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TAPERED WAVEGUIDE TRANSITION SECTION WITH DIELECTRIC
SLEEVE POSITIONED TO REDUCE COUPLING BETWEEN
TE CIRCULAR MODES
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FIG. 1

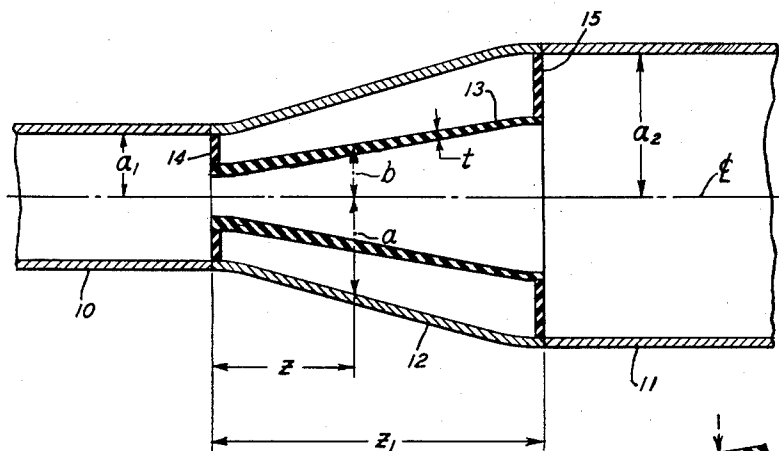


FIG. 2

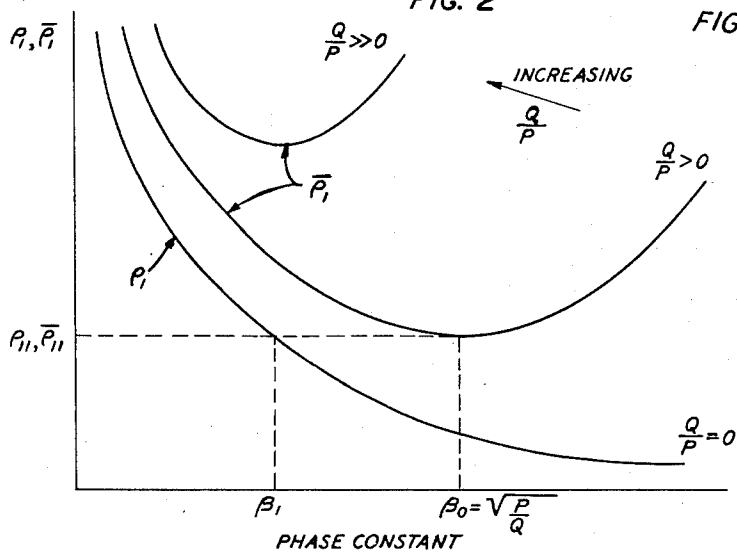
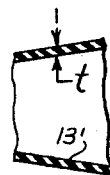


FIG. 3



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1

3,146,414

TAPERED WAVEGUIDE TRANSITION SECTION WITH DIELECTRIC SLEEVE POSITIONED TO REDUCE COUPLING BETWEEN TE CIRCULAR MODES

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This invention relates to electromagnetic wave transmission systems and, in particular, to short, multimode, transition sections to connect waveguides of different cross-sectional dimensions.

In United States Patent 2,938,179, issued to H. G. Unger on May 24, 1960, there are described tapered transition sections of varying cone angle. As explained therein (and in an article by H. G. Unger entitled "Circular Waveguide Taper of Improved Design," published in the July 1958 issue of the Bell System Technical Journal) in a multimode transmission system there is a tendency for transition sections used to connect waveguides of different cross-sectional dimensions to generate higher order propagating modes. In a transmission system propagating the circular electric mode this presents a serious problem since no simple means are known which are capable of suppressing the spurious higher order circular electric modes without affecting the preferred, lowest order circular electric mode. Hence, mode conversion-reconversion distortion in a circular electric mode transmission system can only be avoided by keeping the power in the higher order circular electric modes at an extremely low level. Previously this was accomplished by using extremely long tapers of uniform cone angle. Unger, however, achieved a marked reduction in taper lengths by varying the cone angle of his transition sections in a prescribed manner. Nevertheless, there still remained room for a further reduction in the length of such units.

