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HIGH FREQUENCY AMPLIFIER

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FIG. 1

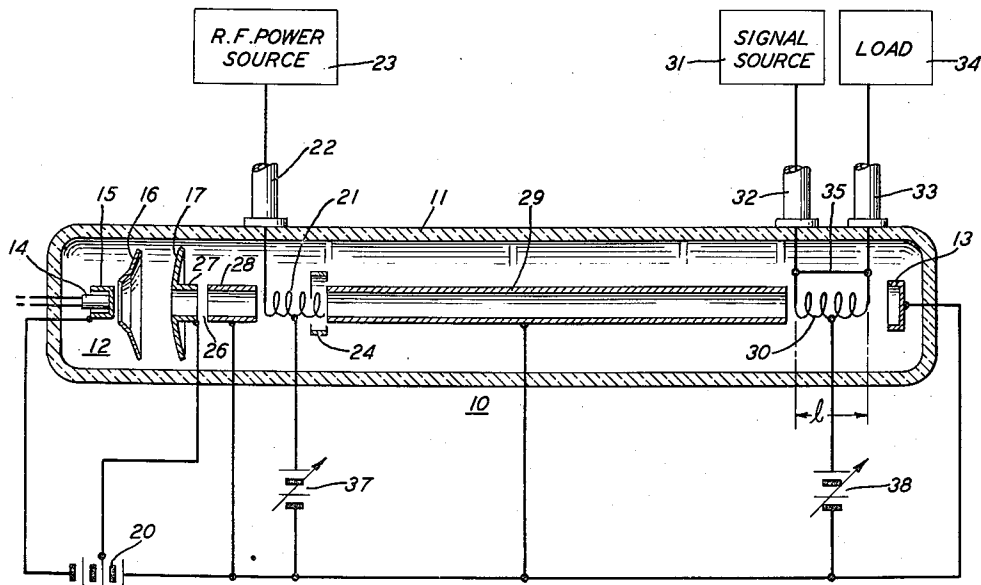
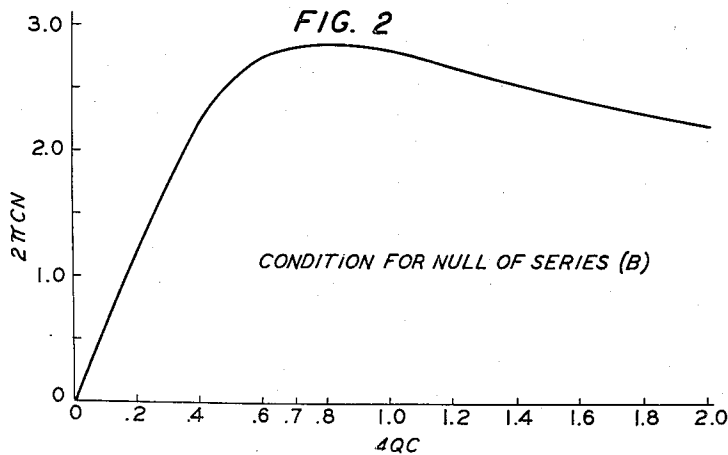


FIG. 2



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2,972,702

HIGH FREQUENCY AMPLIFIER

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This invention relates to high frequency electron discharge devices and, more particularly, to those of the velocity modulation type.

In general, electron discharge devices of this type, such as the traveling wave tube or the klystron, for example, are capable of amplification and oscillations at very high frequencies with reasonably good efficiency and stability. Moreover, in the case of the traveling wave tube, such efficiency and stability are maintained over an exceedingly wide band of frequencies.

In such devices, amplification is usually dependent upon the interchange of energy between an electron beam and electromagnetic wave energy either propagating along a distributed slow wave circuit or contained within a high Q resonant circuit. During such an interchange of energy, the beam is initially velocity modulated, i.e., some of the electrons are slowed down while others are speeded up, which eventually results in the beam becoming bunched or density modulated. This latter characteristic results in a variable beam current at the signal frequency in the form of space charge waves. These space charge waves are then converted into amplified electromagnetic wave energy and abstracted by a suitable output transducer in coupling relation with the beam, such as a resonator circuit in the case of a klystron or a distributed type of delay line circuit in the case of a conventional traveling wave tube.

Detracting from the advantageous characteristics of such devices is the problem of noise that is inherently associated with an electron beam. Considerable effort has been directed toward eliminating or at least reducing this noise, but, because of the number of sources of beam noise together with their interrelated characteristics, such prior efforts directed toward the suppression of one form of noise have not proven too successful in minimizing the other types of noise.

The two basic and independent sources of noise that have proven most difficult to eliminate are commonly referred to as the "current noise" and the "velocity noise." The "current noise" arises from the random rate of emission of, and the Maxwellian distribution of velocities among, the electrons emitted from the thermionic cathode characteristic of such apparatus. Such inhomogeneities in the beam establish what has become known as shot noise or velocity fluctuation noise in the beam after emerging from the gun. The "velocity noise" arises from the thermal energy distribution leading to an effective A.-C. velocity noise component in the beam at the potential minimum near the cathode which is converted into a noise current fluctuation upon emerging from the gun. Thus, it is seen that the original "current noise" is converted into velocity fluctuation noise and the original "velocity noise" is converted into current fluctuation noise along a drift region following the gun. It is known that both the current and velocity noise fluctuations in the beam, after emerging from the gun, cause a noise standing wave pattern on the beam, and that these patterns exhibit maxima and minima along the path of flow between

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the cathode and the point where signal modulations are impressed upon the beam. It is a further characteristic that a point of noise current fluctuation minimum is a point of noise velocity fluctuation maximum and vice versa, at regions of the beam significantly removed from the cathode.

