

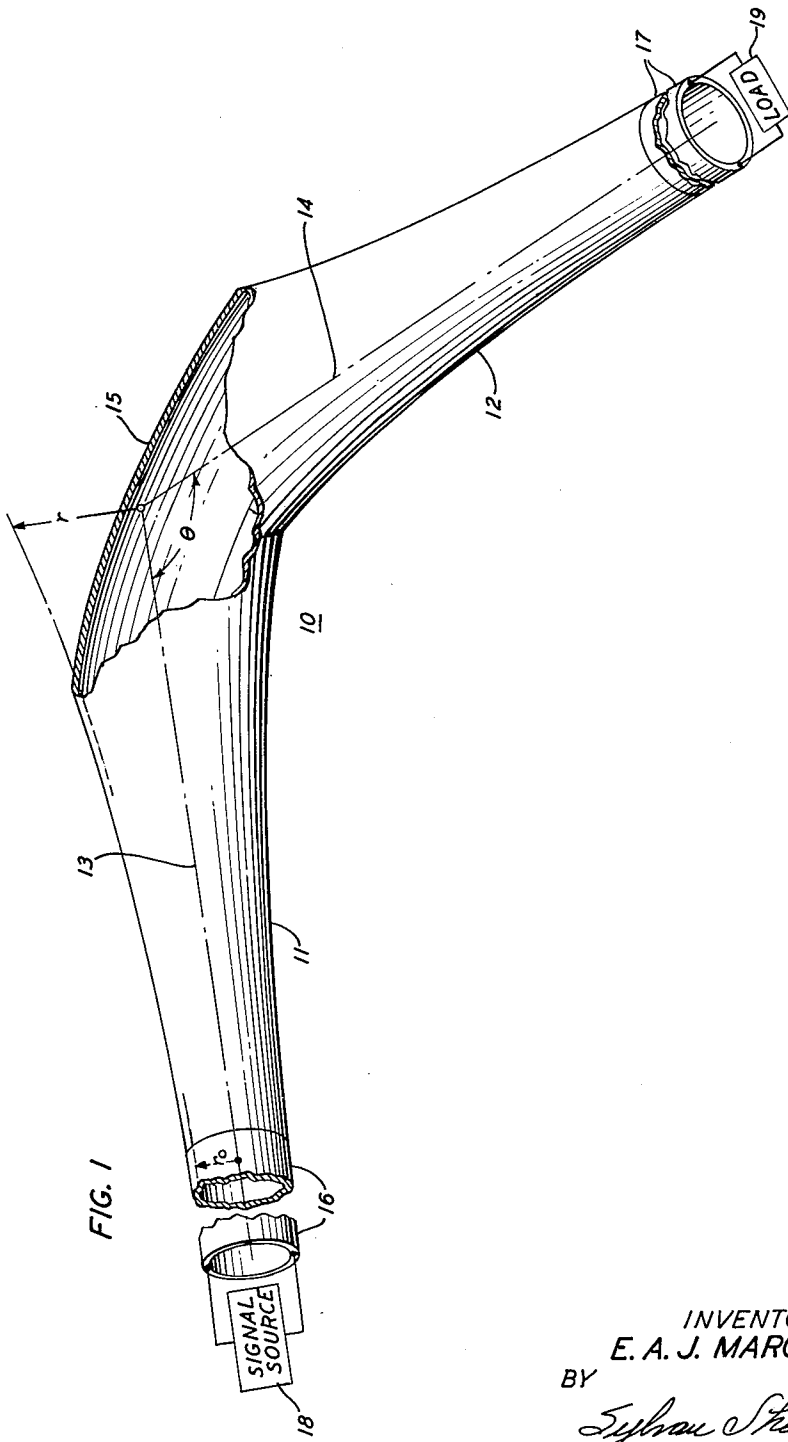
May 21, 1963

E. A. J. MARCATILI
WAVEGUIDE ELBOW

3,090,931

Filed March 8, 1962

3 Sheets-Sheet 1



INVENTOR
E. A. J. MARCATILI
BY
Sydney Sherman
ATTORNEY

May 21, 1963

E. A. J. MARCATILI

3,090,931

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FIG. 2

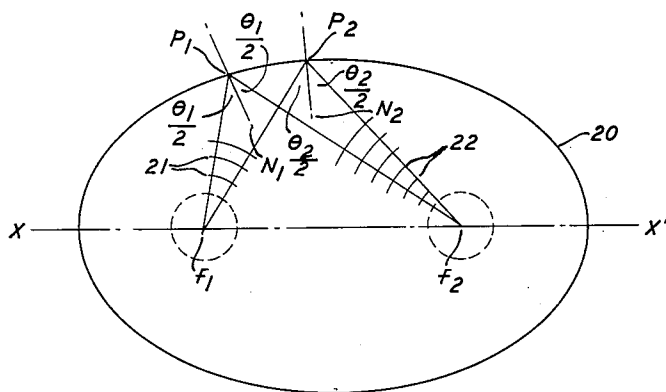
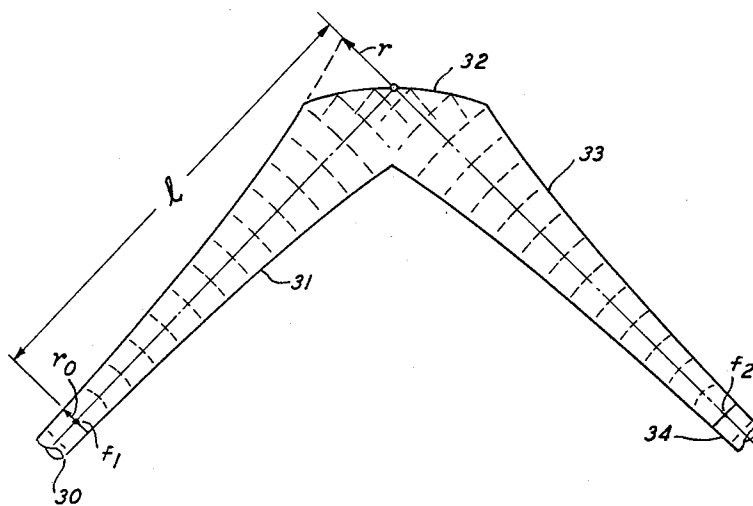


FIG. 3



INVENTOR
E. A. J. MARCATILI
BY *Sylvan Sherman*
ATTORNEY

May 21, 1963

E. A. J. MARCATILI

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FIG. 4

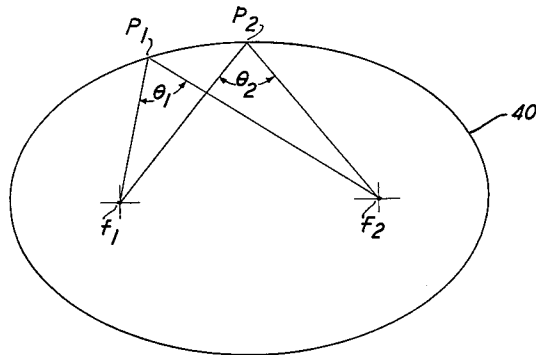
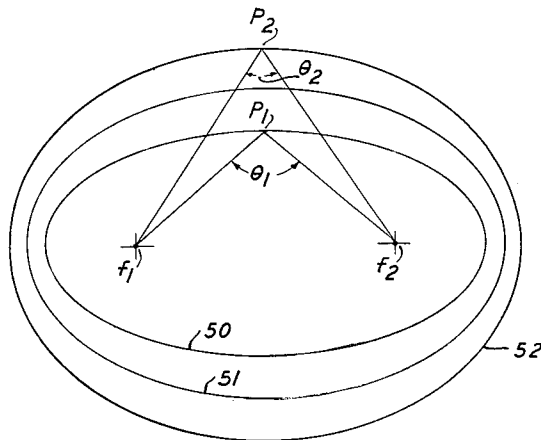


FIG. 5



INVENTOR
E. A. J. MARCATILI
BY
Sylvan Sherman
ATTORNEY

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3,090,931

WAVEGUIDE ELBOW

Enrique A. J. Marcatili, Fair Haven, N.J., assignor to Bell Telephone Laboratories, Incorporated, New York, N.Y., a corporation of New York

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9 Claims. (Cl. 333—98)

This invention relates to electromagnetic wave transmission systems and, more particularly, to miter elbows for use in such systems. The invention has particular application to systems in which the wave energy propagates in the circular electric mode.