In the copending application of C. C. H. Tang, Serial No. 97,602, filed January 31, 1961, now Patent No. 3,050,701, issued August 21, 1962, there is described a method of improving the mode conversion characteristic of tapered sections by the introduction of "new zeros" which have the effect of modifying the shape of the taper and further reducing its length.

Tapers that are currently available, applying the teachings of Unger and Tang and having a TE_{02}° mode conversion of approximately 50 decibels or better, are typically of the order of two to three feet long and operate over a band of frequencies between 37.5 and 75 kmc. per second.

It is an object of this invention to reduce further the length of tapered waveguide transition sections used to connect waveguides of different cross-sectional dimensions.

It is an additional object of this invention to increase the frequency range over which such transition sections can operate and still maintain a given spurious mode level.

In accordance with the invention, the tendency for mode conversion in multimode transition sections is materially reduced by the inclusion within the tapered sections of a low-loss dielectric element which is located in a region of high electric field intensity for one mode and in a region of low electric field intensity for another mode. While this technique can be used in connection with waveguides of various cross-sectional configurations and with all types of propagating modes, it is of particular interest in connection with circular waveguides propagating the circular electric mode. Hence, the invention

2

will be described in more detail hereinbelow in that context. Thus, more specifically, the tendency for mode conversion from the TE_{01}° to the TE_{02}° circular electric mode is substantially reduced by the inclusion within a tapered transition section of a tapered sleeve of low-loss dielectric material. The sleeve is placed in a region of high electric field intensity for the TE_{01}° mode and low electric field intensity for the TE_{02}° mode. Preferably the sleeve conforms to the shape of the taper and, at any point along the taper, has an average radius b which is related to the taper radius a at that point by

$$\frac{b}{a} = \frac{k_1}{k_2}$$

where k_1 and k_2 are roots of the Bessel function of the first kind and order and are equal to 3.832 and 7.016 respectively. The thickness of the sleeve t is a function of the taper radius a , the dielectric constant of the material and the frequency. A dielectric sleeve so proportioned has the effect of eliminating the degeneracy between the TE_{01}° and TE_{02}° modes that tends to occur for signals whose wavelengths are small compared to the radius of the waveguide. That is, the tendency for coupling between the TE_{01}° mode and the TE_{02}° mode is substantially reduced by the presence of the dielectric sleeve. The improvement in operation realized in a tapered section containing a dielectric sleeve can be understood by recognizing that the coupling between the TE_{01}° and TE_{02}° modes can be likened to the coupling between separate transmission lines in that a large exchange of energy occurs if the phase constants for the two lines (or modes) are sufficiently similar. If, on the other hand, the phase constants are sufficiently different, little or no energy is coupled between the lines (or between modes). As an example of the effectiveness of a dielectric sleeve to increase the difference in the phase constants of these two modes, let us consider a wave having a wavelength of one millimeter propagating in a 2 inch diameter waveguide. For such a wave the phase shift per unit wavelength in an unloaded waveguide is 0.0044 radian. The added phase shift per unit wavelength in the same guide but including a dielectric sleeve having a thickness of one millimeter and a dielectric constant of 3, is 0.55 radian. This represents a change of over three orders of magnitude. Compared to prior art tapers, transition sections designed in accordance with the invention are reduced in length and operate over a far wider range of frequencies.

In multimode systems capable of transmitting TE_{03}° mode energy, the thickness of the sleeve can be adjusted to produce related amounts of TE_{02}° and TE_{03}° mode wave energy or a second sleeve can be added to separately limit the TE_{03}° mode level.

These and other objects and advantages, the nature of the present invention, and its various features, will appear more fully upon consideration of the various illustrative embodiments now to be described in detail in connection with the accompanying drawings, in which:

FIG. 1 shows, in cross-sectional view, a tapered transition section including a dielectric sleeve of varying thickness, in accordance with the invention;

FIG. 2, included for purposes of illustration, shows the variation in the factor ρ as a function of β , the free space phase constant; and

FIG. 3 shows a section of a dielectric sleeve of uniform thickness.

Referring to FIG. 1 there is shown a preferred embodiment of the invention in which a pair of circular waveguides 10 and 11 of radii a_1 and a_2 , respectively, are electrically and physically joined together by means of a tapered transition section 12. In general, section 12 can have many shapes and may be either a solid wall taper or a helical taper of the type described in United States

3

Patent 3,126,517, which issued to S. E. Miller on March 24, 1964. Preferably, the taper is designed in accordance with the teachings of Unger and Tang cited hereinabove.

