

May 24, 1960

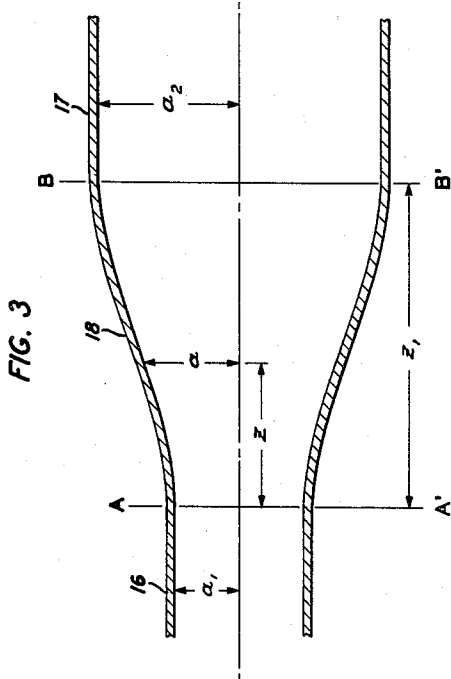
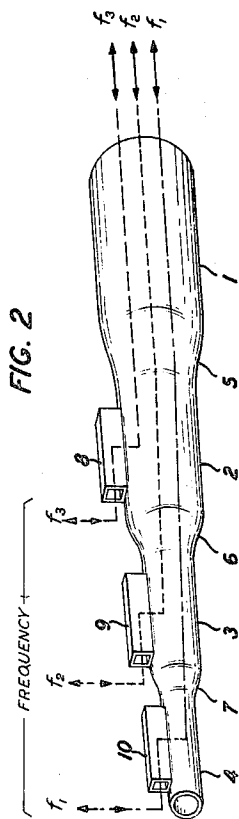
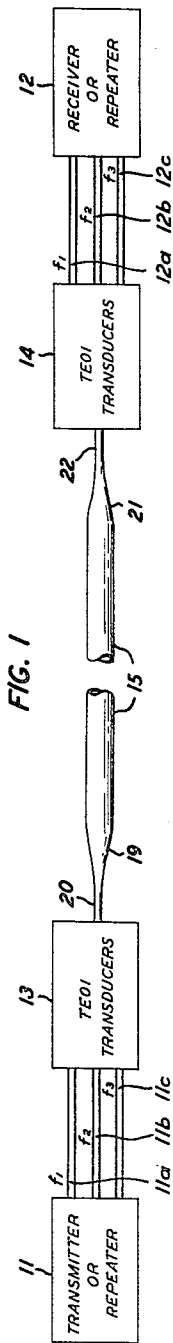
HANS-GEORG UNGER

2,938,179

VARIABLE TAPERED WAVEGUIDE TRANSITION SECTION

Filed Aug. 20, 1957

2 Sheets-Sheet 1



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VARIABLE TAPERED WAVEGUIDE TRANSITION SECTION

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2 Sheets-Sheet 2

MODE CONVERSION PATTERN

WAVEGUIDE TAPER

FIG. 4a

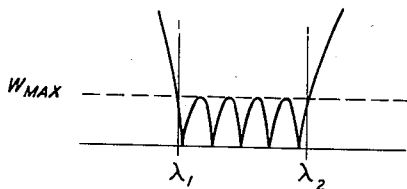
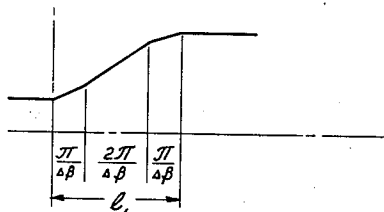


FIG. 4b



MULTISECTION TRANSITION OF OPTIMUM DESIGN

FIG. 5a

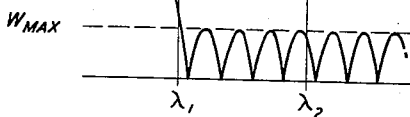
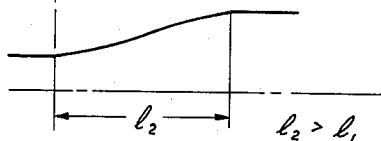


FIG. 5b



GRADUAL TAPER OF OPTIMUM DESIGN

FIG. 6a

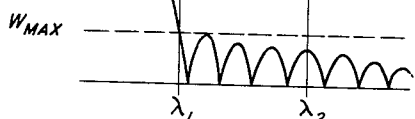
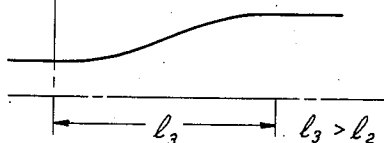


