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BROADBAND TRANSFORMERS  
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3,037,175

FIG. 1

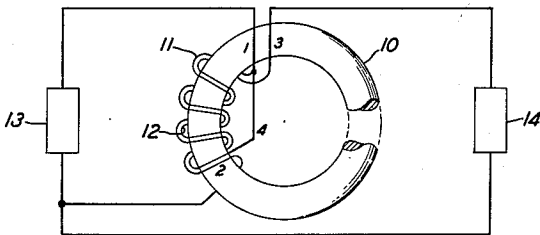


FIG. 2

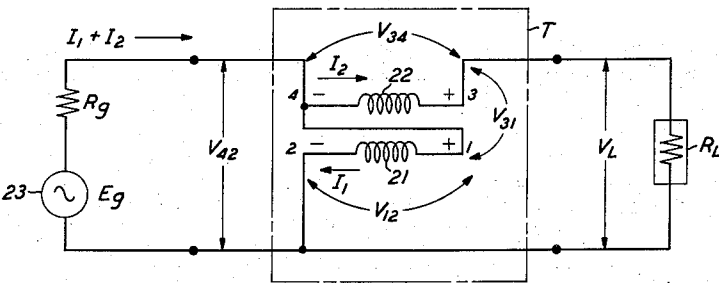


FIG. 3

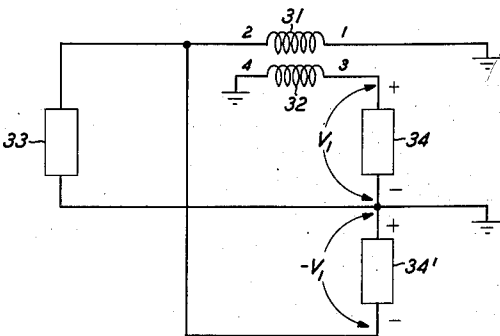
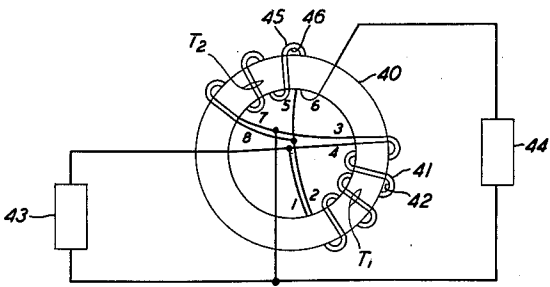


FIG. 4



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## BROADBAND TRANSFORMERS

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This invention relates to impedance transforming devices and, more particularly, to broadband bifilar wound autotransformers.

The problem of distortionless transmission of signals over wires is an old one, encountered often in electrical communications systems. As the range of operating frequencies is extended, as is the current trend, this problem has become more and more acute. For example, in order to supply gain for pulses of millimicroseconds duration, amplifiers and coupling transformers with bandwidths of hundreds of megacycles are needed. While the problem of extending the frequency range of transformers has received the attention of many investigators, and some suggestions and improvements have been made, currently available transformers still fall far short of fulfilling the bandwidth requirements presently encountered in the electronic arts.

The difficulties in providing broadband transformers is due in great part to the series self-inductance and parasitic interwinding capacitance of the transformer windings themselves. For example, in a conventionally wound transformer, the upper end of the pass band is generally determined by the large interwinding capacity which resonates at some relatively low frequency, while the low frequency end of the pass band is limited by a relatively small coil inductance which appears as a low impedance in parallel with the signal source. While these limitations have been somewhat overcome by the use of miniature construction and new and improved magnetic core materials, this type of approach to the problem has enjoyed only limited success.

