

Aug. 21, 1962

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3,050,701

TAPERED WAVEGUIDE TRANSITION SECTION

Filed March 22, 1961

4 Sheets-Sheet 1

FIG. 1

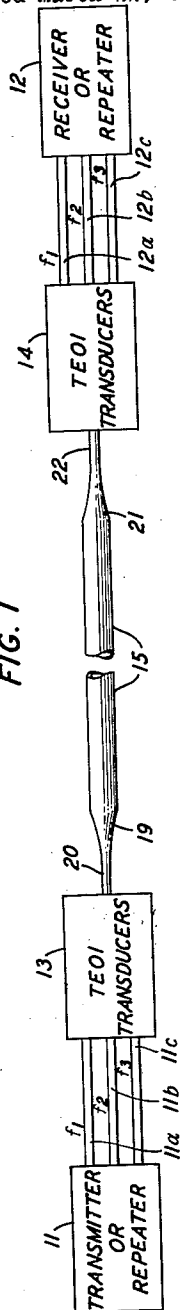


FIG. 2

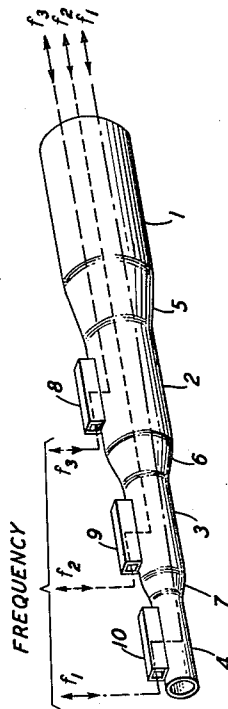
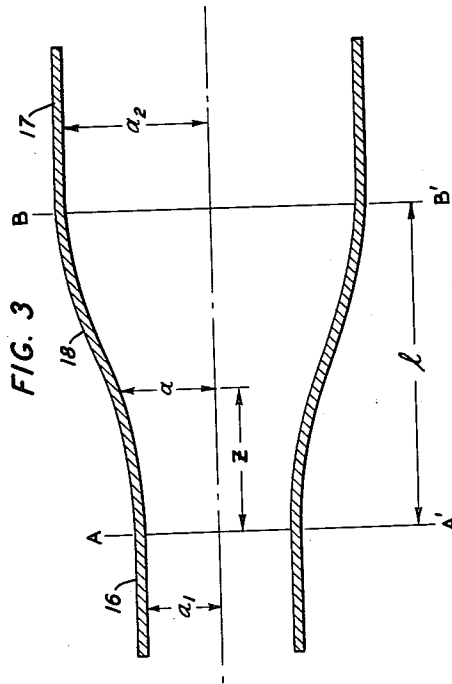


FIG. 3



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FIG. 4

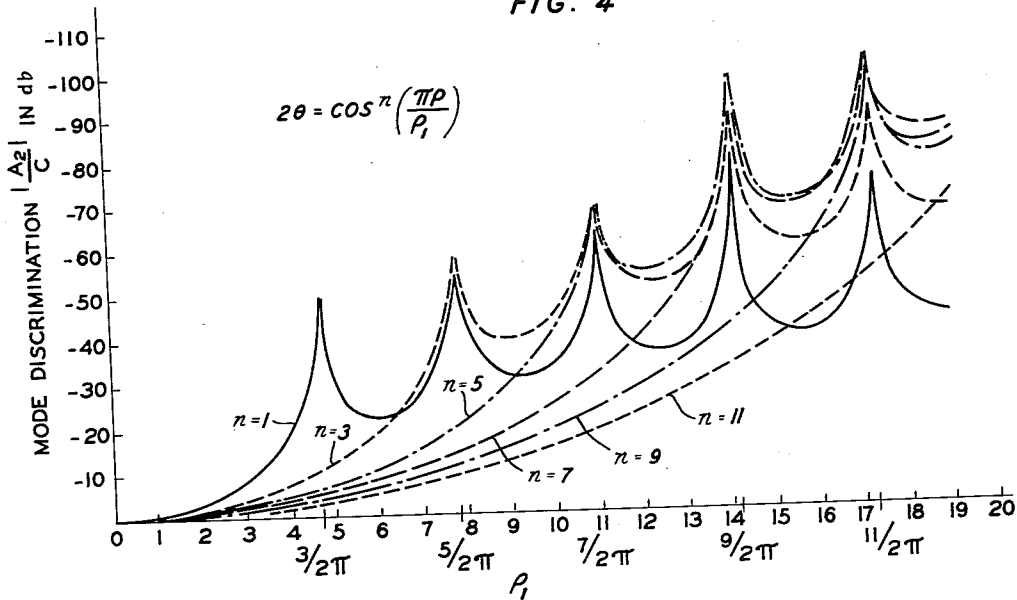
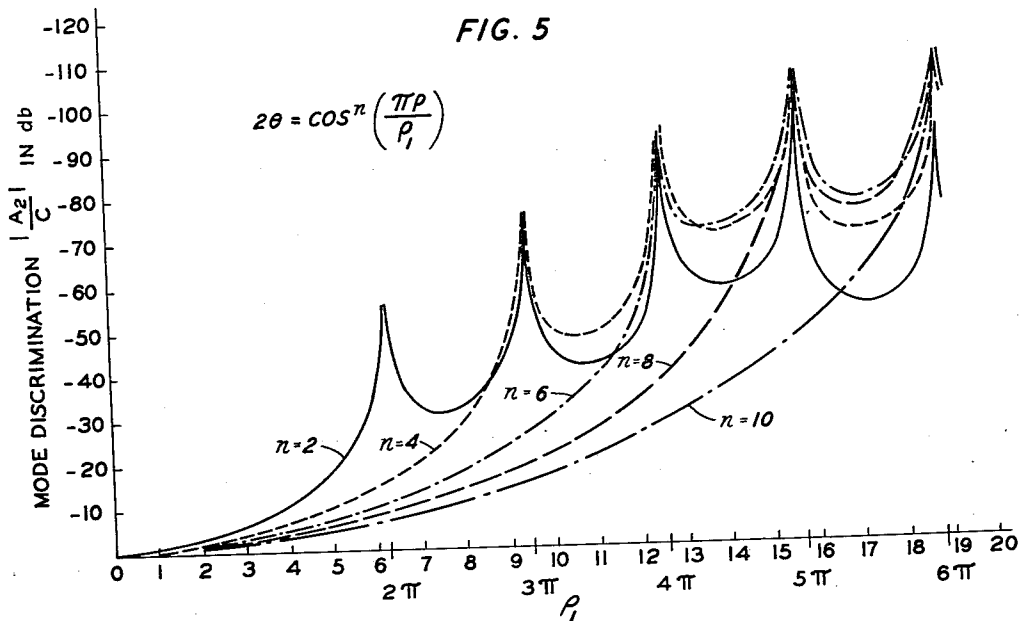


