

Nov. 24, 1959

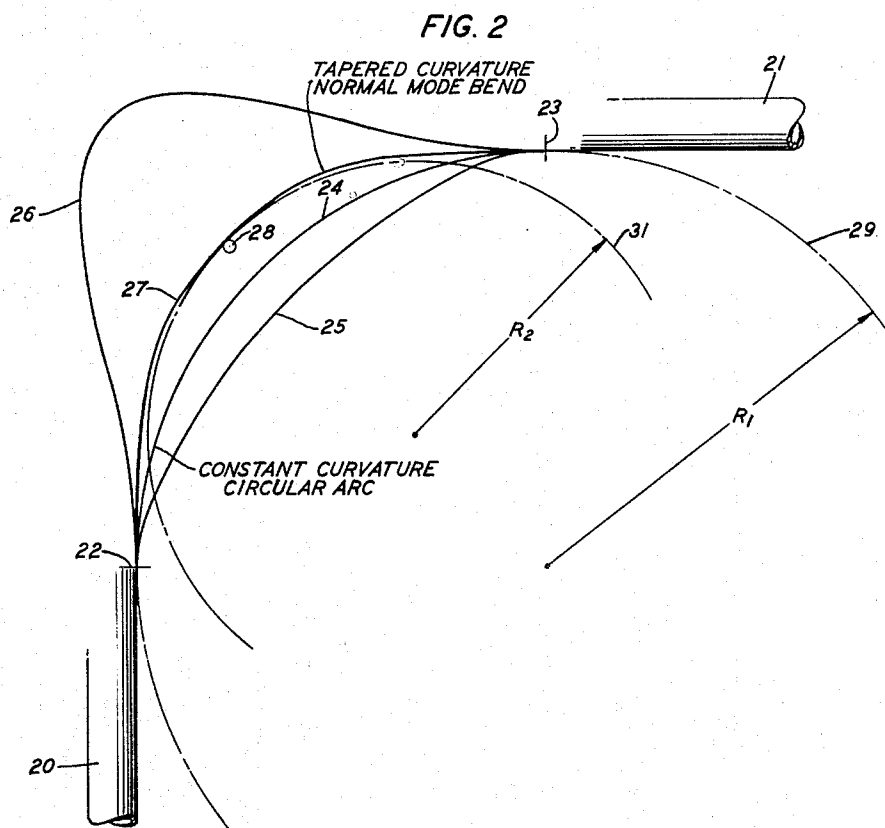
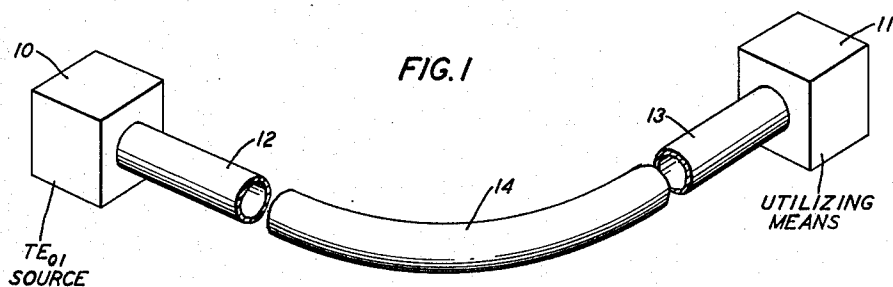
HANS-GEORG UNGER

