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WAVEGUIDE ATTENUATOR

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FIG. 1

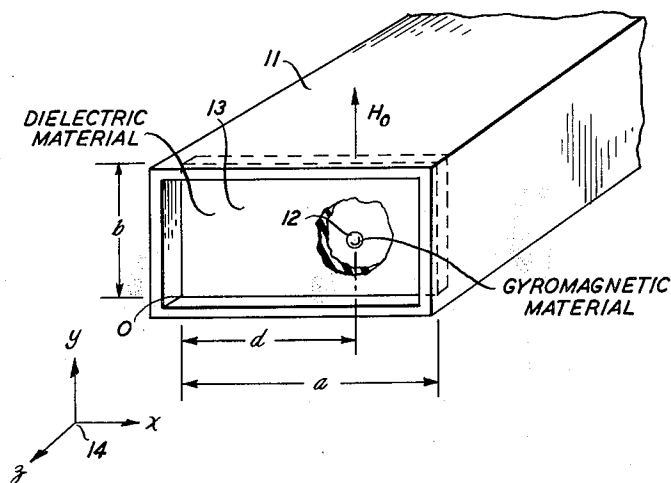


FIG. 2

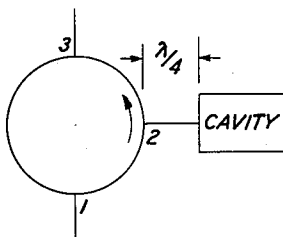
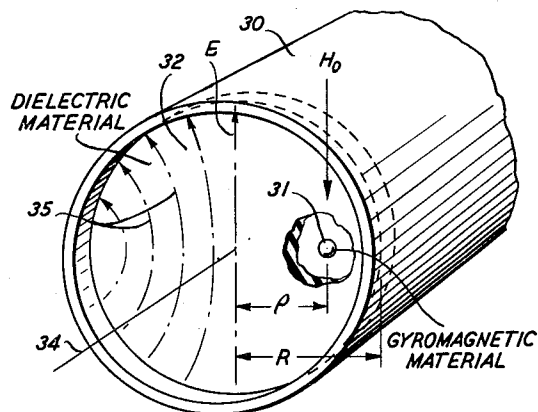


FIG. 3



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WAVEGUIDE ATTENUATOR

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This invention relates to electromagnetic wave transmission systems and more particularly to transmission structures having nonreciprocal attenuation properties for use in such systems.

The use of materials having gyromagnetic properties to obtain both reciprocal and nonreciprocal effects in microwave transmission circuits is widely known and has found numerous and varied applications in electromagnetic wave systems of both the waveguide and the transmission line type.

Included among the new transmission components that have found widespread use in the microwave art is the so-called "isolator." The isolator may be defined as a circuit element which transmits without substantial loss electromagnetic waves propagating therethrough in one direction, designated the forward direction, whereas electromagnetic waves propagating in the opposite, or reverse, direction are attenuated by the isolator to the extent required by the system.

One particular class of isolator makes use of the resonance effect characteristic of all gyromagnetic materials. A typical isolator of this type comprises a vane of resonantly biased gyromagnetic material, asymmetrically located in the electromagnetic wave path. One of the characteristics of such isolators is that the amount of attenuation that can be realized in the reverse or lossy direction is a function of the volume of the gyromagnetic element. The greater the required attenuation, the greater must be the volume of gyromagnetic material used. This basic requirement tends to make such isolators large, thus precluding their use in systems in which small size is an essential requirement. Furthermore, the need for a large volume of gyromagnetic material complicates the biasing problem, making it difficult to obtain a uniform biasing field throughout the gyromagnetic element. In addition, the use of a large volume of material tends to increase the losses in the forward, or low-loss direction of propagation and to substantially increase the permittivity of the wave path.

It is, therefore, an object of this invention to introduce a high order of nonreciprocal attenuation in electromagnetic wave systems using small samples of resonantly biased gyromagnetic material.