There have been numerous techniques utilized in the prior art directed to the suppression or elimination of the above-mentioned types of beam noise. One such technique has comprised projecting an electron beam through a non-resonant gap which sharply accelerates the electrons at a point of noise current fluctuation maximum along the path of flow between the cathode and the start of the slow wave circuit and locating the start of the slow wave interaction circuit at an optimum point with respect to a noise current fluctuation minimum. This technique, generally referred to as velocity jump de-amplification, effectively acts to reduce the noise velocity fluctuation component but tends to increase the noise current fluctuation component in the beam.

A prior art technique specifically directed to the reduction of the noise current fluctuation component, arising from the original "velocity noise" near the cathode, involves positioning a permeable control electrode in close proximity to the cathode and biased with respect to the cathode so as to confine the region affecting the return of electrons to the cathode to a short region of relatively short transit time. Inasmuch as the original "velocity noise" is directly affected by the electron acceleration in a space-charge-limited region, such noise is reduced by accelerating the electrons in a region beyond where they are so limited. In an attempt to reduce further such noise, the control member and cathode have been constructed to form a cavity resonator which presents a high radio-frequency impedance across the space-charge-limited region. While these techniques of noise suppression have afforded an improvement in the noise figure of conventional velocity modulation devices, such an improvement has not proved completely satisfactory with respect to the noise figure requirements encountered in small signal applications.

A completely different approach directed to the suppression of beam noise is disclosed in a copending application of T. J. Bridges-R. Kompfner, Serial No. 666,812, filed June 20, 1957. In that application there is disclosed an electron beam parametric amplifier wherein the reactance of a unique double gap or floating drift tube type of cavity resonator is varied by a bunched electron beam traversing it. The variable reactance characteristic and the particular type of parametric amplification realized therewith are accomplished by modulating an electron beam with radio-frequency energy at a frequency greater than the signal frequency and then projecting the beam through a double gap cavity resonator to which the signal energy to be amplified is introduced. When the reactance of this resonator is varied in a prescribed manner, there is presented to the remainder of the system a frequency-dependent negative resistance, the largest negative value of which occurs at the signal frequency. If the value of the negative resistance is adequate and the load is a positive resistance, the small total resistance of the system results in large signal amplification being realized. An essential feature of this device resides in the fact that noise current modulation in the beam does not introduce noise into the circuit. This is achieved by making the electrical distance between the gaps in the cavity resonator equal to an integral odd number of half wavelengths. More particularly, the spacing of the second gap from the first is such that the interaction between the signal voltage at a frequency f_s and the beam, previously modulated at a frequency $2f_s$, for example, in the first gap, induces a current at the second gap of frequency

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f_s which is in phase quadrature with the voltage at the second gap. This results in effectively shunting the cavity resonator with a reactance which varies at a frequency twice the signal frequency, f_s . The type of reactance shunting the cavity resonator is determined by the specific length of the drift tube separating the two interaction gaps. If the distance between centers of the respective gaps is made such that the drift time corresponds to $n + \frac{1}{2}$ cycles at a frequency f_s , where n is an integer, the shunting reactance is a pure positive (inductive) reactance. Significantly, this results in any current modulation, such as noise in the beam at a frequency f_s , being in antiphase at the two gaps. When the currents are in antiphase, there is substantially complete elimination of noise at a frequency f_s in the output of the cavity resonator. While this device permits a considerable reduction in the noise content of the beam, as compared to the aforementioned prior art arrangements, the utilization of a double gap high Q cavity resonator results in limitations on bandwidth and, thus, is not efficiently operable over a very wide band of frequencies.

Accordingly, it is an object of this invention substantially completely to eliminate or suppress the noise content of the beam in a velocity modulation device.

It is another object of this invention to eliminate or suppress the noise content of the beam of such a device over a wide operating band of frequencies.

It is an additional object of this invention to produce parametric gain of the signal wave along a helical delay line while simultaneously therewith inhibiting the excitation of current fluctuation noise wave energy thereon.

These and other objects of our invention are attained in one illustrative embodiment thereof wherein an electron discharge device comprises an evacuated envelope with an electron gun and collector positioned at opposite ends thereof for forming and projecting an electron beam along an extended path therebetween. A first wave interaction circuit is positioned adjacent at least a portion of the beam path for modulating the beam with radio-frequency power, hereinafter interchangeably referred to as pumping power, at a first frequency. Intermediate the electron gun and first circuit there is provided a so-called velocity jump de-amplification region to provide a change in the magnitude of the noise components in the beam which is particularly conducive to noise suppression in accordance with the principles of the invention. A second wave propagation circuit, modified and dimensioned in accordance with principles of this invention described in greater detail hereinafter, is positioned downstream of the first interaction circuit and separated therefrom by a conductive drift tube region for velocity modulating the beam at the signal frequency. A positive feedback loop is provided with the second circuit to enhance further the gain obtainable. As used hereinafter, the expression downstream signifies a closer proximity to the collector than the electron gun relative to a given reference point therebetween, whereas upstream signifies the converse.

In accordance with the feature of our invention, the wave propagation circuit to which the signal energy is applied comprises a helix, dimensioned to have a predetermined length, described in detail hereinafter, such that current fluctuation noise in the beam will not introduce noise wave power onto the helix. Simultaneously therewith, a helix so dimensioned assures that an electron beam, modulated at a frequency greater than the signal frequency, will couple to and amplify the propagating signal wave on the helix exponentially over a wide band of frequencies by effectively varying several circuit parameters, namely, the self-inductance and capacitance of the beam. Thus, the prerequisite for parametric amplification is satisfied.

In accordance with another feature of this invention, there is provided a velocity jump de-amplification region preceding the first wave propagation circuit so that the noise velocity fluctuation component of the beam is re-

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duced and the noise current fluctuation component increased by an amount which is conducive to matching the noise suppression characteristics of the second amplifying wave propagation circuit.