FIG. 5



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FIG. 6

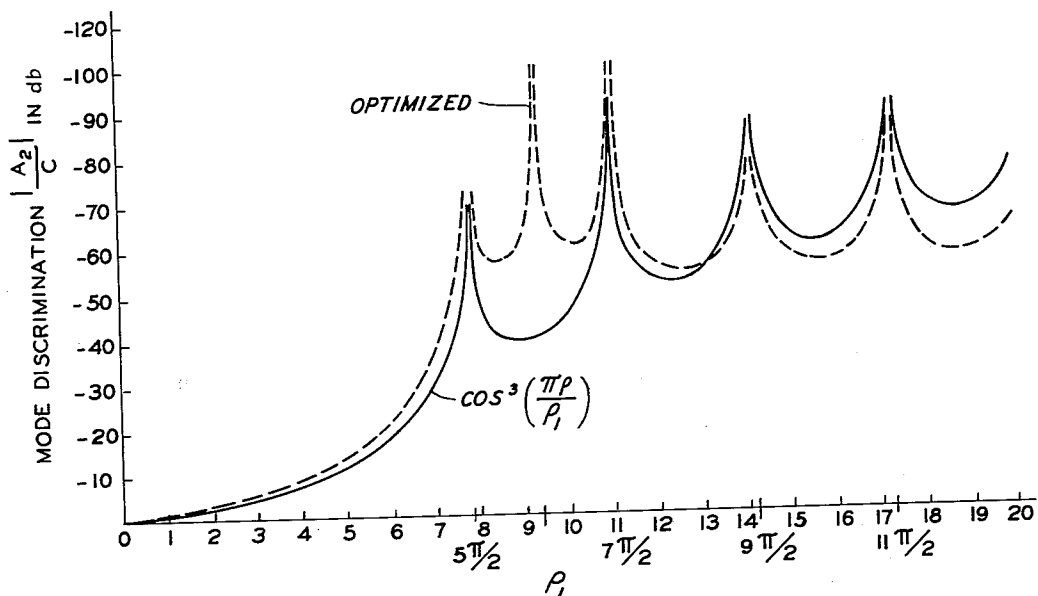
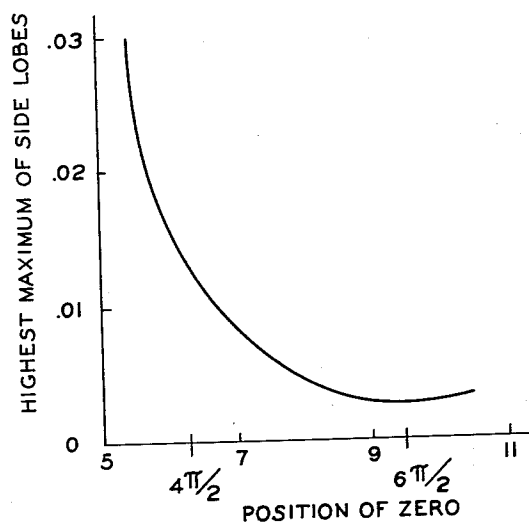


FIG. 7



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TAPERED WAVEGUIDE TRANSITION SECTION

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FIG. 8
COMPARISON OF PROFILE
OF TAPERS FOR SAME
MODE CONVERSION (50 db)

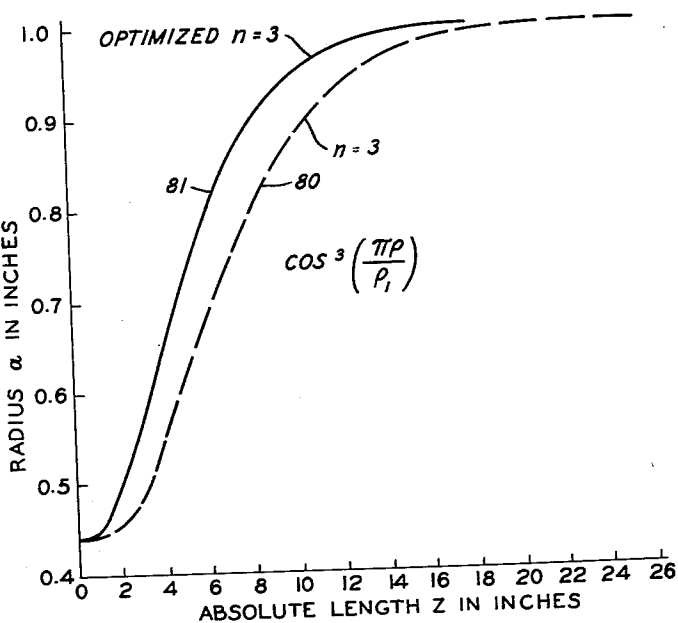
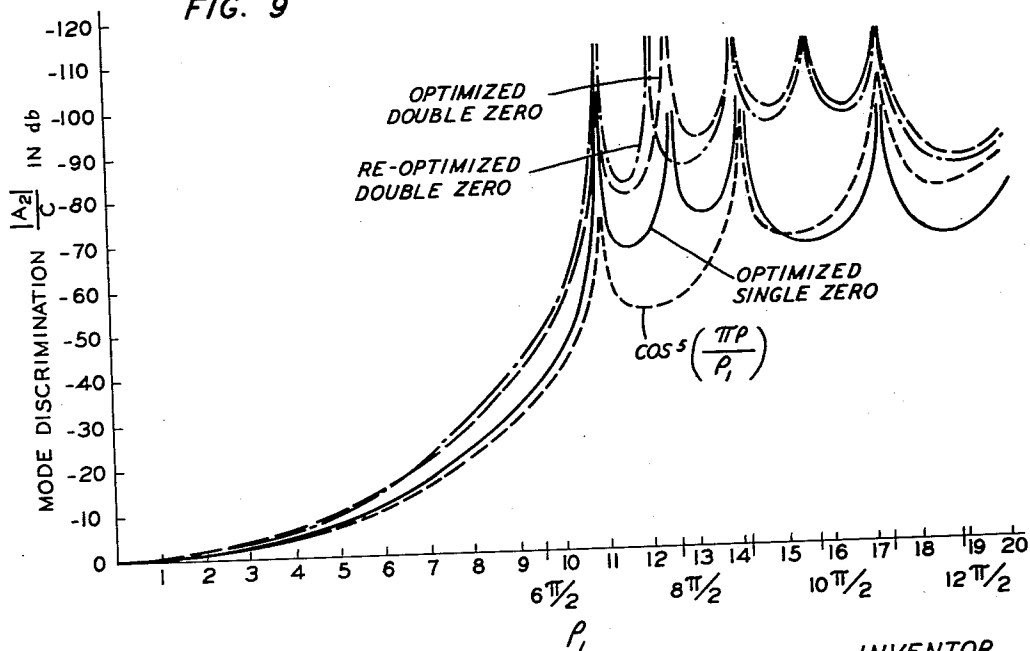


FIG. 9



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TAPERED WAVEGUIDE TRANSITION SECTION
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Filed Mar. 22, 1961, Ser. No. 97,602

6 Claims. (Cl. 333-34)

This invention relates to mode matching devices and more particularly to tapered sections of waveguide having a gradually changing taper angle to connect waveguides of different cross-sectional dimensions for the preferential transmission of a particular mode.

