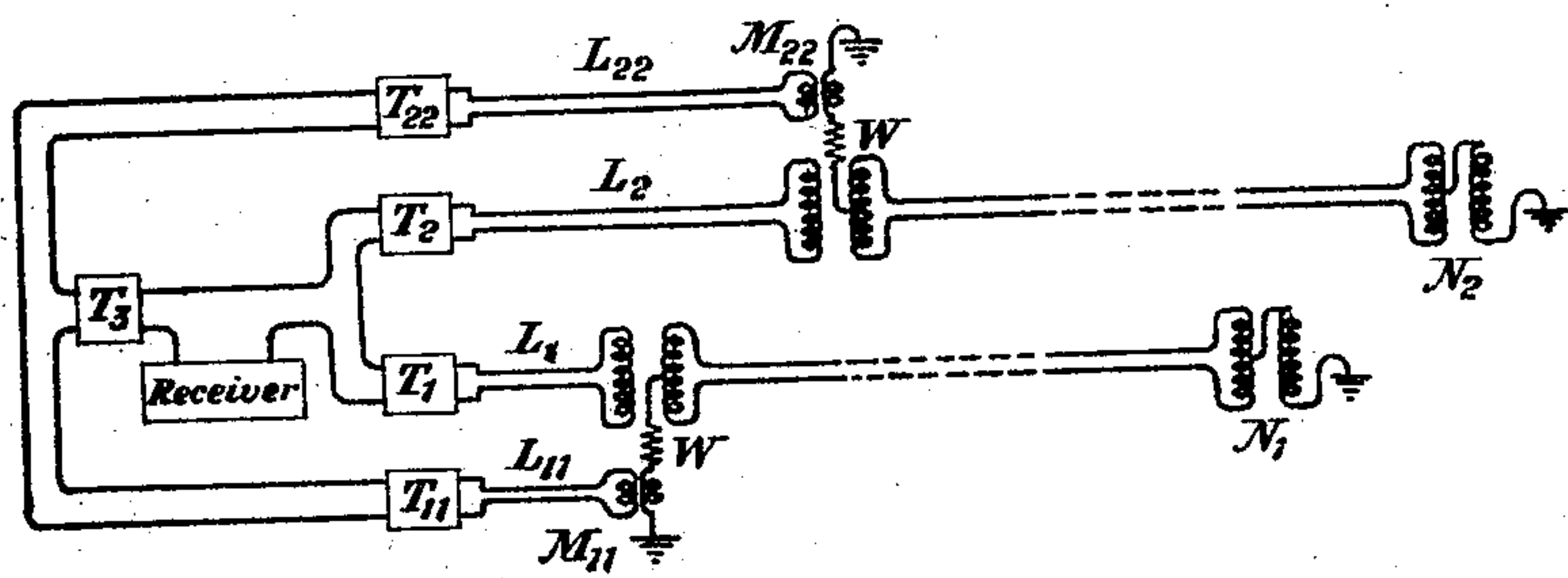
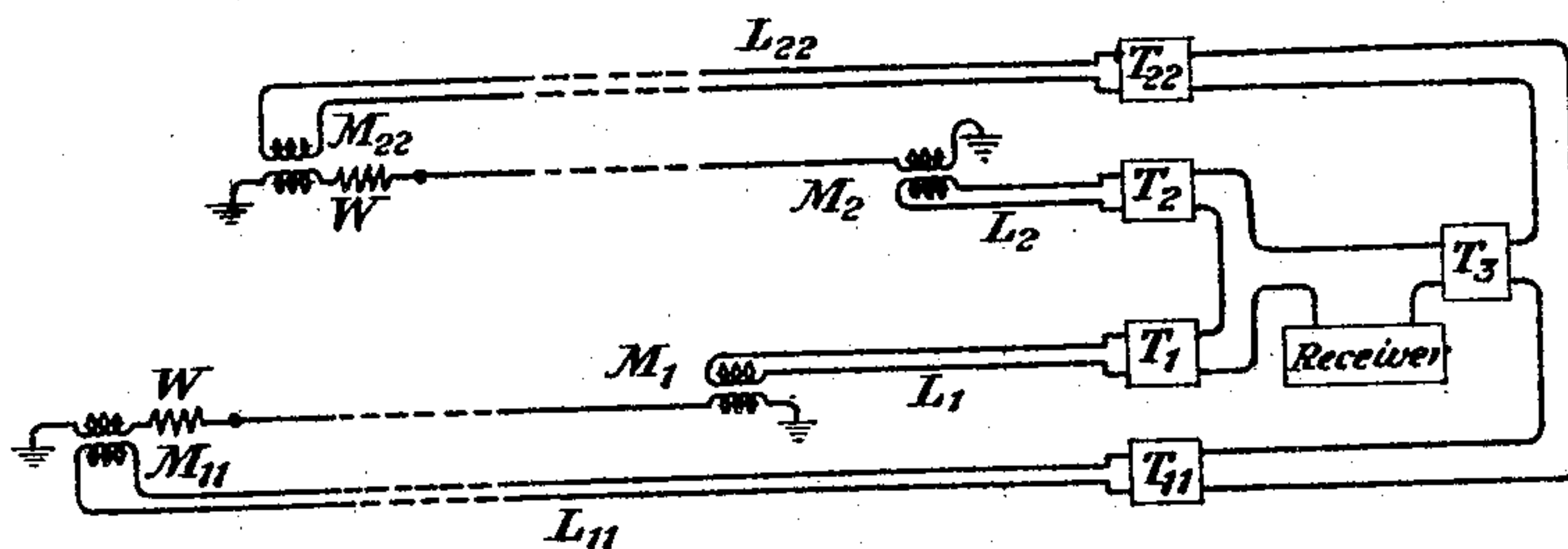