More specifically, in accordance with a feature of our invention, the length of the wave propagation circuit for amplification of the applied signal should be substantially equal to

$$C \sqrt{\frac{\lambda}{b^2 - \frac{4}{b}}}$$

where C is the gain parameter of the wave propagation circuit, λ is the wavelength of a signal wave applied to the circuit and to be amplified, and b is the off-synchronous or velocity parameter which effectively represents the ratio of the difference in velocity of the electron beam and the signal wave to the velocity of the signal wave.

Additionally, we have found that for optimum amplification of the signal wave over the desired frequency band, the length of the signal interaction or wave propagation circuit should be substantially $.459 \lambda/C$. When this relationship is present, in accordance with our invention, the varying effects of the gain of the circuit, including circuit and beam impedance, and the length of the circuit, which may be expressed in terms of N , the number of wavelengths of the signal frequency along the interaction circuit, are balanced to obtain optimum amplification over the desired frequency band with minimum excitation of noise energy appearing in the amplified signal wave due to current fluctuation noise present on the electron beam.

A complete understanding of this invention and of the above-noted and other features thereof may be gained from consideration of the following detailed description taken in conjunction with the accompanying drawing, in which:

Fig. 1 is a schematic view of an illustrative embodiment of this invention; and

Fig. 2 is a graphic representation illustrating the characteristics of certain operating parameters affecting low noise, wide bandwidth amplifying characteristics of a helical wave propagation circuit designed in accordance with the principles of this invention.

Description of structure of the invention

Referring now to Fig. 1, there is depicted schematically an illustrative embodiment of a velocity modulation device 10 embodying the principles of this invention. Positioned within and at opposite ends of an evacuated envelope 11, which, for example, may be of glass or any other suitable material, is an electron gun 12 for projecting an electron beam along an extended path to a collector 13. The electron gun 12, as shown schematically comprises a heater 14, electron emissive cathode surface 15, beam forming electrode 16 and accelerating electrode or anode 17. For convenience and simplicity, the gun support structure and heater lead-in connections have not been shown. The collector 13 is maintained at a suitable positive potential with respect to the cathode 15 by means of suitable lead-in connections from a multi-tapped voltage source 20. Located along the path of flow at a point downstream with respect to the electron gun 12 is a first wave propagation circuit 21, which may, for example, comprise a helix operating in the so-called "Kompfner Dip" condition. The characteristics of such a helix are disclosed in detail in an article entitled "A Coupled Mode Description of the Backward Wave Oscillator and the 'Kompfner Dip' Condition" by R. W. Gould, I.R.E. Transactions on Electron Devices, vol. ED-2, No. 4, October 1955, pages 37-42. Explained briefly, such a helix insures that all of the signal energy applied to the helix will be transferred to the beam in the form of space charge wave modulations. Thus, there is essentially no R-F signal energy appearing on the helix at the output thereof.

Helix 21 has applied thereto through a suitable transmission line 22 radio-frequency pump power from a source 23, preferably at twice the frequency of the signal to be amplified. Helix 21 is terminated at its downstream end by a resistive termination 24, which may take any number of forms, such as a lossy ring surrounding the end of the helix as shown so as to prevent undesired wave reflections from affecting the operation of the device when the helix is operated at some point other than the "Kompfner Dip" condition.

While the first interaction circuit 21 has been shown to comprise a helix, it is to be understood that other suitable interaction circuits, such as a cavity resonator or distributed delay lines of the interdigital, ladder or serpentine types may be utilized with equal effectiveness to modulate the beam with radio-frequency pump power in the manner to be described in greater detail hereinafter. Similarly, the first interaction circuit 21 may comprise a spatial harmonic backward wave oscillator whereby it would constitute an effective internal pump source adaptable for operation in accordance with the principles of this invention.

Intermediate the electron gun 12 and the helix 21 is what may be described as a space charge wave discontinuity region 26 inasmuch as it is utilized to produce a desired change in the magnitude of the space charge wave noise components in the beam. This discontinuity region is established by two conductive drift tube sections 27 and 28, the adjacent ends of which are separated by a short gap defining the discontinuity region 26. As is shown, drift tube section 27 comprises an integral part of the accelerating electrode 17, but could comprise a separate section if desired. Drift tube section 27 is biased less positively than drift tube section 28, as shown by the respective connections to the multi-tapped voltage source 29, and, thus, a velocity jump is established across the discontinuity region 26. This velocity jump is utilized to de-amplify the noise velocity fluctuation component of the beam and increase the noise current velocity fluctuation by an amount which is conducive to matching the noise current fluctuation suppression characteristics of the amplifying circuit of the device further downstream in accordance with the principles of the invention.

Downstream of helix 21 and separated therefrom by a conductive drift tube section 29 is a second helical delay line 30 which, in accordance with the principles of our invention, produces parametric amplification while substantially completely eliminating the affect of current fluctuation beam noise on the output signal. Signal energy is applied from a signal source 31 through a suitable transmission line 32 to the upstream end of helix 30. The downstream end of helix 30 is connected through a suitable coupling connection 33 to a load 34 for utilization. In order to enhance further the gain obtainable with helix 30, a positive feedback loop 35, which may take any of the known forms, may be utilized.

While drift tube sections 28 and 29 together with helices 21 and 30 will generally all be at the same potential so as to prevent undesired beam wave discontinuities along the path of flow, voltage sources 37 and 38 are shown connected to helices 21 and 30, respectively, to act as trimmers permitting the gain to be optimized for a given signal frequency independently of the other biased circuit elements. Voltages are applied to the remaining circuit elements by the multi-tapped voltage source 20.