2,914,741

WAVEGUIDE BEND

Filed Aug. 29, 1957

3 Sheets-Sheet 1



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FIG. 3

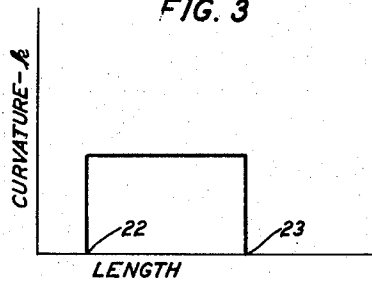


FIG. 4

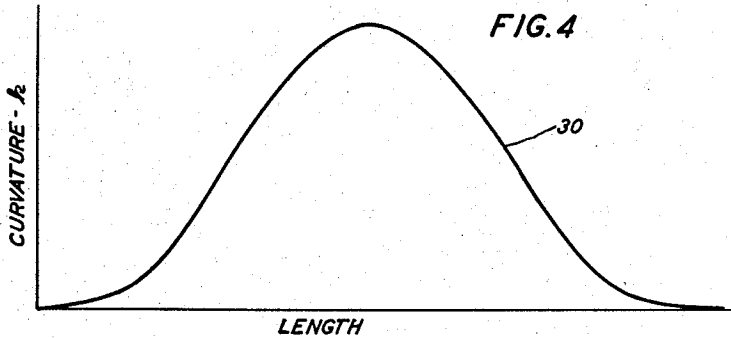
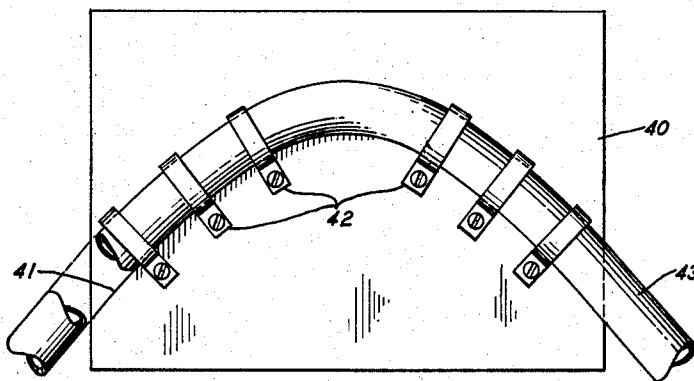


FIG. 5



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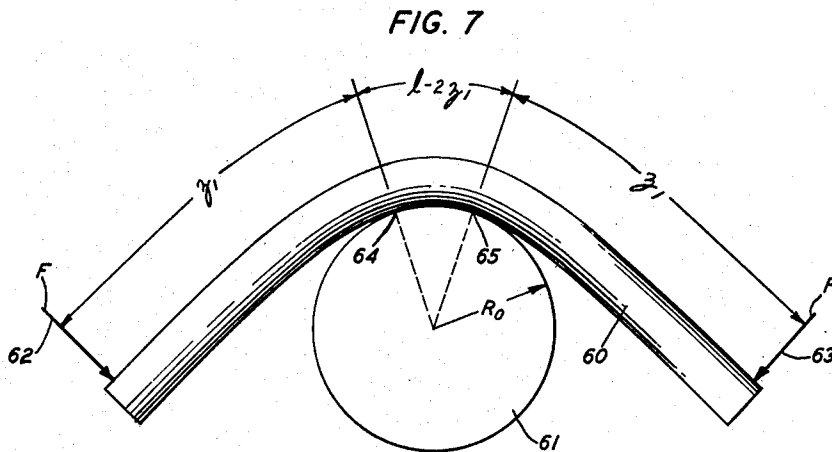
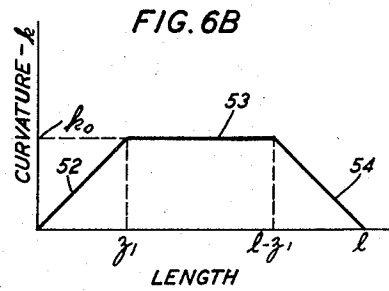
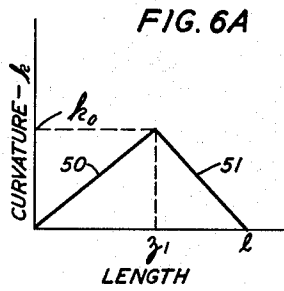
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WAVEGUIDE BEND

Filed Aug. 29, 1957

3 Sheets-Sheet 3



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2,914,741

WAVEGUIDE BEND

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Application August 29, 1957, Serial No. 681,027

5 Claims. (Cl. 333—98)

This invention relates to guided electromagnetic wave transmission and, more particularly, to waveguide configurations for transmitting the TE_{01} circular electric wave mode through curved waveguide sections. In this specification, the term bend will be considered generic to all waveguide configurations which deviate smoothly from axial linearity.

As is now well appreciated in the wave transmission art, the propagation of microwave energy in the form of TE_{01} waves in circular waveguides is especially suited to long distance transmission since the attenuation characteristic of this mode, unlike that of most other modes, decreases with increasing frequency. However, a major difficulty arises in conjunction with this mode of transmission from the fact that the TE_{01} mode is not the dominant transmission mode in the hollow pipe type waveguide of circular cross section. Thus energy may be lost to other wave modes capable of transmission within the guide. Propagation of TE_{01} waves in an ideal round waveguide which is perfectly straight, uniform, and conducting is essentially undisturbed. However, slight imperfections in the guide itself and, more especially, any deviation from straightness of the longitudinal axis of the waveguide may excite waves of other modes and thus produce serious losses. These losses are due mainly to the fact that waveguide curvature induces a coupling between the desired TE_{01} and other wave transmission modes. The major conversion problem is that involving the TE_{01} and the TM_{11} modes. To a much less serious extent, losses occur between the TE_{01} and TE_{12} modes as well as between the TE_{01} and other modes of higher order. The ease with which conversion to TM_{11} occurs is explained by an equality, in perfectly conducting uniform round waveguide, between the propagation constants of the TE_{01} and the TM_{11} modes.