Recognizing that a small sample of resonantly biased gyromagnetic material behaves as a rotating point magnetic dipole, which is energized by the incident microwave field, it is proposed to utilize the reradiated microwave energy produced by such a magnetic dipole to cancel the incident field. Accordingly, a sample of gyromagnetic material is placed in a region of a wave path wherein the high frequency magnetic field component is circularly polarized. In one direction of propagation, for which the direction of rotation of the high frequency magnetic field is compatible with the natural precessional direction of the sample, the gyromagnetic material is strongly coupled to the incident field, and the induced dipole reradiates in the same direction as the incident wave only. In particular, at resonance, the reradiated wave is 180 degrees out of phase with the incident wave and tends to cancel it. For a specific size sample, the reradiated field is such as to completely cancel the incident field with substantially all the power contained in the wave being absorbed in the sample.

An isolator, in accordance with the invention, is thus

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capable of completely absorbing the incident wave in an extremely small sample of gyromagnetic material. Because the diameter of the sample is of the order of only 25 to 50 mils, losses on the forward direction are correspondingly small. Thus, isolators constructed in accordance with the invention have very high reverse-to-forward loss ratios.

These and other objects and advantages, the nature of the present invention, and its various features, will appear more fully upon consideration of the various illustrative embodiments now to be described in detail in connection with the accompanying drawings, in which:

FIG. 1 shows an isolator in accordance with the invention;

FIG. 2 shows, by way of illustration, the equivalent circuit of the embodiment of FIG. 1; and

FIG. 3 is a second embodiment of the invention.

Referring more specifically to FIG. 1, an isolator is shown as an illustrative embodiment of the present invention. The isolator comprises a section 11 of bounded electrical transmission line for guiding wave energy which may be a rectangular waveguide of the metallic shield type having a wide internal cross-sectional dimension a of at least one-half wavelength of the wave energy to be propagated therethrough and a narrow dimension b substantially one-half of the wide dimension.

Located midway between the wide walls and at a distance d from one of the narrow walls is a small sphere 12 of gyromagnetic material suitably supported by means of a slab 13 of low-loss dielectric material. The term "gyromagnetic material" is employed here in its accepted sense as designating the class of magnetically polarizable materials having unpaired spin systems involving portions of the atoms thereof that are capable of being aligned by an external magnetic polarizing field and which exhibit a significant precessional motion at a frequency within the range contemplated by the invention under the combined influence of said polarizing field and an orthogonally directed varying magnetic field component. The spin system is characterized as having an angular momentum, a gyroscopic moment and a magnetic moment. Typical of such materials are ionized gases, paramagnetic materials and ferromagnetic materials, the latter including the spinels such as magnesium aluminum ferrite and aluminum zinc ferrite, and the garnet-like materials such as yttrium-iron garnet.

Element 12 is biased by a steady magnetic field H_0 normal to the direction of propagation of wave energy along the waveguide. This field may be supplied by an electric solenoid, by a permanent magnet structure, or element 12 may itself be permanently magnetized, if desired.

The problem of radiation damping produced by magnetic resonance effects has been treated by S. Bloom (Journal of Applied Physics 28, 1957, page 800) and by N. Bloembergen and R. V. Pound (Physical Review 95, 1954, page 8). They, however, have considered the spin system in the gyromagnetic element when coupled to a lumped resonant circuit containing damping. While these treatments are adequate where the magnetization is coupled to a cavity or resonant circuit, the nonreciprocal properties found in waveguides are not evident, and a solution to the more general boundary-value problem is more appropriate.

For reference purposes, guide 11 and element 12 are located in a coordinate system represented by the mutually perpendicular vectors 14 labeled x , y and z . Vector x indicates a positive sense along the wide transverse dimension of guide 11. The y vector indicates a positive sense along the narrow transverse dimension of guide 11, whereas the z vector indicates a positive sense along the longitudinal direction of the guide, parallel to the direc-

tion of propagation therein. The origin ($x=y=z=0$) is taken at the inside lower left corner of the guide, marked 0.

With the amplitude of the biasing field H_0 adjusted to produce gyromagnetic resonance in the sphere 12, the magnetization will precess such that its projection in the $x=z$ plane rotates in a counterclockwise direction with substantially constant amplitude m . Its interaction with the dominant TE_{10} mode will be considered.