In general, such a device is provided with a magnetic focusing structure or other suitable means, not here shown, for focusing the electron beam throughout its travel along the path of flow.

In accordance with our invention the length of the helix 30 is critically determined so that the current fluctuation noise present on the electron beam when the beam emerges from the drift tube 29 and enters the helix 30 is substantially equal to the current modulation noise on the electron beam on leaving the helix 30 to impinge on the electron collector 13. Specifically, the length of the

helix 30, or the length of any other wave propagation circuit utilized for amplification of the applied signal, is made substantially equal to

$$\frac{\lambda}{C\sqrt{b^2 - \frac{4}{b}}}$$

where C is the gain parameter of the helix, being dependent on the beam impedance and the circuit impedance of the helix and determined by known design data for a given set of operating conditions. λ is the wavelength of the signal wave applied to the circuit and to be amplified, and b effectively represents the ratio of the difference in velocity of the electron beam and the signal wave to the velocity of the signal wave.

In order to obtain optimum amplification, the product CN should be maximized and should be constant over the desired frequency range of the input signal, where N is the number of wavelengths of the signal frequency along the helix 30. This occurs when the length of the helix 30 is substantially equal to $.459\lambda/c$.

Operation of illustrative embodiment of the invention

Having described the structure of one specific illustrative embodiment of our invention wherein low noise, high gain parametric amplification may be obtained in a specifically dimensioned length helix or wave interaction circuit, in combination with the other elements set forth above, the manner of operation involved in the device 10 depicted in Fig. 1 will now be considered.

An electron beam is formed and projected along a rectilinear path from the electron gun 12 to the collector 13, passing axially and successively through the velocity jump de-amplification region 26, wave propagation circuit 21 and wave propagation circuit 30. As previously mentioned, the velocity jump region 26 reduces the noise velocity fluctuation component while simultaneously increasing the noise current fluctuation component by an amount which is conducive to matching the optimum noise current fluctuation suppression characteristics of the modified helix 30 in accordance with the principles of this invention. The electron beam in traversing past helix 21 interacts with radio-frequency pump energy, preferably at twice the signal frequency f_s , supplied from the external pump source 23. It is to be understood, however, that a strong pump harmonic could be utilized in accordance with the principles of this invention, whereby the fundamental pump frequency could be less than the signal frequency. As stated above, other types of wave interaction circuits may be utilized in place of helix 21 with equal effectiveness. Helix 21, shown by way of example, advantageously is adjusted to operate in the so-called "Kompfner Dip" condition, which assures that all of the pump energy applied to the helix is transferred to the beam in the form of velocity modulated space charge waves. In traversing the drift region 29, the velocity modulations impressed upon the beam are converted into density modulations in the form of space charge waves. The electron beam then traverses past and in synchronous coupling relation with signal wave energy propagating along the modified helix 30. As described above, helix 30 is dimensioned to prevent the noise current fluctuation component in the beam from introducing any appreciable power onto the helix and, simultaneously therewith, to insure that beam current modulated at a frequency higher than the signal frequency gives rise to parametric gain. The specific dimensions for the circuit parameters which make these unique characteristics possible are set forth above and in Equations 8, 9 and 10 described below. The signal to be amplified exponentially, in accordance with the principles of parametric phenomenon, is applied to the input of helix 30 from the source 31. As a result of the particular dimensions of helix 30 together with the current modulations impressed upon the beam by the pump source at a fre-

quency higher than the signal frequency, the electrons acquire bunches in positions corresponding to both maximum accelerating and maximum retarding R-F fields along helix 30 substantially all the time. These fields result in the bunches of electrons acquiring a phase difference of approximately one quarter cycle with respect to the signal wave propagating along the helix 30 from the input to the output thereof. It has been found in accordance with an aspect of this invention that this phase difference results in the reactance of the beam being varied at the pump frequency, and thus, the prerequisite condition for parametric amplification is satisfied. Inasmuch as the speeded-up bunches of electrons spend less time in the circuit than the slowed-down ones, as a result of the accelerating and retarding R-F fields, there is a net energy transfer from the beam to the wave. To enhance further this gain, a regenerative positive feedback coupling loop has been incorporated into the helix circuit 30. The amplified signal energy is abstracted at the downstream end of helix 30 by means of a suitable transmission line 33 for utilization in a load 34.

Analysis of the principles of the invention

We have discovered that the objectives sought and advantages realized with the double gap cavity resonator disclosed in the aforementioned copending Bridges-Kompfner application, namely, low-noise, parametric amplification, can be achieved by means of interaction between a bunched electron beam and waves on a wide band distributed circuit. The following discussion will be directed to the design considerations for a unique type of wide band helical wave propagation circuit wherein (1) noise fluctuation current in the beam introduces no power onto the helix and (2) beam current modulated at a frequency greater than the signal frequency gives rise to parametric gain.

For the sake of simplicity, we shall consider only forward waves on helices; that is to say, the wave and the electrons traveling in the same direction. However, the noise reduction principles discussed below are equally applicable to circuits designed for backward-wave interaction.

Consider a helix of length l , phase velocity v , effective coupling impedance K_0 , traversed by an electron beam with beam current I_0 and voltage V_0 .

We then ask the question: Under what circumstances will an A.-C. current component i of the beam current I_0 introduce no voltage V_1 at the output end of the helix of length l ?

