Sept. 19, 1961 B. K. KINARIWALA 3,001,156

ACTIVE IMPEDANCE BRANCH

Filed Dec. 1, 1958 2 Sheets-Sheet 1

FIG. 1

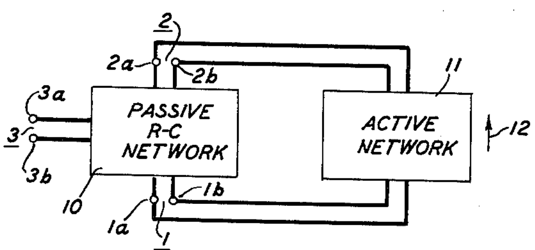

FIG. 3

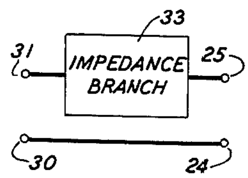

FIG. 4

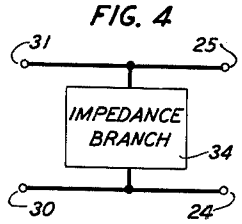

FIG. 2

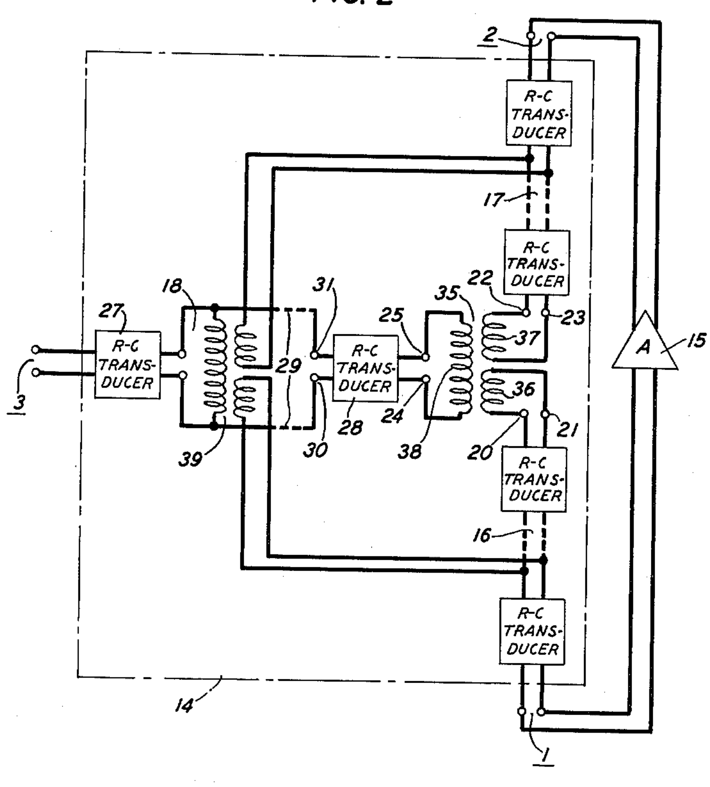

FIG. 5

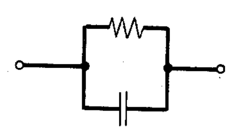

FIG. 6

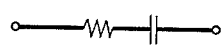

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FIG. 7

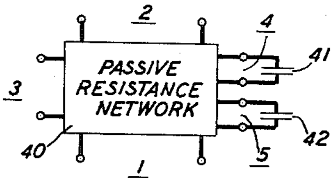


FIG. 9

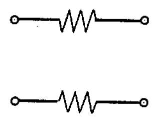


FIG. 10

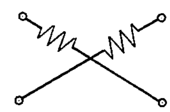


FIG. 8

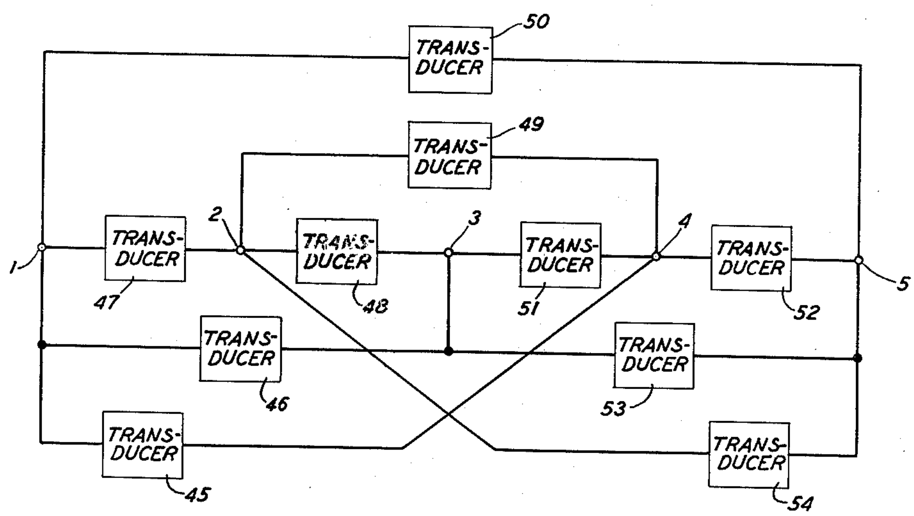
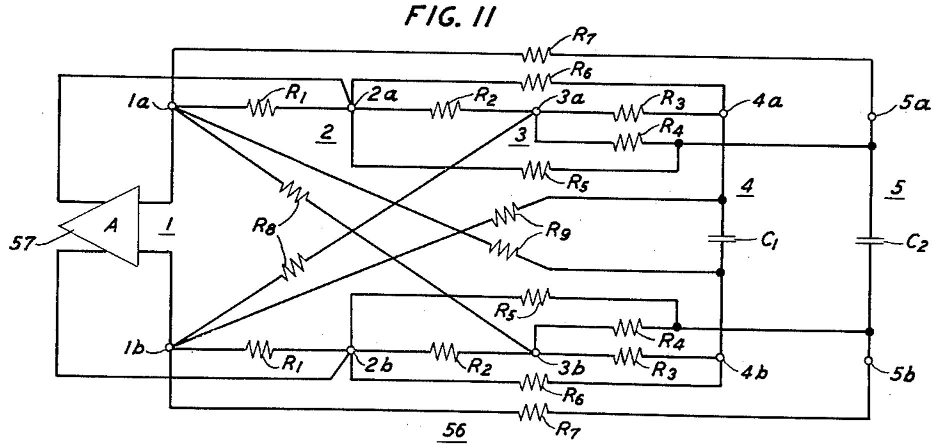


FIG. 11



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United States Patent Office 3,001,156
Patented Sept. 19, 1961

3,001,156

ACTIVE IMPEDANCE BRANCH

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Filed Dec. 1, 1958, Ser. No. 777,402
6 Claims. (Cl. 333—80)

This invention relates to wave transmission networks and more particularly to an active impedance branch for use in such networks.

The principal object of the invention is to eliminate inductors, and also transformers, in a two-terminal impedance branch without restricting the impedance characteristic obtainable. A related object is to increase the quality factor in an impedance branch of this type.

Wave transmission networks often operate at frequencies so low or so high that it is difficult to provide satisfactory inductors for use as component parts thereof. The inductors may be too large physically or too expensive, or there may be too much stray capacitance associated with them. Therefore, it is desirable to eliminate inductors in the impedance branches from which the network is constructed. It is also desirable to eliminate transformers.

The present invention provides a two-terminal impedance branch which requires no inductors but which has an unrestricted driving-point impedance with both zeroes and poles at any desired points in the complex frequency plane. The branch comprises a passive network having at least three ports and a two-port active network. If the passive network has only three ports, it is made up of resistors and capacitors, which may be formed into simple structures, and generally one or more transformers. The transformers may be eliminated in every instance by increasing the number of ports in the passive network to more than three and terminating the added ports in properly chosen capacitors. In this case, the passive network will comprise only resistors interconnecting each port with each of the other ports. The active network may be simply an amplifier. If three ports of the passive network are designated **1**, **2**, and **3**, the active network is connected between ports **1** and **2**. If the impedance is Z_3 at port **3** when the gain of the active network is zero, the zeroes of Z_3 correspond to the poles of the transfer function between ports **1** and **2** when port **3** is short-circuited, and the poles of Z_3 correspond to the poles of the transfer function between ports **1** and **2** when port **3** is open-circuited, then the unrestricted impedance appears at port **3**.

