

March 30, 1965

E. EISNER

3,175,406

MECHANICAL AMPLITUDE TRANSFORMER

Filed March 25, 1963

6 Sheets-Sheet 1

FIG. 1

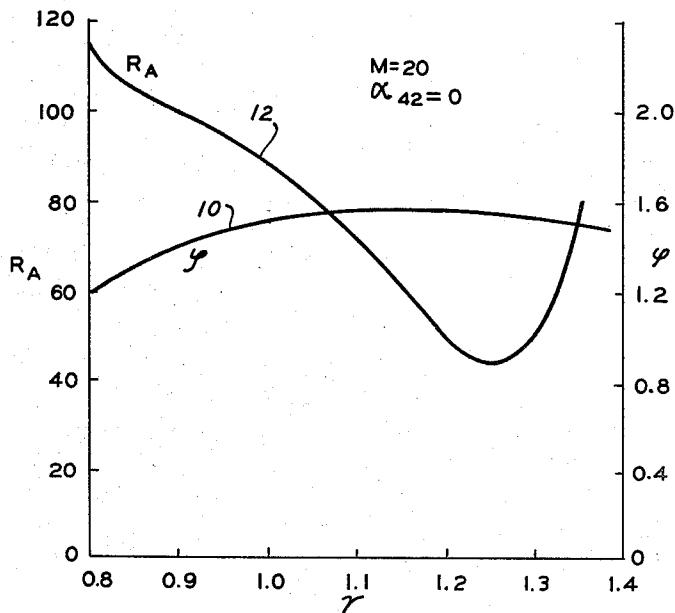
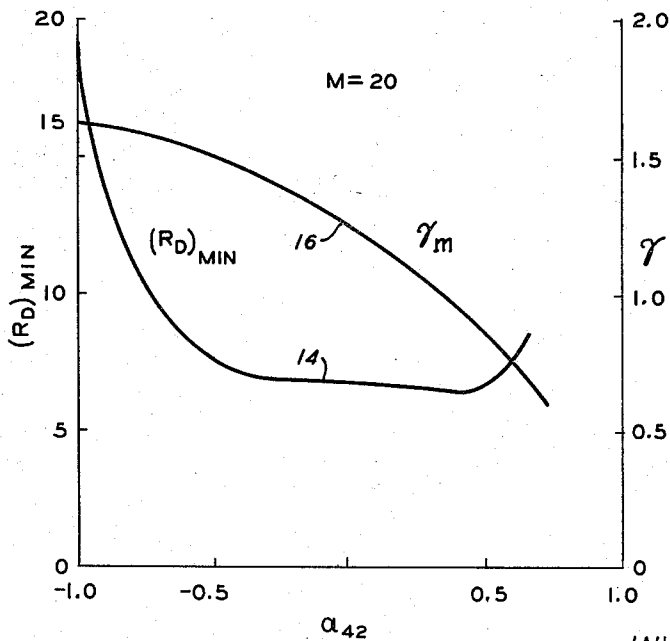