Assuming ideal conditions, this can be solved, in general, using Pierce's theory, in which the solution is represented by three waves which have to be superimposed so that they satisfy the particular boundary conditions. This theory is described in a text by J. R. Pierce, entitled "Traveling Wave Tubes," Van Nostrand Company, 1950. When we take into account, however, the difference between the circuit wave velocity and the beam velocity, and the affect of space charge and circuit loss, the problem becomes too complicated to be solved by this theory. It is possible, however, in the case where an electron beam and a helix are represented by a coupled network, to solve the problem by defining the various parameters, in the case of a short helix, in terms of a rather exhaustive complex power series analysis. Inasmuch as this analysis is of considerable length and involves a large number of complex mathematical equations which are not of direct interest, only those equations which afford a better understanding of the principles of this invention and which are necessary to determine the design considerations of the unique helix embodied therein will be considered.

From this power series analysis, expressions have been derived for the circuit voltage, the beam current, and the

beam velocity at the output end of a short lossless helix as functions of the corresponding quantities at the input of the helix.

More particularly, the power series has been applied to the conditions wherein the three exponential waves on the electron beam, described by Pierce, interact to produce a null point at the output of a forward wave helical amplifier section.

In the power series analysis, the usual transmission line differential equations, which Pierce in his aforementioned text has shown to be valid for traveling wave tubes, have been used. In order to discuss the dynamics of the beam, which represents the effective transmission line in the equations, it was convenient to write the differential equations in terms of the R-F voltage on the circuit using the well-known force equation, the continuity equation, and the definition of current as defined in the Pierce text. The additional equation required for the analysis involved the relation between the circuit voltage V_{1c} and the voltage acting on the beam V_{1b} . This relation may also be found in the aforementioned text of J. R. Pierce. The above-mentioned differential equations, as modified, were then solved in terms of a power series rather than by exponential solutions.

From the analysis, it can be shown that the R-F voltage V_1 acting on the beam at the output of a lossless helix may be defined by the differential expression

$$\frac{\partial^3 V_{1c}}{\partial z^3} + j\beta C b \frac{\partial^2 V_{1c}}{\partial z^2} + \beta^2 C^2 (4QC) \frac{\partial V_{1c}}{\partial z} + j\beta^3 C^3 (1 + 4QCb) V_{1c} = 0 \quad (1)$$

where V_{1c} is the R-F circuit voltage, z is the axial length of the helix, b is the off-synchronous or velocity parameter and β , C and Q are, respectively, the propagation constant, coupling or gain parameter and space charge parameter as defined by J. R. Pierce in his aforementioned text. Similarly, it can be shown that the differential equation for the R-F current i in the beam can be defined by the expression

$$\frac{\partial^3 i_1}{\partial z^3} + j\beta C b \frac{\partial^2 i_1}{\partial z^2} + \beta^2 C^2 4QC \frac{\partial i_1}{\partial z} + j\beta^3 C^3 (1 + 4QCb) i_1 = 0 \quad (2)$$

and the R-F beam velocity v can be defined by the expression

$$\frac{\partial^3 V_1}{\partial z^3} + j\beta C b \frac{\partial^2 V_1}{\partial z^2} + \beta^2 C^2 4QC \frac{\partial V_1}{\partial z} + j\beta^3 C^3 (k + 4QCb) V_1 = 0 \quad (3)$$

Equations 1 through 3 have been solved for the complete set of input boundary conditions whereat the three exponential waves on the beam interact to produce a null condition. Inasmuch as these equations are of the third degree, three separate equations are required to define the voltage, current, and velocity series, respectively. We are particularly concerned with only one of the power series equations, namely, the one derived from the voltage series which answers the question: Is it possible to have a condition in which noise fluctuation current in the beam introduces no power into the circuit? This requires setting the following equation derived from the power series analysis, and referred to hereinafter simply as Series B, equal to zero:

$$V(z) = j \frac{2V_0}{I_0} C^2 i_1(0) \text{ Series B} \quad (4)$$

or

$$\begin{aligned} \text{Series } B = & \left[(2\pi CN) - j \frac{(2\pi CN)^2}{2} b - \frac{(2\pi CN)^3}{3!} (b^2 + 4QC) \right. \\ & + j \frac{(2\pi CN)^4}{4!} (b^3 + 4QCb - 1) \\ & + \frac{(2\pi CN)^5}{5!} (b^4 + (4QC)^2 - 2b + 4QCb^2) \\ & \left. - j \frac{(2\pi CN)^6}{6!} (b^5 + 4QCb^3 - 3b^2 + b(4QC)^2 + 2(4QC) \dots) \right] = 0 \end{aligned} \quad (5)$$

where V is the R-F voltage representative of the noise current fluctuation component on the helix and N is the number of operating wavelengths along the interaction region of the helix. For the case where there is loss along the helix, the above-described analysis is equally applicable by simply replacing the velocity parameter b by $b - Id$, where d is the attenuation parameter.

In the course of computing the conditions for which Series B as defined by Equation 5 is zero, it has been found that these conditions are explicitly described when

$$-b = \frac{1}{4QC} \quad (6)$$

or

$$-b = \frac{u_0 - v}{Cv} \quad (7)$$

and

$$CN = \frac{1}{\sqrt{b^2 - \frac{4}{b}}} \quad (8)$$

where v is the phase velocity of the wave in the absence of electrons, and u_0 is the D.-C. or electron beam velocity. Since N defines the number of wavelengths along the helix, and C defines the gain parameter which is dependent on circuit and beam impedance, the product CN effectively defines the axial length of the interaction region of helix 30 for a given set of operating conditions in accordance with the principles of this invention. Since N has previously been defined as the ratio of

$$\frac{1}{\lambda}$$

the specific length of the helix may be defined in a more tractable form as follows:

$$l = \frac{\lambda}{C \sqrt{b^2 - \frac{4}{b}}} \quad (8a)$$

which distance is designated l in Fig. 1.