Located within section 12 is a low-loss dielectric sleeve 13 whose shape and thickness will be described in greater detail hereinbelow. Sleeve 13 is supported within section 12 in any convenient manner. As shown in FIG. 1, a pair of dielectric rings 14 and 15, located one at each end, are used for this purpose.

As explained hereinabove, the improvement in operation realized in a tapered section containing a dielectric sleeve can be understood by recognizing that the coupling between the TE_{01}° and TE_{02}° modes can be likened to the coupling between separate transmission lines in that a large exchange of energy occurs if the phase constants for the two lines (or modes) are sufficiently similar. If, on the other hand, the phase constants are sufficiently different, little or no energy is coupled between the lines (or between modes).

In the Unger and Tang designs the phase constants of the TE_{01}° and TE_{02}° modes tend to become equal at the higher frequencies. Hence, the tendency to couple between modes is increased. In accordance with the invention, however, this tendency is inhibited by the inclusion of a dielectric sleeve.

In his article cited above, Unger has shown that the equations for a taper of improved design are given by

$$Z = 2 \int_0^{\rho} \frac{d\rho}{\beta_1 - \beta_2} \quad (1)$$

and

$$a = a_1 e^{\left(\frac{\rho}{\rho_1} - \frac{1}{2\pi} \sin 2\pi \frac{\rho}{\rho_1} \right) \ln \frac{a_2}{a_1}} \quad (2)$$

where:

a is the radius of the taper at any point Z ;
 β_1 and β_2 are the phase constants of the TE_{01}° and TE_{02}° modes, respectively, at point Z and are given by

$$\beta_1 - \beta_2 = \frac{k_2^2 - k_1^2}{2\beta a^2} \quad (3)$$

k_1 and k_2 are roots of the Bessel function of the first kind and order and are equal to 3.832 and 7.016, respectively;

$$\beta = \frac{2\pi}{\lambda}$$

is the free space propagation constant; and
 λ is the free space wavelength.

ρ_1 can be expressed as

$$\rho_1 = \frac{1}{2} \int_0^{Z_1} (\beta_1 - \beta_2) dz \quad (4)$$

where Z_1 is the total length of the taper.

The TE_{02}° mode conversion is given by

$$|N_2(\rho_1)| = \frac{2k_1 k_2}{k_2^2 - k_1^2} \left(\ln \frac{a_2}{a_1} \right) \frac{|1 - e^{-j\rho_1}|}{2\rho_1 \left(1 - \frac{\rho_1^2}{\pi^2} \right)} \quad (5)$$

The total length Z_1 of the taper is obtained by integrating Equation 1 between the limits 0 and ρ_1 . Thus,

$$Z_1 = 2 \int_0^{\rho_1} \frac{d\rho}{\beta_1 - \beta_2} \quad (6)$$

From Equation 6 it is seen that the larger the difference between propagation constants $(\beta_1 - \beta_2)$, the smaller the taper length Z_1 .

In accordance with the invention, the value $\beta_1 - \beta_2$ is made and maintained large by the inclusion of the dielectric sleeve 13. The new length $\bar{Z}_1$ is then

$$\bar{Z}_1 = 2 \int_0^{\rho_1} \frac{d\rho}{\bar{\beta}_1 - \bar{\beta}_2} \quad (7)$$

where $\bar{\beta}_1$ and $\bar{\beta}_2$ are the phase constants of the TE_{01}° and TE_{02}° modes, respectively, in a taper having a sleeve.

4

The sleeve has a dielectric constant ϵ_1 and at each cross section it has an average radius b and a thickness t . Hence, the local propagation constant $\bar{\beta}_n$ for the TE_{on}° mode is given approximately by

$$\bar{\beta}_n \approx \beta_n \left[1 + \left(\frac{\epsilon_1}{\epsilon_0} - 1 \right) \frac{tb J_1^2 \left(k_n \frac{b}{a} \right)}{J_0^2(k_n)} \right] \quad (8)$$

where:

β_n is the propagation constant for the TE_{on}° mode in an empty taper;

ϵ_0 is the dielectric constant of the empty taper (typically of air, for which $\epsilon_0 \approx 1$); and

J_0 and J_1 are Bessel functions of the first kind of orders zero and one, respectively.