As is well known, the propagation of electromagnetic wave energy in the form of the circular electric TE_{01}° mode in circular waveguides is ideally suited to the long distance transmission of high frequency, wide band signals since the attenuation characteristic of this transmission mode, unlike that of other modes decreases with increasing frequency. However, since the TE_{01}° mode is not the dominant mode supported in a circular waveguide, energy may be lost to lower order modes also capable of transmission therein. Furthermore, it is desirable from loss considerations to propagate the TE_{01}° mode wave in waveguides whose physical dimensions are substantially larger than those dictated by cut-off considerations. Thus, the transmission medium used to guide TE_{01}° mode wave energy is inherently multimode with respect to modes of higher order than the preferred mode as well as with respect to modes of lower order.

In the January 1961 issue of the Bell System Technical Journal, volume 40, No. 1, there are numerous articles describing various circuit components intended for use in transmission systems propagating wave energy in the circular electric mode. In particular, the article by E. A. J. Marcatili and D. I. Bisbee entitled, "Band-Splitting Filter," pages 197 to 212, describes a miter elbow to change the direction of wave propagation in a transmission system operating in the TE_{01}° mode.

Typically, a miter elbow comprises a pair of intersecting waveguides or arms arranged so that their axes intersect at some given angle. A reflecting planar surface for changing the direction of propagation through said angle is placed so as to pass through the point of intersection of the guide axes and is oriented with its face perpendicular to the bisection of the angle between the guide axes. Sharp bends at any desired angle can thus be made.

It has been found, however, that a miter elbow of the type described in the aforementioned article causes mode conversion from the preferred TE_{01}° mode to other spurious modes. Of all the spurious modes generated, however, the higher order circular electric modes and, in particular, the TE_{02}° mode, are the most troublesome since they cannot be eliminated with simple helical mode filters.

It is, accordingly, the object of this invention to minimize the amount of TE_{02}° mode wave energy induced in a miter elbow transmitting wave energy in the TE_{01}° mode of wave propagation.

It can be shown that both the total TE_{01}° insertion loss and the TE_{02}° conversion loss are functions of the ratio of the guide radius r in the region of the reflecting surface and the free space wavelength λ of the signal. Preferably the ratio r/λ should be as large as possible.

It is, therefore, a more specific object of the invention to increase the cross-sectional dimensions of the intersecting wave paths in a miter elbow.

In any practical transmission system, the diameter of the wave guide used to transmit circular electric mode waves over extended distances will be determined by both technical and economical considerations which need not be considered here. (Currently, a guide diameter of ap-

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proximately two inches is favored.) The problem, therefore, is how to increase the diameter of the intersecting waveguides used to form the miter section to something greater than the guide diameter generally used for long distance runs.

One obvious way to increase the elbow guide diameter in the region of the reflecting surface is to construct an elbow of larger diameter and to insert a tapered transition section between the waveguide and the elbow. The elbow itself can be optimized for minimum mode conversion in the manner explained in my copending application Serial No. 178,427, filed March 8, 1962. However, this would tend to make the over-all arrangement comprising the tapered transition sections and the elbow too long since it would entail the use of two transition sections of the type described, for example, in United States Patent 2,938,179 issued to H. G. Unger and in the copending application of C. C. H. Tang, Serial No. 97,602, filed March 22, 1961. Each of the transition sections would have to be capable of transforming a wave having a planar wavefront into a wave having a spherical wavefront and then reconverting the spherical wave to a planar wave, and doing so without introducing any appreciable amount of higher order circular electric mode waves. A substantial reduction in over-all length can be realized, however, by eliminating the need to reconvert the spherical wave back to a planar wave prior to entering the elbow, and by incorporating the transition sections into, and making them an integral part of, the elbow.

It is, accordingly, a further object of this invention to change the direction of propagation of a wave having a substantially spherical wavefront.

In accordance with the principles of the invention, the reflecting surface used in the miter elbow is a portion of an ellipsoid whose foci are located at the input ends of the elbow arms. Since it is a property of an ellipsoidal surface to reflect a spherical wave and to preserve the spherical wavefront, the transition sections, which now comprise the arms of the elbow, need only be designed to transform the incident planar wave to a spherical wave. Thus, by modifying the shape of the reflecting surface and by combining the transition sections and the elbow into a unitary structure, the diameter of the transmission path at the reflecting surface can be substantially increased in an elbow of practical over-all size.

In a specific embodiment of the invention, the elbow arms are sections of a hyperboloid of revolution, generated by rotating a section of a hyperbola about the guide axis.