FIG. 6b



PERFECTLY SMOOTH TAPER OF OPTIMUM DESIGN

FIG. 7a

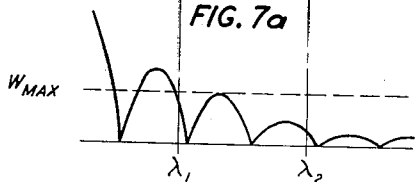
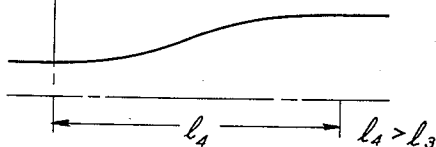


FIG. 7b



RAISED COSINE TAPER

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2,938,179

VARIABLE TAPERED WAVEGUIDE TRANSITION SECTION

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5 Claims. (Cl. 333—34)

This invention relates to mode matching devices and more particularly to tapered sections of waveguide having a gradually changing taper angle to connect waveguides of different cross-sectional dimensions for the preferential transmission of a particular mode.

In the transmission of electromagnetic wave energy through a hollow conductive pipe or other waveguide it is well known that the energy can propagate in one or more transmission modes, or characteristic field configurations, depending upon the cross-sectional size and shape of the particular guide and the operating frequency, and that the larger the cross section of the guide is made the greater is the number of modes in which the energy can propagate at a given operating frequency. Very generally it is desired to confine propagation of the energy to one particular mode, chosen because its propagation characteristics are favorable for the particular application involved. If the desired mode happens to be the so-called dominant mode, it is feasible to restrict the cross-sectional dimensions of the guide so that no modes other than the dominant mode can be sustained therein. This expedient is not available however, if the desired mode is not the dominant mode or if a guide of large cross section is prescribed in order, for example, that advantage may be taken of its relatively low attenuation. This is particularly true of systems employing the TE_{01} circular electric mode. As is well known, the propagation of microwave energy in the form of the TE_{01} mode in circular waveguides is ideally suited for the long distance transmission of high frequency wide band signals since the attenuation characteristic of this transmission mode, unlike that of other modes, decreases with increasing frequency. However, since the TE_{01} mode is not the dominant mode supported in a circular waveguide, energy may be lost to other modes that are also capable of transmission therein.

In an ideal system which has a waveguide that is perfectly straight, uniform and conducting, the propagation of TE_{01} waves therethrough would be undisturbed. In any practical system, however, many changes in waveguide diameter will be made in order to take maximum advantage of the transmission characteristics of the TE_{01} mode. For example, it is known that the transmission loss for the circular electric mode is inversely related to the guide diameter. Hence, long, uninterrupted runs of waveguide will be made with large diameter pipe. Multiplexing of a series of frequency bands into one pipe, on the other hand, is most efficiently done at certain smaller diameters. Thus, numerous changes in guide size will be made, for example, at repeater stations where various bands of frequency are to be coupled into and out of the system. Such diameter changes must of course be made with a minimum of loss. Similarly, sharp intentional bends can be more easily negotiated at smaller diameters, which will necessitate further diameter changes wherever such bends are required. Thus, it is apparent that any practical transmission system will afford ample opportunity to dis-

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turb the TE_{01} circular electric mode unless proper transmission means are provided to connect the different diameter guides encountered. In the absence of such transition sections there will, most certainly, be a conversion of power from the desired mode into other spurious and undesired modes. The latter will, in many instances be propagated within the guide along with the desired mode and thereby introduce deleterious effects.

Presently known transition means, built in the form of a conical taper of constant cone angle to match the waveguide sizes at both ends, tend to excite too high a level of higher order modes. When such sections have been used it has been necessary to provide mode-selective absorbers or attenuators to substantially prevent the transmission of the unwanted modes. These, however, are generally limited in respect to the modes they can accommodate, or tend to be resonant or otherwise to be effective only over a narrow range of frequencies, or tend to present an impedance mismatch to the waves of the desired mode and thereby to cause unwanted reflection of the latter. In addition, since they depend on absorption of wave power the energy in the spurious modes is lost, and this tends toward an inefficient system. Furthermore, in the case of the circular electric mode, no simple means are known to suppress the higher order circular electric modes without affecting the lowest order wave, and thus, as a practical matter, mode conversion-reconversion distortion can best be avoided by keeping the power converted to any of the higher order modes at an extremely low level. To attempt to approach the required low level of mode conversion using the prior art linear tapers, however, would require extremely long sections.