To maximize the phase velocity difference, we select

$$\frac{b}{a} = \frac{k_1}{k_2} \approx \frac{3.8}{7.0} \quad (9)$$

which locates the dielectric material in the region of maximum electric field intensity for the TE_{01}° mode and minimum field intensity for the TE_{02}° mode. Hence, we obtain

$$\bar{\beta}_1 = \beta_1 \left[1 + \left(\frac{\epsilon_1}{\epsilon_0} - 1 \right) \left(\frac{tb}{a^2} \right) \frac{J_1^2 \left(\frac{k_1^2}{k_2} \right)}{J_0^2(k_1)} \right] \quad (10)$$

and

$$\beta_2 = \beta_2 \quad (11)$$

Therefore

$$\bar{\beta}_1 - \bar{\beta}_2 = \frac{k_2^2 - k_1^2}{2\beta a^2} + \beta \left(\frac{\epsilon_1}{\epsilon_0} - 1 \right) \left(\frac{t}{a} \right) \left(\frac{k_1}{k_2} \right) \frac{J_1^2 \left(\frac{k_1^2}{k_2} \right)}{J_0^2(k_1)} \quad (12)$$

Comparing Equations 12 and 3, it is seen that the inclusion of the dielectric sleeve increases the phase velocity difference between the two modes.

For purposes of illustration, we select a sleeve whose thickness t varies as $1/a$ in which case we can write

$$\bar{\beta}_1 - \bar{\beta}_2 = \left(\frac{P}{\beta} + Q\beta \right) \frac{1}{a^2} \quad (13)$$

where P and Q are constants derived from 12 to be

$$P = \frac{k_2^2 - k_1^2}{2} \quad (14)$$

$$Q = \left(\frac{\epsilon_1}{\epsilon_0} - 1 \right) t a \frac{k_1}{k_2} \frac{J_1^2 \left(\frac{k_1^2}{k_2} \right)}{J_0^2(k_1)} \quad (15)$$

From Equation 5 it is deduced that once the extreme radii a_1 and a_2 are given, the TE_{02}° mode conversion depends exclusively on the value ρ_1 as given by Equation 4.

Let us therefore calculate ρ_1 for an empty taper and $\bar{\rho}_1$ for the same taper with a dielectric sleeve. Substituting Equations 3 and 13 in 4 we obtain that

$$\rho_1 = \frac{P}{2\beta} \int_0^{Z_1} \frac{dz}{a^2} \quad (16)$$

and

$$\bar{\rho}_1 = \frac{1}{2} \left[\frac{P}{\beta} + Q\beta \right] \int_0^{Z_1} \frac{dz}{a^2} \quad (17)$$

In FIG. 2, ρ_1 and $\bar{\rho}_1$ are plotted as a function of the phase constant β for a fixed geometry of the metal taper and for different values of

$$\frac{Q}{P}$$

It is seen from FIG. 2 that ρ_1 decreases monotonically for increasing β whereas $\bar{\rho}_1$ decreases to a minimum and then increases. It will be noted that $\bar{\rho}_1$ always has a minimum. This occurs at a phase constant β_0 given by

$$\beta_0 = \sqrt{\frac{P}{Q}} \quad (18)$$

Let it be assumed, for purposes of explanation, that an empty taper has been designed to operate at a minimum $\beta = \beta_1$ having a certain ρ_{11} (FIG. 2). ρ_{11} fixes the maximum amount of mode conversion. At lower frequencies ($\beta < \beta_1$), $\rho_1 > \rho_{11}$ and consequently the mode conversion decreases. At higher frequencies ($\beta > \beta_1$), $\rho_1 < \rho_{11}$ and the mode conversion increases. If now a dielectric sleeve is inserted within the taper such that

$$\rho_{11} = \rho_{11} \quad (19)$$

there is now no value for β for which $\bar{\rho}_1 < \bar{\rho}_{11}$ and consequently the taper with the sleeve can operate at any frequency with less TE_{02}° mode conversion than the empty taper had at β_1 .