In the transmission of electromagnetic wave energy through a hollow conductive pipe or other waveguide it is well known that the energy can propagate in one or more transmission modes, or characteristic field configurations, depending upon the cross-sectional size and shape of the particular guide and the operating frequency, and that the larger the cross-section of the guide is made the greater is the number of modes in which the energy can propagate at a given operating frequency. Very generally it is desired to confine propagation of the energy to one particular mode, chosen because its propagation characteristics are favorable for the particular application involved. If the desired mode happens to be the so-called dominant mode, it is feasible to restrict the cross-sectional dimensions of the guide so that no modes other than the dominant mode can be sustained therein. This expedient is not available however, if the desired mode is not the dominant mode or if a guide of large cross-section is prescribed in order, for example, that advantage may be taken of its relatively low attenuation. This is particularly true of systems employing the TE_{01} circular electric mode. As is well known, the propagation of microwave energy in the form of the TE_{01} mode in circular waveguides is ideally suited for the long distance transmission of high frequency wide band signals since the attenuation characteristic of this transmission mode, unlike that of other modes, decreases with increasing frequency. However, since the TE_{01} mode is not the dominant mode supported in a circular waveguide, energy may be lost to other modes that are also capable of transmission therein.

In an ideal system which has a waveguide that is perfectly straight, uniform and conducting, the propagation of TE_{01} waves therethrough would be undisturbed. In any practical system, however, many changes in waveguide diameter will be made in order to take maximum advantage of the transmission characteristics of the TE_{01} mode. For example, it is known that the transmission loss for the circular electric mode is inversely related to the guide diameter. Hence, long, uninterrupted runs of waveguide will be made with large diameter pipe. Multiplexing of a series of frequency bands into one pipe, on the other hand, is most efficiently done at certain smaller diameters. Thus, numerous changes in guide size will be made, for example, at repeater stations where various bands of frequency are to be coupled into and out of the system. Such diameter changes must of course be made with a minimum of loss. Similarly, sharp intentional bends can be more easily negotiated at smaller diameters, which will necessitate further diameter changes wherever such bends are required. Thus, it is apparent that any practical transmission system will afford ample opportunity to disturb the TE_{01} circular electric mode unless proper transmission means are provided to connect the different diameter guides encountered. In the absence of such transition sections there will, most certainly, be a conversion of power from the desired mode into other spurious and undesired modes. The latter will, in many instances be propagated within the guide along with the desired mode and thereby introduce deleterious effects.

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In United States Patent No. 2,938,179, issued to H. G. Unger on May 24, 1960, it was pointed out that transition sections, built in the form of a conical taper of constant cone angle to match the waveguide sizes at the two ends, tend to excite too high a level of higher order modes. Unger found that substantial improvements could be obtained by continuously varying the cone angle of the tapered section. Compared to a taper with constant cone angle, a taper of continuously varying cone angle can be made much shorter for a specified spurious mode level. The resulting tapers, however, are still too long for many applications.

It is, therefore, the broad objective of this invention to reduce the length of transition sections of waveguide used to connect waveguides of different cross-sectional dimensions.

In the above-cited patent, Unger derives an expression for the spurious mode level at the end of the tapered section. Based upon this expression, the resulting mode conversion curves for typical tapers are given. In general, a typical mode conversion pattern consists of a series of lobes of decreasing maximum amplitude separated by a set of "intrinsic zeros." The minimum length of the taper is thus determined by the rate at which the maxima of the side lobe amplitudes decrease and hence, the maximum amplitude of the first few side lobes is of particular interest.

It is, accordingly, a more specific object of this invention to decrease the amplitude of the first few side lobes of the mode conversion curve for any tapered transition section of prescribed specifications.

In accordance with the principles of the invention, the amplitude of the first few side lobes of any given mode conversion curve is reduced by the creation of one or more "new zeros" at such positions along the mode conversion curve that the side lobes near the origin are leveled and at the same time optimally lowered. The effect of the "new zeros" is to modify the taper profile in a manner to reduce the length of the taper for a given spurious mode level or, alternatively, to reduce the spurious mode level for a given taper length.

These and other objects and advantages, the nature of the present invention, and its various features, will appear more fully upon consideration of the various illustrative embodiments now to be described in detail in connection with the accompanying drawings, in which:

FIG. 1 diagrammatically illustrates a guided microwave communication system employing the circular electric wave and having use for the tapered sections provided by the present invention;

FIG. 2 more specifically shows the use of the tapered sections of the present invention;

FIG. 3 shows a typical taper profile of an illustrative embodiment of the present invention;

FIGS. 4 and 5 show the mode conversion curves obtained in a tapered section for a conversion distribution function of the

$$\cos^n \left(\frac{\pi p}{\rho_1} \right)$$

family;

FIG. 6 shows a mode conversion curve for the conversion distribution function

$$\cos^3 \frac{\pi p}{\rho_1}$$

and the mode conversion curve when optimized in accordance with the invention;

FIG. 7 shows a typical variation of the highest maximum of the side lobe as a function of the position of the "new zero";

FIG. 8 is a comparison of the taper profile of a transi-

tion section in accordance with the prior art and when optimized in accordance with the present invention; and FIG. 9 is a comparison of the mode conversion curves for a taper having a distribution function

$$\cos^5 \frac{\pi \rho}{\rho_1}$$

and for various optimized conditions.

Referring more specifically to FIG. 1, there is shown a portion of a typical long distance guided microwave transmission system in which the invention might be used. Such a system is characterized as long, to distinguish it from the short distances found in terminal equipment, and to define a system in which the factor of transmission attenuation becomes important. This system comprises a terminal station 11 which may be a transmitter, or if this is an intermediate station, a repeater which is connected to a receiver or subsequent repeater station 12. The circular electric TE₀₁ mode is the mode in which energy is transmitted between stations. To do so with a minimum of loss, a large multimode cylindrical waveguide 15 is used as the transmission medium. However, since the TE₀₁ circular mode is not necessarily produced nor utilized directly in the components of a station, transducers 13 and 14 are interposed between stations 11 and 12 and the long distance transmission line 15. The latter is connected to transducers 13 and 14 by sections of waveguide 20 and 22 and tapered sections 19 and 21. Connecting station 11 and transducer 13 are three transmission paths 11a, 11b and 11c carrying three different frequency channels f_1 , f_2 and f_3 which are multiplexed in transducer 13 for the long distance transmission to transducer 14. Connecting transducer 14 and station 12 are three other transmission paths 12a, 12b and 12c carrying the again separated channels f_1 , f_2 and f_3 for utilization in station 12.