This invention, therefore, has as a general objective the propagation of circular electric wave energy between different diameter waveguides with a minimum of mode conversion. How this may be accomplished can be better understood by considering the effect a tapered transition section has upon the circular electric wave as it traverses such a section. This may best be explained in terms of normal modes, where normal mode is defined as that field distribution of wave energy which can be propagated without energy exchange to other modes or field distributions.

To the circular electric family, the normal modes in a cylindrical guide are cylindrical circular electric waves having plane equiphase surfaces whereas in the conical guide they are spherical circular electric waves with spherical equiphase surfaces. At the junction of the cylindrical guide and the conical guide, the TE_{01} cylindrical wave incident from the cylindrical guide excites higher order circular electric modes which combine to form the requisite spherical wave front. If the cone angle of the conical or tapered section is constant, the incident and spurious modes will propagate along the section maintaining the requisite spherical wave front, which is a true normal mode, until the next discontinuity in the waveguide. In the ordinary tapered section the next discontinuity consists of the junction between the tapered section and the second cylindrical guide. At this point each of the circular electric modes present generates a series of lower and higher order modes as the plane wave front is reestablished in the cylindrical guide. In the usual case, the spurious modes generated at the two junction points add to form a relatively high order spurious mode level, the latter constituting a loss in the system. The natural tendency therefore might be to smooth out the discontinuities that occur at the junction of the cylindrical guides and the tapered section. However, it will be shown below that the presence of specific discontinuities within the wave path may be put to good use and do not necessarily represent sources of uncontrolled mode conversion. Further-

more, it will be shown that where such discontinuities are eliminated, it will not suffice to do so arbitrarily, but rather in accordance with those principles to be set forth below.

Another method to minimize undesirable high order spurious mode conversion involves the introduction of a controlled transition at the junction of the cylindrical guide and the tapered section to transform the cylindrical wave into a spherical wave. In United States Patent 2,762,982, issued September 11, 1956, S. P. Morgan, Jr. has suggested and worked out the design of dielectric inserts to be placed near the junction which, acting as a quasi-optical lens, transforms the cylindrical waves into spherical waves. However, a system of this type has many drawbacks. For example, good broad band performance using this technique is difficult to achieve; the dielectric losses at the higher frequencies become appreciable; and the sections themselves are difficult to fabricate.

It is, therefore, a more specific objective of this invention to transform, with a minimum of net mode conversion, circular electric waves having plane equiphase surfaces of a first cross sectional dimension, into circular electric waves having spherical equiphase surfaces and then to reconvert such waves into circular electric waves having plane equiphase surfaces of a second cross sectional dimension.

It is a further object of this invention to perform such transformations over a wide range of frequencies.

In accordance with the present invention it has been found that if the transition between cylindrical waveguides is made in sections, each section having a particular length and cone angle, the net mode conversion may be reduced appreciably. Properly designed, a transition section of this type will have the shortest length for a given level of spurious mode conversion and bandwidth. However, since the length of the several sections and their cone angle are a function of the particular spurious mode being minimized, the usefulness of the multisection transition section is limited to systems having only one higher order mode of importance, i.e., systems capable of only supporting one higher order mode. Thus, such a transition section will not be capable of operating in a multimode system where many higher order modes are capable of being formed and transmitted therein.

In a multimode system it has been found that the transition from cylindrical waves to spherical waves may be realized by continuously varying the cone angle of the tapered section from zero at the first cylindrical guide to some finite value and back to zero at the second cylindrical guide. In a tapered section of changing cone angle, spurious modes are being continuously generated as the signal travels along the length of the transition section. In this type of section, the so called normal mode is not a "true" normal mode as defined above, but rather a local normal mode which is constantly changing, but which nevertheless, maintains its spherical configuration. If the transition is made gradually and in accordance with those principles to be explained hereinafter, the phase and amplitude of all the spurious modes generated by the incident wave energy traveling along the transition section will be such that their sum in the direction of energy flow will be substantially zero. Thus, substantially all the circular electric wave energy incident from the first cylindrical guide will emerge from the transition section in the original mode.