Advantageously, it can be shown that CN has a maximum and substantially stationary value of approximately .459 when

$$b = \sqrt{2} \quad (9)$$

or

$$4QC = \frac{1}{\sqrt{2}} \quad (10)$$

Thus, Equation 8a may be rewritten in the following form:

$$l = \frac{.459\lambda}{C} \quad (10a)$$

Equations 8, 8a and 10a thus give us explicit expressions for the length of the helix at the Series B null condition for which no noise power, derived from the noise current fluctuation component in the beam, is introduced on the helix at the output. Further, this condition of no noise power can be obtained over a relatively

wide range of $4QC$ with a nearly constant value of CN , by dimensioning the helix to satisfy Equations 9 and 10 and, hence, noise reduction can be obtained over a relatively wide range of frequencies. Fig. 2 depicts the substantially uniform wide band operating characteristics for a range of $4QC$, plotted along the abscissa, versus a range of $2\pi CN$, plotted along the ordinate in the vicinity of the null condition whereat Series B is substantially zero. Moreover, Fig. 2 defines the specific relationships between the various circuit parameters which are necessary, in accordance with the principles of the invention, so as to arrive at the critical length of the helix which will insure the realization of an optimum signal to noise ratio over a wide range of frequencies at the output of the helix. By knowing that CN has a maximum value of .459 which is substantially constant when

$$4QC = \frac{1}{\sqrt{2}}$$

or approximately .795, as defined in Equation 10 and depicted by the curve of Fig. 2, and knowing the desired midband operating frequency, the helix diameter, pitch and gain parameter C are then easily determined for a given set of operating conditions from the design data given in the aforementioned text of J. R. Pierce. The value of C so obtained is then inserted into Equation 10a and the unique and critical length of the helix thus ascertained.

In modern traveling wave tubes designed for broad-band applications, the gain in db is proportional to the product CN where $N = l/\lambda$, the number of wavelengths in the total length l of the helix. In general, C decreases as the frequency increases because the coupling between the R-F field and the beam then becomes weaker, while N increases because l , the wavelength on the circuit, decreases. Thus, there is an optimum design condition at which the increase and decrease just cancel, and where CN is approximately constant.

Accordingly, we can assume that the left-hand side of Equation 8 can be made approximately constant over an appreciable range of frequencies. What about the right-hand side of Equation 8? We are interested in the magnitude of Series B, Equation 6, as ω_q , the effective plasma frequency, deviates from its optimum value $\omega_{q \text{ opt}}$, or ω , defining the operating frequency, deviates from its optimum value ω_{opt} inasmuch as QC is dependent on frequency. It can be shown under such conditions that Series B assumes very negligible values over a reasonably wide range of ω_q and, equally significant, that CN is effectively independent of frequency.

Accordingly, the helix design conditions which afford low noise, in accordance with the principles of this invention, namely, the parameters of Equation 8, as defined in Equations 9 and 10, can be satisfied over a substantially wide range of frequencies inasmuch as CN has not only a substantially stationary value with respect to $4QC$ as so defined and, as evidenced from Fig. 2, but also since beam-circuit combinations can be designed such that CN is independent of frequency.

We shall now turn our attention to the value of the noise figure that can be expected in a velocity modulation device utilizing a helix designed in accordance with the principles of this invention.

As previously mentioned, we postulate two independent sources of noise: (1) the randomness of electrons passing the potential minimum in the gun diode, i.e., the so-called "current noise," and (2) the thermal energy distribution leading to an effective A.-C. "velocity noise" at the potential minimum.

Further, we postulate that we are successful in locating the input end of the second helix 30 at a point where the current noise has been completely converted into velocity fluctuations and where the velocity noise has been completely converted into current fluctuations. Both of these processes go on in a drift tube with a spatial periodicity

given by the effective plasma wavelength ω_q . This point can be found either experimentally or by a theoretical calculation.

Before we can consider the specific values of the velocity and current fluctuations in the drift region, we have to account for the passage of the original fluctuations from the potential minimum in the diode, which comprises the focusing and accelerating electrodes of the gun assembly, to the accelerating electrode thereof. It can be shown that the A.-C. "velocity noise" does not change within a fully space-charge-limited cathode-accelerating anode region when the transit time is several periods. Thus, in a gun designed with such characteristics, as is known in the art the original "velocity noise" at the potential minimum appears unchanged at the accelerating anode. The noise current fluctuation due to this "velocity noise" component also turns out to be negligible at the accelerating anode. As a result of the bunching process, however, the original "velocity noise" will be converted into a noise current fluctuation component in the beam along the drift region which exhibits a series of maxima.

The "current noise" at the potential minimum of a space-charge-limited diode gun assembly becomes a noise current fluctuation component at the accelerating anode of the gun. As a result of the bunching process in the drift region, the noise current fluctuation is converted into a velocity fluctuation component exhibiting a series of maxima along the drift region.

In analyzing the above-mentioned sources of beam noise, there will be no appreciable error in assuming that the total noise input to the amplifier section, namely, helix 30 depicted in Fig. 1, consists of the uncorrelated maximum values of the current and velocity fluctuations. Specific theoretical derivations which define these two sources of beam noise may be found in the aforementioned text of J. R. Pierce.

As previously shown, we have found in accordance with the principles of this invention that when Equations 6, 7 and 8 are satisfied, the maximum value of noise current fluctuation in the drift tube, established originally by the "velocity noise" at the potential minimum, does not introduce any noise current fluctuation power at all into the circuit in an infinitesimal frequency interval at the exact frequency at which $CN=0.459$. In a finite bandwidth B, however, Series B, derived from the aforementioned power series analysis, will not be zero, and we therefore have to take the sum of all of the noise contributions arising from this fact and integrate over the bandwidth B of which we are particularly interested. This calculation has been undertaken, with the noise current fluctuation power W of the beam at the input of helix 30 being defined by the following expression

$$W_{\text{noise}} = \frac{\bar{V}^2}{K_0} \approx CkT_c B \left(\frac{2\pi B}{\omega_p} \right)^2 \quad (11)$$

where $\bar{V}^2$ is the mean square noise voltage of the current fluctuation noise, k is Boltzman's constant, T_c is the cathode temperature in degrees centigrade, B is the bandwidth in which the mean square value of the current fluctuation component has frequencies which must be considered, and ω_p is the frequency deviation from the null frequency at which Series B is zero, the remaining coefficients having been previously defined.