A wave moving in the $+z$ direction has components

$$\left. \begin{aligned} E_y &= \sqrt{\frac{\mu}{\epsilon}} \frac{H_m \sin \frac{\pi x}{a}}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} e^{j(\omega t - \Gamma z)} \\ H_x &= -H_m \sin \frac{\pi x}{a} e^{j(\omega t - \Gamma z)} \\ H_z &= \frac{-jH_m \left(\frac{\lambda}{2a}\right) \cos \frac{\pi x}{a}}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} e^{j(\omega t - \Gamma z)} \end{aligned} \right\} \quad (1)$$

where

μ is the permeability of the wave path,
 ϵ is the permittivity of the wave path,
 H_m is the amplitude of H_x in the center of the waveguide,
 λ is the wavelength of the electromagnetic wave in the unbounded dielectric 13 and is equal to

$$\frac{\lambda_0}{\sqrt{\epsilon}}$$

λ_0 is the free space wavelength of the electromagnetic wave,

ω is 2π times the frequency of the electromagnetic wave, and

Γ is the propagation constant along the guide.

It is convenient to think of the magnetic field as the superposition of clockwise and counterclockwise rotating components as seen looking on the $x=z$ plane from $y=\infty$. These components may be expressed as

$$\left. \begin{aligned} H_{e1} &= \frac{H_m \sin \left(\frac{\pi x}{a} + \varphi\right)}{2 \cos \varphi} e^{j(\omega t - \Gamma z)} \\ H_{e2} &= \frac{H_m \sin \left(\frac{\pi x}{a} - \varphi\right)}{2 \cos \varphi} e^{j(\omega t - \Gamma z)} \end{aligned} \right\} \quad (2)$$

In these equations the substitutions

$$\sin \varphi = \frac{\lambda}{2a}, \quad \cos \varphi = \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2} \quad (3)$$

have been made.

For a sample placed in the $x=z$ plane at $x=d$ width H_0 in the $+y$ direction, the counterclockwise component is the (+) component and produces a transverse magnetic dipole moment m , which at resonance is

$$m = (\chi'' - \kappa'') v H_{\text{eff}} \quad (4)$$

where v is the volume of the sample and $(\chi'' - \kappa'')$ is the loss component of the susceptibility of a gyromagnetic substance under the influence of a circularly polarized magnetic field. Here an effective magnetic field, H_{eff} , is used rather than H_{e1} to take into account the reaction of m on the fields within the waveguide. For the time being we shall write

$$H_{\text{eff}} = \frac{KH_m \sin \left(\frac{\pi d}{a} - \varphi\right)}{2 \cos \varphi} \quad (5)$$

and the proportionality factor K will be obtained presently.

The general expressions for the far fields radiated by a rotating magnetic dipole in a waveguide are given as:

$$\left. \begin{aligned} E_y &= -\sqrt{\frac{\mu}{\epsilon}} \frac{2\pi m \sin \frac{\pi x}{a} \sin \left(\frac{\pi d}{a} - \varphi\right)}{a^2 b \sin 2\varphi} e^{j(\omega t - \Gamma z)} \\ H_x &= \frac{\pi m \sin \frac{\pi x}{a} \sin \left(\frac{\pi d}{a} - \varphi\right)}{a^2 b \sin \varphi} e^{j(\omega t - \Gamma z)} \\ H_z &= \frac{j\pi m \cos \frac{\pi x}{a} \sin \left(\frac{\pi d}{a} - \varphi\right)}{a^2 b \cos \varphi} e^{j(\omega t - \Gamma z)} \end{aligned} \right\} \quad (6)$$

for radiation in the $+z$ direction, and

$$\left. \begin{aligned} E_y &= \sqrt{\frac{\mu}{\epsilon}} \frac{2\pi m \sin \frac{\pi x}{a} \sin \left(\frac{\pi d}{a} + \varphi\right)}{a^2 b \sin 2\varphi} e^{j(\omega t + \Gamma z)} \\ H_x &= \frac{\pi m \cos \frac{\pi x}{a} \sin \left(\frac{\pi d}{a} + \varphi\right)}{a^2 b \sin \varphi} e^{j(\omega t + \Gamma z)} \\ H_z &= \frac{j\pi m \cos \frac{\pi x}{a} \sin \left(\frac{\pi d}{a} + \varphi\right)}{a^2 b \cos \varphi} e^{j(\omega t + \Gamma z)} \end{aligned} \right\} \quad (7)$$

for radiation in the $-z$ direction.