From Equations 16 through 19 we deduce that

$$\beta_0 = 2\beta_1 = \sqrt{\frac{P}{Q}} \quad (20)$$

and consequently from Equations 14, 15, and 20 we obtain, as the expression for the sleeve thickness t ,

$$t = \frac{k_2^2 - k_1^2}{8a} \cdot \frac{k_2}{k_1 \left[\frac{\epsilon_1}{\epsilon_0} - 1 \right] \beta_1^2 J_1^2 \left[\frac{k_1^2}{k_2} \right]} \cdot \frac{J_0^2[k_1]}{J_1^2 \left[\frac{k_1^2}{k_2} \right]} \quad (21)$$

From Equations 3 and 6 we obtain for the length of the taper Z_1 , the expression

$$Z_1 = \frac{4\beta_1}{k_2^2 - k_1^2} \int_0^{\rho_1} a^2 da \quad (22)$$

From Equation 22 it is seen that the taper length is proportional to the phase constant β_1 . It is therefore possible, in principle, to design an empty taper with a given TE_{02}° mode conversion level at a β_1 which is as small as desired. The length of the taper Z_1 is then correspondingly as short as desired. If, now, a dielectric sleeve is added, the taper becomes infinitely broadband. However, in practice this is not possible because the thickness t of the dielectric sleeve (as given by Equation 21) is inversely proportional to β_1^2 and, hence, becomes prohibitively large. Nevertheless, substantial improvements over empty tapers are possible.

As an illustration of the improvement that can be realized by the insertion of the dielectric sleeve within the taper, we consider a taper which connects a $\frac{7}{8}$ inch guide to a 2 inch guide. Unger has calculated that such a taper, having a TE_{02}° mode level better than 50 decibels up to 75 kmc. per second, has a length Z_1 of 3 feet. If now following Unger's teachings we design another empty taper for $\frac{7}{8}$ inch to 2 inch guide whose TE_{02}° mode conversion is better than 50 decibels up to 25 kmc., the length of this taper, according to Equation 22, is reduced to

$$Z_1 = \frac{Z_1}{3} = 1'$$

We can now design the dielectric sleeve to make this 1 foot taper infinitely broadband. According to Equation 21, and assuming a

$$\frac{\epsilon_1}{\epsilon_0} = 2.5$$

the sleeve thickness varies between 0.032 inch at the small diameter end to 0.013 inch at the large diameter end. There is thus obtained a taper of reduced length and increased bandwidth.

It should be noted that in the illustrative derivation given above, the sleeve thickness was assumed to vary

inversely as the taper radius. This was done solely to simplify the mathematics. In practice the sleeve can be made of uniform thickness corresponding to the thickness at the small diameter end. A section of sleeve 13' of uniform thickness is shown in FIG. 3. In such a case the TE_{02}° mode conversion is reduced still further than that given above.

Up until now we have considered only the effect upon the TE_{02}° mode. No consideration has been given to the TE_{03}° mode which, if the taper diameter is sufficiently large, could also be generated. It can be shown that if an empty taper is designed (according to Unger's calculations) for a given level of TE_{03}° mode conversion (this entails only changing $k_2 = 7.016$ to $k_3 = 10.17$), the addition of a sleeve of thickness

$$t = \frac{[k_3^2 - k_1^2]}{8a[k_2^2 - k_1^2]} \cdot \frac{k_2}{k_1} \cdot \frac{1}{\left[\frac{\epsilon_1}{\epsilon_0} - 1 \right] \beta_0^2} \cdot \frac{J_0^2[k_1]}{J_1^2 \left[\frac{k_1^2}{k_2} \right]} \quad (23)$$

results in a taper having equal maximum TE_{02}° and TE_{03}° mode conversion. This occurs at $\beta = \beta_0$.

From Unger's calculations (Formula 24, page 908, B.S.T.J., July 1958) the ratio between the lengths of an empty taper and one with sleeve, both operating with the same level of TE_{02}° and TE_{03}° over the same range of frequencies, is

$$\frac{Z_1}{Z_1} \approx \frac{k_3^2 - k_1^2}{k_2^2 - k_1^2} = 2.53 \quad (24)$$

The taper with sleeve is then 1/2.53 times as long as an empty one.

If higher band operation is desired and if both the TE_{02}° and the TE_{03}° modes are to be maintained below a specified maximum level, a second sleeve can be added within the taper, located at a point of minimum field intensity for the TE_{03}° mode and substantial field intensity for the TE_{01}° mode. Since there are two such points, the second sleeve can have a radius c , at any point, given by either

$$\frac{c}{a} \approx \frac{k_1}{k_3}$$

or

$$\frac{c}{a} \approx \frac{k_2}{k_3}$$

The thickness of both sleeves and the shape of the taper are derived following the technique used for a single sleeved taper as described hereinabove.