Various structural modification techniques have been suggested for improving circular electric wave propagation through circular waveguides. For example, some of these structures include a circularly corrugated waveguide, as disclosed in United States Patent 2,751,561, which issued June 19, 1956, to A. P. King; a slotted waveguide as disclosed in United States Patent 2,779,006, issued January 22, 1957, to W. J. Albersheim; a helix waveguide as disclosed in the copending applications of J. R. Pierce, Serial No. 416,315, and S. E. Miller, Serial No. 416,316, both filed March 15, 1954, now United States Patents 2,848,695 and 2,848,696, respectively, both issued August 19, 1958; or a dielectrically lined waveguide as disclosed in applicant's copending application Serial No. 681,054, filed August 29, 1957. Each of the above-noted structures removes the TE_{01} - TM_{11} propagation constant degeneracy and thus reduces mode conversion loss from the circular electric wave mode in curved sections of circular waveguide. In many waveguide applications, necessity for relatively sharp bends arises. In such bends, mode conversion losses are undesirably high even in these modified structures.

It is, therefore, an object of this invention to reduce

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conversion losses suffered by wave energy in the circular electric wave mode in waveguide bends.

It is a further object to define physical waveguide bend configurations which are attractive from an energy transmission viewpoint.

A more specific object is to bend an elastic waveguide section into a configuration, having a tapered curvature through the application of an external force at the center of the section directed radially opposite to the external forces applied at the section extremities.

In accordance with the invention, it has been discovered that ordinary prior art waveguide bends, which are of several general types, do not provide the lowest attainable conversion loss for circular electric wave transmission when utilized in conjunction with one of the modification techniques hereinabove noted. It has been further discovered that TE_{01} wave mode transmission losses in bends may be substantially reduced by tapering the curvature of the curved waveguide section from zero at its junction with the straight waveguide section to a given maximum value and subsequently tapering the curvature from this given value to zero. Such a waveguide curvature characteristic provides a smooth physical and electrical transition at the junction between the angularly related straight waveguide sections and the curved waveguide member joining them. A waveguide bend in accordance with the invention has a radius of curvature associated therewith at a point just beyond the junction considerably greater than that which would normally result from the stresses and strains associated with an ordinary bent member. Such a novel waveguide bend is designated a tapered curvature normal mode bend. As will be more fully explained in a later portion of this specification, a tapered curvature normal mode bend is fabricated by bending a waveguide section while under the influence of at least one external force directed radially outward at the point of maximum curvature. Physically, this indicates that a bend in accordance with the invention is formed by bending a waveguide section around at least one fixed point.

In accordance with a first principal embodiment of the invention, a waveguide section is physically shaped to possess a curvature which conforms with the graphical representation of the Fourier transform of a Tschebyscheff polynomial of infinite degree. Such a configuration represents an optimum tapered curvature normal mode waveguide bend from an electrical transmission viewpoint.

In accordance with a second principal embodiment of the invention, a waveguide section is physically shaped to possess a tapered curvature characteristic which gradually increases from zero over a first portion of its length and a tapered curvature characteristic which gradually decreases to zero over a second portion of its length. These portions may be directly connected or they may be connected by a third portion having constant curvature. Such a configuration represents an optimum tapered curvature normal mode waveguide bend from a mechanical or fabrication viewpoint.

The above and other objects, the nature of the present invention, and its features and advantages will appear more fully upon consideration of the drawings and the detailed description thereof which follow.

In the drawings:

Fig. 1 is a perspective view of a simplified waveguide transmission system;

Fig. 2 illustrates a pair of angularly related straight waveguides connected by several types of waveguide bends;

Fig. 3 is a graphical representation of the length versus curvature characteristic of a prior art waveguide bend;

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Fig. 4 is a graphical representation of the length versus curvature characteristic of one waveguide embodiment of the invention;

Fig. 5 is a waveguide bend having the curvature characteristic of Fig. 4;

Figs. 6A and 6B are graphs illustrating the length versus curvature characteristic of additional waveguide embodiments of the invention; and

Fig. 7 illustrates a method of fabricating a curved waveguide section having the curvature characteristic of Fig. 6B.

Referring more specifically to the drawings, Fig. 1 illustrates a simplified microwave installation comprising microwave source 10 supplying energy in the form of TE₀₁ waves to a microwave utilizing means 11 through a continuous circular waveguiding passage. Utilizing means 11 may be a microwave amplifier, a receiver, or an antenna, for example. The circular waveguiding passage comprises angularly related straight sections 12, 13 which are joined by smoothly curved section 14, the particular design of which will be explained in detail in a later portion of this specification.