Transducers 13 and 14 may be of any suitable well-known types for converting TE₀₁ wave energy to and from a dominant wave mode configuration. For example, they may be structures of the type disclosed in United States Patent No. 2,748,350, granted to S. E. Miller, May 29, 1956, and illustrated in FIG. 2 of said patent.

In FIG. 2 there is shown a mode multiplex system in which a plurality of dominant mode TE₁₀ modulated signals of different frequencies are transformed into TE₀₁ circular electric waves at corresponding frequencies. Thus, a first modulated TE₁₀ wave, f_1 , entering guide 10 is transformed into a first modulated TE₀₁ circular electric wave at the same frequency in cylindrical guide 4. This signal then passes on to guide 1 through guides 3 and 2 and conical tapered sections 7, 6 and 5. Similarly, signals f_2 and f_3 are transformed and pass on to guide 1 through tapered sections 6 and 5.

To minimize the conversion of wave energy from the preferred TE₀₁ mode to higher order circular electric modes as a result of the presence of tapered sections 5, 6, 7, 19 and 21, the shape of the tapered sections is controlled in a prescribed manner. That is, the manner in which the radius of each taper varies as a function of the longitudinal distance along the taper is selected so that the amplitudes of the spurious mode generated by the incident wave energy traveling along the transition section are such that their sum, at the end of the taper in the direction of energy flow, is minimized.

FIG. 3 shows, generally, a transition section in accordance with the invention, in which 16 and 17 represent cylindrical guides of radii a_1 and a_2 , respectively, which are to be joined by means of a tapered transition section 18. The radius a of section 18 is related to the distance z along the taper in a manner to be explained below.

The field excited in any cross-section of the taper by an incident TE₀₁ wave can be expressed as a sum of TE_{0m} waves of a cylindrical guide of the same radius as that cross-section. Using such a representation, the

tapered waveguide appears to be an infinite set of mutually-coupled transmission lines, where each transmission line represents one of the cylindrical TE_{0m} modes. Furthermore, since the two guides are generally large and operate far removed from cut-off, there is little change in characteristic impedance involved in going from one diameter pipe to the other. Therefore, we need consider only the forward traveling waves, as the relative power coupled from the forward waves to the backward waves due to mismatch is quite small. The total wave propagating in such a system reduces to the following set of infinite first order differential equations:

$$\frac{dA_m}{dz} = -j\beta_m A_m + \sum_{n=1}^{\infty} k_{mn} A_n \quad (1)$$

where A_m and A_n are the complex forward wave amplitudes of the cylindrical TE_{0m} and TE_{0n} mode waves, respectively, β_m is the phase constant of the m^{th} mode wave and k_{mn} is the real coefficient of coupling between the m^{th} and n^{th} mode waves with $k_{mn} = k_{nm}$. For a cylindrical taper supportive of the circular electric mode of wave propagation k_{mn} is given as

$$k_{mn} = \frac{1}{a} \frac{da}{dz} \left[\frac{k_m k_n}{k_m^2 - k_n^2} \right] \left[\sqrt{\frac{Z_m}{Z_n}} + \sqrt{\frac{Z_n}{Z_m}} \right] \quad (2)$$

where k_m and k_n are the m^{th} and the n^{th} Bessel function roots and Z_m and Z_n are the wave impedances of the TE_{0m} and TE_{0n} modes, respectively.

Equation 1 can be conveniently cast into the following matrix form:

$$D\{A\} = [M]\{A\}, \{A(0)\} = \{C\} \quad (3)$$

where D , $\{ \}$ and $[\]$ represent, respectively, the first derivative operator

$$\frac{d}{dz}$$

a column matrix and a square matrix. $\{C\}$ is a column matrix of constants representing the initial boundary conditions.

To solve the problem, a nonlinear, nonsingular matrix transformation is introduced to obtain a new orthogonal set of amplitudes $B_m(z)$ along the tapered section 18 in at least a localwise sense. The transformation is of the following form:

$$\{A\} = [P(z)]\{B(z)\} \quad (4)$$

where $[P(z)]$ is the transformation matrix to be sought and is a function of the length of the taper, and $\{B(z)\}$ is the amplitudes of the local orthogonal modes. Substituting Equation 4 into Equation 3 yields

$$D\{B\} = [Q]\{B\}, \{B(0)\} = [P(0)]^{-1}\{C\} \quad (5)$$

$$D[B] = [Q][B], [B(0)] = [I] \quad (6)$$

where

$$[Q] = [P]^{-1}[M][P] - [P]^{-1}D[P]$$

or, more briefly,

$$[Q] = [q] - [\epsilon] \quad (7)$$

The solution of Equation 5 is

$$\{B\} = [B][P(0)]^{-1}\{C\} \quad (8)$$

where

$$[B] = e^{\int_0^z [q] dz} - \int_0^z e^{\int_0^z [q] dz} e^{-\int_0^{z'} [q(z')] dz'} [\epsilon(z')] [B(z')] dz' \quad (9)$$

The formal solution of Equation 3 at the end of a taper of length l is, therefore,

$$\{A(l)\} = [P(l)][B(l)][P(0)]^{-1}\{C\} \quad (10)$$

If the taper is sufficiently gentle, the power in each of the TE_{0m} modes, where $m > 1$, is small compared to the

power in the TE_{01} term. Furthermore, of the higher order modes, the principal one that need be considered is the second order TE_{02} term. Thus, neglecting all modes above the second, we obtain as the expression for the amplitude of the spurious mode level at the end of the taper

$$|A_2(\rho_1)| = \left| \int_0^{\rho_1} (2\theta) \rho^{2i} d\rho \right| \quad (11)$$

where

$$\rho(z) = \int_0^z \sqrt{\left(\frac{\beta_1 - \beta_2}{2}\right)^2 + k^2} dz \quad (12)$$

$$\rho_1 = \rho(l) = \int_0^l \sqrt{\left(\frac{\beta_1 - \beta_2}{2}\right)^2 + k^2} dz \quad (13)$$

and

$$2\theta = \sin^{-1} \frac{k}{\sqrt{\left(\frac{\beta_1 - \beta_2}{2}\right)^2 + k^2}} \quad (14)$$

and (2θ) can be appropriately interpreted as the "mode conversion distribution function" along the taper.

Equation 11 is in a form suitable for computing the spurious mode level when the "conversion distribution function," 2θ , is properly chosen in terms of the parameter ρ . Once the distribution function 2θ is selected, both the radius a of the taper and the distance z along the taper can be obtained as a function of the parameter ρ .