Fig. 7a

Fig. 8

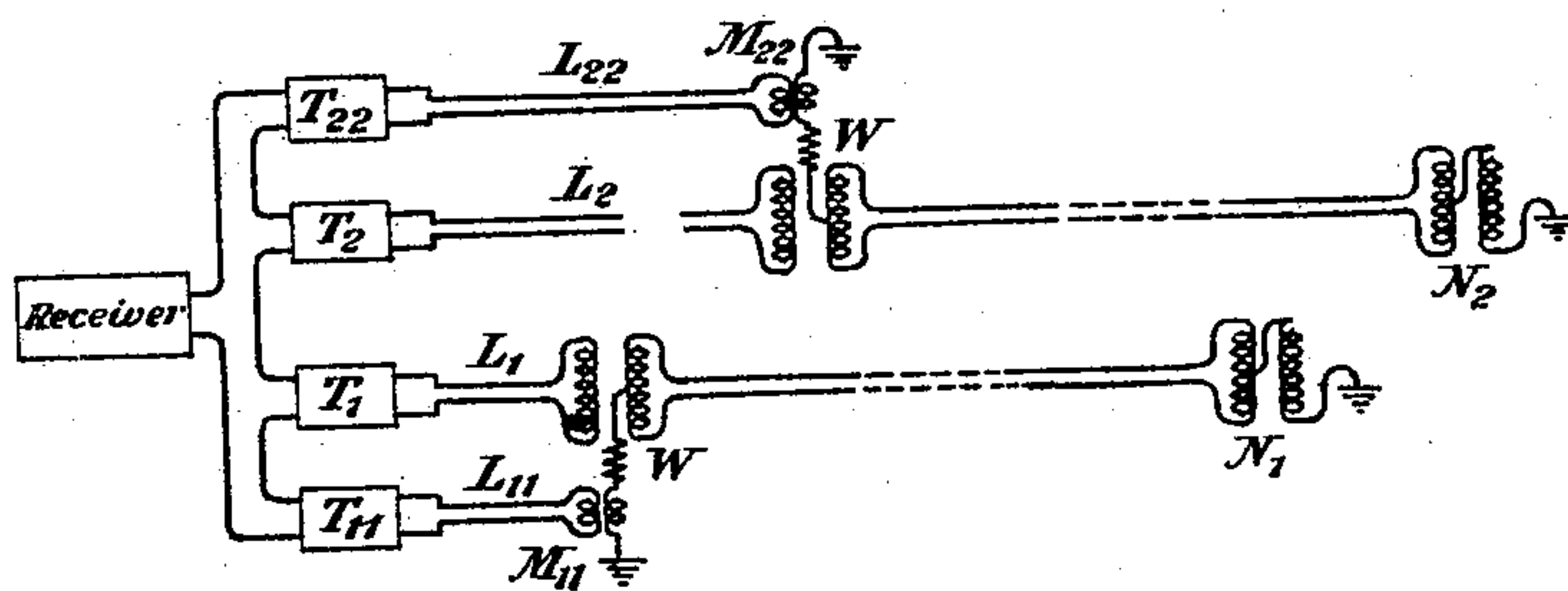
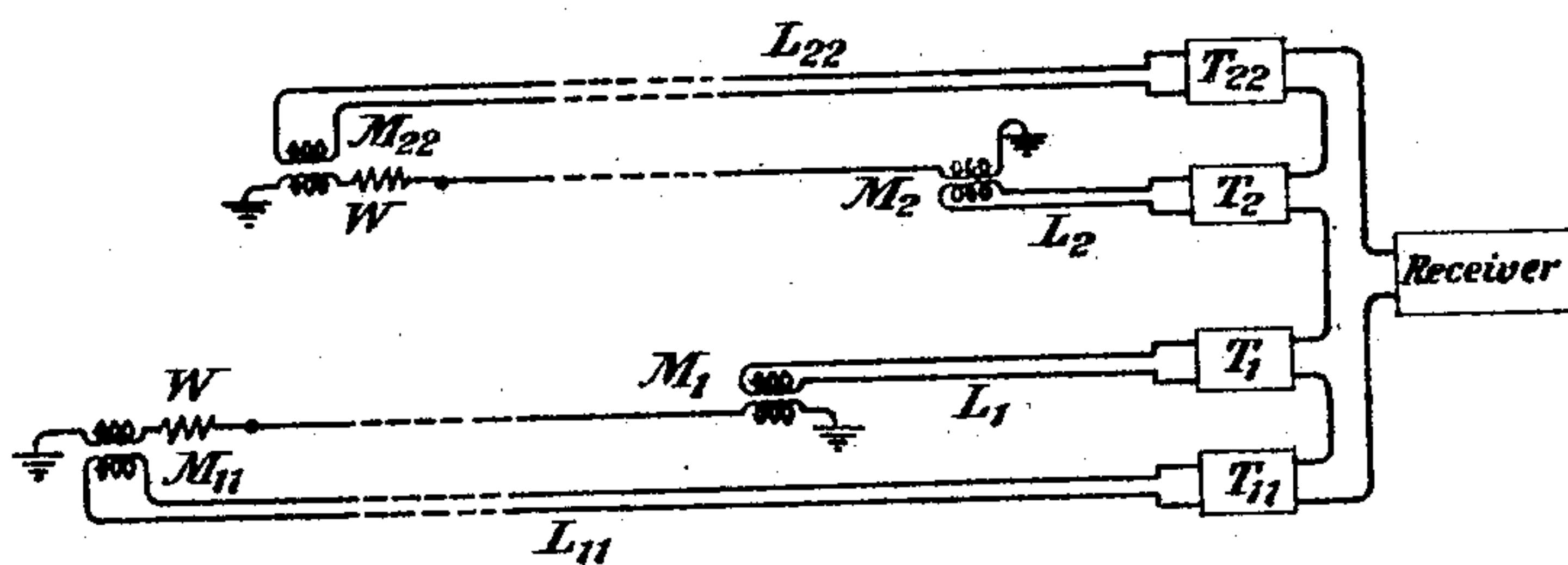


Fig. 8a

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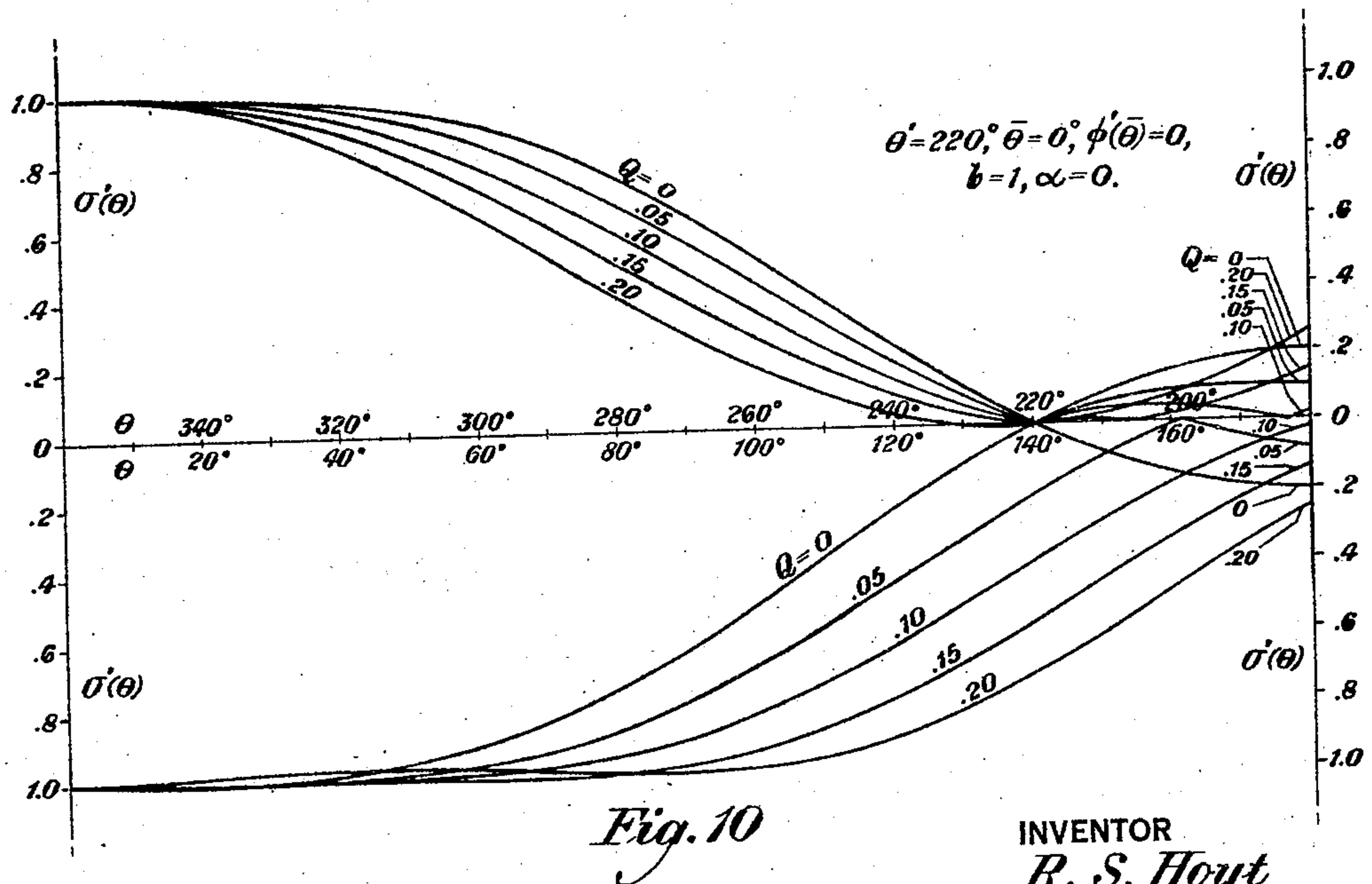
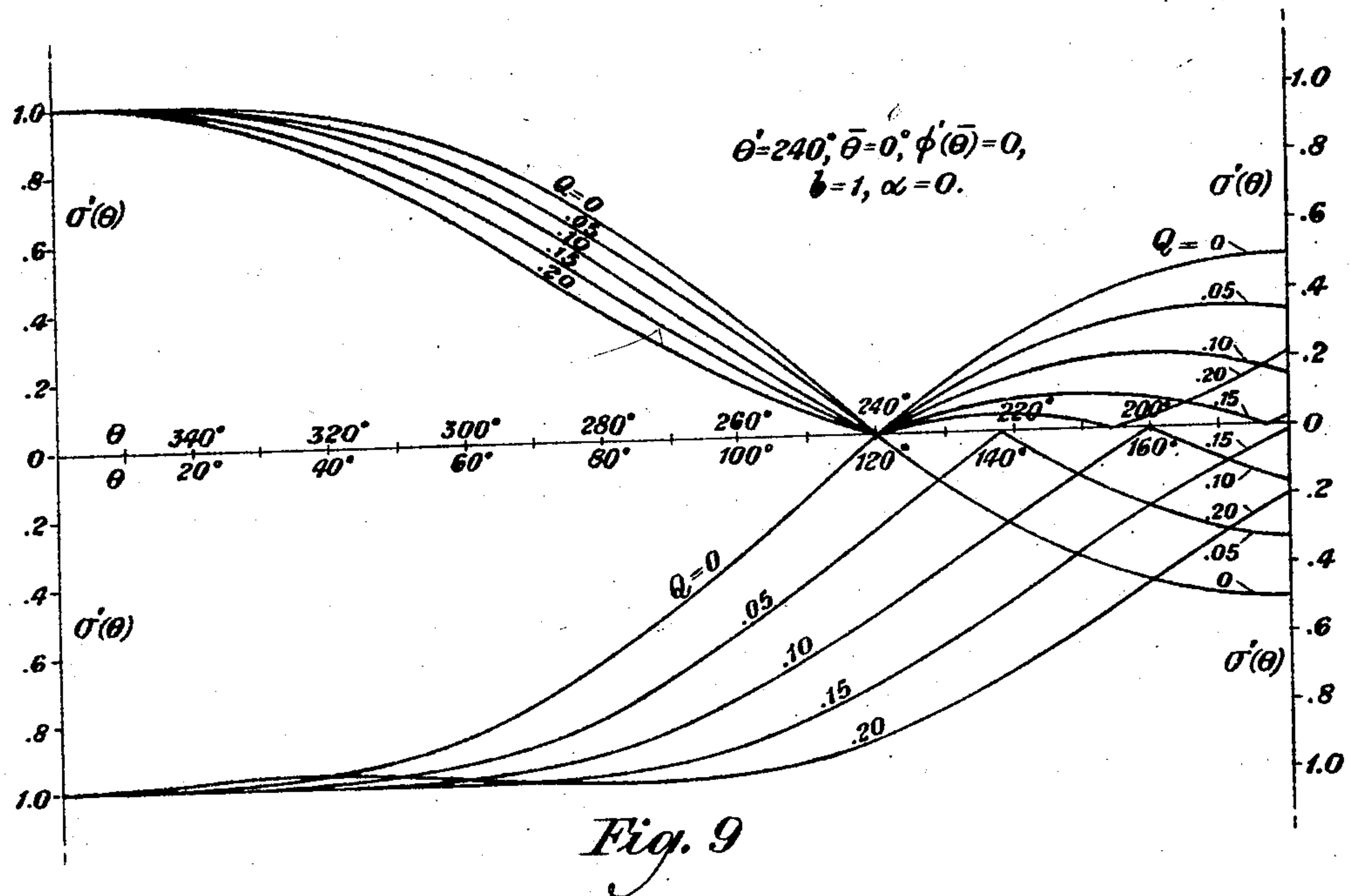
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DIRECTIONALLY SELECTIVE RADIO RECEIVING SYSTEM