Compared to a taper with constant cone angle, a taper of the suggested design can be built much shorter for a specified spurious mode level and may be made to operate properly over a broad frequency range.

These and other objects, the nature of the present invention, and its various features and advantages will appear more fully upon consideration of the various specific illustrative embodiments shown in the accompanying drawings and analyzed in the following detailed description of these drawings.

In the drawings:

Figure 1 diagrammatically illustrates a guided microwave communication system employing the circular electric wave and having use for the tapered sections provided by the present invention;

Figure 2 more specifically shows the use of the tapered sections of the present invention;

Figure 3 shows diagrammatically an embodiment of the present invention;

Figures 4b and 4a show a multisection taper of optimum design and the spurious mode response thereof, respectively;

Figures 5b and 5a show a taper of near optimum design having a gradual taper and discontinuities at each end, and the spurious mode response thereof, respectively;

Figures 6b and 6a show a perfectly smooth taper of optimum design and the spurious mode response thereof, respectively, and

Figures 7b and 7a show a raised cosine taper and the spurious mode response thereof, respectively.

Referring more specifically to Fig. 1, there is shown a portion of a typical long distance guided microwave system in which the invention might be used. Such a system is characterized as long, to distinguish it from the short distances found in terminal equipment, and to define a system in which the factor of transmission attenuation becomes important. This system comprises a terminal station 11 which may be a transmitter, or if this is an intermediate station, a repeater 11 which is connected to a receiver or subsequent repeater comprising station 12. The circular electric TE_{01} mode is the mode in which energy is transmitted between stations. To do so with a minimum of loss, large multimode cylindrical waveguide 15 is used as the transmission medium. However, since the TE_{01} circular mode is not usually produced or utilized directly in the components of a station, transducers 13 and 14 are interposed between stations 11 and 12 and the long distance transmission line 15, which is connected to transducers 13 and 14 by sections of waveguide 20 and 22 and tapered sections 19 and 21. Connecting station 11 and transducer 13 are three channels 11a, 11b and 11c carrying three different frequency channels f_1 , f_2 and f_3 which are multiplexed in transducer 13 for the long distance transmission to transducer 14. Connecting transducer 14 and receiver or repeater 12 are three transmission paths 12a, 12b and 12c carrying the again separated channels f_1 , f_2 and f_3 in dominant mode guides for utilization in station 12.

Transducers 13 and 14 may be of any suitable well-known types for converting TE_{01} wave energy to and from a dominant wave mode configuration. For example, they may be structures of the type disclosed in United States Patent 2,748,350, granted to S. E. Miller, May 29, 1956, and illustrated in Fig. 2.

In Fig. 2 is shown a mode multiplex system in which a plurality of dominant mode TE_{10} modulated signals of different frequencies are transformed into TE_{01} circular electric waves at corresponding frequencies. Thus, a first modulated TE_{10} wave, f_1 , entering guide 10 is transformed into a first modulated TE_{01} circular electric wave at the same frequency in cylindrical guide 4. This signal then passes on to guide 1 through guides 3 and 2 and conical tapered sections 7, 6 and 5. Similarly, signals f_2 and f_3 are transformed and pass on to guide 1 through tapered sections 6 and 5.

As has been mentioned previously, to avoid any appreciable spurious mode conversion as a result of the presence of tapered sections 5, 6, 7, 19 and 21 which are used to connect the multimode sections of waveguide of different diameters, a transition is needed which will transform the cylindrical waves in the guides to spherical waves in the tapered section in a particular manner. If this transition is made in accordance with the present invention, nearly all the power incident in the cylindrical

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wave will traverse the tapered section with a minimum of net mode conversion.

Fig. 3 shows a preferred embodiment of the present invention in which 16 and 17 represent cylindrical guides of radii a_1 and a_2 respectively which are to be joined by transition section 18. The radius a of section 18 is related to the distance Z along the taper in a manner to be explained below.