This remarkably low noise power is, of course, only obtainable when CN is exactly equal to what may be described as the "balance value" 0.459, i.e., where CN is substantially constant with respect to frequency in accordance with the principles of this invention.

Next we must calculate the noise power introduced by the "current noise" of the beam originating at the potential minimum of the gun, which "current noise" appears as a noise velocity fluctuation component in the drift tube preceding the input of the amplifying helix section 30. As noted from the above discussion, setting Series B equal

to zero does not substantially inhibit the excitation of noise velocity fluctuation power on the helix.

By utilizing another one of the complex equations derived from the aforementioned power series analysis, and with the values of

$$4QC = \frac{1}{\sqrt{2}} NC = 0.459 \text{ and } b = \sqrt{2}$$

defining the parameters of helix 30, in accordance with the principles of this invention, it has been found that the noise velocity fluctuation power W' at the input of the amplifying helix 30 is defined by the following expression

$$W'_{\text{noise}} = \frac{\bar{V}^2}{K_0} = 12.3kT_c B C \quad (12)$$

The noise factor NF referred to room temperature is

$$NF \approx 1 + \frac{W + W'}{GkTB} \quad (13)$$

where G is the gain of the amplifying section.

Since the gain will be very nearly unity, this can be written

$$NF = 1 + 12.3C \frac{T_c}{T} \left[1 + \frac{1}{12.3} \frac{(2\pi B)^2}{\omega_p} \right] \quad (14)$$

or

$$NF \approx 1 + 40C \quad (14a)$$

if the cathode is at a temperature characteristic of the usual oxide type.

It would not be advisable to draw the conclusion from Equation 14a that the lowest possible value of C should be sought in order to obtain a low noise figure. We have neglected circuit losses in our analysis; and a very low value of C would require a long circuit, which would result in an appreciable total distributed loss. The thermal agitation voltage generated in such a loss is bound to be amplified and as a consequence, a lowering of C would lead to a deterioration of the noise figure.

Advantageously, since the major portion of the total noise power normally introduced into the circuit stems from the noise velocity fluctuation component at the input of the circuit, this component of beam noise may be reduced by means of velocity jump de-amplification. Such a procedure will lead to a corresponding increase in the current-fluctuation input to the helix, but, in accordance with the noise current fluctuation suppression characteristics of helix 30, we can afford an increase of a factor

$$\sqrt{12.3} \times \frac{\omega_p}{2\pi B}$$

before the two contributions to the noise become equal. While a further reduction in the noise velocity fluctuation component is possible, the corresponding increase in the noise current fluctuation component requires a more limited bandwidth in order to prevent a fractional part of the latter component of noise from appearing as a noise power on the helix output. Typically, we can increase the noise current fluctuation component by a factor of the order of 100 which leads to a noise figure defined as follows:

$$NF_{\text{opt}} \approx 1 + 0.4C \quad (15)$$

While the noise figure defined by Equation 15 was derived without taking into account partition noise of various kinds and higher-order noise waves on the electron beam, nevertheless, it is exceedingly good, and by dimensioning the helical wave propagation circuit with the design parameters defined in accordance with the principles of this invention, noise figures substantially below those previously realized with such circuits will be realized.

Next, it will be shown that beam current modulated at a frequency higher than the signal frequency gives rise to parametric gain in accordance with the principles of this

invention. This characteristic can perhaps be best described by means of the following qualitative analysis.

When practicable values of QC and C are chosen, it turns out that the necessary difference between beam and wave velocities is such as to lead in the course of the time of transit along the helix 30 to approximately one quarter cycle phase difference between the bunches and the wave at the signal frequency f_s . Thus, a bunch of electrons established by density modulation which leads, for example, the point of peak R-F field by one-eighth of a cycle at the input of helix 30 will lag by one-eighth of a cycle at the output of the helix, and there will be no appreciable error if we assume that such a bunch is in a time-constant uniform field continuously.

If the bunches arrive at a rate $2f_s$, for example, there will then be bunches in positions corresponding to both maximum accelerating and maximum retarding fields substantially all the time. To a first order of approximation, the energy supplied to the accelerated bunches by the signal wave along helix 30 will be equal to the energy given to the signal wave by the decelerated bunches and, thus, there is no net energy transfer. To a second order of approximation, the speeded-up bunches spend less time in the circuit than the slowed-down ones and therefore there is a net energy transfer from the beam to the signal wave. A calculation based on a model of a perfectly bunched beam, has lead to a power gain G of the wave given by the expression

$$G=1+C(2\pi NC)^2 \quad (16)$$

Since C may be a small number under certain operating conditions, positive feedback may be employed in order to enhance the parametric gain, such as achieved, by way of example, with the simple regenerative feedback loop 35 depicted in Fig. 1. In arriving at an estimate of the noise figure, we were justified in ignoring such feedback because, as is well known, feedback does not affect the intrinsic noise figure.

It is to be understood that the specific embodiment described herein is merely illustrative of the general principles of this invention. Various other structural arrangements and modifications may be devised in the light of this disclosure by those skilled in the art without departing from the spirit and scope of this invention.