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## UNITED STATES PATENT OFFICE

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## DIRECTIONALLY-SELECTIVE RADIO RECEIVING SYSTEM

Application filed December 29, 1927. Serial No. 243,380.

It is among the objects of my invention to provide new and improved apparatus and a corresponding method for receiving radio signals from a desired direction with avoidance of interference from other directions. Another object is to provide a radio receiving system that shall be adjustable or flexible in design so as to receive efficiently from one particular direction with substantially no interference from another direction or directions. Another object is to adjust the design so as to reduce the effect of random interference over a considerable range of directions other than the desired direction of reception. Another object is to provide a system of receiving wave antennae adjustable or flexible in design for one or more of the purposes stated above. Still another object is to provide a system of elements comprising two parallel wave antennae in staggered and spaced relation, with the dimensions and other determining factors of the system so assigned that there will be good reception in one desired direction and exclusion of interference in another desired direction or directions. All these objects and other objects of my invention will become apparent on consideration of a limited number of specific embodiments of the invention which are disclosed in the following specification taken with the accompanying drawings. It will be understood that the following disclosure relates to these examples of the invention and that the scope of the invention will be indicated in the appended claims.

Referring to the drawings, Figure 1 is a diagrammatic plan view of a system of two parallel staggered wave antennae embodying my invention; Fig. 2 is a corresponding diagrammatic elevation of one of these two wave antennae; Fig. 3 is a diagrammatic plan view with certain dimensions and other data indicated as a basis for discussion; Fig. 4 is a diagrammatic plan showing one way of combining the currents received over the several antennae for the operation of a single receiver; Fig. 5 corresponds with Fig. 4 but has the receiver at the "front end" instead of at the "back end"; Fig. 6 is a diagram for a system such as that of Fig. 4, with com-

pensation currents taken from opposite ends of each wave antenna; Fig. 6a is a diagram showing compensation for Fig. 5 in the manner of Fig. 6; Fig. 7 is a diagram showing another manner of effecting compensation as applied to a system like that of Fig. 4; Fig. 7a shows the same manner of effecting compensation as applied to the system of Fig. 5; Fig. 8 shows still another manner of effecting compensation in a system such as that of Fig. 4; Fig. 8a shows the kind of compensation of Fig. 8 as applied to a system like that of Fig. 5; Fig. 9 is a curve diagram giving the measure of advantage for certain particular examples of my invention as fixed for example by certain definitely assigned parameters, and Fig. 10 is a similar curve diagram in which the parameters are the same except that the direction for null reception is different.

A wave antenna may be a comparatively long horizontal conductor extending in a straight line at a moderate height and in a direction nearly the same as that in which maximum intensity of receiving is desired. The elementary theory of such a wave antenna is presented in the papers by H. H. Beverage et al., in the Journal of the A. I. E. E. for March, 1923 (page 258), and succeeding issues. My invention may be embodied in a pair of wave antennae in spaced parallel relation and staggered; that is, one of them displaced along the direction of their length. Two such wave antennae are shown in Fig. 1, adapted for reception of radio waves incoming on the left. These two antennae are indicated as 1 and 2, and corresponding parts and data associated therewith are distinguished by the subscripts 1 and 2.

I shall employ the term "transducer" in this specification in its usual meaning of any assembly of apparatus having a set of input terminals to which input electromotive forces are applied and a set of output terminals for output currents, these currents being a function of the said electromotive forces. Under this broad term may be mentioned as examples a transformer, a rotary converter, an amplifier, an attenuator, a transmission line, a phase shifter. Some authorities use the



term "quadripole" instead of "transducer."

In this specification I use certain characters, such as T, sometimes to represent a piece of apparatus, say T for a transducer, and sometimes to represent an associated numerical measure such as the complex number value for the transfer factor of the transducer. In each instance the sense intended will be apparent from the context.

As shown in Fig. 1, combining transducers  $T_1$  and  $T_2$  are situated at the back ends of the antennæ 1 and 2. Each transducer enables the ratio  $j/I$  of its output current  $j$  to its input current  $I$  to be adjusted to any value as regards magnitude and phase. The antenna output currents  $I_1$  and  $I_2$  are evidently the transducer input currents. The transducer output currents  $j_1$  and  $j_2$ , after having been adjusted to any desired values by means of the respective transducers  $T_1$  and  $T_2$ , are finally added together in some such manner, for instance, as indicated by Fig. 4. It will be found that either one (but not both) of the transducers can be proportioned arbitrarily as regards its transfer factor T or can even be omitted or be combined with the other; but for symmetry both are retained in all of the figures.

Referring to Fig. 3,  $s$  denotes the length of each wave-antenna,  $p$ , the "staggering" of the wave-antennæ (that is, the amount by which antenna 2 is displaced longitudinally with respect to antenna 1),  $q$  the spacing (transverse separation) of the wave-antennæ, and  $r$  the direct distance between the two front ends (and hence between the two back ends). Wave-antenna 1 is taken as the reference axis, its "front end" O as origin of coordinates, and the direction from front to back as positive direction. Angles are measured counter-clockwise from the reference axis (antenna 1).  $\delta$  denotes the angle from the reference axis to the line  $r$ .

In Fig. 3 there is shown a train of plane electromagnetic waves (radio waves, or space waves) incident at any angle  $\theta$ , measured from the reference axis to the direction of propagation of the radio waves along the earth's surface.  $f_1$  denotes the horizontal component of the electric force of the radio waves at the front end of antenna 1, and  $f_2$  the corresponding simultaneous value at the front end of antenna 2; these electric forces are along the direction of incidence (that is, the direction of propagation in space).

As explicitly indicated by Fig. 1, each antenna is terminated at its front end in an arbitrary impedance W, and at its back end in a transducer which presents to the wave-antenna an arbitrary impedance Z. For simplicity, the front-end terminal impedances are not shown in Fig. 3, but they are to be regarded as present there. (In most practical applications, Z and W would be made equal to the characteristic impedance

K of each wave-antenna, for simplicity and also for securing desirable directional characteristics for the individual wave, antennæ.