Before proceeding, it would be well to examine, briefly, the terms 2θ and ρ for their physical significance. From Equations 12 and 14 it is seen that both these terms are functions of k , the coefficient of coupling between modes and β_1 and β_2 , the mode phase constants. However, k , β_1 and β_2 are functions of a , the guide radius which is a function of the distance z along the taper, and the frequency. It is apparent that the choice of the distribution function is not unique. A theoretical derivation shows that any function which satisfies the end boundary conditions, i.e., that the function itself vanish at both ends of the taper, may be selected as the distribution function. Once a particular function is selected for 2θ , however, the shape of the taper is defined and the spurious mode level can be calculated from Equation 11.

The distribution function (2θ) can be expressed in a general form as a symmetrical Fourier series with unknown coefficients

$$2\theta = \sum_{m=0}^{\infty} D_m \cos m \left(\frac{\rho}{\rho_1} \right) \quad (15)$$

and the expression for the spurious mode level becomes, via Equation 11,

$$|A_2(\rho_1)| = \left| 2(e^{2i\rho_1} - 1) \sum_{m=0}^{\infty} \frac{D_m \cos m\pi/2}{\left(\frac{m\pi}{\rho_1}\right)^2 - 4} \right| \quad (16)$$

for even values of m , and

$$|A_2(\rho_1)| = \left| \frac{\rho_1}{\pi} (e^{2i\rho_1} + 1) \sum_{m=0}^{\infty} \frac{m D_m \sin m(\pi/2)}{m^2 - \left(\frac{2\rho_1}{\pi}\right)^2} \right| \quad (17)$$

for odd values of m .

If, for the purposes of illustration, 2θ is selected to be

$$\cos^n \left(\frac{\pi \rho}{\rho_1} \right)$$

where

$$\cos \left(\frac{\pi \rho}{\rho_1} \right)$$

is considered the "generating function," the expression for the spurious mode level then becomes

$$|A_2(\rho_1)| = \left| c(2 \cdot 4 \cdot 5 \cdot \dots \cdot n)^2 \frac{2j \left(\frac{\pi}{\rho_1} \right)^n (e^{2i\rho_1} - 1)}{\rho_1 \prod_{s=0}^n \left[(2j)^2 + s^2 \left(\frac{\pi}{\rho_1} \right)^2 \right]} \right| \quad (18)$$

for n even, and

$$|A_2(\rho_1)| = \left| c(1 \cdot 3 \cdot 5 \cdot \dots \cdot n)^2 \frac{\left(\frac{\pi}{\rho_1} \right)^{n+1} (e^{2i\rho_1} + 1)}{2 \prod_{s=1}^n \left[(2j)^2 + s^2 \left(\frac{\pi}{\rho_1} \right)^2 \right]} \right| \quad (19)$$

for n odd, where

$$C = \frac{2k_m k_n}{k_n^2 - k_m^2} \log \frac{a_2}{a_1} \quad (20)$$

k_m and k_n are the m^{th} and the n^{th} root of Bessel function J_1 and a_1 and a_2 are the radii of the taper at the respective ends.

It is evident from Equations 18 and 19 that C corresponds to the mode conversion produced by a step discontinuity in the diameter of the waveguide, i.e., a taper of zero length. That is

$$A_2(\rho_1 = 0) = C = \frac{k_1 k_2}{k_1^2 - k_2^2} \log \frac{a_2}{a_1} \quad (21)$$

For cases where n is a noninteger, general solutions for the mode discrimination in closed form are impossible and it is necessary to resort to numerical integration. Equations 18 and 19 are plotted in FIGS. 4 and 5 for integral values of n .

The curves are plotted as a function of ρ_1 which, is itself a function of the taper length, l , and the wavelength, λ . Specifically, for any given taper length, ρ_1 is approximately proportional to the wavelength. Alternatively, at any given frequency, ρ_1 is approximately proportional to the length of the taper.

It is seen from FIGS. 4 and 5 that there is an "optimum" integer n which minimizes the length of the taper for a prescribed discrimination. Similarly, if the taper length is prescribed, there is an "optimum" integer n that provides the highest mode discrimination. From FIG. 4 it is seen that for a taper length defined by $\rho_1 = 18.5$, optimum discrimination occurs for $n = 7$. On the other hand, if a mode discrimination of minus 50 db is prescribed, the shortest taper is obtained when $n = 3$, for which $\rho_1 = 10.4$.

Another simple choice for the distribution function is the following polynomial in ρ/ρ_1 :

$$2\theta = G_n \left(\frac{4\rho}{\rho_1} \right)^n \left(1 - \frac{\rho}{\rho_1} \right)^n \quad (22)$$

where the generating function is

$$\left(\frac{4\rho}{\rho_1} \right) \left(1 - \frac{\rho}{\rho_1} \right)$$

and n may or may not be an integer. The mode discrimination curve obtained with 2θ defined by Equation 22 is similar, in its general appearance, to the mode discrimination curves shown in FIGS. 4 and 5. In general, the mode conversion curves are characterized by a single infinity of "intrinsic zeros" which are imbedded in the mode conversion curves given by Equations 16 and 17 and are determined by the factor

$$(e^{2i\rho_1} - 1)$$

or

$$(e^{2i\rho_1} + 1)$$

respectively. In FIG. 4 these intrinsic zeros occur for values of ρ_1 equal to

$$(n+2)\frac{\pi}{2}$$

where n is an odd integer. Between $\rho_1=0$ and the first intrinsic zero is the first, or major, lobe of the mode conversion curve. Between successive intrinsic zeros are the side, or minor, lobes, each one of which reaches a maximum amplitude which is less than the maximum amplitude of the preceding lobe. In accordance with the principles of the invention, the tapered transition section 18 is optimized by "optimizing" the mode conversion curve defined by Equations 16 or 17. In particular, the mode conversion curve is reshaped in such a way that the maximum amplitude of one or more of the first few side lobes of the mode conversion curve is substantially lowered and leveled. Before carrying out this optimizing procedure, however, it is appropriate to discuss the basis for this optimization in some detail.