The field excited in any cross-section of the taper by an incident TE_{01} wave can be expressed as a sum of TE_{0m} waves of a cylindrical guide of the same radius as that cross-section. With this representation, the tapered waveguide appears to be an infinite set of mutually-coupled transmission lines, where each transmission line represents one of the cylindrical TE_{0m} waves. The wave propagation in such a system is described by an infinite set of first order differential equations. If the taper is sufficiently gentle, the power in each of the TE_{0m} terms with $m > 1$ will be small compared to the power in the TE_{01} term. We may then consider the coupling between the TE_{01} mode and each of the TE_{0m} modes taken one at a time. Furthermore since the two guides are generally large and are operated far removed from cut-off, there is little change in characteristic impedance involved in going from one diameter pipe to the other. Therefore we need consider only the forward traveling waves, as the relative power coupled from the forward waves to the backward waves due to mismatch is quite small. Thus the infinite system reduces to the well known coupled line equations:

$$\begin{aligned} \frac{dA_1}{dz} &= -j\beta_1 A_1 + k_{21} A_2 \\ \frac{dA_2}{dz} &= -j\beta_2 A_2 + k_{12} A_1 \end{aligned} \quad (1)$$

in which A_1 and A_2 are the complex amplitudes of the cylindrical TE_{01} and TE_{02} waves respectively, β_1 and β_2 are the phase constants of these cylindrical modes and k_{12} and k_{21} are the coupling coefficients. Both the phase constants and coupling coefficients are functions of z , the distance along the taper.

Power conservation requires that

$$\text{and that } k_{21}k_{12} = -k^2 \quad (2)$$

Because of the taper, the normal mode in the transition section is a function of z . We may however, express A_1 and A_2 in terms of W_1 and W_2 , the local normal spherical modes, as

$$\begin{aligned} A_1 &= \left[W_1 \cos \frac{1}{2}\xi - W_2 \sin \frac{1}{2}\xi \right] e^{-j \int_0^z \beta dz} \\ A_2 &= \sqrt{\frac{k_{12}}{k_{21}}} \left[W_1 \sin \frac{1}{2}\xi + W_2 \cos \frac{1}{2}\xi \right] e^{-j \int_0^z \beta dz} \end{aligned} \quad (3)$$

$$\beta = 1/2(\beta_1 + \beta_2)$$

$$\tan \xi = 2 \frac{k}{\beta_1 - \beta_2} = 2 \frac{k}{\Delta\beta} \quad (4)$$

Substituting Equations 3 into Equations 1, the local normal modes must satisfy

$$\begin{aligned} \frac{dW_1}{dz} + j\Gamma(z)W_1 &= \frac{1}{2} \frac{d\xi}{dz} \\ \frac{dW_2}{dz} - j\Gamma(z)W_2 &= \frac{1}{2} \frac{d\xi}{dz} \\ \Gamma(z) &= \frac{1}{2} \sqrt{\Delta\beta^2 + 4k^2} \end{aligned} \quad (5)$$

The spherical waves represented by Equations 5 are coupled only through the terms proportional to $d\xi/dz$.

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If ξ is a constant, they are uncoupled and W_1 and W_2 are true normal modes. For a gentle change in taper angle

$$\left| \frac{1}{2\Gamma(z)} \frac{d\xi}{dz} \right| \ll 1 \quad (6)$$

and the approximate solution of 5 may be obtained in terms of powers of $d\xi/dz$.

The taper begins with zero cone angle, and the initial conditions are $A_1(0)=1$ and $A_2(0)=0$. From (3) it therefore follows that $W_1(0)=1$ and $W_2(0)=0$, where W_1 is the wanted local normal mode and W_2 is the unwanted mode. Substituting these boundary conditions in the solution of Equations 5, we obtain the expression for the unwanted mode at the end of the taper to be

$$|W_2(z_1)| = \frac{1}{2} \left| \int_0^{z_1} \frac{d\xi}{dz} e^{-2i\Gamma(z)} dz \right| \quad (7)$$

Since the cone angle at the end of the taper is also zero $\xi(z_1)=0$, Equation 7 reduces to

$$|W_2(z_1)| = \left| \int_0^{z_1} \Gamma(z) \xi(z) e^{-2i\Gamma(z)} dz \right| \quad (8)$$

$$\rho(z) = \int_0^z \Gamma(z) dz \quad (9)$$

The mode conversion in any smooth but otherwise arbitrary tapered section can be calculated from either (7) or (8).

Design of a taper

An optimally designed taper by definition has the smallest possible length for a given difference in diameters at both ends, and a specified unwanted mode level in a given frequency band. The frequency band of interest may be specified, as shown in Fig. 4a, by the free space wavelengths λ_1 and λ_2 at the band limits. The maximum spurious mode level is specified as $W_{\max}$.