In a microwave system for the transmission of TE₀₁ waves, such as, for example, the system of Fig. 1, the inside radius a of the circular pipe guide selected for the propagation of these waves must be greater than the critical or cut-off radius a_c for the TE₀₁ mode. For the TE₀₁ mode, a_c is equal to $0.61\lambda_0$ where λ_0 is the wavelength in free space of the lowest frequency wave in the transmission band. In practice, a is made greater than a_c and may vary in different systems from $1.5\lambda_0$ to $15\lambda_0$. For illustrative purposes, a suitable inner radius for the waveguide structures to be described herein may be $7.7a_c$ or $4.7\lambda_0$. Thus if a hollow pipe guide two inches in diameter were chosen for transmission of the TE₀₁ waves, λ_0 in accordance with the above would be 0.213 inch or 5.4 millimeters.

Curved circular waveguide section 14 serves to connect, both physically and electrically, angularly related straight waveguide sections 12 and 13. TE₀₁ wave propagation through curved section 14 is most easily explained in terms of normal modes; that is, those waves for a particular waveguiding structure which propagate without loss except for dissipation. Normal modes represent solutions of the wave equations in the particular waveguiding structure being investigated. In a straight waveguide excited by a source of TE₀₁ wave energy, the normal mode of interest is the TE₀₁ mode. There are other normal modes capable of propagating in straight waveguide sections but, in the absence of waveguide discontinuities therein which would cause conversion to these other modes, they need not be considered. The normal modes of a curved waveguide section are not so simple as those of a straight section. They may, however, be expressed as a sum of the straight guide normal modes. In the present invention, the normal mode in the curved guide which is of interest is that one which, when represented as a sum of straight guide modes, has the greatest part of its power in the TE₀₁ mode portion of the sum. Thus the curved guide normal mode of interest is very similar to the TE₀₁ mode of the straight guide.

At a transition from a straight waveguide section to a curved waveguide section, such as between sections 12 and 14 of Fig. 1, the normal mode of straight guide section 12 (TE₀₁) will excite not only this normal mode but a series of other normal modes in curved section 14. This combination of normal modes propagates through section 14 and, at the transition between section 14 and straight section 13 excites not only the TE₀₁ mode but a series of other normal modes of the straight waveguide. The total power in modes other than the TE₀₁ mode in section 13 thus represents mode conversion loss introduced by curved section 14.

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As stated above, this mode conversion may be reduced by modifying section 14 in such a way as to remove the propagation constant equality between the TE₀₁ and TM₁₁ wave modes. However, mode conversion to TM₁₁ wave energy is not completely eliminated by such modification. In addition, mode conversion to modes other than TM₁₁ persists.

It is clear that a curved waveguide transition between angularly related straight waveguide sections which transforms the TE₀₁ normal mode of the straight waveguide into only one of the normal modes of the curved guide and, by the same token, transforms a particular normal mode of the curved waveguide into only the TE₀₁ normal mode of the straight guide will avoid all mode conversion losses. In accordance with the invention, such a waveguide transition can be realized by tapering the curvature of the curved waveguide section from zero at its junction with the straight waveguide. In Fig. 1, for example, if the curvature of bend 14 is tapered over its length from zero to a finite value and then back to zero, the normal TE₀₁ mode incident from straight section 12 will be gradually transformed into the particular normal mode of bend 14 which is most similar in field configuration to the circular electric wave. At each point along the taper, there is only one normal mode consistent with the value of curvature presented by the bend to the propagating wave energy as well as being similar in field configuration to the TE₀₁ mode. By gradually tapering the curvature of the bend the propagating wave energy is retained in the desired normal wave mode throughout the bend. Such a bend is the tapered curvature normal mode bend.

It is clear that the waveguide sections under consideration in this specification have physical dimensions large enough to permit more than one wave mode to propagate therein. In such a situation, the two or more wave modes which may propagate together have differing phase constants. This is true even for the TE₀₁-TM₁₁ phase constants in the modified bend section 14 from which their inherent degeneracy has been removed. Since the phase constant of the TM₁₁ as well as those of other modes capable of propagating in the circular waveguide are different from that of the TE₀₁, there will be a periodic phasal reinforcement or addition and a periodic phasal destruction or subtraction between the TE₀₁ and each of the unwanted higher order modes. Associated with each mode pair comprising the TE₀₁ and one of the unwanted higher order modes is a periodic distance between consecutive points of equal amplitude and phase. This distance is called the beat wave length, λ_B , for that particular mode pair. In order for the tapered curvature portion of the normal mode bend to function properly, its length z_1 should be of the order of or longer than the longest beat wave length associated with the structure. That is, since

$$\lambda_B = \frac{2\pi}{\Delta\beta} = \frac{2\pi}{\beta_{TE_{01}} - \beta_{TM_{mn}}} \quad (1)$$

$$z_1 \geq \frac{2\pi}{\beta_{TE_{01}} - \beta_{TM_{mn}}} \quad (2)$$

It can be seen that, for plain circular waveguide bends; that is, those in which the degeneracy between the TE₀₁ and TM₁₁ phase constant has not been removed, the normal mode taper would have to be infinite in length. Thus, a nondegenerate waveguide becomes an essential condition for a normal mode bend in accordance with the present invention.