The counterclockwise or $+$ component of the exciting magnetic field leads m by $\pi/2$; therefore the radiated fields in the $+z$ direction are out of phase with the incident waves. The voltage transmission coefficient is unity plus the ratio of a component of the wave reradiated in the $+z$ direction to the analogous component of the incident wave in the $+z$ direction. From the components given by Equations 1 and 6, the transmission coefficient is

$$T = 1 - \frac{K\pi v(\chi'' - \kappa'') \sin^2 \left(\frac{\pi d}{a} - \varphi\right)}{a^2 b \sin 2\varphi} \quad (8)$$

Similarly, the reflection coefficient is

$$R = -\frac{K\pi v(\chi'' - \kappa'') \sin \left(\frac{\pi d}{a} - \varphi\right) \sin \left(\frac{\pi d}{a} + \varphi\right)}{a^2 b \sin 2\varphi} \quad (9)$$

In (8) and (9) m has been eliminated with the use of (4) and (5). The power absorbed by the magnetization at resonance is

$$P_a = \omega m H_{\text{eff}} \quad (10)$$

which becomes

$$P_a = \frac{2\pi K^2}{\lambda} \sqrt{\frac{\mu}{\epsilon}} \frac{v(\chi'' - \kappa'') H_0^2 \sin^2 \left(\frac{\pi d}{a} - \varphi\right)}{4 \cos^2 \varphi} \quad (11)$$

The power flow for the incident wave is

$$P_i = \frac{ab}{4} \sqrt{\frac{\mu}{\epsilon}} \frac{H_0^2}{\cos \varphi} \quad (12)$$

so that the power absorption coefficient is

$$A = \frac{K^2 2\pi v(\chi'' - \kappa'') \sin^2 \left(\frac{\pi d}{a} - \varphi\right)}{a^2 b \sin 2\varphi} \quad (13)$$

We may now compute K . Because of conservation of energy

$$T^2 + R^2 + A = 1 \quad (14)$$

A substitution of Equations 8, 9 and 13 in 14 gives

$$K = \frac{1}{\pi v(\chi'' - \kappa'') (1 - \cos \left(\frac{2\pi d}{a} \cos 2\varphi\right) + \frac{2a^2 b \sin 2\varphi}{\pi v(\chi'' - \kappa'') (1 - \cos \left(\frac{2\pi d}{a} \cos 2\varphi\right))}} \quad (15)$$

For convenience, set

$$F = \frac{\pi v(\chi'' - \kappa'')}{2a^2 b} \quad (16)$$

Note that F is independent of frequency insofar as $(\chi'' - \kappa'')$ is frequency independent. Its significance will become apparent shortly.

The transmission, reflection and absorption coefficients now become

$$T = \frac{\left(1 + F \sin \frac{2\pi d}{a}\right) \sin 2\varphi}{\sin 2\varphi + F \left(1 - \cos \frac{2\pi d}{a} \cos 2\varphi\right)} \quad (17)$$

$$R = \frac{F \left(\cos 2\varphi - \cos \frac{2\pi d}{a}\right)}{\sin 2\varphi + F \left(1 - \cos \frac{2\pi d}{a} \cos 2\varphi\right)} \quad (18)$$

and

$$A = \frac{2F \sin 2\varphi \left[1 - \cos \left(\frac{2\pi d}{a} - 2\varphi\right)\right]}{\left[\sin 2\varphi + F \left(1 - \cos \frac{2\pi d}{a} \cos 2\varphi\right)\right]^2} \quad (19)$$

Let us now consider several situations. It is seen from Equation 18 that the reflection coefficient is the same for positions equidistant from the center of the waveguide, and becomes zero when

$$\frac{\pi d}{a} = \begin{cases} \varphi \\ \pi - \varphi \end{cases} \quad (20)$$

