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QUARTER SQUARE ELECTRIC VOLTAGE MULTIPLIER

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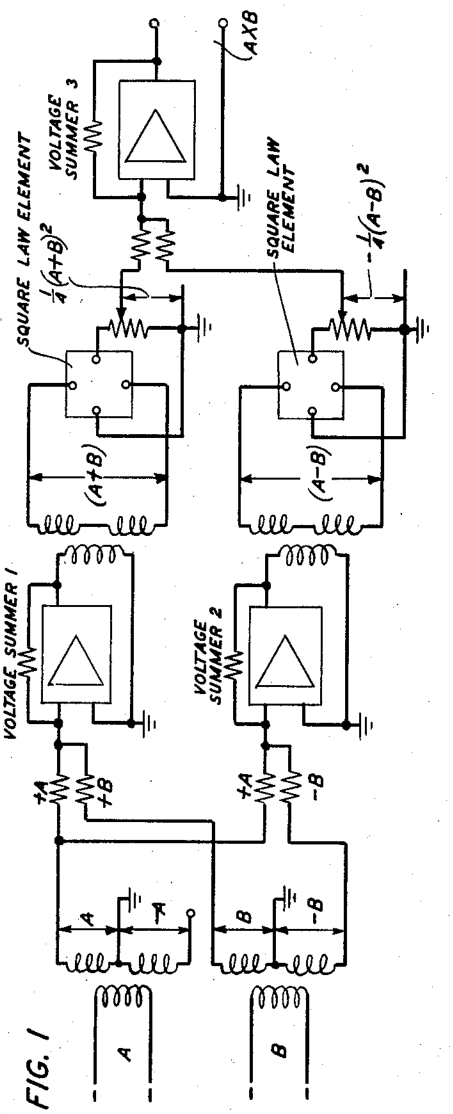

FIG. 1

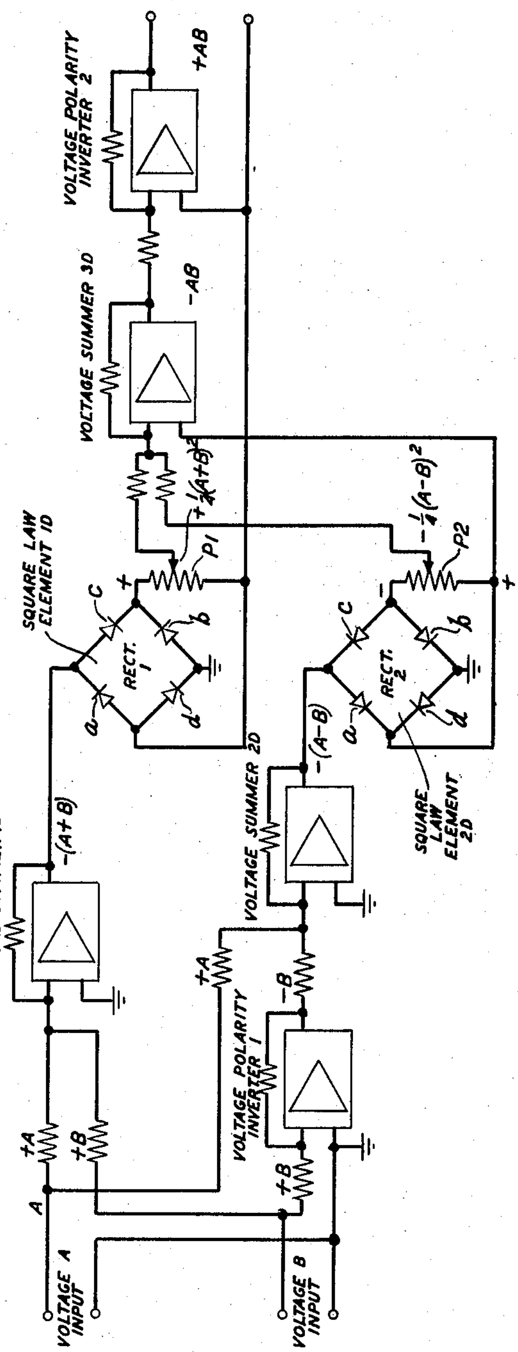

FIG. 2

INVENTOR
C. A. LOVELL
BY
John E. Cassidy
ATTORNEY

United States Patent Office

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QUARTER SQUARE ELECTRIC VOLTAGE MULTIPLIER

Clarence A. Lovell, Summit, N.J., assignor to Bell Telephone Laboratories, Incorporated, New York, N.Y., a corporation of New York

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7 Claims. (Cl. 235—194)

This application is a continuation-in-part of application Serial No. 1,471, filed January 9, 1948, now Patent No. 2,658,189, which application was copending with the present application, and claim is made to all of the equitable and legal benefits which flow from such fact.