The theory of transmission lines, which has been well developed in the communications art, on the other hand, has indicated how this problem may be attacked by means of a more fundamental and promising approach. It is known that a pulse can be propagated down a transmission line without distortion, and that the input impedance of any length of properly terminated line is a pure resistance. Thus, for instance, a Lecher wire line having the desired characteristic impedance can be wound on a core form so that the conductors are distributed over the surface of the core with the two conductive members forming a bifilar wound transformer having a 1:1 turns ratio. One end of the bifilar coil comprises the input or primary end of the transformer, and the other end the output or secondary end. In such an arrangement, the series inductances and interwinding capacitances, which normally set the frequency limits for the conventional lumped parameter transformer, are now part of a distributed transmission line system. If the line is properly terminated with a pure resistance, which may readily be computed in terms of the line constants, a 1:1 impedance transformer of unusually broad bandwidth is obtained.

Such distributed transformers have been used in the past, but since the impedance transformation ratio is only 1:1, it is necessary to use several such coils where impedance transformations greater than 1:1 are sought. In particular, where an impedance transformation of 4:1 is desired, two bifilar coils have to be used. Furthermore, where an impedance transformation of 4:1 is sought to connect two unbalanced networks, an additional, or third coil, has to be added to the two above-mentioned coils. In such multiple coil arrangements,

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some saving in space and material is affected by winding all the coils on a common core. This may be done since the net magnetic field resulting from the flow of signal current is zero, and consequently there is no magnetic coupling among the several coils even though they share a common magnetic core. The magnetizing current, however, does produce a net magnetic field. Consequently, by winding the several coils series aiding for the magnetizing current, there is a net increase in the number of coupled turns, and the corresponding increase in the effective transformer inductance. This has the effect of extending the low frequency end of the transformer pass band. The difficulty with such an arrangement, however, resides in the care which must be taken in winding the transformer so as to minimize the interwinding capacitance among the coils since this capacitance will adversely affect the high frequency response.

It is, therefore, an object of this invention to produce broadband impedance transformation ratios greater than 1:1 using a single bifilar wound coil.

It is a further objective of this invention that said transformation ratios be obtainable in a transformer used to couple two unbalanced systems.

If the length of the conductors making up the bifilar coil is an appreciable part of a wavelength of the signal transmitted through the transformer, the input impedance to the transformer will be a function of the length of the conductors as well as the load impedance. Since it is desirable for matching purposes that the terminal impedances of the transformer be only a function of the load impedance and the transformer turns ratio, it is an additional object of this invention to provide broadband impedance transformation means which are substantially independent of the signal frequency over an extended range of operating frequencies.

A transformer constructed in accordance with the invention comprises a pair of insulated conductive wires, uniformly spaced from each other and wound together in a substantially helical form. A coil so wound has the distributed properties of a uniform transmission line and the corresponding broadband capabilities when used as a transformer. An impedance transformation ratio of 4:1 is obtained by serially connecting the coils by conductively joining one end of one of the conductors to the other end of the other conductor and by making connections to and from the transformer whereby one of the external circuits is connected across only one of the coiled conductors and the other external circuit is connected across both of the coiled conductors.

It is a feature of the present invention that by merely rearranging the ground connections, the transformer may be used either to connect two unbalanced networks or to connect an unbalanced network to a balanced network without the further cascading of an additional coil, which addition tends to decrease the overall bandwidth.

It is a further feature of the invention that by using core material of sufficiently high permeability, the number of turns may be reduced for a given low frequency response, and the length of line constituting the bifilar coil made small. Since the transformer output is zero at the high frequency end of the transformer pass band when the electrical line length is a half of a wavelength of the signal frequency, using fewer turns has the effect of extending the upper end of the transformer response characteristic. Furthermore, since the input impedance of a short length of transmission line is approximately equal to the load impedance terminating the line, the transformer is substantially independent of the line characteristics, and free of transmission line effects over a major portion of the transformer pass band. A transformer constructed and operated in accordance with the teachings of the invention has the broadband character-

istics of a parallel wire transmission line and the impedance transformation characteristics of a center-tapped autotransformer.

These and other objects, the nature of the present invention, and its various features and advantages will appear more fully upon consideration of the various specific illustrative embodiments shown in the accompanying drawings and analyzed in the following detailed description of these drawings.