What is claimed is:

1. In an electron discharge device, means including an electron gun for forming and projecting an electron beam along an extended path, means positioned adjacent at least a portion of said path for modulating said beam at a first frequency, a wave transmission circuit downstream of said modulating means forming an interaction region with said beam, and means for applying a signal wave of a frequency different from said first frequency to said wave transmission circuit, the length of said interaction region being given by the expression

$$l = \frac{\lambda}{C\sqrt{b^2 - \frac{4}{b}}}$$

where C is the gain parameter, λ is the wavelength of the propagating signal wave on the wave transmission circuit and b effectively represents the ratio of the difference in velocity of the electron beam and the signal wave to the velocity of the signal wave.

2. In an electron discharge device, means including an electron gun for forming and projecting an electron beam along an extended path, means positioned adjacent at least a portion of said path for modulating said beam at a first frequency, a wave transmission circuit comprising a helix downstream of said modulating means forming an interaction region with said beam and means for applying a signal wave of a frequency different from said first frequency to said helix, the length of said interaction region satisfying the relationship $CN=0.459$ where C is

the gain parameter and N is the number of wavelengths of the signal frequency in the length of the helix.

3. In an electron discharge device, means including an electron gun for forming and projecting an electron beam along an extended path, first means positioned adjacent at least a portion of said path for modulating said beam at a first frequency, means positioned intermediate said electron gun and said first means for de-amplifying the noise velocity fluctuation component of the beam, a wave transmission circuit downstream of said modulating means forming an interaction region with said beam, and means for applying a signal wave at a frequency different from said first frequency to said wave transmission circuit, the length of said interaction region being given by the expression

$$l = \frac{\lambda}{C\sqrt{b^2 - \frac{4}{b}}}$$

where C is the gain parameter, λ is the wavelength of the propagating signal wave on the helix and b effectively represents the ratio of the difference in velocity of the electron beam and the signal wave to the velocity of the signal wave.

4. An electron discharge device in accordance with claim 3 wherein said first means comprises a helix operated in the "Kompfner Dip" condition and said wave transmission circuit comprises a helix.

5. An electron discharge device in accordance with claim 3 wherein said means intermediate said electron gun and said first means comprises two collinear conductive drift tube sections separated by a short longitudinal gap with means for establishing a potential difference between said drift tube sections whereby there is established an electric field which produces an abrupt predetermined change in the velocity of said beam.

6. In an electron discharge device, means including an electron gun for forming and projecting an electron beam along an extended path, first means positioned adjacent at least a portion of said path for modulating said beam at a first frequency, means positioned intermediate said electron gun and said first means for de-amplifying the noise velocity fluctuation component of the beam, a wave transmission circuit comprising a helix downstream of said modulating means forming an interaction region with said beam, and means for applying a signal wave at a frequency one half said first frequency to said helix, the length of said interaction region being given by the expression

$$l = \frac{0.459\lambda}{C}$$

where λ is the wavelength of the propagating signal wave on the helix and C is the gain parameter.

7. An electron discharge device in accordance with claim 6 wherein said first means comprises a helix operated in the "Kompfner Dip" condition and said means intermediate said electron gun and said "Kompfner Dip" helix comprises two collinear conductive drift tube sections separated by a short longitudinal gap with means for establishing a potential difference between said drift tube sections whereby there is established an electric field which produces an abrupt predetermined change in the velocity of said beam.

8. An electron discharge device in accordance with claim 6 wherein said wave transmission circuit comprising a helix includes a positive regenerative feedback loop for enhancing the amplification of the helix.

9. An electron discharge device comprising electron gun means for projecting an electron beam having noise thereon in the form of velocity and current fluctuations, an electron collector opposite said gun means and velocity jump de-amplification means, a first interaction circuit, means defining a drift space and a second interaction circuit positioned in that order between said gun

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means and said collector, said velocity jump de-amplification means removing a substantial portion of said velocity fluctuation noise, means for applying a wave at a first frequency to said first interaction circuit, means for applying a signal wave of a frequency different than said first frequency to one end of said second interaction circuit, and means for obtaining an output from the other end of said second interaction circuit, said second interaction circuit having a length equal to $.459 \lambda/C$ where λ is the wavelength of the signal frequency and C is the gain parameter of said second interaction circuit.

10. An electron discharge device comprising electron gun means for projecting an electron beam having noise thereon in the form of velocity and current fluctuations, an electron collector opposite said gun means, means adjacent said gun means for removing a substantial portion of said velocity fluctuation noise, a first interaction circuit, means for applying a wave at a first frequency to said first interaction circuit, a second interaction circuit positioned adjacent said collector, means defining a drift space intermediate said first and second interaction circuits, said second interaction circuit being of a length such that the current modulation noise on said electron beam entering said second interaction circuit is substantially the same as the current modulation noise on said electron beam leaving said second interaction circuit, means for applying a signal wave of a frequency different than said first frequency to one end of said second interaction circuit, and means connected to the other end of said second interaction circuit for removing said signal wave after parametric amplification thereof in said second interaction circuit and without the transfer of substantial noise power thereto from said electron beam in said second interaction circuit.

11. An electron discharge device in accordance with claim 10 wherein said second interaction circuit has a length substantially equal to

$$C \sqrt{\frac{\lambda}{b^2 - \frac{4}{b}}}$$

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where C is the gain parameter of said second interaction circuit, λ is the wavelength of said signal wave, and b effectively is the ratio of the difference in velocity of said electron beam and said signal wave to the velocity of said signal wave.

12. An electron discharge device in accordance with claim 10 wherein said second interaction circuit has a length equal to $.459 \lambda/C$.

13. An electron discharge device in accordance with claim 12 wherein said second interaction circuit is a helix and further comprising feedback means directly connecting said one and said other ends of said second interaction circuit.

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