FIG. 2



INVENTOR
E. EISNER
BY
H. D. Wright
ATTORNEY

March 30, 1965

E. EISNER

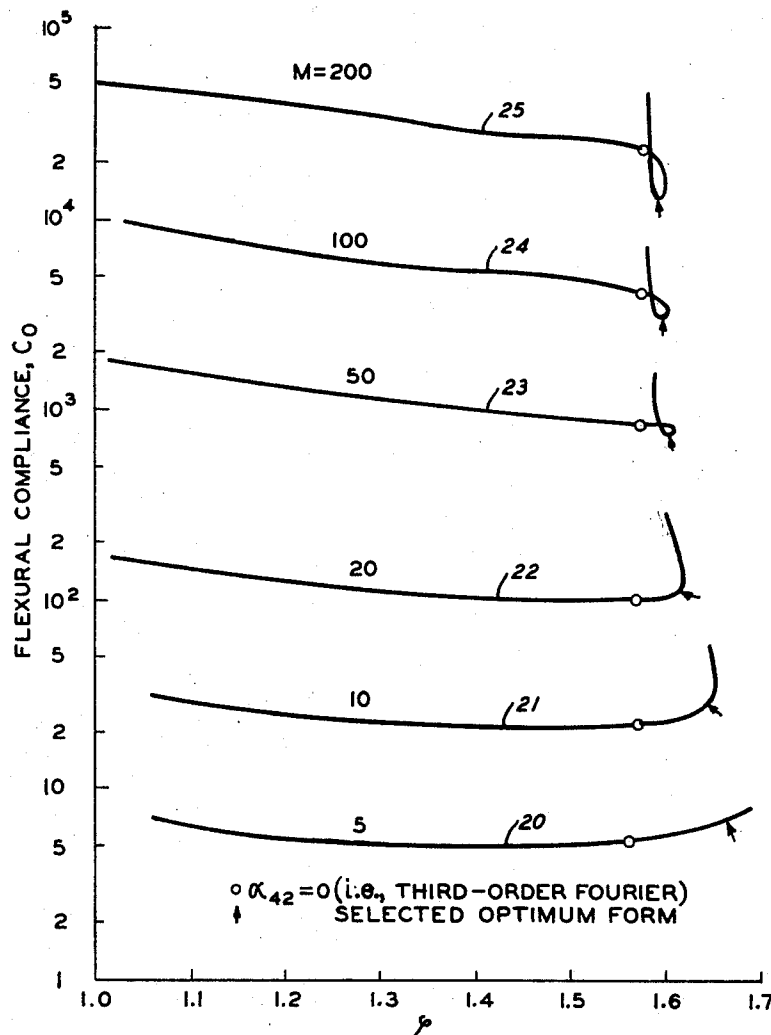
3,175,406

MECHANICAL AMPLITUDE TRANSFORMER

Filed March 25, 1963

6 Sheets-Sheet 2

FIG. 3



March 30, 1965

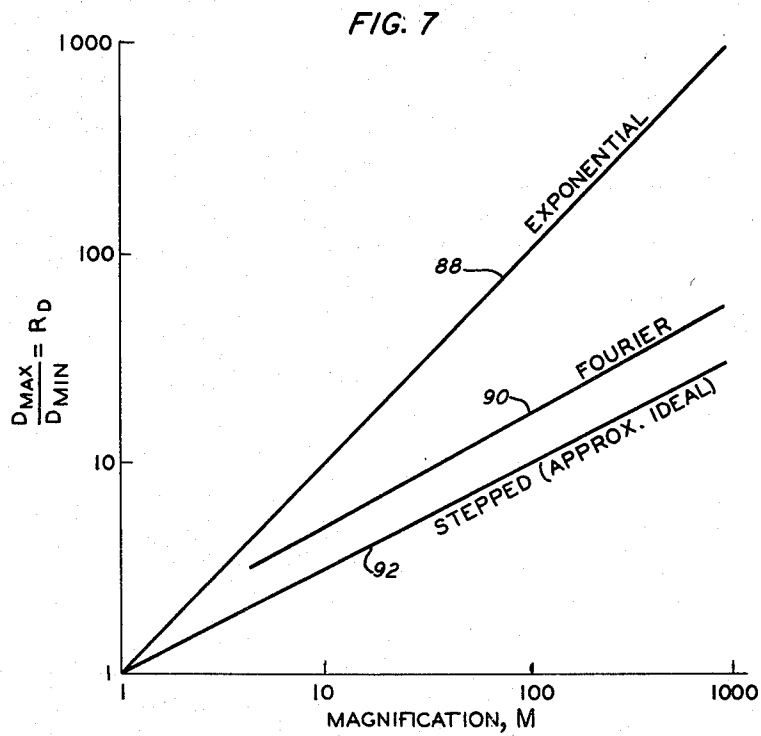
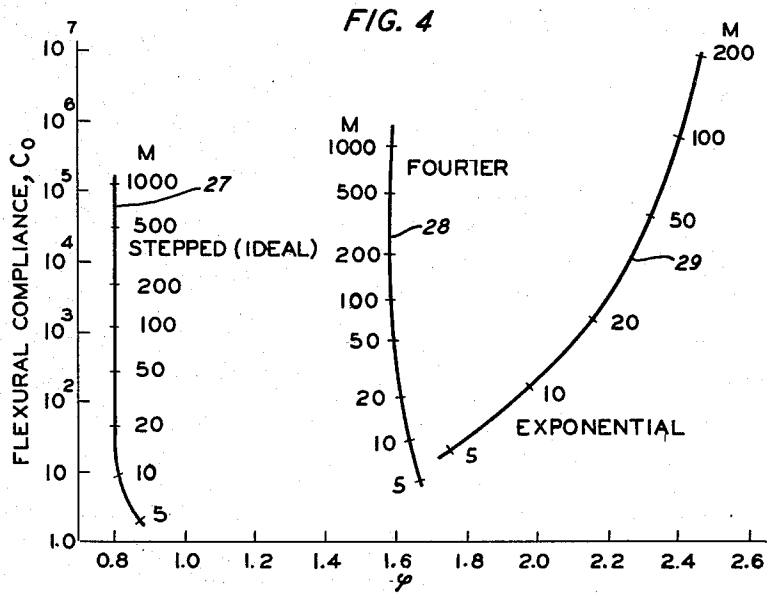
E. EISNER

3,175,406

MECHANICAL AMPLITUDE TRANSFORMER

Filed March 25, 1963

6 Sheets-Sheet 3



March 30, 1965

E. EISNER

3,175,406

MECHANICAL AMPLITUDE TRANSFORMER

Filed March 25, 1963

6 Sheets-Sheet 4

FIG. 5A

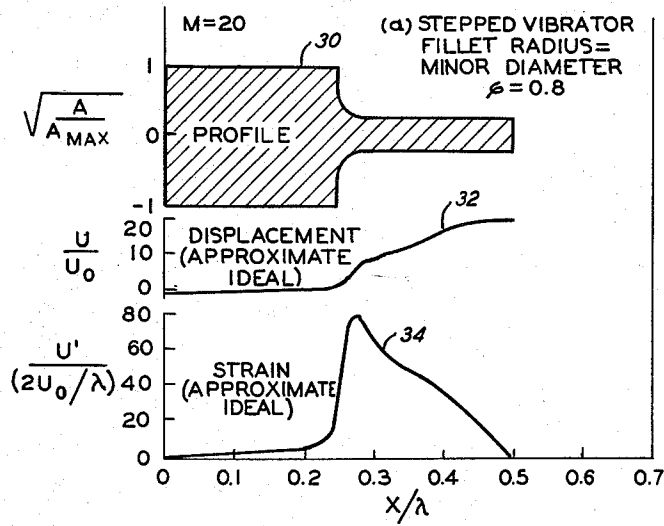
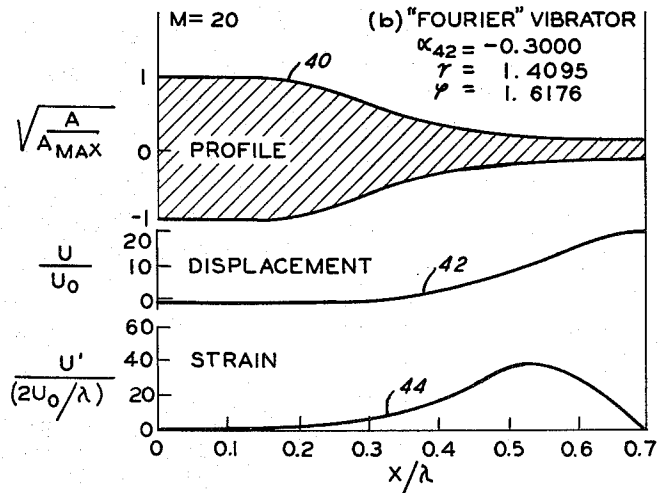