The class of distribution functions that have been considered has the property that the distribution function and all its derivatives are single-valued, uniformly bounded, and continuous in the interval of interest. Any distribution function of this class can be transformed to a general form in terms of the zeros of the function. With the coordinate origin at the center of the taper, the function of interest has the form:

$$2\theta=f(\rho)\left[\rho^2-\left(\frac{\rho_1}{2}\right)^2\right]^n \quad (23)$$

where $f(\rho)$ is an even function [due to the symmetry of the function (2θ)] which neither vanishes at the taper ends

$$\left(\rho=\pm\frac{\rho_1}{2}\right)$$

nor for any other value of ρ in the interval. T. T. Taylor in his paper entitled "Design of Line-Source Antenna for Narrow Bandwidth and Low Side Lobes" (Trans. IRE, PGAP-3, January 1955, pp. 16-28) has shown that the Fourier transform of Equation 23 has the following asymptotic form as ρ_1 approaches infinity:

$$F(\rho_1)\sim F\left(\frac{\rho_1}{2}\right)\Gamma(n+1)\frac{\cos\left(\rho_1-\frac{n+1}{2}\pi\right)}{(\rho_1)^{n+1}} \quad (24)$$

It is seen from Equation 24 that the mode discrimination of very long tapers is only trivially different, no matter what form the distribution function has. On the other hand, the initial slope of the function, and accordingly the value of n , is of considerable importance in optimizing the taper. The initial slope of the function (2θ) determines the positions of the zeros near the origin of the mode discrimination curve $A_2(\rho_1)$. The smaller the initial slope, the further the first intrinsic zero is from the origin. Similarly, the value of n alters the zero positions since n changes the slope of the function. Equation 24 indicates clearly that the spurious mode conversion, or discrimination, decays as

$$\frac{1}{|\rho_1|^{n+1}}$$

and that the zeros appear at

$$\rho_1=n\frac{\pi}{2}$$

for large values of ρ_1 . Equations 16 and 17 confirm this decay rate and the position of the zeros for large values of ρ_1 . At this point it is particularly appropriate to compare this decay rate with the nondecaying characteristics of the Tchebycheff polynomial of infinite degree. If we are to simulate the Tchebycheff polynomial of infinite degree by the function of Equation 24, we see that the only way to make Equation 24 nondecaying is to make

$n=-1$. However, for this choice of n the function of Equation 23 has poles at the taper ends and is no longer uniformly bounded. This violates one basic requirement. It is clear, therefore, that a smooth transition taper with its mode conversion characteristics described by a Tchebycheff polynomial of infinite degree is unrealizable. In fact this unrealizability is simply a consequence of the law of conservation of energy.

From the above exposition we see that in order to further optimize, we need only to "flatten" or "level" the decay rate of the first few side lobes of the mode conversion curve as much as permissible after choosing the optimum value of n for a prescribed discrimination level. Knowing that the shape of the mode conversion curve depends very much on the density of the distribution of zeros near the origin, optimization, in accordance with the principles of this invention, is achieved by the creation of one or more extra "zeros" at such positions that the side lobes near the origin are leveled and at the same time optimally lowered. It is important to note that the new zeros are introduced by properly choosing the coefficients D_m in Equations 16 and 17, while the original or intrinsic zeros were determined only by the terms

$$(e^{2\rho_1}\pm 1)$$

For purposes of illustration, this process of optimizing will be carried out for

$$2\theta=\cos^3\left(\frac{\pi\rho}{\rho_1}\right) \text{ and } 2\theta=\cos^5\left(\frac{\pi\rho}{\rho_1}\right)$$

The Fourier expansion of

$$\cos^3\left(\frac{\pi\rho}{\rho_1}\right)$$

has two terms, the coefficients of which are given with the aid of Equation 19, as

$$D_1=\frac{9}{8}\frac{\pi C}{\rho_1} \text{ and } D_3=\frac{1}{3}\frac{9}{8}\frac{\pi C}{\rho_1}$$

In accordance with the invention, the distribution function 2θ is modified by selecting a new pair of coefficients D_1' and D_3' . In particular the coefficients D_1' and D_3' are determined by requiring firstly that $A_2(\rho_1)$ vanishes for some properly selected value of ρ_1 , and secondly that $A_2(\rho_1=0)$ is equal to unity (due to normalization of the mode conversion function A_2). From Equation 17, for $m=1$ and $m=3$, we obtain

$$D_1\left[3^2-\left(\frac{2}{\pi}\right)^2\right]-3D_3\left[1^2-\left(\frac{2}{\pi}\right)^2\right]=0 \quad (25)$$

and for $\rho_1=0$

$$\left(D_1'-\frac{D_3'}{3}\right)=1 \quad (26)$$

These two simultaneous equations can be solved for D_1' and D_3' when the position of the new zero is intelligently selected. Inspection of FIG. 6, which is a plot of the mode discrimination curve for the

$$\cos^3\left(\frac{\pi\rho}{\rho_1}\right)$$

distribution function, shows that it is advisable to locate the new zero at about

$$\rho_1=6\frac{\pi}{2}$$

in order to achieve the desired results. With this value of ρ_1 we get $D_1'=1.09375$ and $D_3'=0.28125$ from Equations 25 and 26. A plot of Equation 17 using these values of D_1' and D_3' , shown also in FIG. 6, indeed gives the desired results. FIG. 7, showing the relation between the position of the new zero and the corresponding maximum of the maxima of the side lobes also confirms the

fact that the optimum location of the new zero should be in the vicinity of

$$\rho_1 = 6\frac{\pi}{2}$$

Once the form of the distribution function 2θ is explicitly determined in the manner explained above, the shape of the tapered section can be calculated for the circular waveguide case using the following parametric equations:

$$a = a_1 e^{\left(\frac{k_2^2 - k_1^2}{2k_2 k_1} \right) \cdot \int_{-\frac{\rho_1}{2}}^{\rho} 2\theta d\rho} \quad (27)$$

$$z = \frac{1}{k_2^2 - k_1^2} \left\{ 4\beta \int_{-\frac{\rho_1}{2}}^{\rho} a^2 d\rho - \left[\rho - \frac{\rho_1}{2} \right] \left[\frac{k_2^2 + k_1^2}{\beta} \right] - \left[\frac{k_2^4 + k_1^4}{4\beta^3} \right] \int_{-\frac{\rho_1}{2}}^{\rho} \frac{1}{a^2} d\rho \right\} \quad (28)$$

where k_1 and k_2 are the Bessel function roots equal to 3.832 and 7.016, respectively, and β is the intrinsic propagation constant of the dielectric material within the transition section for the highest frequency in the band of interest is equal to $\omega\sqrt{\mu_0\epsilon_0}$ in air.

The actual taper profile calculated from Equations 27 and 28 are plotted in FIG. 8 for comparison. Curve 80 shows the taper profile for a distribution function

$$\cos^5 \left(\frac{\pi\rho}{\rho_1} \right)$$

whereas curve 81 shows the taper profile when optimized in accordance with the invention. The curves are for a 50 db prescribed mode discrimination in a taper connecting a $\frac{1}{8}$ inch inside diameter waveguide to a 2 inch inside diameter waveguide operating at frequencies below 75 kmc./sec. and shown an overall reduction in length of approximately 30 percent for the optimized taper.