$I_1$  denotes the output current of antenna 1 and hence the input current of transducer  $T_1$ , and  $j_1$  denotes the corresponding output current of transducer  $T_1$ ; and similarly for  $I_2$  and  $j_2$ . Evidently these currents are functions of  $\theta$ ; this fact will usually be explicitly indicated by employing the functional symbols  $I_1(\theta)$ ,  $I_2(\theta)$ ,  $j_1(\theta)$ ,  $j_2(\theta)$ ; thus the symbols  $I_1$ ,  $I_2$ ,  $j_1$ ,  $j_2$  are to be regarded as abbreviations of the corresponding functional symbols. The fact that these currents depend on the incidence angle  $\theta$  of the radio waves, may be indicated by saying that these currents have "directional properties or characteristics".

The "transfer factors"  $T_1$  and  $T_2$  of the combining transducers are to be regarded as unlike, in general. They are merely the current ratios defined by the equations

$$T_1 = j_1(\theta)/I_1(\theta), \quad (1)$$

$$T_2 = j_2(\theta)/I_2(\theta), \quad (2)$$

which, of course, are independent of  $\theta$ .

Finally, the received currents  $j_1(\theta)$  and  $j_2(\theta)$  are regarded as being directly combined by simple addition so that, if  $J(\theta)$  denotes the resultant current, then

$$J(\theta) = j_1(\theta) + j_2(\theta) = T_1 I_1(\theta) + T_2 I_2(\theta), \quad (3)$$

any desired transformations having been previously accomplished in the transducers, as symbolized by the transfer factors  $T_1$  and  $T_2$ , in accordance with Equations (1) and (2).

This disclosure is concerned with the directional characteristics of the resultant received current  $J(\theta) = j_1(\theta) + j_2(\theta)$ . To establish a formula for  $J(\theta)$ , let  $Y_1(\theta)$  and  $Y_2(\theta)$  denote the "directional admittance" of antennæ 1 and 2, respectively, as defined by the equations

$$Y_1(\theta) = I_1(\theta)/sf_1, \quad (4)$$

$$Y_2(\theta) = I_2(\theta)/sf_2. \quad (5)$$

Since the two antennæ are alike and are terminated alike (as regards the impedances W and Z), it is evident that

$$Y_1(\theta) = Y_2(\theta) = Y(\theta), \text{ say, } (6)$$

whence

$$I_1(\theta) = sf_1 Y(\theta), \quad (7)$$

$$I_2(\theta) = sf_2 Y(\theta). \quad (8)$$

$\gamma$  denoting the propagation constant of the radio waves along their direction of propagation over the earth's surface (per unit length), Fig. 3 shows that

$$f_2 = f_1 e^{-\gamma r \cos(\theta - \delta)}. \quad (9)$$

Hence, by Equations (3), (7), (8), (9), the resultant received current

$$J(\theta) = j_1(\theta) + j_2(\theta)$$



is given by the equation

$$J(\theta) = sf_1 T_1 Y(\theta) G(\theta), \quad (10)$$

where

$$G(\theta) = 1 + \frac{T_2}{T_1} e^{-\gamma r \cos(\theta - \delta)} \quad (11)$$

$$= 1 + \frac{j_2(\theta)}{j_1(\theta)}. \quad (12)$$

array of the two wave-antennæ is represented by the "resultant directional ratio"  $\rho_a(\theta)$  defined by the equation

$$\rho_a(\theta) = |J(\theta)/J(\theta_r)|, \quad (17)$$

whence by (10) and (16) successively, it is seen that

$$\rho_a(\theta) = \left| \frac{Y(\theta) G(\theta)}{Y(\theta_r) G(\theta_r)} \right| = \rho(\theta) \left| \frac{G(\theta)}{G(\theta_r)} \right|, \quad (18)$$

where  $\rho(\theta) = \rho_1(\theta) = \rho_2(\theta)$  is the "directional ratio" of each wave-antenna. By introducing the "group factor ratio"  $\sigma(\theta)$  defined by the equation

$$\sigma(\theta) = |G(\theta)/G(\theta_r)|, \quad (19)$$

(18) can be written

$$\rho_a(\theta) = \sigma(\theta) \rho(\theta), \quad (20)$$

which expresses the "resultant directional ratio"  $\rho_a(\theta)$  as the product of the "directional ratio"  $\rho(\theta)$  of each wave-antenna by the "group factor ratio"  $\sigma(\theta)$ . The directional properties of  $\rho(\theta)$  need not be derived nor discussed at much length in this specification as they are sufficiently known in the art; reference may be made to the paper by Beverage et al. mentioned heretofore, where it is shown that the "directional ratio"  $\rho(\theta)$  of a single wave antenna can be made decidedly directional by suitable choice of its length; an alternative exposition of the fundamental theory and formulas for a single wave antenna will also be found in the article by Carson and Hoyt, "Propagation of periodic currents over a system of parallel wires", in the Bell System Technical Journal for July, 1927, pages 532 to 535. The system disclosed in the present specification in accordance with my invention, affords improvement in that the "resultant directional ratio"  $\rho_a(\theta)$  of the spaced staggered array of the two wave-antennæ can be made decidedly more directional than the "directional ratio"  $\rho(\theta)$  of each wave-antenna alone; this improvement results from the presence in Equation (20) of the "group factor ratio"  $\sigma(\theta)$  which, as will be shown in the present specification can be given particularly desirable directional properties by suitably proportioning the array of the two wave-antennæ as regards the staggering  $p$ , spacing  $q$ , and ratio  $U = T_2/T_1$  characterizing the combining transducers, so that the product  $\sigma(\theta)\rho(\theta)$  will be decidedly more directional than is  $\rho(\theta)$  alone.

By reference to Equation (11) it is seen that the "group factor ratio"  $\sigma(\theta)$ , defined by (19) represents the dependence of the directional selectivity of the system constituting my invention, on the staggering  $p$  and spacing  $q$  of the array of the two wave-antennæ (or, in other words, on  $r$  and  $\delta$ ), and on the ratio  $T_2/T_1$  of the transfer factors of the combining transducers. Therefore the properties of the "group factor ratio"  $\sigma(\theta)$  will now

The function  $G(\theta)$  will be termed the "group factor" (or, more fully, the "group directional factor"). This term is adopted because  $G(\theta)$  does not depend on the wave-antennæ themselves, but only on their "grouping" as represented by the relative positions of their front ends (or of their back ends), as specified by  $r$  and  $\delta$ , and on the ratio  $T_2/T_1$  of the transfer factors of the combining transducers.

The functional notation  $G(\theta)$  denotes that  $\theta$  is regarded as the independent variable and hence that the other quantities are regarded as parameters. It may be observed, however, that  $\theta - \delta$  could be taken as the independent variable, which would correspond to taking the line along  $r$  as reference axis and would apparently reduce the number of independent parameters by one; but this reduction is only apparent, because, when the line along  $r$  is taken as reference axis, the parameter  $\delta$  then appears in the formula for the directional characteristic of each wave-antenna alone, whereas it is absent from that formula when the direction of the wave-antennæ is taken as the reference axis. On the whole it seems best to take the wave-antenna as the reference axis, so that  $\delta$  then occurs as a parameter in the formula for  $G(\theta)$ .

If  $Y_a(\theta)$  denotes the "resultant directional admittance" of the array of the two wave-antennæ, combined as aforesaid by means of the transducers  $T_1$  and  $T_2$ , and referred to the electric force  $f_1$ , so that  $Y_a(\theta)$  is defined by the equation

$$Y_a(\theta) = J(\theta)/sf_1, \quad (13)$$

then, by (10),

$$Y_a(\theta) = T_1 Y(\theta) G(\theta). \quad (14)$$

(Subscript "a" denotes "array").

The "directional selectivity" of a single wave-antenna, say antenna 1, with respect to any arbitrary reference value  $\theta_r$  of  $\theta$ , is represented by the "directional ratio"  $\rho_1(\theta)$  defined by the equation

$$\rho_1(\theta) = |L_1(\theta)/I_1(\theta_r)|, \quad (15)$$

a polar graph of which is the so-called "polar diagram" representing the directional characteristic of a single wave antenna. (Usually  $\theta_r = 0$ ). From (15), (4), (5), (6) it is seen that

$$\rho_1(\theta) = \rho_2(\theta) = \rho(\theta) = |Y(\theta)/Y(\theta_r)|. \quad (16)$$

Similarly the directional selectivity of the



be set forth in some detail. This will be accomplished by studying the "group factor"  $|G(\theta)|$ , since  $\sigma(\theta)$  is proportional to  $|G(\theta)|$ , by Equation (19); it is seen that  $\sigma(\theta)=1$  when  $\theta=\theta_r$ .