FIG. 5B



March 30, 1965

E. EISNER

3,175,406

MECHANICAL AMPLITUDE TRANSFORMER

Filed March 25, 1963

6 Sheets-Sheet 5

FIG. 5C

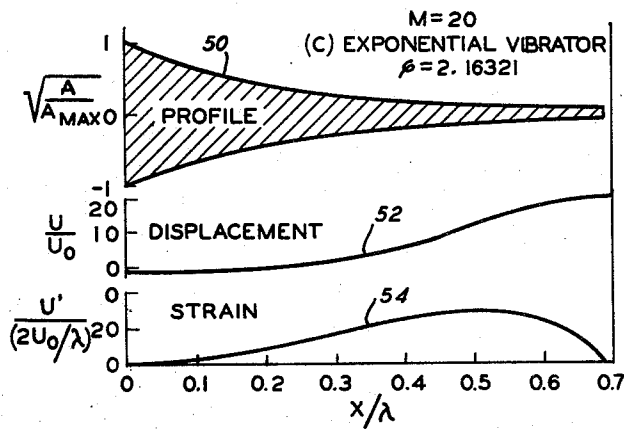
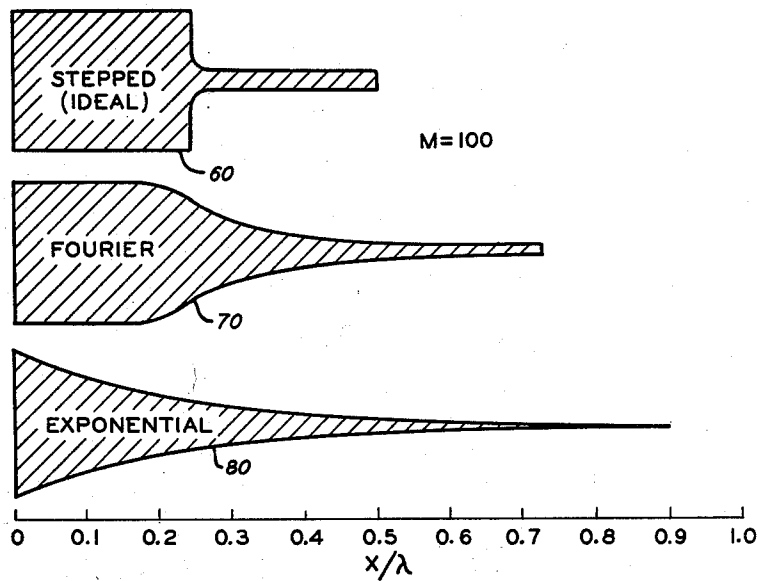


FIG. 6



March 30, 1965

E. EISNER

3,175,406

MECHANICAL AMPLITUDE TRANSFORMER

Filed March 25, 1963

6 Sheets-Sheet 6

FIG. 8

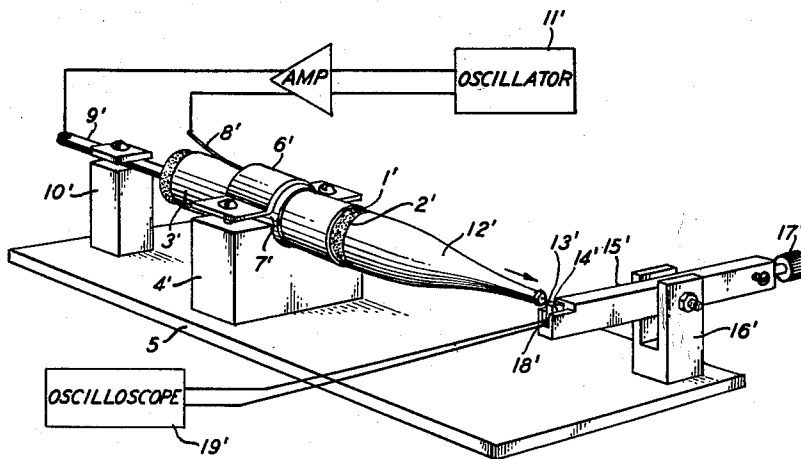
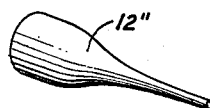


FIG. 9



1

3,175,406

MECHANICAL AMPLITUDE TRANSFORMER

Edward Eisner, Bernardsville, N.J., assignor to Bell Telephone Laboratories, Incorporated, New York, N.Y., a corporation of New York

Filed Mar. 25, 1963, Ser. No. 267,718

5 Claims. (Cl. 74—1)

This invention relates to elongated mechanical vibratory members for amplifying the magnitude of repetitive physical displacements.

Reference may be had to United States Patent No. 2,573,168, granted October 30, 1951, to W. P. Mason and R. F. Wick, and to W. P. Mason's book entitled "Physical Acoustics and the Properties of Solids," published by D. Van Nostrand and Co., Princeton, New Jersey, 1958, Chapter VI, pages 156-178, for information relating to elongated mechanical vibratory members exponentially tapered to effect an amplification of longitudinally vibratory motion.

It is also known in the art that a "stepped" vibratory member having two sections of similar transverse cross-sectional shape but radically differing cross-sectional areas, each section being substantially one-quarter wavelength long, can be employed to effect an amplification of longitudinally vibratory motion. "Stepped" members are discussed, for example, by L. Balamuth in "The Transactions of the Institute of Radio Engineers, Professional Group on Ultrasonics Engineering" (2), November 1954; see particularly section 8 on page 29.

All such members as are of interest in connection with the present invention have a maximum transverse dimension which is a fraction of a wavelength, for example, less than one-half and preferably less than one-quarter wavelength of the energy being amplified. Such members are further in the order of one-half to three-quarters of a wavelength long.

For reasons which will become more readily apparent from a perusal of the descriptive material given hereunder, the "stepped" type of vibratory member mentioned above is limited as a practical matter by the strains generated in the member to relatively much smaller amplitude ratios or "magnifications" of vibratory amplitude than those which are frequently desirable. As used for comparative purposes throughout this specification, the "ideal" characteristics of the "stepped" type member are indicated. Accordingly, the "Fourier" type will, in actual practice, be "better" as compared with an actual "stepped" type than indicated by the present application.

Chapter VI of Mason's book, mentioned above, discusses numerous uses for which devices of the invention and Mason's exponentially tapered members are both readily adapted.

While Mason's exponentially tapered members and the related conically tapered members, and the like, can, as a practical matter, provide much higher amplitude ratios or "magnifications" of vibratory amplitude than the "stepped" members, the exponentially tapered and similarly shaped members, for large amplitude ratios, become so slender over a considerable fraction of their total length that they may be difficult to manufacture and difficult to use since unwanted flexural vibration or motion of the slender portion of the member can scarcely be avoided. Stated in other words, they lack sufficient stiffness at large amplitude ratios to function in a completely satisfactory manner.