Fig. 2 illustrates, in comparative relationship, the physical shape of an embodiment of the present invention together with physical shapes of the general types of possible prior waveguide bends. In Fig. 2, segments 20,

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21 represent angularly related straight waveguide sections to be joined by a curved waveguide section at junctions 22, 23. Sections 20, 21 are illustrated as being angularly related by 90 degrees but the invention may be utilized to connect waveguide sections of any angular relationship. In prior art waveguide bends, a curved section of constant bending radius was thought to be the ideal waveguide bend configuration for joining segments 20, 21. For example, in United States Patent 2,774,945, entitled "Methods and Apparatus for Transmitting Circular Electric Waves in Wave Guides," issued December 18, 1956, to S. E. Miller, bends of constant curvature, or constant bending radius, are utilized throughout. Such a bend is illustrated in Fig. 2 as arc 24 of circle 29 which has a radius R_1 .

In the prior art, almost exclusive attention was directed to the problem of maintaining a smooth change of direction of the longitudinal axis of curved waveguide sections. What was not realized is that the curvature associated with the waveguide bend should also be changed smoothly, that is, a smooth change in the degree of curvature is also desirable. Thus, in Fig. 2, even though straight sections 20, 21 are tangent to arc 24, thus providing a smooth change in direction, such an arc is of constant curvature while straight sections 20, 21 with which it joins are of zero curvature. An abrupt joinder of these sections results in a sharp change in the degree of curvature or a curvature discontinuity at junctions 22, 23. Such a discontinuity will cause the normal straight guide TE_{01} wave mode, propagating in straight section 20 for example, to be transformed into a plurality of curved waveguide normal modes upon entering arc 24 and into a plurality of straight waveguide normal modes upon exiting arc 24 and entering section 21. Such a transformation results in prohibitively high transmission loss.

Curved segments 25, 26 represent physical shapes of curved waveguide sections for which the linear dimension of the wave path is too short and too long, respectively, to form the constant curvature bend represented by arc 24. Segment 25 presents a curvature discontinuity at junctions 22, 23 even more severe than that of arc 24. Both arc 24 and segment 25 have a radius of curvature associated therewith at a location just beyond their junction points with the straight sections considerably less than does the tapered curvature normal mode bend in accordance with the present invention. Curved segment 26 presents, in addition to a curvature discontinuity at junctions 22, 23, a curvature characteristic which reverses sense twice over its longitudinal extent. That is, as curved segment 26 is traversed between angularly related straight sections 20, 21, proceeding, for example, in the direction from section 20 to section 21, the rotational sense associated with the curvature changes from counterclockwise to clockwise and subsequently from clockwise back to counterclockwise. A bend in accordance with the invention is described by a direction of curvature of constant sense.

In Fig. 2 curved segment 27 represents the physical shape of a tapered curvature normal mode bend in accordance with the invention. From Fig. 2, it may be seen that, given a pair of angularly related straight waveguide sections, the length of a tapered curvature bend or transition section joining them is greater than the length of a bend of constant curvature. However, the tapered curvature transition is not one into which a waveguide section will deform under the influence of the normal stresses and strains associated with the application of bending moments at the extremities of the section. In order that a tapered curvature bend in accordance with the invention be produced, it is essential that at least one fixed point apart from the section extremities be fixed. Such a fixed point is represented by element 28 in Fig. 2. Fixed point or support 28 provides an externally applied force to section 27 outwardly

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directed with respect to the curvature at its area of contact with the waveguide section.

The midpoint of this area of contact represents a point of maximum curvature of the bend. Circle 31, which has a radius R_2 is seen to be tangent to segment 27 at point 28. Since $R_2 < R_1$, it is clear that the tapered curvature bend has a radius of curvature at its center point considerably less than that of circular arc 24 which is tangent to straight waveguide sections 20, 21 at junctions 22, 23. As will become more evident hereinafter, a tapered curvature waveguide bend may be formed about a single fixed point support, about a plurality of such supports, or about a single circular form of constant radius. In all cases, the additional force or forces provided by these supports produces the novel curvature by which the present invention is distinguished from random prior art waveguide bends.

Fig. 3 illustrates the length versus curvature characteristic of a circular arc of constant bending radius such as for example, arc 24 of Fig. 2. At points 22, 23, representing junctions 22, 23 of Fig. 2, the curvature suffers an abrupt discontinuity. It is this type of discontinuity and its associated mode conversion effects which the present invention eliminates.

Fig. 4 illustrates the length versus curvature characteristic of the tapered curvature normal mode bend embodiment of optimum design from an electrical transmission viewpoint. Such a characteristic may be derived mathematically by considering coupling in a waveguide bend between the TE_{01} and only one of the curved section normal mode composite modes at a time. The optimum curvature function derived is the Fourier transform of a Tschebyscheff polynomial of infinite degree.