The same procedure can be applied for $n=5$. The Fourier expansion of

$$\cos^5 \left(\frac{\pi\rho}{\rho_1} \right)$$

has three terms, the coefficients of which are given, with the aid of Equation 19, as

$$D_1 = \left(\frac{75}{64} \right) \frac{\pi C}{2\rho_1}, \quad D_3 = \frac{1}{2} \left(\frac{75}{64} \right) \frac{\pi C}{2\rho_1}$$

and

$$D_5 = \frac{1}{10} \left(\frac{75}{64} \right) \frac{\pi C}{2\rho_1}$$

The mode discrimination curve for $n=5$ is shown in FIG. 9. Optimization, in accordance with the invention, is obtained by evaluating a new set of coefficients D_1' , D_3' and D_5' such that

$$D_1' \left[3^2 - \left(\frac{2}{\rho_1 \frac{\pi}{\pi}} \right)^2 \right] \left[5^2 - \left(\frac{2}{\rho_1 \frac{\pi}{\pi}} \right)^2 \right] - 3D_3' \left[1^2 - \left(\frac{2}{\rho_1 \frac{\pi}{\pi}} \right)^2 \right] + \left[5^2 - \left(\frac{2}{\rho_1 \frac{\pi}{\pi}} \right)^2 \right] + 5D_5' \left[1^2 - \left(\frac{2}{\rho_1 \frac{\pi}{\pi}} \right)^2 \right] \left[3^2 - \left(\frac{2}{\rho_1 \frac{\pi}{\pi}} \right)^2 \right] = 0 \quad (29)$$

and

$$\left[D_1' - \frac{D_3'}{3} + \frac{D_5'}{5} \right] = 1 \quad (30)$$

at $\rho_1 = 0$.

Inspection of FIG. 9 shows that the logical first choice for the location of a new zero is at

$$\rho_1 = 8\frac{\pi}{2}$$

Substituting

$$\rho_1 = 8\frac{\pi}{2}$$

in Equation 29, however, leaves us three unknowns with but two equations. If only a single new zero is sought, it then becomes necessary to assume an appropriate value for D_1' so that D_3' and D_5' can be determined. A suitable choice might be

$$D_1' = \frac{75}{64}$$

which is the normalized Fourier coefficient obtained from Equation 19. The resulting mode discrimination curve obtained for the optimized single zero case for

$$D_1' = \frac{75}{64} \text{ and } \rho_1 = 8\frac{\pi}{2}$$

is also shown in FIG. 9 and may readily be compared with the simple

$$\cos^5 \frac{\pi\rho}{\rho_1} \text{ curve}$$

If further optimization is desirable, we may create a second new zero. It will be recalled that D_1' was arbitrarily assigned the value

$$\frac{75}{64}$$

If, instead, a second new zero is located at

$$\rho_1 = 10\frac{\pi}{2}$$

in the center of the second side lobe, a third, independent equation is obtained and the three coefficients D_1' , D_3' and D_5' can be uniquely determined. The resulting optimized double zero mode discrimination curve is also plotted on FIG. 9 and can be compared with the unmodified and with the single zero case.

A further reduction in the spurious mode level can be effected by relocating the first new zero from

$$\rho_1 = 8\frac{\pi}{2}$$

to a point

$$\rho_1 = 7.72\frac{\pi}{2}$$

The resulting improvement is shown by the re-optimized double zero curve which is also plotted in FIG. 9.

Accordingly, it is seen that a taper can be optimized by creating new zeros between the first few intrinsic zeros of the mode conversion curve. It will be noted that the number of new zeros that can be created increases directly as the number of undetermined coefficients and will be greater for larger values of n .

In the discussion above, particular reference was made to a cylindrical tapered transition section to connect cylindrical waveguides of different diameters. It is to be understood, however, that the principles of the invention are equally applicable to waveguides of other cross-sectional configurations and to other types of transmission media generally. While there will be differences in the parameters of the equations used to describe the other types of transmission media, the same techniques can, nevertheless, be applied to minimize the taper length and to reduce the level of the higher order spurious modes. As an illustration of these differences, consider a rectangular taper slowly flaring in a single plane. The total wave propagating in such a taper is, as before, given by Equation 1. However, k_{mn} , the coefficient of coupling between the m^{th} and the n^{th} mode waves, is now

$$k_{mn} = \frac{mn(-1)^{m+n}}{m^2 - n^2} \frac{1}{a} \frac{da}{dz} \left(\sqrt{\frac{Z_m}{Z_n}} + \sqrt{\frac{Z_n}{Z_m}} \right) \quad (31)$$

where a is the varying dimension which, in the case of a rectangular taper, may be either the guide width or the

guide height. The taper profile is given by the following set of parametric equations:

$$\begin{aligned} a &= a_1 e \\ z &= \frac{4\beta}{\pi^2(m^2-n^2)} \int_{-\rho_1}^{\rho} a^2 d\rho + \frac{(m^2+n^2)}{\beta(m^2-n^2)} \left(\frac{\rho_1}{2} - \rho \right) \\ &\quad - \frac{\pi^2(m^4+n^4)}{4\beta^3(m^2-n^2)} \int_{-\rho_1}^{\rho} \frac{d\rho}{a^2} \end{aligned} \quad (32)$$

If, for example, the desired propagating mode in the rectangular guide is TE_{10} , then $n=1$ and m can only be odd due to symmetry.

In all cases it is understood that the above-described arrangements are illustrative of a small number of the many possible specific embodiments which can represent applications of the principles of the invention. Numerous and varied other arrangements can readily be devised in accordance with these principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. In an electromagnetic wave transmission system supportive of at least the first and the second order modes of a family of modes of wave propagation, a tapered transition section for the preferential transmission of said first order mode to connect a first multimode waveguide supportive of said modes to a second larger multimode waveguide of a similar cross-sectional configuration supportive of said modes, said taper having a first order mode to second order mode conversion characteristic including a plurality of intrinsic zero points and characterized by at least one new zero point at a preselected point between two of said intrinsic zero points.

2. In an electromagnetic wave transmission system supportive of at least the TE_{01} and the TE_{02} circular electric modes of wave propagation, a conical tapered section for the preferential transmission of said TE_{01} mode to connect a first multimode cylindrical waveguide of radius a_1 supportive of said modes to a second multimode cylindrical waveguide of radius a_2 supportive of said modes, said taper having a TE_{01} to TE_{02} mode conversion characteristic including a plurality of intrinsic zero points and characterized by at least one new zero point at a preselected point between two of said intrinsic zero points.

3. The taper according to claim 2 wherein the amplitude of said mode conversion characteristic is given in terms of a parameter ρ_1 of said taper as

$$|A_2(\rho_1)| = \left| 2(e^{2i\rho_1} - 1) \sum_{m=0}^{\infty} \frac{D_m \cos m\frac{\pi}{2}}{\left(\frac{m\pi}{\rho_1}\right)^2 - 4} \right|$$

for even values of m , and wherein the summation

$$\sum_{m=0}^{\infty} \frac{D_m \cos m\frac{\pi}{2}}{\left(\frac{m\pi}{\rho_1}\right)^2 - 4}$$

is zero for at least one predetermined value of ρ_1 .