With a view to writing Equation (11) for  $G(\theta)$  in more useful and more significant forms we introduce the ratios  $P$ ,  $Q$ ,  $R$ , defined by the following equations, in which  $\lambda$  denotes the wave length of the radio waves in the direction of propagation along the earth's surface:

$$P=p/\lambda, \quad (21)$$

$$Q=q/\lambda, \quad (22)$$

$$R=r/\lambda, \quad (23)$$

and from Fig. 3 we note that

$$R=P/\cos \delta=Q/\sin \delta. \quad (24)$$

Also, we let  $\Gamma$  denote the propagation constant of the radio waves per wave length, so that

$$\Gamma=\lambda\gamma=\alpha+i2\pi, \quad (25)$$

$\alpha$  therefore denoting the attenuation constant of the radio waves per wave length. Furthermore, we let  $U$  denote the ratio of  $T_2$  to  $T_1$ , so that

$$T_2/T_1=U=|U|e^{iu} \quad (26)$$

$u$  therefore denoting the phase-angle of  $U$ . By means of the substitutions (23), (25), (26) Equation (11) for  $G(\theta)$  becomes

$$G(\theta)=1+Ue^{-\Gamma R \cos(\theta-\delta)}, \quad (27)$$

$$=1+|U|e^{-\alpha R \cos(\theta-\delta)}e^{i\phi}, \quad (28)$$

where

$$\phi=\phi(\theta)=u-2\pi R \cos(\theta-\delta). \quad (29)$$

Comparison with (12) shows that

$$j_2(\theta)/j_1(\theta)=Ue^{-\Gamma R \cos(\theta-\delta)}, \quad (30)$$

$$|j_2(\theta)/j_1(\theta)|=|U|e^{-\alpha R \cos(\theta-\delta)}, \quad (31)$$

$$\text{ag}[j_2(\theta)/j_1(\theta)]=\phi=\phi(\theta), \quad (32)$$

"ag" denoting "angle of" or, more fully "phase angle of".

Since  $\cos(\theta-\delta)=\cos(\delta-\theta)$ , it follows from (27) that

$$G(\theta)=G(2\delta-\theta), \quad (33)$$

$$G(\delta+\theta)=G(\delta-\theta). \quad (34)$$

Similarly Equation (29) shows that

$$\phi(\theta)=\phi(2\delta-\theta), \quad (35)$$

$$\phi(\delta+\theta)=\phi(\delta-\theta). \quad (36)$$

Thus the geometrical line for which  $\theta=\delta$ , that is, the line passing through the front ends of the two wave-antennæ, is an axis of symmetry for each of the functions  $G(\theta)$  and  $\phi(\theta)$ , and hence for  $\sigma(\theta)$  defined by (19).

As just noted, the function  $\sigma(\theta)$  is symmetrical with respect to a line along the directions  $\theta=\delta$ . The function  $\rho(\theta)$ , pertaining to a single wave-antenna, is well-known to be symmetrical with respect to a line along the

direction of the wave-antenna ( $\theta=0$ ). The product  $\rho_a(\theta)$ , defined by (20) and pertaining to the array of the two wave-antennæ, is not symmetrical with respect to either of these lines, but the major lobe of its polar diagram may be roughly symmetrical with respect to some intermediate line. It may be advantageous to so choose the direction of the array of wave-antennæ that the desired signal will come in approximately along this "intermediate line," so that the major lobe of the polar diagram will be at least roughly symmetrical with respect to the direction of the desired signal. But, in many cases, considerable dissymmetry may be desirable (to minimize the effects of static and other interference); such desired dissymmetry can be at least partially secured by suitably choosing the direction of the wave-antennæ.

The effect of changing  $\delta$  to  $\delta+\xi$  is to rotate the polar diagram of  $|G(\theta)|$  through the angle  $\xi$ . This follows from Equation (27), since  $\theta-\delta$  is unchanged when  $\theta$  and  $\delta$  are each increased by  $\xi$ . In particular, changing  $\delta$  to  $-\delta$  rotates the polar diagram through the angle  $-2\delta$  (since  $-\delta=\delta-2\delta$ ).

Thus far we have dealt with the "group factor"  $G(\theta)$  in a general manner. We now proceed to show how the system can be so proportioned that the absolute value  $|G(\theta)|$  of the group factor will have desirable directional properties. (We are not concerned with the phase angle of the received current  $J(\theta)$ , and hence not with the phase angle of  $G(\theta)$ .)

On referring back to Equations (28) and (29), it is seen that  $|G(\theta)|$  depends on no less than five parameters, namely the set  $\alpha$ ,  $|U|$ ,  $u$ ,  $R$ ,  $\delta$  or the equivalent but more convenient set  $\alpha$ ,  $|U|$ ,  $u$ ,  $P$ ,  $Q$  (since  $P=R \cos \delta$  and  $Q=R \sin \delta$ ). The parameter  $\alpha$ , namely the attenuation constant (per wave length) of the radio waves in space, is of course supposed to be known, at least approximately. The four remaining parameters  $|U|$ ,  $u$ ,  $P$ ,  $Q$  are at our disposal and, in fact, are to be so chosen as to yield so far as possible the desired directional properties for  $|G(\theta)|$ . The considerations leading up to the evaluation of these parameters will now be set forth.

In most practical applications it is desirable that  $|G(\theta)|$  be zero, or at least very small, at some specified value of  $\theta$ , say  $\theta'$ . Therefore the first condition to be imposed will be that  $|G(\theta')|=0$ . It will be found that this condition suffices to determine both  $|U|$  and  $u$ , that is, the absolute value and the phase angle of  $U=T_2/T_1$ .

For securing approximately maximum sensitivity at a particular value of  $\theta$ , say  $\theta$  (usually  $\theta=0$ ), it is evidently necessary that the output currents  $j_1(\theta)$  and  $j_2(\theta)$  shall not be greatly out of phase for  $\theta=\theta$ . Of course



the sensitivity is a maximum when these two currents are exactly in phase; moreover the practical operation of the system is considerably facilitated if they are in phase. From Equation (32), it is seen that the condition that they be in phase for  $\theta = \theta'$  is that  $\phi(\theta) = 0$ ,  $\phi(\theta)$  being the angle by which  $j_2(\theta)$  leads  $j_1(\theta)$ . However, a value of  $\phi(\theta)$  somewhat different from zero may in some applications lead to appreciably better directional characteristics for  $|G(\theta)|$ . Consequently  $\phi(\theta)$  will be regarded as specified, but not necessarily equal to zero. Specification of the values of  $\theta$  and  $\phi(\theta)$  enables either P or Q to be eliminated; it will be found advantageous to eliminate P rather than Q, for a practical reason appearing later.

As a result of applying the foregoing considerations it will be found that  $|G(\theta)|$  is expressed in terms of the parameters Q,  $\theta'$ ,  $\theta$ ,  $\phi(\theta)$ . For any particular application, the most suitable values of these parameters can best be determined by plotting sets of curves of  $\sigma(\theta) = |G(\theta)/G(\theta_r)|$  for various sets of values of these parameters, as described near the end of this specification and illustrated by Figs. 9 and 10.

Having thus outlined the proposed steps in the evaluation of the parameters fixing the system, we shall now indicate the details of those steps, and the resulting design-procedure and design-formulas.

Before applying the condition  $|G(\theta')| = 0$ , it may be noted from Equation (33) that if  $G(\theta)$  is zero for  $\theta = \theta'$  it is also zero for  $\theta = \theta''$  such that

$$\theta' + \theta'' = 2\delta, \quad (37)$$

but (as shown in connection with Equation (50)), as long as  $\alpha$  is not zero, there are no other values of  $\theta$  for which  $G(\theta)$  is zero. In general,  $\theta'$  and  $\theta''$  are distinct; but they are coincident when  $\theta'$  is chosen equal to  $\delta$ , for then  $\theta'' = \delta$  by Equation (37).