Mathematically, such a curvature may be expressed as a function of its length x as

$$k = \frac{k_0}{2} \left(1 + \frac{1}{\cosh A} \left[A^2 \int_0^x \frac{I_1(A\sqrt{1-y^2})}{A\sqrt{1-y^2}} dy \right] \right)$$

where

I_1 = modified Bessel function, and

$$A = \Delta\beta \frac{l}{4}$$

where $\Delta\beta$ is the smallest difference in phase constants and l is the total axial length of the tapered curvature portions of the bend.

The curvature characteristic shown in Fig. 4 as curve 30 is substantially a graphical representation of this Fourier transform.

Fig. 5 shows a physical waveguide embodiment of a tapered curvature bend having the optimum curvature characteristic of Fig. 4. Since this curvature characteristic is embodied in a structure which has a physical shape different from those described above in connection with Fig. 2, it is essential to the fabrication of such a specialized bend configuration to utilize a physically constraining bending form.

In Fig. 5, such a form comprises rectangular sheet or slab 40 which has template 41 and clamping supports 42 disposed on its surface. Template 41 is designed to have a physical shape corresponding to the particular bend shape desired. In Fig. 5, template 41 is shaped to possess the curvature characteristic of Fig. 4. Waveguide section 43 is placed over template 41 and physically deformed to conform to the shape thereof. Clamping supports 42 are then fastened to slab 40 in order to preserve the bend configuration. Template 41 and supports 42 apply external forces to waveguide section 43 different from the internal stresses and strains which are associated with unconstrained bending.

Figs. 6A and 6B are length versus curvature characteristics of tapered curvature normal mode bend embodi-

ments of optimum design from a mechanical or fabrication viewpoint. Referring specifically to Fig. 6A, a waveguide section described by the curvature characteristic represented thereon comprises two distinct regions. The first of these regions, represented by segment 50, is described by a curvature which tapers linearly from zero to a finite value k_0 . The second region, represented by segment 51, is the reverse of segment 50 and is described by a curvature which tapers linearly from value k_0 to zero. The total length of the curved section comprising curvature segments 50, 51 is defined as l and each of the regions of tapered curvature has a length z_1 . Referring now to Fig. 6B, a waveguide section described by the curvature characteristic represented thereon comprises three distinct regions. The first of these regions, represented by segment 52 is described by a curvature which tapers linearly from zero to a finite value k_0 . The second region, represented by segment 53, is described by a constant curvature k_0 over its length. The third region, represented by segment 54, is the reverse characteristic of segment 52 and is described by a curvature which tapers linearly from value k_0 to zero. The total length of the curved section is again defined as l and each of the regions of tapered curvature has a length z_1 . For a given bending angle, that is the angle through which the direction of the waveguide line is desired to be changed, an optimum total length l may be derived. The derivation of such an optimum relationship will be presented in a later portion of this specification.

The curvature characteristic of Fig. 6B is realized in the waveguide embodiment illustrated in Fig. 7. In Fig. 7, waveguide section 60 comprises a conductively bounded hollow round pipe from which the TE_{01} - TM_{11} phase constant degeneracy has been removed. This pipe is bent around circular form 61 in a manner not exceeding the elastic deformation limits of the physical waveguide. Form 61 is illustrated as being circular. The important property of form 61 is that it has a constant curvature over its area of contact with waveguide 60. The radius R_0 of form 61 thus becomes the minimum bending radius of waveguide section 60. Bending is preferably accomplished by applying forces F , indicated by arrows 62, 63 at the extremities of waveguide section 60. These inwardly directed forces, which are caused to act upon both ends of section 60 produce, with form 61, a torque which bends section 60 in a manner producing the curvature versus length characteristic of Fig. 6B. Essentially, form 61 provides the outwardly directed force noted above which imparts the tapered curvature characteristic to section 60. Thus, from the left end of section 60 to the point of contact 64 between guide 60 and form 61, a distance 31, the pipe curvature increases linearly from zero (descriptive of straight waveguide) to the constant value associated with form 61 over its area of contact with section 60. Between points 64 and 65 waveguide 60 and form 61 are in mutual contact. Thus the curvature of waveguide 60 has a constant value over this interval. From point of contact 65 to the right end of section 60, again a distance z_1 , the curvature decreases linearly from the constant value associated with form 60 to zero. For an embodiment of the invention in which the constant curvature of form 61 is defined as k_0 and the point of contact between form 61 and section 60 along the length l of said section is defined as z_1 , the curvature k at any point along the linear taper is

$$k = k_0 \frac{z}{z_1} \quad (3)$$

The curvature expressed in (3) is also descriptive of that associated with a waveguide bend having the curvature characteristic of Fig. 6A. The fabrication of the embodiment of Fig. 6A is substantially similar to that

shown for Fig. 6B. That is, in Fig. 7, the region of contact of guide 60 and form 61 is reduced substantially to zero. A point force is outwardly applied in place of the distributed force provided by form 61. In practice this could be accomplished by bending a waveguide section over a fixed pin having a radius negligible with respect to the length of the waveguide bend. Bending would be accomplished through the application of forces, identical to forces 62, 63 of Fig. 7, at the extremities of the waveguide section to be bent.