4. The taper according to claim 2 where the amplitude of said mode conversion characteristic is given in terms of a parameter ρ_1 of said taper as

$$|A_2(\rho_1)| = \left| \frac{\rho_1}{\pi} (e^{2i\rho_1} + 1) \sum_{m=0}^{\infty} \frac{m D_m \sin m\frac{\pi}{2}}{m^2 - \left(\frac{2\rho_1}{\pi}\right)^2} \right|$$

for odd values of m , and wherein the summation

$$\sum_{m=0}^{\infty} \frac{m D_m \sin m\frac{\pi}{2}}{m^2 - \left(\frac{2\rho_1}{\pi}\right)^2}$$

is zero for at least one predetermined value of ρ_1 .

5. In an electromagnetic wave transmission system supportive of at least the TE_{01} and the TE_{02} circular electric modes of wave propagation over a given frequency band, a conical tapered section of length l for the preferential transmission of the TE_{01} mode to connect a first multimode cylindrical waveguide of radius a_1 supportive of said modes to a second multimode cylindrical waveguide of radius a_2 supportive of said modes wherein the radius a of said taper and the distance z from said first guide vary parametrically as

$$\begin{aligned} a &= a_1 e \\ z &= \frac{1}{k_2^2 - k_1^2} \left[4\beta \int_{-\rho_1}^{\rho} a^2 d\rho \right. \\ &\quad \left. \left(\rho - \frac{\rho_1}{2} \right) \left(\frac{k_2^2 + k_1^2}{\beta} \right) - \left(\frac{k_2^4 + k_1^4}{4\beta^3} \right) \int_{-\rho_1}^{\rho} \frac{1}{a^2} d\rho \right] \end{aligned}$$

where k_1 and k_2 are Bessel function roots equal to 3.832 and 7.016, respectively, β is the intrinsic propagation constant of the dielectric material within the transition section for the highest frequency within said band equal to $\omega\sqrt{\mu_0\epsilon_0}$ in air,

$$\rho = \int_0^z \left[\left(\frac{\beta_1 - \beta_2}{2} \right) + k^2 \right]^{1/2} dz$$

$$\rho_1 = \int_0^l \left[\left(\frac{\beta_1 - \beta_2}{2} \right) + k^2 \right]^{1/2} dz$$

β_1 and β_2 are the phase constants of the TE_{01} and TE_{02} modes respectively, k is the coefficient of coupling between said modes and wherein the term 2θ is judiciously chosen consistent with all boundary conditions and expressed in generality as

$$2\theta = \sum_{m=0}^{\infty} D_m \cos m \left(\frac{\pi \rho}{\rho_1} \right)$$

with specific coefficients D_m , said taper having a TE_{01} to TE_{02} mode conversion characteristic, A_2 , given by

$$|A_2(\rho_1)| = \left| 2(e^{2i\rho_1} - 1) \sum_{m=0}^{\infty} \frac{D_m \cos m\frac{\pi}{2}}{\left(\frac{m\pi}{\rho_1}\right)^2 - 4} \right|$$

for even values of m , and

$$|A_2(\rho_1)| = \left| \frac{\rho_1}{\pi} (e^{2i\rho_1} + 1) \sum_{m=0}^{\infty} \frac{m D_m \sin m\frac{\pi}{2}}{m^2 - \left(\frac{2\rho_1}{\pi}\right)^2} \right|$$

for odd values of m , characterized in that said summation

$$\sum_{m=0}^{\infty} \frac{D_m \cos m\frac{\pi}{2}}{\left(\frac{m\pi}{\rho_1}\right)^2 - 4}$$

is zero for at least one predetermined value of ρ_1 .

$$\sum_{m=0}^{\infty} \frac{m D_m \sin m\frac{\pi}{2}}{m^2 - \left(\frac{2\rho_1}{\pi}\right)^2}$$

for odd values of m , are zero for at least one predetermined value of ρ_1 .

6. In an electromagnetic wave transmission system supportive of at least the TE_{10} and TE_{30} mode of wave propagation over a given frequency band, a tapered transition section of length l for the preferential transmission of the TE_{10} mode to connect a first rectangular waveguide having cross-sectional dimensions a_1 and b_1 supportive of said modes to a second rectangular waveguide having cross-sectional dimensions a_2 and b_1 supportive of said modes, wherein the dimension a of said taper and the distance z from said first guide vary parametrically as

$$a = a_1 e^{\frac{4}{3} \int_{-\frac{\rho_1}{2}}^{\rho} (2\theta) d\rho}$$

$$z = \frac{\beta}{2\pi^2} \int_{-\frac{\rho_1}{2}}^{\rho} a^2 d\rho - \frac{5}{4\beta} \left(\rho - \frac{\rho_1}{2} \right) - \frac{41\pi^2}{16\beta^3} \int_{-\frac{\rho_1}{2}}^{\rho} \frac{1}{a^2} d\rho$$

where β is the intrinsic propagation constant of the dielectric material within the transition section for the highest frequency within said given band equal to $\omega\sqrt{\mu_0\epsilon_0}$ in air,

$$\rho = \int_0^z \left[\left(\frac{\beta_1 - \beta_2}{2} \right) + k^2 \right]^{1/2} dz$$

$$\rho_1 = \int_0^l \left[\left(\frac{\beta_1 - \beta_2}{2} \right) + k^2 \right]^{1/2} dz$$

β_1 and β_2 are the phase constants of the TE_{10} and TE_{30} modes respectively, k is the coefficient of coupling between said modes and wherein the term 2θ is judiciously chosen consistent with all boundary conditions and ex-

pressed in generality as

$$2\theta = \sum_{m=0}^{\infty} D_m \cos m \left(\pi \frac{\rho}{\rho_1} \right)$$

5 with specific coefficients D_m , said taper having a TE_{10} to TE_{30} mode conversion characteristic, A_2 , given by

$$|A_2(\rho_1)| = \left| 2(e^{2j\rho_1} - 1) \sum_{m=0}^{\infty} \frac{D_m \cos m \frac{\pi}{2}}{\left(\frac{m\pi}{\rho_1} \right)^2 - 4} \right|$$

10 for even values of m , and

$$15 \quad |A_2(\rho_1)| = \left| \frac{\rho_1}{\pi} (e^{2j\rho_1} + 1) \sum_{m=0}^{\infty} \frac{m D_m \sin m \frac{\pi}{2}}{m^2 - \left(\frac{2\rho_1}{\pi} \right)^2} \right|$$

for odd values of m , characterized in that said summation

$$20 \quad \sum_{m=0}^{\infty} \frac{D_m \cos m \frac{\pi}{2}}{\left(\frac{m\pi}{\rho_1} \right)^2 - 4}$$

25 for even values of m , and said summation

$$\sum_{m=0}^{\infty} \frac{m D_m \sin m \frac{\pi}{2}}{m^2 - \left(\frac{2\rho_1}{\pi} \right)^2}$$

30 for odd values of m , are zero for at least one predetermined value of ρ_1 .

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