From Equation (27), the necessary and sufficient condition that  $G(\theta)$  be zero for any specified value  $\theta'$  of  $\theta$  (and hence for  $\theta'' = 2\delta - \theta'$ ), is that U have the value  $U'$  such that

$$U' = -e^{\Gamma R \cos(\theta' - \delta)} \quad (38)$$

$$= e^{\Gamma R \cos(\theta' - \delta) + ib\pi}, (\pm b = 1, 3, 5, \dots) \quad (39)$$

since

$$-1 = e^{ib\pi}. \quad (40)$$

Hence, by aid of Equations (25) and (24),

$$|U'| = e^{\alpha R \cos(\theta' - \delta)}, \quad (41)$$

$$= e^{\alpha(P \cos \theta' + Q \sin \theta')}, \quad (42)$$

$$u' = 2\pi R \cos(\theta' - \delta) + b\pi \quad (43)$$

$$= 2\pi(P \cos \theta' + Q \sin \theta') + b\pi. \quad (44)$$

These will be employed as design-formulas for  $|U'|$  and  $u'$ .

A few comments regarding the quantity  $b$ , first occurring in Equation (39), are per-

haps desirable: From (40) it is seen that  $b\pi$  is the angle of the number  $-1$  regarded as a complex number; hence  $b$  can be assigned any odd positive or negative integral value, as represented in (39). Thus the exponent  $ib\pi$  in (39), or the term  $b\pi$  in (43) and (44), corresponds to  $b$  reversals of phase in the transducers. It will be found that  $b$  occurs in most of the following equations, and hence that the form and properties of the physical system depend on the choice of  $b$ ; it will be sufficiently illustrative of my invention to assume  $b = 1$  for the applications to be hereinafter described.

Since  $1/\lambda = f/v$  and since  $v$ , the velocity of phase propagation of the radio waves along the earth's surface, is approximately independent of the frequency  $f$ , it is seen from Equations (44), (21), (22) that  $u' - b\pi$  is approximately independent of the frequency. It is advantageous that the requisite value of  $u' - b\pi$  is thus (approximately) independent of the frequency, since that is a necessary condition for preserving the wave form of a composite wave, such for instance as a carrier wave modulated by speech.

If  $\phi' = \phi'(\theta)$  denotes the value of  $\phi = \phi(\theta)$  when  $U = U'$ , then by Equation (29),

$$\phi' = u' - 2\pi R \cos(\theta - \delta), \quad (45)$$

whence, by aid of (43) and (44),

$$\phi' = 2\pi R [\cos(\theta - \delta) - \cos(\theta' - \delta)] + b\pi \quad (46)$$

$$= 2\pi P [\cos \theta' - \cos \theta] + 2\pi Q (\sin \theta' - \sin \theta) + b\pi \quad (47)$$

$$= 4\pi R \sin \frac{1}{2}(\theta - \theta') \sin \frac{1}{2}(\theta + \theta' - 2\delta) + b\pi, \quad (48)$$

where  $\pm b = 1, 3, 5, \dots$  as above.

For use below, it will be convenient to introduce the function  $\psi' = \psi'(\theta)$  defined by the equation

$$\psi' = \phi' - b\pi, (\pm b = 1, 3, 5, \dots). \quad (49)$$

If  $G'(\theta)$  denotes the value of  $G(\theta)$  when  $U = U'$ , then from Equations (27), (38), (46), (49), (25) it is found that

$$G'(\theta) = 1 - e^{(\alpha\psi'/2\pi)} e^{ib\psi'}. \quad (50)$$

This equation shows that, so long as  $\alpha$  is not zero,  $G'(\theta)$  is zero when and only when  $\psi'$  is zero; and hence, from Equation (49) together with (46), it is seen that  $G'(\theta)$  is zero for  $\theta = \theta'$  and also for  $\theta = 2\delta - \theta' = \theta''$  but for no other values of  $\theta$ , so long as  $\alpha$  is not zero.

Although, so long as  $\alpha$  is not zero, there are two and only two values of  $\theta$  at which  $G'(\theta)$  is zero, designated by  $\theta'$  and  $\theta''$  and such that  $\theta' + \theta'' = 2\delta$ , it will now appear that when  $\alpha = 0$  there are additional values of  $\theta$  at which  $G'(\theta) = 0$ . For, when  $\alpha = 0$ , Equation (50) reduces to

$$G'(\theta) = 1 - e^{ib\psi'} \quad (51)$$

$$= 2 \sin(\psi'/2) e^{i(\psi' - \pi)/2}, \quad (52)$$



each of which shows that  $G'(\theta)$  is zero for those values of  $\theta$  such that  $\psi' = n(2\pi)$ , where  $\pm n = 0, 1, 2, 3, \dots$ . If any one of these critical values of  $\theta$  is denoted by  $\theta_n$ , then from

$$\cos(\theta' - \delta) - \cos(\theta_n - \delta) = \frac{n}{R}, (\pm n = 0, 1, 2, 3, \dots). \quad (53)$$

Since the value of the left side of this equation necessarily lies between  $\pm 2$ , the applicable values of  $n$  lie in the range

$$-2R \leq n \leq 2R. \quad (54)$$

Hence, when

$$R < 1/2,$$

the only applicable value of  $n$  is 0; whence, by (53), there are then only two values of  $\theta_n$ , namely  $\theta'$  and  $\theta''$  such that

$$\theta' + \theta'' = 2\delta. \quad (55)$$

For values of

$$R \geq 1/2$$

there may be additional values of  $\theta_n$ ; in general, the complete set of values of  $\theta_n$  includes  $\theta'$  and  $\theta''$ , these being the values of  $\theta_n$  for  $n = \pm 0$ .

It is desirable, particularly for engineering applications, to have some knowledge regarding the nature and shape of the graph of  $|G'(\theta)|$  in the neighborhood of the values of  $\theta_n$ , which it will be recalled are the values of  $\theta$  where  $|G'(\theta)|$  is zero. For this purpose a formula for the slope of  $|G'(\theta)|$  is useful, namely a formula for  $d|G'(\theta)|/d\theta$ ; however, as will appear below, the absolute value of the slope will suffice, namely  $|dG'(\theta)/d\theta|$ . For the case  $\alpha = 0$ , it is found from Equations (52), (49), (46) that

$$|dG'(\theta)/d\theta| = 2\pi R |\sin(\theta - \delta) \cos(\psi'/2)|. \quad (56)$$

In particular, when  $\theta = \theta_n$ ,

$$|dG'(\theta)/d\theta| = 2\pi R |\sin(\theta_n - \delta)|, \quad (57)$$

because, when  $\theta = \theta_n$ ,  $\sin(\psi'/2) = 0$  and therefore  $\cos(\psi'/2) = 1$ , by Equation (52). Since, in general, the right side of (57) is not zero, it is seen that the graph of  $|G'(\theta)|$  has at  $\theta = \theta_n$  a cusp minimum, not a stationary minimum; but when  $\theta_n = \delta + n\pi$  (where  $\pm n = 0, 1, 2, 3, \dots$ ), then the graph of  $|G'(\theta)|$  has a stationary minimum, not a cusp minimum. It is geometrically evident that at each  $\theta_n$  where  $|G'(\theta)|$  has a cusp minimum the slope of  $|G'(\theta)|$  must be negative at  $\theta = \theta_n -$  and positive at  $\theta = \theta_n +$ ; and the absolute values of these two slopes must be equal. It is for these reasons that a knowledge of the absolute value of the slope suffices.

For engineering applications it is also desirable to know the values of  $\theta$ , say  $\theta_s$ , for which  $|G'(\theta)|$  has stationary values (that is, a horizontal tangent)—thus not including cusp extrema. Equation (56), by its factor

$\sin(\theta - \delta)$ , shows that one set of values of  $\theta_s$  is given by the equation

$$\theta_s = \delta + n\pi, (\pm n = 0, 1, 2, 3, \dots). \quad (58)$$

From Equations (49) and (46) it is seen that the factor  $\cos(\psi'/2)$  in (56) contributes the additional set of values of  $\theta_s$  given implicitly by the equation

$$\cos(\theta' - \delta) - \cos(\theta_s - \delta) = \frac{b}{2R}, (\pm b = 1, 3, 5, \dots), \quad (59)$$

or, explicitly,

$$\theta_s = \delta + \cos^{-1}[\cos(\theta' - \delta) - b/2R], \quad (60)$$

which gives two values of  $\theta_s$  for each applicable value of  $b$ . Equation (59) shows that the applicable values of  $b$  lie in the range

$$-4R \leq b \leq 4R. \quad (61)$$

Hence, when  $R < 1/4$ , there is no applicable value of  $b$  and therefore Equation (60) yields no value for  $\theta_s$ ; thus, when  $R < 1/4$ , the factor  $\cos(\psi'/2)$  in (56) yields no value of  $\theta_s$ , and hence the only values of  $\theta_s$  are those represented by Equation (58).