In the drawings:

FIG. 1 shows diagrammatically a bifilar wound autotransformer connected in accordance with the principles of the invention;

FIG. 2 is a schematic illustration of the transformer of FIG. 1 when used to connect two unbalanced networks;

FIG. 3 is a schematic illustration of the transformer of FIG. 1 when used to connect an unbalanced network and a balanced network;

FIG. 4 shows diagrammatically a bifilar autotransformer using two cascaded coils wound on a common magnetic core.

Referring to the accompanying drawings, and more specifically to FIG. 1, there is diagrammatically shown a transformer wound and connected in accordance with the present invention. The transformer comprises a pair of insulated conductive filaments 11 and 12, wound together in a substantially helical form over coil form 10. Insulated filaments 11 and 12 are arranged so that their insulated coverings are in close juxtaposition substantially throughout their entire lengths. The juxtaposition or contiguous arrangement of these two wires is such as to produce substantially unity coupling between the two windings and, in addition, to produce the equivalent of a uniform parallel wire transmission line from one end of the coil to the other end thereof. The double threaded spiral or helical coils described above are known in the art as bifilar coils and will be referred to as such hereinafter. The actual spacing of the conductive portions of members 11 and 12, and the diameter of said conductors, will be considered in greater detail below.

The bifilar coil is mounted upon a coil form 10 which may be composed of any suitable high permeability, low loss core material. For example, a number of transformers using nickel-zinc ferrite cores have been constructed and have given very satisfactory results. While coil form 10 has been shown as a toroidal member, it may assume any convenient shape consistent with the electrical requirements of the transformer windings.

An impedance transformation ratio of 1:4 is obtained by serially connecting coils 11 and 12 by joining one end 4 of coil 12 to the other end 1 of coil 11, and by connecting external circuit 13 to ends 1 and 2 of coil 11, and by connecting the other external circuit 14 to ends 2 and 3. Thus connected, circuit 13 is across one of the bifilar coils, and circuit 14 is across the two serially connected coils, and the transformed impedance seen by circuit 13 is one-fourth the impedance of circuit 14.

In FIG. 2 there is shown a schematic diagram of the transformer of FIG. 1 and its associated external circuits in which network 13 of FIG. 1 more specifically comprises a signal generator 23 and its equivalent internal impedance  $R_g$ , and network 14 is represented by load resistor  $R_L$ . Resistor  $R_L$  is equal to  $4R_g$  and is to be matched to the generator impedance by means of transformer T. The transformer comprises the serially connected bifilarly wound coils 21 and 22, of which end 1 of coil 21 is connected to end 4 of coil 22. The generator 23 and resistance  $R_g$  are connected across ends 1 and 2 of coil 21, and load resistor  $R_L$  is connected across ends 3 and 2 of the serially connected coils 21 and 22. So connected, generator 23, resistance  $R_g$ , and coil 21 form a first current mesh, and generator 23, resistance  $R_g$ , coil 22 and load resistor  $R_L$  form a second current mesh.

An analysis of the operation of the invention may best be had by considering the low frequency and high fre-

quency operation separately. At low and at intermediate frequencies, ordinary lumped parameter network analysis is used. In the high frequency region, transmission line techniques are resorted to.

Designating the reactance of each half of the bifilar winding as  $X = \omega L$ , and the mutual inductances as  $-kX$ , where  $k$  is the coefficient of coupling, the low frequency and intermediate frequency mesh equations are given as follows:

$$E_g = (R_g + jX)I_1 + (R_g - jkX)I_2 \quad (1)$$

$$E_g = (R_g - jkX)I_1 + (R_g + R_L + jX)I_2 \quad (2)$$

where

$I_2$  is the load current

and

$I_1 + I_2$  is the generator current.

Solving for  $I_1$  and  $I_2$  gives:

$$I_1 = \frac{E_g[jX(1+k) + R_L]}{\Delta} \quad (3)$$

$$I_2 = \frac{E_g[jX(1+k)]}{\Delta} \quad (4)$$

where  $\Delta = X^2(k^2 - 1) + jX[2R_g(1+k) + R_L] + R_gR_L$ .