These and other objects and advantages, the nature of the present invention, and its various features, will appear more fully upon consideration of the various illustrative embodiments now to be described in detail in connection with the accompanying drawings, in which:

FIG. 1 shows, in perspective, an elbow in accordance with the invention inserted along a waveguide interconnecting a signal source and a load;

FIG. 2, given by way of explanation, shows an ellipse and illustrates certain properties thereof;

FIG. 3, given by way of explanation, shows the wavefront in various portions of the elbow of FIG. 1; and

FIGS. 4 and 5, given by way of explanation illustrates the manner in which the angle θ and the length of the elbow arm can be varied.

Referring to FIG. 1, there is shown an elbow 10, in accordance with the invention, comprising a pair of intersecting tapered sections of circular waveguide 11 and 12, whose respective longitudinal axes 13 and 14 intersect at some angle θ , and a reflecting surface 15. Surface 15 passes through the point of intersection of the guide axes

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13 and 14 and extends completely across and terminates both taper sections 11 and 12.

The intersecting tapered waveguides 11 and 12 are the "arms" of the elbow and will be referred to as such hereinafter. In addition, the smaller ends of the intersecting tapered waveguides will be hereinafter referred to as simply the "ends" of the elbow. Thus, the designation "end of arm 11" will be understood to mean the portion of waveguide 11 of minimum radius.

Electromagnetic wave energy derived from a signal source 18 and propagating in the TE_{01}° circular electric mode of propagation, is coupled to elbow 10 by means of a circular cylindrical waveguide 16 of radius r_0 . The output from elbow 10 is coupled by means of a circular cylindrical guide 17 of radius r_0 to a load 19 adapted to utilize the wave energy in the TE_{01}° mode.

It can be shown that the TE_{01}° insertion loss S_{01} in a miter elbow is given by

$$S_{01} = \frac{P_{01(in)}}{P_{01(out)}} = 20 \log_{10} \left[1 + 0.279 \left(\frac{\lambda}{r} \right)^{3/2} \right] \quad (1)$$

where

$P_{01(in)}$ is the TE_{01}° power in,

$P_{01(out)}$ is the TE_{01}° power out,

r is the guide radius at the reflecting surface, and

λ is the free space wavelength of the applied wave.

Similarly, the conversion loss S_{02} to the TE_{02}° circular electric mode is given by

$$S_{02} = \frac{P_{01(in)}}{P_{02(out)}} = 20 \log_{10} 4 \left(\frac{r}{\lambda} \right)^{3/2} \quad (2)$$

where

$P_{02(out)}$ is the TE_{02}° power out of the elbow.

It is apparent that in both instances it is preferable that the ratio of the elbow radius r to the operating wavelength λ be as large as possible. Accordingly, in the elbow 10 shown in FIG. 1, the radius of each of the arms 11 and 12 increases from r_0 at the small or input end to a larger radius r in the vicinity of the reflecting surface 15. More specifically, the radius r is the radius of the arms as measured at the point of intersection of the guide axes 13 and 14.

Surface 15, for reasons which will be explained in greater detail hereinafter, is in the shape of a portion of an ellipsoid whose foci are located on the axes 13 and 14 at the ends, respectively, of arms 11 and 12.

The arms 11 and 12 are tapered in a manner to minimize the conversion of energy from the preferred TE_{01}° mode to higher order TE_{0n}° modes. In the embodiment of FIG. 1, the tapered sections are single sheet hyperboloids generated by rotating a portion of a hyperbola about the guide axes 13 and 14. The relationship between length l of the elbow arms, the arm radius r at the reflecting surface 15 and the amount of spurious TE_{02}° mode wave energy generated in the tapered sections will be considered in greater detail hereinafter.

The mode of operation of the miter elbow shown in FIG. 1 can best be understood by considering some of the properties of an ellipsoid. In FIG. 2 there is illustrated, for purposes of explanation, an ellipse 20 with foci f_1 and f_2 .

It can be shown that a line drawn perpendicular to the ellipse at any point bisects the angle between the rays drawn from the two foci to that point. This is indicated in FIG. 2 where the angle θ_1 between rays f_1P_1 and f_2P_1 is bisected by the normal to the ellipse N_1P_1 and where the angle θ_2 between rays f_1P_2 and f_2P_2 is bisected by the normal to the ellipse N_2P_2 .