The nature of the invention and its various objects, features, and advantages will appear more fully in the following detailed description of the typical embodiments illustrated in the accompanying drawing, of which

FIG. 1 is a block diagram of an active impedance branch in accordance with the invention using a three-port, passive, R-C network;

FIG. 2 is a circuit showing one embodiment of the branch of FIG. 1 in greater detail;

FIGS. 3 and 4 show alternative arrangements of the two-port R-C transducers used in the partial networks of FIG. 2;

FIGS. 5 and 6 show arrangements of resistors and capacitors which may be used in the impedance branches of FIGS. 3 and 4;

FIG. 7 shows an alternative passive network of resistors with five ports and two terminating capacitors;

FIG. 8 is a block diagram showing the structure of the five-port network for the general case;

FIGS. 9 and 10 are schematic circuits of two forms of transducers suitable for use in one embodiment of the network of FIG. 8; and

FIG. 11 is a schematic circuit of an example of an impedance branch in accordance with the invention using a five-port resistance network of the type shown in FIG. 8.

The impedance branch of FIG. 1 comprises a three-port passive network **10** and a two-port active network **11**. The passive network **10** requires no inductors. It is of the type called an R-C network since it is made up of resistors and capacitors and, in general, one or more transformers. The network **10** has three pairs of terminals *1a*—*1b*, *2a*—*2b*, and *3a*—*3b* which constitute the three ports designated **1**, **2**, and **3**. The network **11**, connected between ports **1** and **2**, transmits only in the direction of the arrow **12** and may be simply an amplifier. An unrestricted driving-point impedance Z appears at the port **3**.

The impedance Z is given by the expression

$$Z=Z_3\left[\frac{1-\mu T_1}{1-\mu T_2}\right] \tag{1}$$

where Z_3 is the impedance at port **3** when the active network **11** is placed in a reference condition of zero transmission in the direction of the arrow **12**, μ is the gain of the network **11**, and T_1 and T_2 are the transfer functions of the network **10** from port **1** to port **2** when port **3** is short-circuited and open-circuited respectively. Since an R-C network such as **10** can have transmission zeroes anywhere in the complex frequency plane, both T_1 and T_2 can have any desired zeroes. Furthermore, the poles of T_1 correspond to the zeroes of Z_3 and the poles of T_2 correspond to the poles of Z_3. Therefore, Z can have both zeroes and poles at any desired points in the complex frequency plane.

It will now be shown that, for any given driving point impedance Z, it is possible to determine an impedance Z_3 and transfer functions T_1 and T_2 which represent a physically realizable passive R-C network **10**. For convenience, it will be assumed that the active network **11** has a gain of unity ($\mu=1$) and input and output impedances each equal to one ohm pure resistance. If the gain and impedances have other values, the element values of the network **10** are merely rescaled accordingly.

Each of the parameters Z, T_1 and T_2 may be expressed as a rational function of the frequency f. However, the analysis is simplified by the introduction of the parameter p, called the complex frequency and defined as

$$p=\delta+j\omega \tag{2}$$

where δ is the real part, $j\omega$ is the imaginary part, and ω is the radian frequency $2\pi f$. If

$$Z=N/D \tag{3}$$

$$T_1=P/Q \tag{4}$$

$$T_2=R/S \tag{5}$$

then

$$Z_3=Q/S \tag{6}$$

and, substituting in (1),

$$Z=\frac{Q}{S}\,\frac{1-P/Q}{1-R/S}=\frac{Q-P}{S-R} \tag{7}$$

where N, D, P, Q, R, and S are polynomials in p, and Q and S have only negative real roots. When N and D are given, it is quite easy to determine P, Q, R, and S. It can be shown that P, Q, R, and S have only negative real roots.