Examining the load current,  $I_2$ , more carefully, it follows that:

$$I_2 = \frac{E_g[jX(1+k)]}{X^2(k^2 - 1) + jX[2R_g(1+k) + R_L] + R_gR_L}$$

Since  $4R_g = R_L$  and  $k \approx 1$ :

$$I_2 = \frac{2E_g(jX)}{jX(8R_g) + 4R_g^2}$$

and

$$I_2 = \frac{E_g}{4R_g - j\frac{2R_g^2}{X}} \quad (5)$$

From Equation 5, the low frequency half power point is obtained when:

$$4R_g = \frac{2R_g^2}{X} \quad (6)$$

or

$$X = \frac{R_g}{2}$$

Thus, if the low frequency half power point is designated as frequency  $f_1$ , the individual coil inductance  $L$  is given as:

$$L = \frac{R_g}{4\pi f_1} \quad (7)$$

The ratio of generator to load current is:

$$\frac{|I_1 + I_2|}{|I_2|} = \sqrt{4 + \left[\frac{R_L}{X(1+k)}\right]^2} \quad (8)$$

Since over the major portion of the pass band  $X(1+k)$  is much greater than  $R_L$ , the transformer has a 2:1 current ratio, and  $I_1 = I_2$ .

The output voltage  $V_L$  is given as:

$$V_L = V_{31} + V_{12}$$

However, since  $k \approx 1$ ,  $V_{34} = V_{12} = V_{42}$  and  $V_L = 2V_{42}$ , the output voltage is twice the input voltage, and the transformer has a 1:2 voltage step-up ratio. As the ratio of input voltage to input current is one-fourth the load voltage to current ratio, the transformer is matched when  $R_L = 4R_g$ , as was indicated above.

As the operating frequency is raised, the output voltage is again expressed as the sum of  $V_{31}$  and  $V_{12}$ ; thus:

$$V_L = V_{31} + V_{12}$$

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The voltage  $V_{12}$ , however, is the same as the input voltage  $V_{42}$ , which voltage appears across one end of the bifilar winding, and drives the bifilar as a transmission line.  $V_{31}$ , then is merely the voltage  $V_{42}$  which has passed through a length,  $l$ , of transmission line. The load voltage may then be rewritten as:

$$V_L = V_{42} + V_{42}e^{-j\beta l}$$

where  $\beta$  is the phase constant, and  $l$  the length of each of the conductors making up the bifilar winding. The ratio of the input voltage  $V_{42}$ , to the load voltage  $V_L$  is then:

$$\left| \frac{V_L}{V_{42}} \right| = |1 + e^{-j\beta l}| = \sqrt{2(1 + \cos \beta l)} \quad (9)$$

From Equation 9 it is evident that the transformer output is zero when the conductor lengths,  $l$ , are one-half of a wavelength long.

The important equations to note are Equations 7 and 9, since it is these equations which fix the low frequency and high frequency cut-off points and thus define the transformer band pass.

While the transformer operation over a major portion of the pass band is substantially independent of the transmission line characteristics of the bifilar coil, the line characteristics do influence the transformer operation at those frequencies where the line lengths,  $l$ , are comparable to a quarter of a wavelength of the operating frequencies. At these high frequencies, the power transfer through the transformer is affected by the characteristic impedance of the bifilar coil. It can be shown, that for maximum power transfer, the characteristic impedance of the transmission line, formed by the two conductors comprising the bifilar coil, is equal to the geometric mean of the input and output impedances:

$$Z_0 = \sqrt{R_{in} R_{out}} \quad (10)$$

Thus, where the input impedance is  $R_g$ , and the output impedance is  $4R_g$ , the characteristic impedance of the coil is given as:

$$Z_0 = 2R_g$$

In terms of the wire size and spacing:

$$Z_0 = 2R_g = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\pi} \cosh^{-1} \left( \frac{b}{a} \right) \quad (11)$$

where

$\mu$  is the effective permeability

$\epsilon$  is the effective dielectric constant

$b$  is the distance between wire centers

and

$a$  is the wire diameter.