An object of the present invention is to provide means and method for multiplying voltages so as to obtain their product.

A more particular object of the invention is to provide an arrangement for multiplying voltages so as to obtain their product by employing electrical apparatus to implement the algebraic identity

$$\frac{1}{4}[(A+B)^2-(A-B)^2]=AB$$

A basic feature of this invention is a method of multiplying voltages; more accurately expressed perhaps, a method of causing equipment to produce a voltage which is at least a close approximation to the product of two applied voltages.

A feature of the invention is an electronic voltage multiplier which may be used, for example, in a system of electrical transmission disclosed in Patent No. 2,658,189, issued November 3, 1953 upon the application above identified. The electronic voltage multiplier may be applied generally wherever there is need of multiplying voltages and comprises one species of apparatus for executing the method. The voltages which may be multiplied may be either direct-current voltages or alternating-current voltages.

The invention may be understood from the following description when read with reference to the associated drawing in which:

Fig. 1 shows a circuit for multiplying alternating-current voltages; and

Fig. 2 shows a circuit for multiplying direct-current voltages.

The circuit of Fig. 1 herein corresponds to the circuit of Fig. 3 of the application identified in the foregoing.

Electronic multiplier

Most modulators are approximate multipliers in which the pure product, although not accurately scaled, is included among the outputs. For most purposes they are made satisfactory through the process of using frequency selective networks to pass the desired product and to exclude all other terms which appear in the output.

The present invention is a method of electronic multiplication which involves a voltage adder or voltage summer, as it is sometimes called, and a non-linear element. The element is chosen so that it has substantially a square law characteristic, so that the multiplication is substantially precise.

The operation of the circuit of the drawing is based on the algebraic identity

$$A\times B\equiv\frac{1}{4}[(A+B)^2-(A-B)^2]$$

Refer now to Fig. 1 of the drawing. In Fig. 1 an alternating-current voltage A is impressed on the primary of the transformer shown at the upper left. The mid-point of the secondary winding is grounded. A voltage +A is produced in the upper winding of the secondary and impressed on voltage summer 1, which is a device for summing up a plurality of voltages. This device is well known in the art being disclosed, for instance, in Patent No. 2,408,081 granted to Lovell-Parkinson-Swartzel-Weber, September 24, 1946, and more particularly in Fig. 12 of said patent. See also Vacuum Tubes and Amplifiers, Radiation Laboratory Series, vol. XVIII by McGraw-Hill Book Company 1948, edited by Valley and Wallman; and Electronic Instruments, Massachusetts Institute of Technology, Radiation Laboratory Series, vol. 21, McGraw-Hill Book Company 1948, Greenwood, Holdam and MacRae, page 32.

The voltage +A is also applied to the input of an identical voltage summer, voltage summer 2. The alternating-current voltage B is impressed on the primary of the transformer shown at the lower left in the drawing, the mid-point of the secondary of which is also grounded. A voltage +B is produced in the upper portion and a voltage —B is produced in the lower portion of the secondary of the lower left-hand transformer. The voltage +B is applied to the input of the voltage summer 1 and the voltage —B is applied to the input of the voltage summer 2. Thus, voltage +A and voltage +B are applied to the input of voltage summer 1 and voltage +A and voltage —B are applied to the input of voltage summer 2. Voltage $A+B$ is produced across the secondary of the transformer connected to the output of voltage summer 1 and the voltage $A-B$ is produced across the secondary of the transformer connected to the output of voltage summer 2. These voltages $A+B$ and $A-B$ are each applied to the input of an individual non-linear conducting device selected to have a square law characteristic, a number of which are well known in the art. This serves as a voltage squarer. The output of each voltage squaring device is impressed across an individual potentiometer. The polarities of the non-linear elements as disposed in the voltage squarers are arranged so that the polarities of the potentials across the potentiometers are reversed, one with respect to the other. The associated sliding contact is adjusted to provide a voltage of positive ¼ $(A+B)^2$ from the upper potentiometer and negative ¼ $(A-B)^2$ from the lower potentiometer. These two voltages are impressed on the input of voltage summer 3, which provides a voltage equal to the sum of the input voltages which is equal to the product of the original voltages A and B or AB.