The transfer function T_3 between ports **1** and **2** when port **3** is terminated in a resistor of one ohm is given by

$$T_3=\frac{T_2+T_1Z_3}{Z_3+1} \tag{8}$$

Substituting from (4), (5), and (6) gives

$$T_3 = \frac{R/S + (P/Q)(Q/S)}{Q/S + 1} = \frac{P+R}{Q+S} \qquad (9)$$

It is obvious from (8) that T_3 refers to a physical R-C network if Z_3 is an R-C impedance. In order to ensure that Z_3 is an R-C impedance, Q and S must be properly chosen. As seen from (7), there is complete freedom in the choice of Q and S.

The network **10** may be determined by a scattering matrix made up of nine scattering parameters. Two of these parameters are determined by T_1 and Z_3. The other elements of the matrix are chosen arbitrarily so as to give a physically realizable scattering matrix. Synthesis of the three-port network from this matrix is then carried out by any one of several known ways. An appropriate one is described by W. Cauer in his book entitled "Theorie der linearen Wechselstromschaltungen," published by Becker und Erler in Leipzig in 1941.

FIG. 2 shows a typical embodiment of the impedance branch of FIG. 1 when the network **10** is synthesized in accordance with the Cauer method. In FIG. 2, the portion enclosed in the broken-line box **14** corresponds to the passive three-port network **10** in FIG. 1. The ports **1**, **2**, and **3** are given the same designations in both figures. The amplifier **15**, connected between ports **1** and **2**, corresponds to the active two-port network **11**.

The network **14** comprises three partial networks **16**, **17**, and **18**. The partial network **16** extends between port **1** and the terminals **20—21**, network **17** between port **2** and the terminals **22—23**, and network **18** between port **3** and the terminals **24—25**. Each of the partial networks is made up of a number of two-port, R-C transducers connected in tandem. In the partial network **18**, for example, two R-C transducers **27** and **28** are shown. Others may be included, as indicated by the broken lines **29**. Each of the R-C transducers such as **28**, between the terminals **24—25** and **30—31**, may comprise a series impedance branch **33**, as shown in FIG. 3, a shunt impedance branch **34**, as shown in FIG. 4, or combinations of these branches. Each of the branches **33** and **34** may comprise a resistor and a capacitor connected in parallel, as shown in FIG. 5, or in series, as shown in FIG. 6. In either of these branches, one of the elements may, in some cases, be omitted.

At their inner ends, the partial networks **16**, **17**, and **18** are connected together through a transformer **35** having three inductively coupled windings **36**, **37**, and **38**. Also, at one or more intermediate points, the partial networks may be coupled by additional transformers such as **39**. If impedance transformation is not required at any particular point, the transformer coupling may be replaced by a direct connection between the partial networks.

In a second embodiment of the invention, all transformers may be eliminated from the passive R-C network **10**. This is done by increasing the number of ports to more than three and terminating the additional ports in capacitors. For example, FIG. 7 shows a network **40** with five ports in which the ports **4** and **5** are terminated in the capacitors **41** and **42**, respectively. The active network **11** is connected between the ports **1** and **2**, as shown in FIG. 1, and the unrestricted impedance appears at the port **3**.

It can be shown that, in the general case, the network **40** will have the configuration shown in FIG. 8. This is a single-line, block diagram in which each of the circles 1 to 5 represents a port with two terminals, each of the blocks **45** to **54** represents a two-port transducer comprising resistors only, and each line connecting a port to a transducer represents a two-wire transmission path. It is seen that transducers connect each of the ports **1** to **5** with each of the other ports. For the five-port case, it can be shown that the transducer **52** interconnecting ports **4** and **5** will have infinite loss and may be omitted. For this case, then, transducers will only be required to interconnect each of the ports **1**, **2**, and **3** with each other and with each of the additional ports **4** and **5**, respectively.

Each of the transducers **45** to **54** comprises two resistors connected directly between corresponding terminals, as shown in FIG. 9, or diagonally, as shown in FIG. 10. Thus, it is seen that the network **40** comprises only internal resistors and the terminating capacitors **41** and **42**. No transformers are required.