These distances correspond to the two positions of circular polarization within the guide, as is evident from Equation 2. For $\pi d/a = \varphi$, however, resonance is not excited in the gyromagnetic material. In the other position, for which

$$\frac{\pi d}{a} = (\pi - \varphi)$$

resonance is strongly excited. Since, from Equation 18, $R=0$ in this position, the dipole can only radiate in the forward direction. The transmission coefficient for

$$\frac{\pi d}{a} = (\pi - \varphi)$$

reduces to

$$T = \frac{1 - F \sin 2\varphi}{1 + F \sin 2\varphi} \quad (21)$$

For the particular case where $F \sin 2\varphi = 1$, T also equals zero, and all the power is absorbed by the sample. The relationship between the electrical and physical properties of the gyromagnetic material and its position within the guide for complete absorption, is obtained by substituting from Equation 16 for F , and from Equation 20 for φ . The resulting expression is

$$\sin 2\pi \left(\frac{d}{a}\right) = \left(\frac{2a^2b}{\pi v}\right) \left(\frac{1}{\chi'' - \kappa''}\right) \quad (22)$$

The minimum value of F for which all the incident power can be transferred to the sample is unity. From Equation 3 the frequency at which this occurs is $\sqrt{2}$ times the cutoff frequency. For single crystal material for which $\chi'' - \kappa'' = 4\pi M_s / \Delta H$, where ΔH is the intrinsic half-width at half power, this is possible only when

$$\frac{\pi v}{2a^2b} \frac{4\pi M_s}{\Delta H} \geq 1 \quad (23)$$

The scattering matrix shows the nonreciprocal nature of the system.

$$S = \frac{1}{1 + F \left(\frac{1 - \cos \frac{2\pi d}{a} \cos 2\varphi}{\sin 2\varphi} \right)} \begin{bmatrix} F \left(\frac{\cos 2\varphi - \cos \frac{2\pi d}{a}}{\sin 2\varphi} \right) 1 - F \sin \frac{2\pi d}{a} \\ 1 + F \sin \frac{2\pi d}{a} F \left(\frac{\cos 2\varphi - \cos \frac{2\pi d}{a}}{\sin 2\varphi} \right) \end{bmatrix} \quad (24)$$

Since the diagonal elements are identical, the reflection coefficient from either end of the waveguide is the same for a given sample position. When the sample is in the center of the guide or on either wall, the off-diagonal elements are identical and the device is reciprocal. For all other positions the system is nonreciprocal.

At the position of pure circular polarization a useful equivalent circuit is a three-arm circulator with a resonant cavity on one arm as shown in FIG. 2. The quarter wave section in arm 2 provides proper phasing at the output. From Equation 16 it is seen that for a given size guide F is a function of the volume of the sample and its susceptibility at resonance. In terms of the equivalent circuit, the device acts as an isolator when the cavity is critically coupled to the transmission line connected to arm 2 of the circulator. Under this condition, power at the resonant frequency entering arm 1 is completely absorbed in the cavity. However, power entering arm 3 leaves by way of arm 1 unattenuated. Critical coupling is obtained by adjusting the volume of the gyromagnetic sample in accordance with the equation $F \sin 2\varphi = 1$.

When the cavity in the equivalent circuit is overcoupled, that is, $F \sin 2\varphi > 1$, a portion of the power entering arm 1 will be reflected at the cavity and will leave by way of arm 3. The remaining portion of the incident energy will be absorbed in the cavity. At resonance there will be a differential phase shift of 180 degrees for wave energy propagating from 1 to 3 as compared to wave energy propagating from 3 to 1.

Near resonance, but not exactly at resonance, the differential phase shift will deviate from 180 degrees in the manner typical of a simple tuned circuit.

In the embodiment of FIG. 1 the gyromagnetic element 12 is shown as a sphere. It is to be understood, however, that this element may have any other convenient physical configuration without adversely affecting the operation of the invention. What is important, however, is that the volume of element 12 satisfy the conditions set forth in Equation 22 and that the dimensions of the sample in the plane of the radio frequency magnetic field be small compared to a wavelength.

In a second embodiment of the invention, shown in FIG. 3, circular cylindrical waveguide is utilized. The isolator comprises a section 30 of cylindrical wave guide of radius R , proportioned to support wave energy in the TE_{11} mode of propagation.