The algebra is quite simple:

Squaring each of the sums

$$+(A+B)^2=A^2+2AB+B^2$$
$$-(A-B)^2=-A^2+2AB-B^2$$

Dividing by 4 or quartering

$$\frac{A^2}{4}+\frac{2AB}{4}+\frac{B^2}{4} \qquad (a)$$

$$-\frac{A^2}{4}+\frac{2AB}{4}-\frac{B^2}{4} \qquad (b)$$

Adding (a) and (b)

$$\begin{array}{r}\frac{A^2}{4}+\frac{2AB}{4}+\frac{B^2}{4}\\ -\frac{A^2}{4}+\frac{2AB}{4}-\frac{B^2}{4}\\ \hline +AB\end{array}$$

No circuit elements are available in which the outputs E_0 are related to the input E_i precisely by the relation

$$E_o=kE_i^2$$

where k is a constant. However a large number of elements are found having substantially this response. The utilization of such elements as square law detectors, such as described herein, is well known in the art, as illustrated in Torrey and Whitmer, Crystal Rectifiers, 15 M.I.T. Radiation Laboratories Series, 3,333 (New York 1948). The present full wave rectification circuit with a voltage divider output, permits adjustments to improve the approximations inherent in such elements. Typical of such adjustments are those which may be made to make the output zero when one of the inputs is made zero or one which will make the output precise when A and B are set in given values.

For the non-linear elements, copper oxide, vacuum tubes and so forth, offer fairly good approximations over limited ranges. Thus, the circuit of the drawing may be constructed from conventional components.

In the arrangement shown in the circuit per Fig. 1, the voltages A and B as mentioned are alternating-current voltages and are shown as applied through transformers.

Refer now to Fig. 2 which shows a circuit for multiplying two direct-current voltages A and B. The circuits per Fig. 1 and Fig. 2 functionally, and with respect to the apparatus elements employed, are essentially the same, in that they both perform multiplication by implementing the operations implicit in the algebraic formulas presented in the foregoing and they both employ circuits similarly arranged and having voltage summers, voltage squarers and voltage quarterers disposed in the same manner. One important difference is in the grounding arrangements. Other differences are that in Fig. 2 an electronic device, voltage polarity inverter **1**, is indicated for reversing the polarity of voltage B and another, voltage polarity inverter **2**, for reversing the polarity of the output of voltage summer 3D. It will be observed that in a direct-current circuit the polarity of the output of the summers is the opposite from that of the input. It will be observed also that the polarity of the output of each of the summers 1D and 2D is negative.

Fig. 2 shows in detail the manner in which the polarity of the output of two squaring devices may be reversed one with respect to the other. It will be observed that the polarity of each dry rectifier element of rectifier **2** has its polarity reversed with respect to each corresponding dry rectifier element of rectifier **1**. The elements as thus disposed afford a path for the negative potential applied to the respective top terminals of the rectifiers. In the case of rectifier **1** the circuit extends from the top terminal of rectifier **1** through rectifier element a, potentiometer P1 and rectifier element b to ground. The bottom terminal of potentiometer P1 is negative with respect to its upper terminal. For rectifier **2** the path for negative potential applied to its top terminal is through rectifier c, potentiometer P2 and rectifier d to ground, so that the polarity of the top terminal of potentiometer P2 is negative with respect to its lower terminal. Therefore, the polarities of the potentials between the slider of each potentiometer and the common lead forming the input to voltage summer 3D will be positive from potentiometer P1 and negative from potentiometer P2.

The voltage summer 3D will add these voltages and reverse the polarity of the added voltages. This gives the negative product of voltage A and voltage B as a result. This negative product is inverted in voltage polarity inverter **2** to afford the positive desired product voltage AB.

What is claimed is:

1. A voltage multiplier, said multiplier comprising a first summing device for adding voltages A and B, a second summing device for adding voltage A and voltage —B, a first square law element which substantially squares voltage $(A+B)$, a second square law element which substantially squares voltage $(A-B)$, means for reversing the polarity of one of said squared voltages with respect to the other, means for quartering each of said squared voltages, one having its sign so reversed, and a third summing device for adding said quartered voltages to provide a voltage substantially equal to the product of voltage A and voltage B.

2. Apparatus for multiplying electrical voltages A and B comprising means for impressing voltages +A and +B upon the input of a first electronic voltage summer to produce voltage $(A+B)$ in the output of said voltage summer, means for impressing voltages +A and —B upon the input of a second electronic voltage summer to produce voltage $(A-B)$ in the output of said second voltage summer, a first voltage squaring element for squaring voltage $(A+B)$, a second voltage squaring element for squaring voltage $(A-B)$, means for reversing the sign of the polarity of one of said squared voltages with respect to the other, a first voltage quartering device for quartering one of said squared voltages, a second voltage quartering device for quartering the other of said squared voltages having its sign so changed and a third electronic voltage summer for adding said quartered voltages to obtain a voltage equal to the product of voltage A and voltage B.