Accordingly, in accordance with the principles of the present invention, a new class of elongated vibratory members is proposed. The members of this new class are capable of providing very large magnifications of the vibratory energy and at the same time have a degree of

2

stiffness which is entirely sufficient for the contemplated uses of the members. Since, as will presently appear, the longitudinal shaping of members of this new class is determined by the employment of Fourier series of third and fourth orders, the members are referred to throughout this application as "Fourier" vibrators.

A principal object of the present invention therefore is to eliminate difficulties encountered with prior art types of elongated mechanical vibratory members for amplifying the magnitude of repetitive physical displacements.

A further principal object is to provide vibratory members with greater ability to withstand large amplitudes.

A still further object is to provide vibratory members of enhanced stiffness at large values of magnification.

Other and further objects, features and advantages of the invention will become apparent from a perusal of the following detailed description of illustrative devices and from the accompanying drawing, in which:

FIGS. 1 and 2 illustrate the variation of the ratio of maximum to minimum transverse areas of a member of the invention for a specific magnification with several design parameters of the members;

FIG. 3 illustrates the effect upon flexural compliance of variations in the ratio of maximum particle velocity to maximum permissible strain for six values of the "magnification" M of a member of the invention;

FIG. 4 presents comparative data relative to the magnification, flexural compliance and ratio of maximum particle velocity to maximum permissible strain for vibrators of the "stepped," "exponential" and "Fourier" types;

FIGS. 5A, 5B and 5C present comparative profiles and displacement and strain characteristics for corresponding vibrators of the "stepped," "Fourier" and "exponential" types, respectively, for a "magnification" M of 20;

FIG. 6 presents comparative profiles of members of the "stepped," "exponential" and "Fourier" types for a "magnification" M of 100;

FIG. 7 presents in graphic form the comparative relations of "magnification" M to the ratio of maximum to minimum diameters for members of the "stepped," "exponential" and "Fourier" types;

FIG. 8 is a perspective showing providing a direct comparison between a member of the invention and an exponentially tapered member of the above-mentioned patent to Mason et al., both members providing a magnification of ten; and

FIG. 9 is a perspective showing of a member of the invention providing a magnification of 100.

It can be shown that in most acoustically resonant solid bodies the maximum particle velocity, v_m , is related to the speed of sound, c , in the material, and to the maximum permissible strain S_m in the body by:

$$\frac{v_m}{c} = \varphi S_m \quad (1)$$

where φ is, in general, a (nondimensional) function of the geometry and of Poisson's ratio of the material.

We shall assume restrictions on the type of resonant system, sufficient for (1) to be true; namely, that the strains at any two points in the system be proportional to each other and that the speed of sound in the material be independent of amplitude of displacement or of strain. These conditions are necessary for most analyses of wave propagation, and do not seriously restrict the utility of Equation 1.

The dependence of φ on Poisson's ratio expresses the fact that, in general, a single speed of sound does not describe the wave propagation, and a single component does not describe the strain. However, in many practical cases, including those of interest in the present application, either the stress or the strain can be described in

terms of a single component and in that case φ is a function of geometry only. Indeed, in many important cases the dependence of φ on shape is also restricted, for instance, if the sound propagates as plane, unidirectional waves, then the ratio of dimensions parallel and perpendicular to the propagation direction does not affect φ as long as the lateral dimensions are much smaller than the wavelength, for example, less than one-quarter wavelength.

Further, for bodies, such as those of interest in connection with the present invention, that do not contain serious stress concentrators, and in which most of the mass is not in unstressed, "passenger" material, φ does not in many instances differ greatly from 1. For example, for a uniform bar in longitudinal resonance, $\varphi=1$; for a uniform cantilever of circular section, $\varphi=0.50$; for a uniform tube (including a rod) in torsion, $\varphi=1$; for an exponential "horn" such for example as is disclosed in United States Patent No. 2,573,168, granted October 30, 1951, to W. P. Mason and R. F. Wick, with a 10:1 ratio of end diameters, $\varphi=2.0$. Even for special shapes designed specifically for large φ , a value as large as 5 can be attained only by very careful design.

Thus, not only is φ an excellent figure of merit for assessing the ability of a particular body to resonate with large amplitude without overstraining, but Equation 1 also provides a ready means of deciding how easily a particular particle velocity can be obtained. If $v_m/(cS_m)$ is less than or equal to 0.3, almost any sensible design will do; if 0.3 is less than $v_m/(cS_m)$ which is less than or equal to 2.0, there is considerable choice among specially designed, tapered shapes; if 2.0 is less than $v_m/(cS_m)$ which is less than or equal to 5.0 only the most carefully designed shapes will do; and if $v_m/(cS_m)$ is greater than 5.0 it is very improbable that any suitable shape can be found.

Consider the longitudinal resonance of a slowly tapering rod whose lateral dimensions are small compared with the wavelength, λ , for extensional waves; the differential equation for the amplitude of longitudinal displacement, $u(x)$, at coordinate x is

$$u'' + u' \cdot \frac{d}{dx} (\ln A) \left(\frac{2\pi}{\lambda} \right)^2 \cdot u = 0 \quad (2)$$

$$\ln A = \log_e A \quad (3)$$

(i.e., the natural logarithm of A)

$$u' = \frac{du}{dx}, u'' = \frac{d^2u}{dx^2} \quad (4)$$

where $A(x)$ is the cross-sectional area.

The conventional way of tackling such a problem is to express A as a function of x and to attempt the solution of the resulting differential equation for u , given the boundary conditions. Algebraic solution is possible in only a few cases, and numerical solution is cumbersome. Furthermore, the object is usually to design a body that will resonate so that the displacement and its derivatives satisfy given conditions. There is no clear way of guessing the function that will do this.

By reversing the procedure and selecting a wave-function $u(x)$, the function $A(x)$ can be unambiguously deduced from Equation 2. If we devise a function $u(x)$ that satisfies the boundary conditions of the problem and gives the values of displacement, strain, et cetera, that we require at specific points, then we get a differential equation for A from Equation 2. $A(x)$ can be obtained by simple integration; and, by judicious choice of the form of $u(x)$, this integration can often be done algebraically in terms of known functions.