It should be noted, as also stated hereinabove, that while the illustrative embodiment is particularly directed to the TE_{01}° mode and circular waveguides, the principles of the invention are equally applicable to other mode and waveguide configurations. Thus, in all cases it is understood that the above-described arrangements are illustrative of only some of the many possible specific embodiments which can represent applications of the principles of the invention. Numerous and varied other arrangements can readily be devised in accordance with these principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. In an electromagnetic wave transmission system for transmitting the TE_{01}° circular electric mode, means comprising:

a tapered transition section for coupling said mode from a first circular waveguide having a first radius to a second circular waveguide having a second radius,

and means for suppressing the generation of spurious TE_{0n}° mode wave energy within said section comprising,

a sleeve of low-loss dielectric material disposed within said section along a region of high electric field intensity for said TE_{01}° mode and low electric field intensity for said TE_{0n}° mode.

2. In an electromagnetic wave transmission system for transmitting the TE_{01}° circular electric mode, means comprising:

a tapered transition section for coupling said mode from a first circular waveguide having a first radius to a second circular waveguide having a second radius,

and means for suppressing the generation of spurious TE_{02}° mode wave energy disposed within said section comprising,

a sleeve of low-loss dielectric material whose average radius b at any point therealong is related to the radius of said section at said point by

$$\frac{b}{a} = \frac{k_1}{k_2}$$

where k_1 and k_2 are roots of the Bessel function of first kind and order and are equal to 3.832 and 7.016, respectively.

3. The combination according to claim 2 wherein said sleeve has a uniform thickness.

4. The combination according to claim 2 wherein the thickness t of said sleeve varies inversely as the radius a and is given by

$$t = \frac{k_2}{k_1} \cdot \frac{(k_2^2 - k_1^2)}{\left(\frac{\epsilon_1}{\epsilon_0} - 1\right)} \cdot \frac{J_0^2(k_1)}{J_1^2(k_1^2/k_2)} \cdot \frac{1}{8a} \cdot \frac{1}{\beta_1^2}$$

where:

ϵ_1 is the dielectric constant of the sleeve;

ϵ_0 is the dielectric constant of the empty taper;

k_1 and k_2 are roots of J_1 ;

J_0 and J_1 are Bessel functions of the first kind, of orders zero and one, respectively; and

β_1 is the free space propagation constant in an empty taper for which the mode conversion has a specified maximum value.

5. The combination according to claim 2 wherein the thickness t of said sleeve is selected for equal TE_{02}° and TE_{03}° mode wave energy at $\beta = \beta_0$ and is given by

$$t = \frac{k_2^2 - k_1^2}{8(k_2^2 - k_1^2)} \cdot \frac{J_0^2(k_1)}{J_1^2(k_1^2/k_2)} \cdot \frac{1}{\left(\frac{\epsilon_1}{\epsilon_0} - 1\right)} \cdot \frac{k_2}{k_1} \cdot \frac{1}{a\beta_0^2}$$

where:

ϵ_1 is the dielectric constant of the sleeve;

ϵ_0 is the dielectric constant of the empty taper;

k_1 and k_2 are roots of J_1 ; and

J_0 and J_1 are Bessel functions of the first kind, of order zero and one, respectively.

6. In an electromagnetic wave transmission system for transmitting the TE_{01}° circular electric mode, means comprising:

a tapered transition section for coupling said mode from a first circular waveguide having a first radius to a second circular waveguide having a second radius,

and means for suppressing the generation of spurious high order mode wave energy within said section comprising,

a plurality of low-loss dielectric sleeves each disposed with said section along a region of low electric field intensity for one of said spurious modes and high electric field intensity for said TE_{01}° mode.

7. In an electromagnetic wave transmission system for transmitting a preferred mode of wave propagation, means comprising:

a tapered transition section for coupling said preferred mode from a first waveguide having first cross-sectional dimensions, to a second waveguide having second cross-sectional dimensions,

and means for suppressing the generation of spurious mode wave energy within said section comprising,

at least one element of low-loss dielectric material disposed along a region of high electric field intensity for said preferred mode and low electric field intensity for said spurious mode.

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