The next step which will be taken is to impose the condition that the phase angle  $\phi(\theta)$  between the output currents  $j_1(\theta)$  and  $j_2(\theta)$  shall have a specified value  $\phi(\theta)$  for a specified value  $\theta$  of  $\theta$ . From Equation (47) it is then found that

$$P = \frac{\phi'(\theta) - 2\pi Q (\sin \theta' - \sin \theta) - b\pi}{2\pi (\cos \theta' - \cos \theta)}, (\pm b = 1, 3, 5, \dots) \quad (62)$$

which will be employed as a design-formula for  $P$ . (This formula shows that by choosing a sufficiently large value for  $b$ , the staggering ratio  $P = p/\lambda$ , could be made so large that interaction between the two wave-antennae would be practically nil, even for small values of the transverse spacing ratio  $Q = q/\lambda$ ; but the directional characteristics would be less desirable for most applications).

Since

$$R^2 = P^2 + Q^2 \quad (64)$$

and

$$\delta = \tan^{-1}(Q/P), \quad (64)$$

$\psi'(\theta)$  can now be calculated by means of the formula

$$\psi'(\theta) = 2\pi R \cos(\theta' - \delta) - \cos(\theta - \delta) \quad (65)$$

obtained from Equations (49) and (46). A direct but less simple formula for  $\psi'(\theta)$  could be obtained by substituting (62) into (47) and employing (49).

By means of Equation (65) together with (52) and (19), the curves in Figs. 9 and 10 were computed. The ordinates of these curves represent the values of the "group factor ratio"  $\sigma'(\theta) = |G'(\theta)/G'(\theta_r)|$  when  $\theta_r$  (the reference value of  $\theta$ ) is taken as zero, and  $b = +1$ .  $\sigma'(\theta)$  denotes, of course, the



value of the function  $\sigma(\theta)$  when  $U$  is so evaluated that  $G(\theta')=0$ , as described in connection with Equations (38) to (44) inclusive. With  $\theta$  as the independent variable, the parameters in Figs. 9 and 10 are  $Q, \theta', \theta, \phi'(\theta)$ , in addition to  $b=1$  and  $\alpha=0$ . On any one figure,  $\theta', \theta$  and  $\phi'(\theta')$  have the fixed values indicated, while the various curves thereon correspond to the various values of  $Q$  affixed to the curves. The curves are all for the limiting case of  $\alpha=0$ ; for illustrative purposes and even for most applications, this limiting case represents the actual case with sufficient closeness, because  $\alpha$  is very small. It will be noted that the shape of these curves depends very considerably on the spacing of the wave-antennae as fixed by the parameter  $Q=q/\lambda$ . This fact influences the choice of  $Q$ ; but  $Q$  must be chosen large enough so that the spacing  $q=\lambda Q$  is sufficient to prevent troublesome interaction between the two wave-antennae. It is for this reason that the curves have been plotted with  $Q$ , instead of  $P$ , as parameter; for  $P$  would seldom be subject to any practical restriction.

Inspection of Figs. 9 and 10 shows that certain of the curves (particularly for  $Q=0.15$  and  $Q=0.20$  in Fig. 9, and  $Q=0.10$  in Fig. 10), have very small values of  $\sigma'(\theta)$  over a considerable range of  $\theta$ , namely a range extending approximately from  $180^\circ$  to  $\theta'$ . For certain practical applications, this is a valuable property and is an improvement accomplished by my invention; it is obtained by a proper choice of the spacing ratio  $Q=q/\lambda$ . For smaller values of  $Q$ , the values of  $\sigma'(\theta)$  over this range of  $\theta$  are on the whole much larger.

Of course each curve in Figs. 9 and 10 corresponds to a definite design to a spaced staggered array of two parallel wave-antennae, so far as the spacing and the staggering and the combining transducers are concerned. The following table gives the values of the parameters pertaining to the ten designs corresponding to the ten curves in Figs. 9 and 10. The values of  $\alpha, b, \theta, \phi'(\theta), \theta, Q$  were pre-assigned; the resulting values of  $P, R, \delta, \theta'', \theta_s, |U'|, u'-b\pi$  were computed by means of Equations (62), (63), (64), (37), (58), (42), (44) respectively.

$\alpha=0, b=1, \theta=0, \phi'(\theta)=0, |U'|=1, \theta_s=\delta$  and  $\delta+180^\circ$

| $\theta'$ | $Q$ | $P$  | $R$  | $\delta$ | $\theta''$ | $-(u'-b\pi)$ |
|-----------|-----|------|------|----------|------------|--------------|
| 240°      | 0   | .333 | .333 | 0.00°    | 120.00°    | 60.2°        |
| 240°      | .05 | .305 | .309 | 9.32°    | 138.63°    | 70.5°        |
| 240°      | .10 | .276 | .294 | 19.93°   | 159.87°    | 80.8°        |
| 240°      | .15 | .247 | .289 | 31.30°   | 182.60°    | 91.2°        |
| 240°      | .20 | .218 | .296 | 42.57°   | 205.13°    | 101.6°       |
| 220°      | 0   | .283 | .283 | 0.00°    | 140.00°    | 78.6°        |
| 220°      | .05 | .265 | .270 | 10.76°   | 161.40°    | 84.9°        |
| 220°      | .10 | .247 | .267 | 22.07°   | 184.13°    | 91.2°        |
| 220°      | .15 | .229 | .274 | 33.23°   | 206.57°    | 97.9°        |
| 220°      | .20 | .210 | .290 | 43.57°   | 227.13°    | 104.4°       |

The sets of curves in Figs. 9 and 10, though

fairly comprehensive, are furnished primarily for illustrative purposes in this patent specification. If needed in practical applications, similar sets of curves for other values of the parameters can be readily computed by means of the formulas furnished in this patent specification.

From the remarks in the paragraph following Equation (36), it is seen that any set of curves such as those in Figs. 9 and 10 remain valid when the sign of  $\delta$  is changed provided also the sign of  $\theta$  is changed; this fact renders it unnecessary to construct separate sets of curves for the case of  $-\delta$ .

With  $\alpha$  supposed known (at least approximately), and with  $b$  chosen, and  $\theta', P, Q$  evaluated, the corresponding requisite values for  $|U'|$  and  $u'$  can be calculated by means of Equations (42) and (44) respectively; and  $p$  and  $q$  by means of (21) and (22),  $\lambda$  being supposed known, of course, since it is the wave length of the radio waves.

It is seen that the foregoing procedure constitutes a systematic system-method for proportioning the system as regards the staggering  $p$  and the separation  $q$  of the two wave-antennae, and the ratio  $U=T_2/T_1$  pertaining to the terminal transducers, with the object of securing desirable directional characteristics.

To design and adjust a wave antenna system that will embody my invention in a particular instance, the procedure may be summarized as follows: Two parallel spaced and staggered wave antennae will be planned, with the spacing  $q$  and the stagger distance  $p$  and the ratio  $T_2/T_1$  as yet undetermined. These two antennae are to lie in approximately the direction of desired good reception for signals, and each is to have the lengths that will give the best polar diagram for that antenna alone for the frequency range of the waves desired to be received; also the selectivity for that frequency range may be enhanced by interposing appropriate filters between the antennae and the associated receiving apparatus. Appropriate values will be chosen and assigned at the outset for  $\theta', \theta, \phi'(\theta), b$  and  $\alpha$ . Thereupon, one can proceed readily to construct a diagram such as in Fig. 9 or 10, making a family of curves by varying the value of  $Q$ , the ratio of the lateral spacing  $q$  to the wave length  $\lambda$ . By means of such a diagram the best value for  $q$  will be chosen.

Next, the stagger distance  $p=\lambda P$  is determined by Equation (62), and thereupon the value of  $U'=T_2/T_1$  becomes definite and ascertainable, as expressed in Equation (39).

The value chosen for the angle  $\theta'$  will be one particular direction of especially bad interference for which it is desired to make the reception null. Incidentally, there will result another direction of null reception, which together with the assigned direction, will be



symmetrical about the axis passing through the front ends of the antennæ.

Moreover, a low intensity of interference over a considerable range of directions may be had by suitable choice of  $Q$  as, for example, in Fig. 9, where it will be seen that from  $180^\circ$  to  $240^\circ$  the ordinates are very much less for  $Q=0.15$  than for  $Q=0.05$ .