It is apparent, in view of this fact, that if the ellipsoidal surface generated by rotating ellipse 20 about its major axis $x-x'$ is constructed of a reflective material and if a source of radiant energy is placed at either focal point, all waves incident upon the inner surface of the ellipsoid will be reflected so as to pass through the other focal

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point. In addition, since the sum of the distances from the two foci to any point on the ellipsoid is a constant by definition, all waves reflected by the ellipsoid arrive at the other focal point in phase. Accordingly, the spherical wavefront associated with waves emanating from either focus is preserved in the reflected waves. This property of the ellipsoid is indicated by the circular sections 21 and 22 intersecting the rays f_1P_1 , f_1P_2 and f_2P_1 , f_2P_2 , respectively.

In the embodiment of the invention illustrated in FIG. 1, however, the wave applied to the input of elbow 10 does not emanate from a point source and hence does not have a spherical wavefront. To the contrary, the normal modes associated with the circular electric family of modes propagating in a circular, cylindrical guide of constant diameter have planar wavefronts. Hence, a transition section is needed to transform, with minimum net mode conversion, circular electric waves having plane, equiphase wavefronts of a first cross-sectional dimension, into circular electric waves having spherical equiphase wavefronts of a second, larger cross-sectional dimension.

In the above-mentioned United States patent issued to H. G. Unger and in the copending application of C. C. H. Tang, there are described tapered sections for coupling circular electric mode wave energy between circular waveguides of different diameters. This involves the design of a tapered section which is capable of transforming waves having a planar wavefront into waves having a spherical wavefront and the reconversion of such waves into plane waves. In the present situation, however, the waves are utilized in their spherical form and, hence, the tapered sections need not reconvert the wavefront from spherical to planar. This is illustrated in FIG. 3, which shows the wavefront in various regions of the elbow.

For example, in guide 30 the wavefront is planar, as indicated by the broken lines. Upon entering and traversing the tapered section 31, the wavefront is gradually converted from a planar to a spherical wavefront. The reflective surface 32, being a portion of an ellipsoid whose foci f_1 and f_2 lie on the guide axes at the junctions of guides 30 and 34 and tapered sections 31 and 33, respectively, reflects the incident wave and because of the properties of an ellipsoid discussed above, the spherical wavefront is preserved. Accordingly, the wavefront associated with the reflected wave is also spherical. Taper 33 gradually transforms the spherical wavefront of the reflected wave back to a planar wavefront for propagation along guide 34.

The precise length and shape of the tapered sections are a function of the spurious mode level that can be tolerated within the system. Changes in guide diameter tend to generate higher order circular electric mode waves which are not readily eliminated by means of a helical filter. Consequently, the shape and length of the tapered section are designed to meet some maximum permissible spurious mode level over the range of operating frequencies. In the embodiment of FIG. 1 the tapers are sections of a hyperbola of revolution generated by revolving about the guide axis a portion of a hyperbola extending from its transverse axis to a given point along the hyperbola. In particular, the hyperbola defining each tapered section has its conjugate axis coincident with the guide axis and its semi-transverse axis equal to r_0 . This simple geometric configuration is thus tangent to the input (and output) waveguide at its smaller end and asymptotically approaches a cone at the larger end. The normal mode, in the conical region, has a spherical wavefront.

For a single sheet hyperboloid, the relationship between the TE_{02}° scattering coefficient γ_{02} and the dimensions of the taper is given by

$$\frac{\lambda}{r_0^2} = \left\{ \frac{\lambda_{02}}{1.628} \left[\left(\frac{r}{r_0} \right)^2 - 1 \right] \right\}^{1/2} \quad (3)$$

where

λ is the free space wavelength at the operating frequency,

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l is the length of the taper measured along the guide axis from its input end to the reflecting surface,
 r_0 is the guide radius at the input end,
 r is the guide radius at the reflecting surface, and
 γ_{02}^2 is equal to the ratio of the incident TE_{01}^o power to the TE_{02}^o output power.

Thus, for any given maximum spurious mode level, the guide diameter at the reflecting surface and the length of the elbow arms can be computed from Equations 2 and 3.

While the elbow arms have been characterized as hyperboloids, it is evident that other tapers capable of transforming a plane wave to a spherical wave can be used. A hyperboloid was described in connection with the illustrative embodiment only because it lends itself to a simple mathematical analysis and was not intended to limit in any way the invention to that particular shape arm.

In the embodiment of FIG. 1, the tapered arms are substantially identical, being equal in length and tapering through the same range of radii. Various modifications are possible, however. For example, the waveguides 16 and 17 can have different radii. In such a situation the elbow arms would taper down to different radii to match the guide radii. A second modification would involve making the arm lengths unequal in order to change the angle θ .