Mathematically, a curve described by the curvature expressed in (3) is known as Cornu's spiral. As already noted, the length of the tapered curvature sections in a waveguide embodiment of the invention must be of the order of or greater than the largest beat wave length associated with the curved waveguide section.

Practically, there is a minimum total length of the curved section imposed by the requirement that the bending may not exceed the elastic deformation limits of the guide for the fabrication method shown in Fig. 7. That is, for a given pipe, the shortest allowable bending radius $R_{\min}$ for form 61 is

$$R_{\min} = \frac{Er_m}{f_{\max}} \quad (4)$$

where

$f_{\max}$ = flexural stress at elastic limit

E = modulus of elasticity of pipe

r_m = outside radius of pipe

For a given specified angle θ_0 through which the direction of a waveguide line is desired to be changed, the minimum bending radius defined in (4) above requires a minimum total length of waveguide $l_{\min}$ of

$$l_{\min} = 2\theta_0 R_{\min} \quad (5)$$

However, in the event that it is desired that the elastic deformation limits of the guide be exceeded, a bending method similar to that shown in Fig. 5 may be employed. Thus, a constraining template and clamping supports would be mounted on a solid sheet of material and the waveguide section to be bent would be placed within the constraints. The general relationship of forces necessary to the invention, that is, outwardly directed at the center of the curved member and inwardly directed at its extremities, would remain.

When a constraining form is used and the elastic limit of the waveguide is exceeded, the bending form may be removed after bending is completed. However, there will be a certain "spring back" factor associated with the metal pipe and this must be taken into account when the layout of the constraints is planned.

As above noted, it has been found that, for a given bending angle θ_0 , a bend geometry which minimizes total bend loss can be determined. In general, the total bend loss A in a normal mode bend can be expressed as

$$A = C_1 l + C_2 \left[\frac{1-4u}{3} \right] + C_3 \left[\frac{1}{l^4(u-u^2)^2} \right] \quad (6)$$

in which C_1 , C_2 , and C_3 are quantities depending on total bending angle, physical waveguide parameters and frequency; and

$$u = \frac{z_1}{l}$$

where z_1 is the length of one tapered curvature transition section and l is total bend length.

Since the quantities C_1 , C_2 and C_3 depend upon the physical parameters of the waveguide bend it is evident that they are in part dependent upon the choice of a particular one of the above-mentioned techniques for removing the TE_{01} - TM_{11} phase constant degeneracy in plain

waveguide. Thus, for example, when waveguide with a dielectric liner of thickness t is used to form the normal mode bend

$$\left. \begin{aligned} C_1 &= \alpha_{01} \left[1 + (\epsilon' - 1) \frac{p_{01}^2 \delta^2}{v_{01}^2} \right] + \frac{p_{01}^2 \epsilon'' \beta_{01}}{3(1 - v_{01}^2)} \delta^3 \\ C_2 &= \sum \frac{c'^2}{\Delta \beta^2} \theta_0^2 (\alpha_{1m} - \alpha_{01}) \\ C_3 &= \sum \frac{c'^2}{\Delta \beta^4} \theta_0^2 \end{aligned} \right\} \quad (7)$$

where

α_{01} = attenuation constant of the TE₀₁ wave mode in plain waveguide,

α_{1m} = attenuation constant of the TE_{1m} or TM_{1m} wave mode in plain waveguide,

$\epsilon = \epsilon' - j\epsilon''$, the real and imaginary parts respectively of the relative dielectric permittivity ϵ of the dielectric liner, $p_{01} = 3.83170$,

$$v_{01} = \frac{\lambda}{\lambda_c} = \frac{p_{01} \lambda}{2\pi a}$$

the cut off factor in a plain waveguide of radius a ,

$$\delta = \frac{t}{a}$$

relative thickness of dielectric coat,

β_{01} = plain waveguide phase constant of TE₀₁ waves,

$\Delta\beta$ = difference in phase constant between the TE₀₁ and any of the coupled modes, and

c' = factor in the curvature coupling coefficient $c = c'k$ which depends on physical waveguide parameters and frequency.

The summation signs in Equations 7 indicate that all coupled modes should be taken into account.