3. In an electrical circuit, a first circuit branch, a first non-linear element having a substantially square law characteristic in said branch, a first potentiometer connected to said non-linear element, a second circuit branch in parallel with said first circuit branch, a second non-linear element having a substantially square law characteristic in said second branch, a second potentiometer connected to said second non-linear element, a common circuit path having a potential responsive device connected to each of said potentiometers, and instrumentalities in said non-linear elements for reversing the polarity of one of the potentials derived from one of said potentiometers with respect to the other of said potentials derived from the other of said potentiometers.

4. An electrical circuit for multiplying two direct-current voltages, voltage +A and voltage +B, to obtain voltage AB, said circuit having a first and a second branch connected in parallel, said branches connected to a third common series branch, a first voltage summer, a first square law element and a first potentiometer connected in series in said first branch, a first voltage polarity inverter, a second voltage summer, a second square law element and a second potentiometer connected in series in said second branch, a third voltage summer and a second voltage polarity inverter connected sequentially in series in said third branch, means for impressing said voltage +A and said voltage +B on said first voltage summer and means responsive thereto for producing a voltage $-(A+B)$, means for impressing said voltage $-(A+B)$ on said first square law element and said first potentiometer and means responsive thereto for producing a voltage $+\frac{1}{4}(A+B)^2$, means for impressing said voltage +B on said first voltage polarity inverter and means responsive thereto for producing a voltage —B, means for impressing said voltages +A and —B on said second voltage summer and means responsive thereto for producing a voltage $-(A-B)$, means for impressing said voltage $-(A-B)$ on said second square law element and said second potentiometer and means responsive thereto for producing a voltage $-\frac{1}{4}(A-B)^2$, means for impressing said voltages $+\frac{1}{4}(A+B)^2$ and $-\frac{1}{4}(A-B)^2$ on said third voltage summer and means responsive thereto for producing a voltage —AB, means for impressing said voltage —AB on said second voltage polarity inverter, and means responsive thereto for producing a voltage +AB, each of said square law elements a bridge having four arms and a dry rectifier in each of said arms, each dry rectifier in each of said arms of said second square law element having its polarity reversed with respect to each dry rectifier in the corresponding arm of said first square law element.

5. An electrical circuit for multiplying two alternating voltages, voltage +A and voltage +B, to obtain voltage +AB, said circuit comprising a source of voltage A connected to the input of a first transformer, the output of said first transformer connected in parallel to a first voltage summer in a first parallel circuit branch and to a second voltage summer in a second parallel circuit branch, a source of voltage +B connected to the input of a second transformer, the output of said second transformer having two windings with a common terminal grounded, means for producing a voltage +B and −B in the output of said second transformer, means for impressing said voltage +B on the input of said first voltage summer, means for impressing said voltage −B on the input of said second voltage summer, a first square law element connected to the output of said first voltage summer, a first potentiometer connected to the output of said first square law element, a second square law element connected to the output of said second voltage summer, a second potentiometer connected to the output of said second square law element, the output of said first potentiometer and of said second potentiometer connected in parallel to the input of a third voltage summer, means responsive to the impressing of said voltages +A and +B on said first voltage summer for producing a voltage $+(A+B)$, means responsive to the impressing of said voltage $+(A+B)$ on said first square law element for producing a voltage $+(A+B)^2$, means for adjusting said first potentiometer to produce a voltage $+\frac{1}{4}(A+B)^2$, means responsive to the impressing of said voltages +A and −B on said second voltage summer for producing a voltage $+(A-B)$, means responsive to the impressing of said voltage $+(A-B)$ on said second square law element for producing a voltage $+(A-B)^2$, means in said second potentiometer for producing a voltage $-\frac{1}{4}(A-B)^2$, and means in said third voltage summer responsive to the impressing thereon on said voltages $+\frac{1}{4}(A+B)^2$ and $-\frac{1}{4}(A-B)^2$ for producing a voltage AB.

6. An electrical circuit having inputs to first and second parallel circuit branches, each of said branches comprising in series, an individual voltage summer, a rectifier bridge circuit having four arms, each arm having a square law dry rectifier element therein, an individual voltage divider, and means for reversing the polarity of the output of one of said branches with respect to the polarity of the output of the other; said parallel branches connected in series with a single voltage summer in a common circuit.

7. A circuit in accordance with claim 6 having a voltage polarity inverter connected to the output of said single voltage summer in said common circuit.

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