In FIGS. 1 and 2, both external networks 13 and 14 were single ended or unbalanced with respect to ground. In FIG. 3, the transformer is used to connect an unbalanced network 33 to a double ended or balanced network 34, 34'. This is accomplished simply by grounding the interconnected terminals 1 and 4 of coils 31 and 32 respectively. As in FIGS. 1 and 2, one of the external circuits is connected between terminals 2 and 1, while the second external network is connected between terminals 2 and 3. Since the latter circuit is balanced with respect to ground, it is shown as comprising two equal portions, 34 and 34'.

In FIG. 4 a pair of cascaded bifilar autotransformers  $T_1$  and  $T_2$  for producing a 1:16 impedance transformation ratio are shown wound on a common core 40. Each of the bifilarly wound coils comprises a parallel wire transmission line, each of which is wound into a double threaded spiral or helical form on said core. Transform-

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er  $T_1$  consists of winding 41, having terminals 1 and 3, and winding 42, having terminals 2 and 4. Adjacent terminals 1 and 2 make up one end of the transformer and adjacent terminals 3 and 4 the other end. Coils 41 and 42 are serially connected by connecting end 1 of coil 41 to the other end 4 of coil 42.

Transformer  $T_2$  consists of winding 45, having terminals 6 and 8, and winding 46, having terminals 5 and 7. Adjacent terminals 7 and 8 make up one end of transformer  $T_2$  and adjacent terminals 5 and 6 the other end. Coils 45 and 46 are serially connected by connecting end 8 of coil 45 to the other end 5 of coil 46.

External network 43 is connected across coil 41 by connecting to terminals 1 and 3 of coil 41. Transformer  $T_2$ , comprising the second external network for transformer  $T_1$ , is attached to transformer  $T_1$  by joining terminal 5 to terminal 2, and joining terminal 7 to terminal 3. So connected, coil 46 of transformer  $T_2$  is attached across the serially connected coils 41 and 42. External network 44, constituting the second impedance across transformer  $T_2$ , is connected across the serially connected coils 45 and 46 by attaching network 44 to terminals 6 and 7. Since each of the transformers  $T_1$  and  $T_2$  has a 1:4 impedance step-up ratio, for matched conditions, network 44 should have an internal impedance sixteen times the impedance of network 43.

As indicated above, the characteristic impedance of the bifilar windings for optimum high frequency operation, is equal to the geometric mean of the impedances of the external networks connected across the transformer. If the impedance of network 43 is  $R$ , then the matched impedances across transformer  $T_1$  are  $R$  and  $4R$ , and the characteristic impedance of the bifilar winding of transformer  $T_1$  is  $2R$ . Similarly, the matched impedances across transformer  $T_2$  are  $4R$  and  $16R$ , and the characteristic impedance of the bifilar winding of transformer  $T_2$  is  $8R$ .

In all cases, it is understood that the above-described arrangements are simply illustrative of a small number of the many possible specific embodiments which can represent applications of the principles of the invention. Numerous and varied other arrangements can readily be devised in accordance with these principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. In combination, a two-element transmission line having a characteristic impedance  $Z_0$ , comprising a pair of conductively insulated wires wound together in a substantially helical form, said wires being contiguous and parallel from a first end of each to the second end of each to form a pair of coils, said coils being serially connected with the first end of one being connected to the second end of the other, an input circuit and an output circuit, one of said circuits having an impedance  $R_1$  connected across both of said serially connected coils, and the other of said circuits having an impedance  $R_2$  connected across one of said coils, wherein said characteristic impedance and said circuit impedances are related by

$$Z_0 = \sqrt{R_1 R_2}$$

2. The combination according to claim 1 wherein  $R_1 = 4R_2$ .

3. The combination according to claim 2 wherein  $R_2 = 4\pi f_1 L$ , where  $L$  is the inductance of each of said coils and  $f_1$  is the low frequency half power point.

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