The optimally designed taper has a mode conversion pattern as a function of frequency as shown in Fig. 4a. Aside from the effects of interference minima, the spurious mode level is constant over the frequency band and rises to higher levels beyond the band limits. The taper that has this mode conversion pattern is the multi-section transition mentioned above and shown in Fig. 4b. The sections of this particular taper have lengths of

$$\frac{\pi}{\Delta\beta}, \frac{2\pi}{\Delta\beta} \text{ and } \frac{\pi}{\Delta\beta} \text{ respectively,}$$

where $\Delta\beta$ is the difference in phase constant between the desired mode and the spurious mode. Each of the several sections has a constant cone angle over its length. The change in cone angle between sections is determined by specifying the constant level mode conversion pattern.

The optimum taper of Fig. 4a can be designed where mode conversion to only one spurious mode is considered, and the conversion to other spurious modes is deemed small enough to be neglected. If, in fact, this is not the case, mode conversion to other modes will distort the conversion pattern of Fig. 4a and the maximum spurious mode level may well be exceeded. Possible significant mode conversion to more than one spurious mode will be more safely handled in a taper of gradually changing cone angle.

An optimally designed taper, which is designed to include a section of gradually changing cone angle, will have the mode conversion pattern of Fig. 5a. Aside from the interference minima, the spurious mode level in such a taper is constant not only over the band of interest, but beyond the band on the long wavelength side. It is only beyond the short wavelength limit that the conversion level rises to a higher value.

While the taper of this near-optimum design has a gradually changing cone angle over most of its length, the cone angle is seen to change abruptly, by a very

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slight amount, at the two ends. Even though these changes are small, they still present a design problem in that their design is predicated upon a single significant spurious mode, and gives rise to the same problems mentioned in regard to the multisection taper.

While the taper of Fig. 5a is a less critical design than the optimum taper, the gradual tapered section affording a degree of control over the unwanted modes of higher order, there still remains the possibility that certain higher order modes may increase the unwanted mode level above the specified upper limit. It should also be pointed out that for the same conversion level and bandwidth, the gradual tapered section has a slightly longer length than the multisection taper. It will be seen that, in general, as the design is simplified or extended to include other modes the length of the transition section for a given set of conditions will increase.

The difficulties inherent in the design of the tapers of Fig. 4a and Fig. 5a are minimized in the smooth taper of Fig. 6a. The perfectly smooth taper of Fig. 6a, designed optimally, will have the mode conversion pattern of Fig. 6a. This pattern is similar to that of Fig. 5a except that the spurious mode level within the band and beyond the long wavelength limit is not a constant but decreases as λ^{-1} for large values of λ . In addition the section itself is larger than either the multisection taper or the gradual taper of Fig. 5a.

The design of the tapers of Figs. 4a, 5a and 6a requires rather involved numerical calculations for which no simple design formula can be given. There is, however, a taper which represents a preferred embodiment and which comes very close to the optimum design of the perfectly smooth taper and can be designed very easily. If the wave-guide modes under consideration are far enough from cut-off and the change in diameter is small compared to the diameters themselves, the design formula for a near optimum taper is simply;

$$\frac{da}{dz} = 2 \left(\frac{a_2 - a_1}{l_4} \right) \sin^2 \pi \frac{z}{l_4} \quad (10)$$

that is, the cone angle

$$\frac{da}{dz}$$

follows a raised cosine law along the taper. a_2 and a_1 are the radii at the ends and l_4 the length of the section. a is the radius at any distance, z , along the section.

The mode conversion pattern of the raised cosine taper is given by

$$W_2 = c(a_2 - a_1) \frac{|1 - e^{-i\Delta\beta l_4}|}{\Delta\beta l_4 \left(1 - \frac{\Delta\beta^2 l_4^2}{4\pi^2}\right)}$$

where c is a constant determined by the waveguide cross-section and the modes under consideration.

The conversion pattern of the raised cosine function is shown in Fig. 7a. It resembles that of Fig. 6a in general. It should be noted that as anticipated in order to maintain the same spurious mode level, the raised cosine taper has to be longer than any one of the optimum design tapers.