AMPLITUDE TRANSFORMERS

An "amplitude transformer" for substantially sinusoidal repetitive physical displacements is a body which, when in "free" resonance, has a pair of strain-free surfaces (ends) at one of which the amplitude is much larger than

at the other. The body can then be driven at the low-amplitude surface (end) by a resonating transducer, as shown for example in the above-mentioned patent to Mason et al., to give magnified vibration at the high-amplitude surface. Two well known types exist for the magnification of longitudinal vibration, namely, the "exponential" (see Mason et al. patent) and the "stepped" (described in detail, for example, by L. Balamuth, as noted hereinabove).

By considering the conservation of momentum, it can be seen that in any useful amplitude transformer, the cross-sectional area near the low-amplitude end must be much bigger than that near the high-amplitude end. We shall call the ratio of the maximum to the minimum cross-sectional areas R_A , and the corresponding ratio of diameters for members of circular cross section, R_D . Thus

$$\frac{D_{\max}}{D_{\min}} = R_D = \left(\frac{A_{\max}}{A_{\min}} \right)^{1/2} = R_A^{1/2} \quad (5)$$

where D indicates diameter and A indicates area. If Equation 2 is to apply with good accuracy to a longitudinal vibrator, the maximum cross-sectional dimension must be small compared to a wavelength, say, preferably less than $\lambda/4$, although approximate results, that is, satisfactory where optimum performance is not required, can be obtained for values up to one-half wavelength. The length of the vibrator will be of the order of $\lambda/2$, for example, in some instances it may be as great as three-quarters of a wavelength. R_D , therefore, for members of circular cross section (and R_A for members of square or other cross section) determines the "slenderness" of the transformer. Whether the transformer is used to drive another vibratory member, or is used directly as an ultrasonic drill or other tool, flexural rigidity is obviously desirable. Indeed, if the flexural rigidity is too low, the transformer will not only be difficult to make accurately but will also be difficult to use since unwanted flexural vibrations are most likely to occur. Thus, too large a value of R_D (or of R_A) is undesirable.

An exponentially tapered vibrator of amplitude magnification M has the diametral ratio $(R_D)_e = M$. If M is large this gives a very slender rod. Values of M up to 20 or more may, for example, be needed. The "stepped" vibrator, on the other hand, has the diametral ratio $(R_D)_s = \sqrt{M}$, and this clearly gives a stiff shape. Unfortunately, the highest value of φ obtainable with the stepped vibrator is about 0.8. This is not only rather low for the majority of purposes, but, in particular, makes it entirely unsuitable when the higher amplitude ratios or magnifications are required, since stresses sufficient to cause damage to the member may be generated.

"FOURIER" TYPE AMPLITUDE TRANSFORMERS

Equation 2 can be rewritten in nondimensional (that is, "normalized") form:

$$U'' + U' \cdot \frac{d}{dX} (\log_e A) + \Omega^2 \cdot U = 0 \quad (6)$$

where

$$U = u/l \quad (7a)$$

$$X = x/l \quad (7b)$$

$$\Omega = 2\pi l/\lambda \quad (7c)$$

and U' is

$$\frac{dU}{dX}$$

U'' is

$$\frac{d^2U}{dX^2}$$

and l is the length of the vibrator.

We shall now look for a function $U(X)$ that satisfies boundary conditions that express the required characteristics of our vibrator.

As boundary conditions, we have

(1) The two ends of the vibrator are to be stress-free and therefore strain-free; thus, at the low-amplitude end

$$U'(0)=0 \quad (8a)$$

and at the high amplitude end

$$U'(1)=0 \quad (8b)$$

(2) The ratio of the amplitudes of vibration at the two ends is to be $-M$, the negative sign expressing opposed phases. Thus,

$$U(0)=U_0 \quad (8c)$$

$$U(1)=-M \cdot U_0 \quad (8d)$$

where U_0 is the value of U at the larger or lower amplitude end of the vibratory member.

(3) The gradient of the lateral surface (or slope of the longitudinal cross-sectional contour of the vibratory member) is to be finite everywhere.

We may rewrite Equation 6 as

$$\frac{d}{dX}(\log_e A) = -\frac{U'' + \Omega^2 U}{U'} \quad (6a)$$

We must then ensure that, wherever U' approaches zero, $(U'' + \Omega^2 U)$ approaches zero at least as rapidly. We therefore choose a function for $U'(X)$ that has no zeros in the range "zero is less than or equal to X which is less than or equal to 1," except for the essential, first order, zeros at $X=0$ and $X=1$. Then, two necessary conditions are:

$$U''(0) = -\Omega^2 U_0 \quad (8e)$$

$$U''(1) = M\Omega^2 U_0 \quad (8f)$$

A design giving a particularly low value of R_A results if the surface gradient at the low-amplitude end is not merely restricted to being finite, but is set to zero. Now, since both U'' and $(U'' + \Omega^2 U)$ are zero at $X=0$, we can see from Taylor's Theorem that

$$\begin{aligned} \lim_{X \rightarrow 0} \frac{d}{dX}(\ln A) &= -\lim_{X \rightarrow 0} \frac{U'' + \Omega^2 U}{U'} \\ &= -\lim_{X \rightarrow 0} \frac{U''' + \Omega^2 U'}{U''} = \frac{U'''(0)}{U''(0)} \end{aligned} \quad (9)$$

where U''' is

$$\frac{d^3 U}{dX^3}$$

$U''(0)$ is finite, and, if the gradient of the surface is to vanish,

$$U'''(0)=0 \quad (8g)$$

This immediately settles the difficulty that (8e) is not a sufficient condition for finite gradient. Since we can similarly complement condition (8f) at $X=1$ by setting

$$U'''(1)=0 \quad (8h)$$

and since zero gradient at $X=1$ has no disadvantages, we shall adopt this condition, too.