Furthermore, still another arbitrary direction of null interfering reception may be assigned and embodied in the design by practicing compensation, as disclosed in connection with Figs. 6 to 8a, as will be pointed out presently. Also, in this connection it will be seen that this assignment will fix still another direction of null interfering reception, which together with the assigned directions, will be symmetrical about an axis parallel to the wave antennæ.

Referring to Fig. 4, this is the same as Fig. 1 except that in Fig. 4 the transducers  $T_1$  and the  $T_2$  and the receiver are situated at a convenient common point more or less remote from the back end of the wave-antennæ, to which they are connected by means of metallic transmission lines  $L_1$  and  $L_2$  (each suitably transposed) and transformers  $M_1$  and  $M_2$ . Any differences in these transmission lines can be made up in one of the transducers, or by means of a supplementary artificial line inserted in series with the shorter transmission line. In setting forth the fundamental theory of such systems, it is convenient to regard the transmission lines  $L_1$  and  $L_2$  (including any supplementary artificial line) and the transformers  $M_1$  and  $M_2$  as constituting parts of the transducers  $T_1$  and  $T_2$ .

Fig. 5 represents a system which does not differ from that of Fig. 4 or of Fig. 1 in fundamental principles, but possesses the practical advantage of enabling the combining apparatus to be located near the front end instead of at the back end. This is accomplished by employing two wires in multiple for each wave-antenna and terminating the back end of each wave-antenna with a three winding transformer ( $N_1$  and  $N_2$ , respectively) so connected that the wave-antenna currents coming in over the two wires in multiple are propagated from the back end to the front end over the metallic circuit composed of the two wires. Of course the same results could be obtained by means of the system of Fig. 4 employing separate metallic transmission lines for propagating the wave-antenna output currents  $I_1$  and  $I_2$  from the back end to the combining transducers and the receiver all situated at a common point near the front end, but the method of Fig. 5 is simpler and more economical and is free from possible interaction between the metallic transmission lines and the wave-antennæ.

It is known that a single wave-antenna can be "compensated" to secure null reception

from any specified angle of incidence of the radio waves, by suitably combining a small portion of the front-end current with the back-end output current. (See, for instance, Fig. 19 of the above-cited paper by Beverage et al.). This feature of construction and operation is involved in the systems of Figs. 6 to 8a.

Figs. 6, 7, 8 show three ways for "compensating" the system of Fig. 4 so as to secure null reception from any specified angle of incidence of the radio waves, by combining a small portion of the front-end currents with the back-end output currents, by means of metallic transmission lines ( $L_{11}$  and  $L_{22}$ ), transformers ( $M_{11}$  and  $M_{22}$ ), and supplementary transducers ( $T_{11}$  and  $T_{22}$  and  $T_3$ ). In Fig. 6 the individual wave-antennæ are first separately compensated (by adjusting the transducers  $T_{11}$  and  $T_{22}$  respectively), and then the so-compensated wave-antennæ are suitably combined by adjusting the transducers  $T_1$  and  $T_2$ . In Fig. 7 the front-end currents are combined with each other by means of  $T_{11}$  and  $T_{22}$ , and the back-end currents with each other by means of  $T_1$  and  $T_2$ , and finally the resultant front-end current is combined with the resultant back-end current by means of  $T_3$ . Briefly stated, in Fig. 6 the two antennæ are first individually compensated, and then are combined; whereas in Fig. 7 the two antennæ are first combined (front end with front end, back end with back end), and then the combination is compensated as though it were a single wave-antenna. In Fig. 8 the four currents, consisting of the two output currents and the two compensating currents, are added directly together after having been adjusted to suitable values by means of the transducers  $T_1$ ,  $T_2$ ,  $T_{11}$ ,  $T_{22}$  respectively. The systems in Figs. 6, 7, 8 are effectively equivalent; but economically and operatively they differ somewhat from each other.

Figs. 6a, 7a, 8a show the same three different ways of compensation applied to the system of Fig. 5. Thus Figs. 6a, 7a, 8a correspond to Figs. 6, 7, 8 respectively in the same manner as Fig. 5 corresponds to Fig. 4.

In some of the systems represented schematically in these figures, transformers would evidently be necessary at certain points in order to secure a state of balance with respect to earth, as is well known in the communication art. These have been omitted from the figures for the sake of simplicity of description and exposition, but they may be regarded as implied. In some instances, the transducers would have to be unilateral in order to secure the contemplated results.

It will readily be understood from the foregoing exposition that the diagrams of Figs. 9 and 10 relate not to the design of the individual antennæ but to a factor based on the



relative arrangement and combination of two antennæ with a receiving system. If each of these two antennæ is directionally selective, then they should be placed so that their polar diagrams will be oriented alike, and preferably so that the maximum radii of the polar diagrams will be pointed approximately toward the direction from which good signal reception is desired.

I claim:

1. The method of directionally selective radio receiving which consists in building up from the radio waves separate, parallel, horizontal waves, spaced and staggered, and combining them with cumulative effect for a desired direction of reception and with low effect for an undesired range of directions other than the said desired direction and with null effect for a plurality of particular directions other than said desired direction.

2. The method of securing maximum reception in one direction and minimum interference in another direction with a pair of parallel wave antennæ, which consists in adjusting them at a certain spacing  $q$  and a certain stagger distance  $p$ , and also adjusting the ratio of the transfer factors from said antennæ to a common receiver at a certain value  $U$ , the values of  $q$ ,  $p$ , and  $U$  being optimum for the said character of reception.

3. The method of securing maximum reception in one direction and minimum interference in some other directions with a pair of parallel wave antennæ, which consists in making them of the best length for receiving at the desired frequency in the direction of their length individually, and also adjusting them at a certain spacing  $q$  and a certain stagger distance  $p$ , and also adjusting the ratio of the transfer factors from said antennæ to a common receiver at a certain value  $U$ , and also drawing off compensating currents through filters from the opposite ends of the respective antennæ; thereby getting the desired character of reception with low intensity over a considerable range of directions other than the said one direction, and null reception in two assigned directions other than said one direction, and null reception in two other directions dependent on said assigned directions.

4. The method of directionally selective radio receiving which consists in building up from the radio waves separate, parallel, horizontal wire waves, spaced and staggered each to the magnitude of best individual directional selectivity at the desired frequency, and combining them with cumulative effect for a desired direction of reception and with low effect for a range of undesired directions, and with null effect for at least one particular undesired direction.

5. In a radio receiving system, in combination, two parallel wave antennæ spaced apart

a certain distance  $q$  and staggered a certain distance  $p$ , a common receiver, and transducers between each antenna and the receiver with a certain ratio  $U$  between their transfer factors  $T_1$  and  $T_2$ , the values  $q$ ,  $p$ , and  $U$  being optimum for maximum reception in one certain direction and also optimum for minimum interference in another certain direction.

6. In a radio receiving system, in combination two parallel wave antennæ spaced apart a certain distance  $q$  and staggered a certain distance  $p$ , a common receiver, transducers between each end of each antenna and the receiver with their transfer factors and the values  $q$  and  $p$  at optimum for good reception in one certain direction and for minimum interference in a range of directions different therefrom and also for null reception in two other assigned directions and two additional directions dependent on the assigned directions.

7. In a radio receiving system, in combination two parallel wave antennæ, a receiver, and transducers between each end of each antenna and the receiver, said antennæ being spaced apart and staggered by optimum distances, and said transducers having their transfer factors related at optimum values for good reception in one assigned direction and minimum interference in a range of other directions and null interference in several particular other directions, of which at least one is assigned.

8. In a radio receiving system, in combination two parallel wave antennæ each of best length for directional selectivity at the desired frequency, a receiver, and transducers between each antenna and the receiver, said transducers comprising filters appropriate to currents of said frequency, and said antennæ being spaced apart and staggered by distances optimum for maximum reception in one assigned direction and minimum interference in an assigned range of other directions and null interference in at least one assigned other direction and at least one other direction dependent on the last mentioned assigned direction.

9. In a radio receiving system, in combination, two parallel wave antennæ, a receiver, and transducers between each antenna and the receiver, said antennæ being spaced apart and staggered by optimum distances and the ratio of the transfer factors of said transducers being chosen at optimum for maximum reception in one assigned direction and minimum interference in another assigned direction.

10. In a radio receiving system, two parallel wave antennæ, a receiver, and transducers between said antennæ and the receiver, said antennæ being spaced and staggered at proper distances and said transducers having the



proper ratio between their transfer factors to make reception maximum in one assigned direction and interference minimum in another assigned direction.

5 In testimony whereof, I have signed my name to this specification this 28th day of December, 1927.

RAY S. HOYT.

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