In the more general case where the difference in diameters is not small compared to the diameters themselves, the relation between a and z is expressed parametrically as

$$\ln \frac{a}{a_1} = \ln \frac{a_2}{a_1} \left(\frac{\rho}{\rho_1} - \frac{1}{2\pi} \sin 2\pi \frac{\rho}{\rho_1} \right) \quad (11)$$

$$z = \frac{4ba_1^2}{k_2^2 - k_1^2} \int_0^\rho \exp \left[2 \ln \frac{a_2}{a_1} \left(\frac{\rho}{\rho_1} - \frac{1}{2\pi} \sin 2\pi \frac{\rho}{\rho_1} \right) \right] d\rho \quad (12)$$

where b is the phase constant of the dielectric within the transition section, equal to

$$\omega\sqrt{\mu_0\epsilon_0}$$

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in air, and k_1 and k_2 are Bessel function roots equal to 3.83 and 7.016 respectively.

$$\text{Setting } \frac{1}{\pi} \ln \frac{a_2}{a_1} = \alpha \text{ and } 2\pi \frac{\rho}{\rho_1} = x$$

the integrand can be expanded in a series such that

$$z = \frac{2ba_1^2\rho_1}{\pi(k_2^2 - k_1^2)} \int_0^x e^{\alpha x} \left(1 - \alpha \ln x + \frac{\alpha^2}{2} \sin^2 x \pm \dots \right) dx \quad (13)$$

The overall conversion will be given as

$$W_2(\rho_1) = \frac{C|1 - e^{-2i\rho_1}|}{2\rho_1 \left(1 - \frac{\rho_1^2}{\pi^2}\right)} \quad (14)$$

where C is a constant equal to

$$\frac{2k_1k_2}{k_2^2 - k_1^2} \ln \frac{a_2}{a_1} \quad (15)$$

A numerical example will show the advantage of a properly tapered transition. If, for example, a transition for the TE₀₁ wave from a 7/8-in. I.D. pipe to a 2-in. I.D. pipe is required to have a spurious mode level of less than -50 db for all frequencies up to 75 kmc., a tapered section designed in accordance with the present invention would have a length of only 3 feet. A taper of constant cone angle, satisfying the same requirements, however, would have to be 58 feet long.

In all cases it is understood that the above-described arrangements are illustrative of a small number of the many possible specific embodiments which can present applications of the principles of the invention. Numerous and various other arrangements can readily be devised in accordance with these principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. An electromagnetic wave transmission system for the transmission of circular electric wave energy over a broad band of operating frequencies, means for producing said energy, means for utilizing said energy located at a distance from said producing means, cylindrical waveguide sections of different internal diameters extending between said producing means and said utilizing means, each of said sections having substantially the same characteristic impedance over said operating frequencies, and means for coupling said guides comprising a conical tapered section of changing cone angle.

2. The combination according to claim 1, wherein said coupling means comprises a multisection taper having a plurality of sections each section having a constant cone angle greater than zero for the controlled generation of a single higher order circular electric mode.

3. The combination according to claim 1, wherein said coupling means comprises a conical tapered section of gradually changing cone angle for the controlled generation of a plurality of higher order circular electric modes, said cone angle being zero at both ends of said section.

4. The combination according to claim 1 wherein said coupling means comprises a conical tapered section of gradually changing cone angle said cone angle being finite at both ends of said section.

5. In an electromagnetic wave transmission system supportive of the circular electric mode of wave propagation, a conical tapered section of gradually changing cone angle to connect a first multimode cylindrical waveguide of radius a_1 supportive of said mode to a second multimode cylindrical waveguide of radius a_2 supportive of said mode, wherein the radius a and the distance z from said first guide vary parametrically as

$$\ln \frac{a}{a_1} = \ln \frac{a_2}{a_1} \left(\frac{\rho}{\rho_1} - \frac{1}{2\pi} \sin 2\pi \frac{\rho}{\rho_1} \right)$$

and

$$z = \frac{4ba_1^2}{k_2^2 - k_1^2} \int_0^{\rho} \exp \left[2 \ln \frac{a_2}{a_1} \left(\frac{\rho}{\rho_1} - \frac{1}{2\pi} \sin \frac{2\rho}{\rho_1} \right) \right] d\rho$$

where b is the intrinsic propagation constant of the dielectric within the transition section, equal to

$$\omega \sqrt{\mu_0 \epsilon_0}$$

in air, k_1 and k_2 are Bessel function roots equal to 3.83 and 7.016 respectively,

$$\rho = \int_0^z \frac{1}{2} \sqrt{(\beta_1 - \beta_2)^2 - 4k_{12}k_{21}} dz$$

β_1 and β_2 are the phase constants of the TE_{01} and TE_{02} modes, and k_{12} and k_{21} are the coupling coefficients between said modes.

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