The necessary conditions for $A(u, l)$ to be a minimum are

$$\frac{\partial A}{\partial u} = \frac{2(1-2u)}{l(1-u^2)} \left[\frac{C_2}{3} - \frac{C_3}{l^2 u^3} \right] = 0 \quad (8)$$

and

$$\frac{\partial A}{\partial l} = C_1 - C_2 \left[\frac{1 - \frac{4}{3}u}{(1-u^2)^{2/3}} \right] - C_3 \left[\frac{4}{(u-u^2)^{2/3}} \right] = 0 \quad (9)$$

Sufficient conditions for $A(u, l)$ to be a minimum are

$$\frac{\partial^2 A}{\partial u^2} \frac{\partial^2 A}{\partial l^2} - \left[\frac{\partial^2 A}{\partial u \partial l} \right]^2 > 0 \quad (10)$$

$$\frac{\partial^2 A}{\partial u^2} > 0; \quad \frac{\partial^2 A}{\partial l^2} > 0$$

Defining

$$w = \frac{l}{2} \left[\frac{C_1}{C_2} \right]^{1/2} \quad (11)$$

$$y = \left[3 \frac{C_3}{C_2} \right]^{1/3} \left[\frac{C_1}{C_2} \right]^{1/2} \quad (12)$$

the solutions of (9) become the roots of

$$w^3(3w^2 - 1) - 2y^3 = 0 \quad (13)$$

To find the optimum bend geometry for a given dielectrically lined normal mode bend having a specified bending angle, y is first calculated from (12). If $y > 1$, the optimum geometry is

$$l = 2 \sqrt{\frac{C_2}{C_1}} w$$

$$z_1 = \sqrt{\frac{C_2}{C_1}} w$$

where w is the positive root of (13) for the calculated y . If $y < 1$, the optimum geometry is

$$\left. \begin{aligned} l &= \left[3 \frac{C_3}{C_2} \right]^{1/3} + \left[\frac{C_2}{C_1} \right]^{1/2} \\ z_1 &= \left[3 \frac{C_3}{C_2} \right]^{1/3} \end{aligned} \right\}$$

The normal mode bend is inherently a broad band device. Some terms contributing to total bend loss decrease with frequency while others increase. The over-all frequency dependence, other than the oscillations of the mode conversion portion of the total loss as caused by spurious mode phasing, is of the same order as is the frequency dependence of the loss in the straight waveguide.

In all cases, it is understood that the above-described arrangements are illustrative of only some of the many possible specific embodiments that represent application of the principles of the invention. Numerous and varied other arrangements can readily be devised in accordance with these principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. A tapered curvature normal mode waveguide bend having first and second terminal ends for transmitting the circular electric wave mode between angularly related waveguide paths, said bend comprising a section of bounded waveguide which is curved with a constant sense as the bend is traversed and in which the TE₀₁ and TM₁₁ wave modes have different propagation constants, the tapered curvature portions of said section each having a length greater than the longest beat wavelength associated with said section and each tapering between zero curvature at said terminal ends and a given curvature.

2. A transmission system having first and second terminal ends for electromagnetic wave energy in the circular electric wave mode, a source of said wave energy connected to said first terminal end, utilizing means for said energy connected to said second terminal end, and at least one curved section of nondegenerate circular waveguide between said source and said utilizing means, said section having at least one region of curvature tapering smoothly in one sense from zero over a length greater than the longest beat wavelength associated with said section.

3. A transmission system for electromagnetic wave energy in the circular electric wave mode comprising first and second angularly related straight waveguide sections joined by a smoothly curved waveguide section which is curved in one sense between said straight sections, said curved section having a radius of curvature at a location just beyond the junction with said first straight section considerably greater than that associated with a circular arc mutually tangent to said straight sections, said curved section having a radius of curvature at the center thereof considerably less than that associated with said arc, said curved section having a radius of curvature at a location just before the junction with said second straight section considerably greater than that associated with said arc, said curved section having a length between said junction and said center greater than the longest beat wavelength associated with said section.

4. The waveguide bend according to claim 1 in which said tapered curvature portions are joined by a section of waveguide having said given curvature over its length.

5. The waveguide bend according to claim 1 in which said tapered curvature portions have a physical contour whose curvature is described by the Fourier transform of a Tschebyscheff polynomial of infinite degree.

References Cited in the file of this patent

UNITED STATES PATENTS

2,649,578 Albersheim ----- Aug. 18, 1953
 2,774,945 Miller ----- Dec. 18, 1956

FOREIGN PATENTS

732,443 Great Britain ----- June 22, 1955

OTHER REFERENCES

Collin: "The Optimum Tapered Transmission Line Matching Section" in Proceedings of the IRE, April 1956, vol. 44, No. 4, pages 539-548.

Klopfenstein: Proceedings of the IRE, January 1956, pages 31-35.