As indicated hereinabove, in connection with the discussions of FIG. 2, a wave emanating from either focus and reflected by the ellipsoidal surface will pass through the other focus. Thus, any portion of the ellipsoid can be used as the reflecting surface. It is also apparent, however, that the angle θ between the arm axes will vary depending upon the portion of the ellipsoid used. This is illustrated in FIG. 4 wherein the rays f_1P_1 and f_2P_1 , representing the arm axes, make an angle θ_1 with respect to each other that is smaller than the angle θ_2 between rays f_1P_2 and f_2P_2 . If point P_2 is on the minor axis of ellipse 40, then rays f_1P_2 and f_2P_2 are equal and the elbow arms are equal in length. However, for any other point of intersection of the arm axes, such as point P_1 , the elbow arms are unequal.

While there is no reason why unequal arms cannot be used, it is preferable that the elbow arms be substantially the same.

It is known that an infinite number of ellipses can be constructed using the same pair of foci. As the so-called eccentricity of the ellipse decreases, the ellipse becomes larger. This is illustrated in FIG. 5, in which three ellipses 50, 51 and 52 are constructed about the same foci f_1 and f_2 . Locating the point P_1 along the minor axis of ellipse 50 and the point P_2 along the minor axis of ellipse 52, it can be seen that the angle θ_1 between rays f_1P_1 and f_2P_1 is greater than the angle θ_2 between rays f_1P_2 and f_2P_2 . In addition, because the points P_1 and P_2 are located along the minor axes of the ellipses 50 and 52, respectively, the rays from the foci to the respective points are equal.

Thus, one method of changing the angle θ in an elbow that is to have equal arms, is to change the eccentricity of the ellipse used to generate the ellipsoidal reflecting surface. It will be noted, however, that as the angle decreases, the arm lengths increase.

Heretofore, elbows for changing the direction of propagation of circular electric mode waves propagating in circular waveguides have been discussed. However, precisely the same techniques can be used for waves propagating in other modes. Thus, in all cases it is understood that the above-described arrangements are illus-

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trative of a small number of the many possible specific embodiments which can represent applications of the principles of the invention. Numerous and varied other arrangements can readily be devised in accordance with these principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. In a guided wave transmission system supportive of wave energy in a first preferred mode of wave propagation and in at least a second, higher order mode of wave propagation;
 - an elbow for changing the direction of propagation of said wave energy from a first direction of propagation to a second direction of propagation comprising;
 - a pair of tapered transition sections whose axes intersect and whose cross-sectional dimensions increase from a first value at a smaller end to a second larger value in the region of intersection, and
 - an ellipsoidal reflecting surface passing through the point of intersection of said axes and having its foci located at the smaller ends of said sections.
2. In a guided wave transmission system supportive of wave energy in the TE_{01}^o mode of wave propagation and in at least the TE_{02}^o mode of wave propagation;
 - an elbow for changing the direction of propagation of said wave energy from a first direction of propagation to a second direction of propagation comprising;
 - a pair of tapered transition sections whose axes intersect at an angle θ and whose diameters increase from a first value at an end to a second larger value in the region of intersection; and
 - an ellipsoidal reflecting surface passing through the point of intersection of said axes and extending across both of said sections, said surface having its foci located along said axes at the ends respectively of each of said sections.
3. The elbow according to claim 2 wherein each of said sections is a portion of a single sheet hyperboloid whose conjugate axes are colinear with a direction of propagation of said wave energy.
4. The combination according to claim 2 wherein said sections are of equal length.
5. The combination according to claim 2 wherein said sections are of unequal length.
6. The combination according to claim 2 wherein the diameters of said sections at their input ends are equal.
7. The combination according to claim 2 wherein the diameters of said sections at their input ends are unequal.
8. An elbow for changing the direction of propagation of electromagnetic wave energy comprising;
 - a pair of conical waveguides whose axes intersect at an angle and whose cross-sectional dimensions taper from a larger value at the region of intersection to a smaller value at their other ends; and
 - means for changing the direction of propagation from along one of said guides to along the other of said guides comprising an ellipsoidal reflecting member whose surface passes through the point of intersection of said axes and whose foci are located along said axes at the input ends of said guides.
9. Means for changing the direction of propagation of a plane wave comprising;
 - an ellipsoidal reflecting surface;
 - and means for converting a plane wave to a spherical wave disposed between said surface and its foci.

No references cited.