If we choose a function $U(X)$ that satisfies conditions (8a-8h), we can integrate (6a) to get the shape of a vibrator that will have the characteristics we have specified. In particular, for given M and Ω^2 , we can evaluate R_A , the ratio of maximum to minimum areas. We can also evaluate φ , since the strain is $U'(X)$ and since from Equations 1 and 7a-7c,

$$\varphi = M \Omega \frac{U_0}{S_m} = \frac{M \Omega U_0}{|U'|_{\max}} \quad (10)$$

If $U(X)$ is expressed as polynomial, it has to be at least seventh order to satisfy the eight conditions (8a-8h). As this makes the problem complicated, we take instead

$$U(X) = \sum_{k=0}^N \alpha_k \cos k\pi X \quad (11)$$

The Fourier series of Equation 11 above is employed in accordance with the present invention in truncated form, that is, its expansion is carried at most to the fourth order (that is, $N=4$) though, as will presently appear, for some purposes the third order (that is, $N=3$) will result in a good vibrating member. The fourth order is, however, preferable since it can provide a still smaller area ratio R_A (or for a vibrator of circular cross section a small diametral ratio R_D) which, as taught in the present application, is particularly advantageous since the vibratory member will then have greater stiffness (lower flexural compliance). Expanded to the fourth order (that is, $N=4$) Equation 11 will, of course, take the form

$$U(X) = \alpha_0 + \alpha_1 \cos \pi X + \alpha_2 \cos 2\pi X + \alpha_3 \cos 3\pi X + \alpha_4 \cos 4\pi X \quad (11a)$$

The virtue of this form is that conditions (8a), (8b), (8g) and (8h) are identically satisfied, leaving only four conditions to which to fit the values of α_k . To do this, $N=3$ is sufficient, and with this form for $U(X)$ the integration (6a) can be carried out algebraically. Using the notation

$$\gamma = \Omega / \pi = 2l = \lambda \quad (12a)$$

and

$$\mu = (M+1)/(M-1) \quad (12b)$$

and computing $U'(X)$ and $A(X)$ for a given M and γ , we can find φ and R_A (the ratio of maximum to minimum cross-sectional areas). We can thus plot $\varphi(\gamma)$ and $R_A(\gamma)$ for any given M , and this is done for $M=20$ in FIG. 1, curves 10 and 12 respectively. The graphs for other values of M (as long as M does not closely approach 1) are essentially similar. The important feature is that $R_A(\gamma)$ has a sharp minimum (as illustrated by curve 12 of FIG. 1) and that $\varphi(\gamma)$ changes slowly in the neighborhood of the corresponding value of γ , curve 10, FIG. 1.

Since the flexural compliance is approximately proportional to R_A^2 , there is nothing to be gained by using any value of γ other than that corresponding to minimum R_A . Thus, while in principle the third order Fourier form of $U(X)$, using $N=3$ in Equation 11, gives us a choice, for given M , of a variety of combinations of φ and R_A , one of these combinations has such overwhelming advantage over the others that we have really found a single, useful shape for each M . While it will later be seen that this shape is an interesting one, the design has turned out to be essentially a "single parameter" type, like the stepped and exponential vibrators.

A "two-parameter" design, in which both M and φ or M and R_A are at choice can be obtained with the Fourier series of Equation 11, using a value of $N=4$ and α_4 , the coefficient of $\cos 4\pi X$, being left as a free parameter. Defining an arbitrary constant α_{42} as

$$\alpha_{42} = \frac{32\alpha_4}{U_0(M-1)} \quad (13a)$$

and using the boundary conditions (8c) through (8f), the following expressions are obtained for the coefficients of the terms of Equation 11

$$\alpha_0 = \frac{1}{32}(M-1)[3\alpha_{42} - 4(4-\gamma^2)]U_0 \quad (13b)$$

$$\alpha_1 = \frac{1}{16}(M+1)(9-\gamma^2)U_0 \quad (13c)$$

$$\alpha_2 = \frac{1}{6}(M-1)(\alpha_{42} + \gamma^2)U_0 \quad (13d)$$

$$\alpha_3 = \frac{1}{16}(M+1)(\gamma^2 - 1)U_0 \quad (13e)$$

This determines $U(X)$ and its derivatives, and it follows that:

$$\frac{d(\ln A)}{d(\cos \pi X)} = \frac{(4-\gamma^2)(\gamma^2 + \alpha_{42}) - \mu(9-\gamma^2)(\gamma^2 - 1) \cos \pi X - (16-\gamma^2)\alpha_{42} \cos^2 \pi X}{\mu(3-\gamma^2) - 2(\gamma^2 + 2\alpha_{42}) \cos \pi X + 3\mu(\gamma^2 - 1) \cos^2 \pi X + 4\alpha_{42} \cos^3 \pi X} \quad (14)$$

The denominator of relation (14) is a cubic in $\cos \pi X$ and could therefore be factorized to allow algebraic integration for $(\ln A)$. But the resulting expressions would be so complicated that nothing would be gained over numerical integration.

A program was therefore written for an International Business Machines Model 7090 computer to compute $U'(X)$ and $A(X)/A(0)$, and hence φ and R_A , for given values of M , α_{42} and γ . This showed that, for all M and α_{42} in the ranges giving practicable values, $R_A(\gamma)$ had a sharp minimum, as had already been found for $\alpha_{42}=0$ (that is, for the third order Fourier form, discussed above). The program was therefore modified to enable it to find the minimum value, $(R_D)_{\min}$, of $R_D(\gamma)$, which as mentioned above is equal to $\sqrt{R_A(\gamma)}$, and the corresponding value γ_m of γ , for given values of M and α_{42} . Thus for any selected values of m and α_{42} , tables can be obtained of the relevant parameters and of the functions of X , in particular $(R_D)_{\min}$, φ and

$$(A(X)/A(0))^{1/2}$$

either for all given values of γ , or for γ_m .

FIG. 2 shows $(R_D)_{\min}$, curve 14, and γ_m , curve 16, as functions of α_{42} for $M=20$. For the range of M investigated, 5 to 200, the curves are essentially similar. The important feature is that, in the approximate range $-0.5 \leq \alpha_{42} \leq +0.5$, $(R_A)_{\min}$ changes rather little, and outside that range it increases rapidly. If the flexural stiffness of the vibrator were a function of cross section only, there would be little to choose among values of α_{42} in this range. But this is true only if the length (which is proportional to γ) changes little, and FIG. 2 shows that this is not the case here. Therefore a subroutine was added to the programme of the computer to compute the static flexural compliances of these nominally "half wave" vibrators, for given loading conditions. Compliance is, of course, the inverse of stiffness.