FIG. 11 is a schematic circuit of a typical two-terminal R–C impedance branch in accordance with this second embodiment. The passive network **56** has five pairs of terminals designated, respectively, *1a—1b*, *2a—2b*, *3a—3b*, *4a—4b*, and *5a—5b*, which constitute five ports numbered **1** to **5**. The desired unrestricted impedance Z appears between the terminals *3a* and *3b*. The amplifier **57**, connected between the ports **1** and **2**, has input and output impedances which are substantially zero and thus the amplifier constitutes a current-controlled voltage source. This type of amplifier may be obtained by employing a large shunt feedback. The capacitors C_1 and C_2 terminating the ports **4** and **5** correspond to the capacitors **41** and **42** in FIG. 7.

The passive network **56** is a symmetrical structure made up only of the pairs of resistors R_1, R_1 through R_9, R_9 which interconnect each of the ports **1**, **2**, and **3** with each other and with each of the other ports **4** and **5**. The resistors R_1 through R_7 are connected directly, as shown in FIG. 9, and the resistors R_8 and R_9 diagonally, as shown in FIG. 10. Thus, one resistor R_1 is connected between the terminals *1a* and *2a*, and the other resistor R_1 between *1b* and *2b*. But one resistor R_8 interconnects *1a* and *3b* and the other resistor R_8 interconnects *1b* and *3a*. It will be noted that no resistors directly connect the ports **4** and **5**. In this example, the expression for the impedance is

$$Z = 1{,}122 \left[\frac{p^2 + 2.25 \times 10^{10}}{p^2 + 2 \times 10^5 p + 3 \times 10^{10}} \right] \qquad (10)$$

The resistances of the resistors in ohms and the capacitances of the capacitors in micromicrofarads are as follows:

$R_1 = 23{,}809$	$R_7 = 5{,}635$
$R_2 = 7{,}510$	$R_8 = 25{,}556$
$R_3 = 3{,}611$	$R_9 = 5{,}732$
$R_4 = 1{,}444$	$C_1 = 1{,}007$
$R_5 = 7{,}232$	$C_2 = 1{,}260$
$R_6 = 19{,}098$	

It is to be understood that the above-described arrangements are only illustrative of the application of the principles of the invention. Numerous other arrangements may be devised by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. A two-terminal impedance branch of unrestricted impedance comprising a passive network, an active network, and a capacitor, the passive network having four ports and including only resistors interconnecting each port with each of the other ports, respectively, the capacitor terminating one of the ports, and the active network being connected between two of the other ports.

2. A two-terminal impedance branch of unrestricted impedance comprising a passive network having three ports and at least one additional port, capacitors equal in number to the number of additional ports, and an active network, the passive network including only resistors interconnecting each of the three ports with each other and with each of the additional ports, the capacitors terminating the additional ports, and the active network being connected between two of the three first-mentioned ports.

3. In combination, a passive network and an active network, the passive network comprising only one type of reactor and having three ports designated **1**, **2**, and **3**, the

the active network being connected between ports **1** and **2** and the impedance at port **3** being unrestricted and equal to $Z_3(1-\mu T_1)/(1-\mu T_2)$, where Z_3 is the impedance at port **3** when the active network is placed in a reference condition of zero transmission from port **1** to port **2**, μ is the gain of the active network, and T_1 and T_2 are the respective transfer functions of the passive network from port **1** to port **2** when port **3** is short-circuited and open-circuited, respectively.

4. The combination in accordance with claim 3 in which the passive network comprises three partial networks extending between a common point and each of the network ports, respectively, and the partial networks are coupled together at the common point.

5. The combination in accordance with claim 4 in which the partial networks are also coupled together at intermediate points.

6. An impedance branch comprising a passive network and an amplifier, the passive network including capacitors as the only type of reactor and having three ports designated **1**, **2**, and **3**, the amplifier being connected between ports **1** and **2** and the impedance at port **3** being unrestricted and equal to $Z_3(1-\mu T_1)/(1-\mu T_2)$, where Z_3 is the impedance at port **3** when μ, the gain of the amplifier, is zero, and T_1 and T_2 are the respective transfer functions of the passive network from port **1** to port **2** when port **3** is short-circuited and open-circuited, respectively.

References Cited in the file of this patent

UNITED STATES PATENTS

2,549,065	Dietzold	Apr. 17, 1951
2,817,822	Meyers	Dec. 24, 1957