A portion of the electrical field configuration is also shown in FIG. 3. The electric field pattern for the TE_{11} mode is transverse to the guide axis 34 and is indicated, for the left-hand side of guide 30, by the vectors 35. The field pattern for the right-hand side is essentially the mirror image of the left-hand side. The center vector E , passing through the guide axis 34, defines the direction of polarization of the wave and is referred to as the principal electrical field vector.

Located a distance ρ from the guide axis 34, along a radius normal to the principal vector E , is a small sphere 31 of gyromagnetic material, suitably supported by means of a disk of low-loss dielectric material 32.

Element 31 is biased by means of a steady magnetic field H_0 , directed normal to the direction of wave propagation along the waveguide and parallel to the direction of the principal vector E .

Applying the same analytical technique as was applied to the embodiment of FIG. 1, it can be shown that complete absorption is obtained in the reverse direction when

$$1 = \frac{2v(k_{11})^2[\chi'' - \kappa''] \left[J_1 \left(k_{11} \frac{\rho}{R} \right) \right]^2}{\lambda R^2 [(k_{11})^2 - 1] [J_1(k_{11})]^2} \quad (25)$$

where

v is the volume of gyromagnetic material;

ρ is the radial distance of the gyromagnetic material from the guide axis;

λ is the guide wavelength of the electromagnetic wave for the TE₁₁ mode;
 J_1 is Bessel's function of the first kind and first order;
 J'_1 is the first derivative of Bessel's function;
 R is the guide radius;
 $(\chi'' - \kappa'')$ is the susceptibility at resonance of the gyromagnetic material; and
 k'_{11} is the constant 1.8412, such that $J'_1(k'_{11}) = 0$.

In all cases it is understood that the abovedescribed arrangements are illustrative of a small number of the many possible specific embodiments which can represent applications of the principles of the invention. Numerous and varied other arrangements can readily be devised in accordance with these principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. In an electromagnetic wave transmission system supportive of dominant mode wave energy at a given frequency, an attenuator comprising a rectangular waveguide having a wide transverse dimension a and a narrow transverse direction b , an element of gyromagnetic material having a volume v and susceptibility $(\chi'' - \kappa'')$ at resonance disposed within said guide at a distance d along said wide dimension where the volume of said element and said location within said guide are related by

$$\sin 2\pi \left(\frac{d}{a} \right) = \left(\frac{2a^2b}{\pi v} \right) \left(\frac{1}{\chi'' - \kappa''} \right)$$

and means for magnetically biasing said element to gyromagnetic resonance at said given frequency.

2. The combination according to claim 1 wherein

$$\left(\frac{1}{\chi'' - \kappa''} \right) = \frac{\Delta H}{4\pi M_s}$$

where ΔH is the intrinsic half line width at half power of said element and M_s is the saturation magnetization of said element.

3. The combination according to claim 1 wherein said gyromagnetic material is located in a region of circular polarization for which

$$d = a \left[1 - \frac{1}{\pi} \sin^{-1} \frac{\lambda_0}{2a} \right]$$

where λ_0 is the free space wavelength at said given frequency.

4. An electromagnetic wave transmission path supportive of wave energy in the TE₁₁ mode at a given frequency, an isolator comprising a circular cylindrical waveguide of radius R , an element of gyromagnetic material having a volume v and a susceptibility $(\chi'' - \kappa'')$ at resonance disposed within said guide a distance ρ from the guide axis, where the volume of said element and said location within said guide are related by

$$1 = \frac{2v}{\lambda R^2} [\chi'' - \kappa''] \left[J'_1 \left(k'_{11} \frac{\rho}{R} \right) \right]^2 \left[\frac{(k'_{11})^2}{J_1(k'_{11})^2 [(k'_{11})^2 - 1]} \right]$$

where

λ is the guide wavelength of said wave energy;

J_1 is the Bessel function of the first kind and order;
 J'_1 is the first derivation of said Bessel function; and
 k'_{11} is a constant equal to 1.8412,

and means for magnetically biasing said element to gyromagnetic resonance at said given frequency.

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