In FIG. 3, curves 20 through 25, inclusive, the values of flexural compliance C_0 and φ corresponding to γ_m for given α_{42} are plotted against each other for several values of M as marked on each curve.

In general, it may be assumed that there is no point in using a design with reduced stiffness unless thereby a higher value of φ is gained, that is, that only those parts of the curves 20 through 25, inclusive, on which $(\partial C_0 / \partial \varphi) > 0$ are important. Now, from FIG. 3, it is apparent that, except for the lowest values of M (which are of little interest), $(\partial C_0 / \partial \varphi) > 0$ for only very small ranges of C_0 or φ . Therefore we have gained very little freedom of independent choice of M and φ by our addition of a free parameter to $U(X)$. In fact, for each value of M there is a value of α_{42} that gives nearly the greatest bending stiffness and nearly the greatest φ . However, it is also clear that this value is not $\alpha_{42}=0$, but $\alpha_{42} \approx -0.3$. While the difference between these two configurations is not great for $M < 50$, for large values of M the difference may be considerable. Thus, for $M=200$, the shape for $\alpha_{42} = -0.34$ is twice as stiff as that for $\alpha_{42}=0$, and this ratio continues to increase with increasing M .

From the curves of FIG. 3 values of α_{42} that give good compromises between stiffness and φ can be selected. The points corresponding to the third order Fourier series (that is, $N=3$) are indicated by small circles and the selected optimum fourth order Fourier series ($N=4$) points are indicated by the small arrows.

The numerical data computed for the four higher values of M for the corresponding vibrators represented by curves 22 through 25 are shown in Table I.

Table I

PROFILES OF FOURTH-ORDER FOURIER TRANSFORMERS

M = Magnification
 Φ = Figure of merit for ability to withstand large amplitude
 Φ = Φ is chosen to give a good compromise between flexural stiffness and Φ
 λ = Wavelength of extensional waves
 $\Gamma = 2 \times \text{length} / \lambda$. It is chosen to give minimum section ratio
 $Z = (\text{Distance from low-amplitude end}) / \lambda$
 D/D_0 = Square root of the ratio of the sectional area at Z to that at $Z=0$.

$M=20.000$ Alpha42=-0.30000 Gamma= 1.40950 Phi= 1.61755		$M=50.000$ Alpha42=-0.30000 Gamma= 1.43210 Phi= 1.60161	
Z	D/D0	Z	D/D0
0.	1.00000	0.	1.00000
0.03524	1.00078	0.03580	1.00030
0.07047	1.00378	0.07160	1.00282
0.10571	1.00985	0.10741	1.00973
0.14095	1.01882	0.14321	1.01566
0.17619	1.01437	0.17901	0.99566
0.21142	0.98057	0.21481	0.90763
0.24666	0.89268	0.25062	0.74477
0.28190	0.75589	0.28642	0.56502
0.31714	0.60093	0.32222	0.41892
0.35237	0.47813	0.35802	0.31509
0.38761	0.37940	0.39383	0.24407
0.42285	0.30738	0.42963	0.19539
0.45809	0.25558	0.46543	0.16154
0.49332	0.21836	0.50124	0.13766
0.52856	0.19158	0.53704	0.12066
0.56380	0.17246	0.57284	0.10853
0.59904	0.15910	0.60864	0.10017
0.63427	0.15029	0.64445	0.09464
0.66951	0.14528	0.68025	0.09150
0.70475	0.14366	0.71605	0.09048

$M=100.000$ Alpha42=-0.34000 Gamma=1.46275 Phi=1.59287		$M=200.000$ Alpha42=-0.34000 Gamma=1.46725 Phi=1.58941	
Z	D/D0	Z	D/D0
0.	1.00000	0.	1.00000
0.03657	0.99465	0.03668	0.98943
0.07314	0.98529	0.07336	0.97124
0.10971	0.98513	0.11004	0.97098
0.14627	0.99740	0.14672	0.99506
0.18284	0.98835	0.18341	0.97622
0.21941	0.87805	0.22009	0.75787
0.25598	0.66408	0.25677	0.52658
0.29255	0.46251	0.29345	0.34101
0.32912	0.32287	0.33013	0.23085
0.36569	0.23408	0.36681	0.16528
0.40226	0.17731	0.40349	0.12455
0.43883	0.13996	0.44017	0.09811
0.47540	0.11406	0.47686	0.08030
0.51196	0.09710	0.51354	0.06793
0.54853	0.08474	0.55022	0.05933
0.58510	0.07603	0.58690	0.05323
0.62167	0.07000	0.62358	0.04902
0.65824	0.06605	0.66026	0.04625
0.69481	0.06381	0.69694	0.04468
0.73138	0.06308	0.73363	0.04417

In FIG. 4 the values of compliance, that is, the inverse of stiffness, for the selected cases (that is, various values of M) are plotted against φ , curve 28, together with the corresponding curves 27 and 29 for the "stepped" and the "exponential" types of mechanical transformers, or vibrators, respectively.

In FIGS. 5A, 5B and 5C, the profiles 30, 40 and 50 of the stepped, the Fourier and the exponential amplitude transformers (or vibrators) with $M=20$ are shown, respectively, together with their displacement and strain functions, as illustrated by curves 32, 42 and 52 and 34, 44 and 54, respectively. The cross-sectional shapes of the three types of transformers are similar, and the maximum cross-sectional dimension and the wavelength, λ , are the same for all the transformers (or vibrators). The trends in flexural stiffness (inverse of compliance) and in maximum strain are clearly apparent.

In FIG. 6 the clear advantages of the "Fourier" transformer or vibrator at a high magnification (M of 100) is apparent from an inspection of the profiles 60, 70 and 80 of the "stepped," "Fourier" and "exponential" types, respectively.

FIG. 7 compares the ratios of maximum to minimum "diameters" $R_D (= \sqrt{R_A})$ needed to produce given magnifications "M" with the exponential, curve 88; the Fourier, curve 90; and the stepped, curve 92, amplitude transformers. This shows that for the Fourier transformer the ratio for a given magnification is only about sixty percent greater than for the stepped. This comparison is particularly favorable to the Fourier transformer because the stepped transformer here considered is an idealized, one-dimensional vibrator. The magnification produced in practice by a stepped transformer of given R_D is probably very considerably smaller than that shown in FIG. 7, particularly at high magnification.

FIG. 8 is substantially an exact reproduction of the perspective showing of FIG. 1 of the above-mentioned patent to Mason et al. except that member 12' is tapered in accordance with a truncated Fourier series as taught in the present application instead of being exponentially tapered as taught in the Mason et al. patent. Both members, that is, member 12 of the Mason et al. patent and member 12' of FIG. 8, are proportioned to provide the very modest magnification of ten. All other parts of FIG. 8 are intended to be identical with and as described in the Mason et al. patent for the corresponding designation numbers (not primed in the patent drawing and description). Assuming as a fair representative example, that transducer or motor element 1' of FIG. 8 and its corresponding element 1 of FIG. 1 of the patent both provide a particle velocity of 50 centimeters per second to the larger end of the tapered member, since both members have the same magnification ten, the smaller ends of both members will have particle velocities of 500 centimeters per second.

It is obvious from inspection and comparison of the two figures (FIG. 8 of this application and FIG. 1 of the patent) that the Fourier tapered member 12' of the present application is much stiffer mechanically than the exponentially tapered member of the patent.

In FIG. 9, member 12'' is a perspective showing of the member whose profile is designated 70 in FIG. 6 of this application. This member has a truncated Fourier taper and is proportioned to provide a magnification of 100. Assuming again that the transducer driving the larger end of the member imparts a particle velocity of 50 centimeters per second to said larger end, the smaller

end will have a particle velocity of 5000 centimeters per second.

As shown in FIG. 6, and as described hereinabove, the corresponding exponentially tapered member would have the profile designated 80 and would obviously be extremely slender and lacking in stiffness for a major portion thereof toward the smaller end.

Contrasted with this, member 12'' of FIG. 9 is still mechanically superior to member 12 of the Mason et al. patent from the standpoint of stiffness, notwithstanding the fact that member 12'' provides ten times the magnification of member 12 of the patent.

To recapitulate, a primary object of the invention is to produce a type of amplitude transformer with greater ability to withstand large amplitudes (measured by the "figure of merit," φ) than that of the stepped transformer, but with only slightly lower flexural stiffness. The standard of comparison is the exponential transformer, the only other well known type of transformer, but which is very slender and hence of rather low stiffness at large magnification.

This object is achieved by deriving the family of profiles corresponding to displacement functions of third-order truncated Fourier series, Equation 11 with $N=3$. Owing to the special form of the result, these displacement functions essentially give a single useful solution

for each value of magnification, M. To examine the possibility of providing a useful choice of stiffness versus φ , and of finding a design with even higher stiffness, a fourth-order Fourier series or displacement function is analyzed, Equation 11, $N=4$, with the following results. For $M < 10$, it is possible to achieve a useful increase in stiffness by sacrificing φ for $10 \leq M \leq 40$ the stiffness varies very little over a wide range of φ , and the third-order form, which is shown by the points indicated by small circles in FIG. 3, offers nearly the maximum possible φ and the advantage of an analytical solution; for $M > 40$ it is still not possible to trade φ for stiffness, but the optimum form is stiffer than the third-order form, the advantage increasing with increasing M.

Thus, the primary object of the invention is very satisfactorily achieved. A type of vibrator is disclosed having a value of $\varphi (\approx 1.6)$ about twice that of the stepped vibrator and about three-quarters of that of the exponential vibrator (2.0-2.4). This has been achieved in a flexurally very stiff form, suitable for use as a tool or for numerous other purposes such as driving other vibrators. By way of example, for 100-fold magnification the Fourier form is only 2.6 times less stiff than the stepped form, but 130 times stiffer than the exponential form.

Numerous and varied modifications and rearrangements of devices of the invention can be readily devised by those skilled in the art without departing from the spirit and scope of the principles of the invention. These principles are, in view of the disclosure of the present specification, broadly applicable to numerous arrangements in which mechanically vibratory members are employed.

What is claimed is:

1. A mechanical transformer for magnifying the amplitude of a repetitive, substantially sinusoidal, physical displacement by a factor M, M having a value exceeding 2, said transformer comprising an elongated acoustically resonant member having one end larger than the other end, a length l less than a wavelength λ for extensional waves of the member's resonant frequency propagated along the member, a maximum transverse dimension not greater than one-half of said wavelength, the cross-sectional area A of the member varying between the larger and smaller ends of the member in accordance with the integral of the relation

$$\frac{d(\ln A)}{d(\cos \pi X)} = \frac{(4-\gamma^2)(\gamma^2+\alpha_{42})-\mu(9-\gamma^2)(\gamma^2-1)\cos \pi X-(16-\gamma^2)\alpha_{42}\cos^2 \pi X}{\mu(3-\gamma^2)-2(\gamma^2+2\alpha_{42})\cos \pi X+3\mu(\gamma^2-1)\cos^2 \pi X+4\alpha_{42}\cos^3 \pi X}$$

where

$$\gamma = \frac{2l}{\lambda}, \mu = \frac{M+1}{M-1}, X = \frac{x}{l}$$

where x represents the distance along the member from its larger end of the cross section for which the area A is being instantly determined and α_{42} is a constant at choice having a value between minus one and plus one.

2. The transformer of claim 1 in which γ is chosen to give substantially the lowest ratio of maximum to minimum cross-sectional areas for the selected value of α_{42} .

3. The transformer of claim 2 in which α_{42} has a value of substantially -0.3.

4. The transformer of claim 1 in which α_{42} has a value of substantially -0.3.

5. The transformer of claim 1 in which α_{42} is zero.

References Cited by the Examiner

UNITED STATES PATENTS

2,017,695	10/35	Hahneemann	181—0.5
2,044,807	6/36	Noyes	181—0.5
2,779,880	1/57	Malherbe	73—71.5 X
2,948,154	8/60	Kleesattel	

BROUGHTON G. DURHAM, *Primary Examiner*.