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TRAVELING WAVE TYPE ELECTRON DISCHARGE DEVICES

Filed June 15, 1951

3 Sheets-Sheet 1

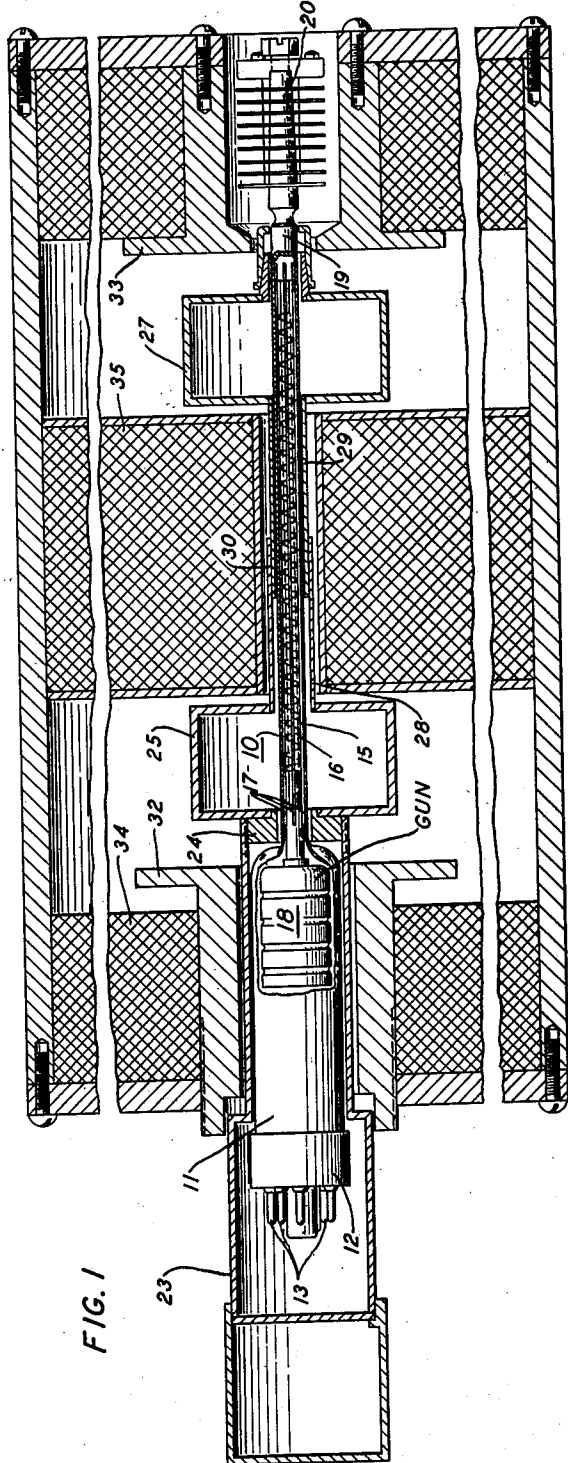


FIG. 1

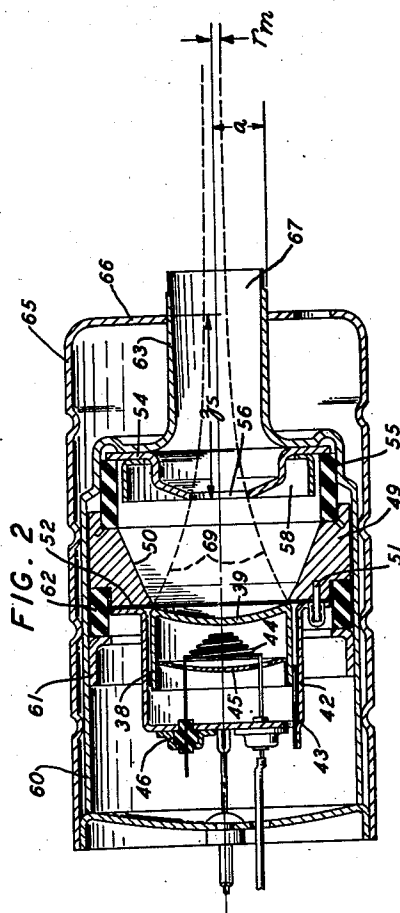


FIG. 2

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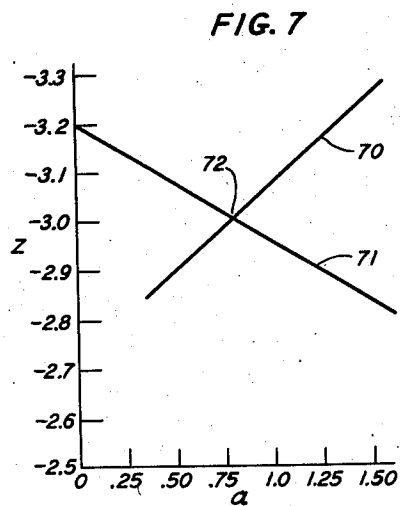
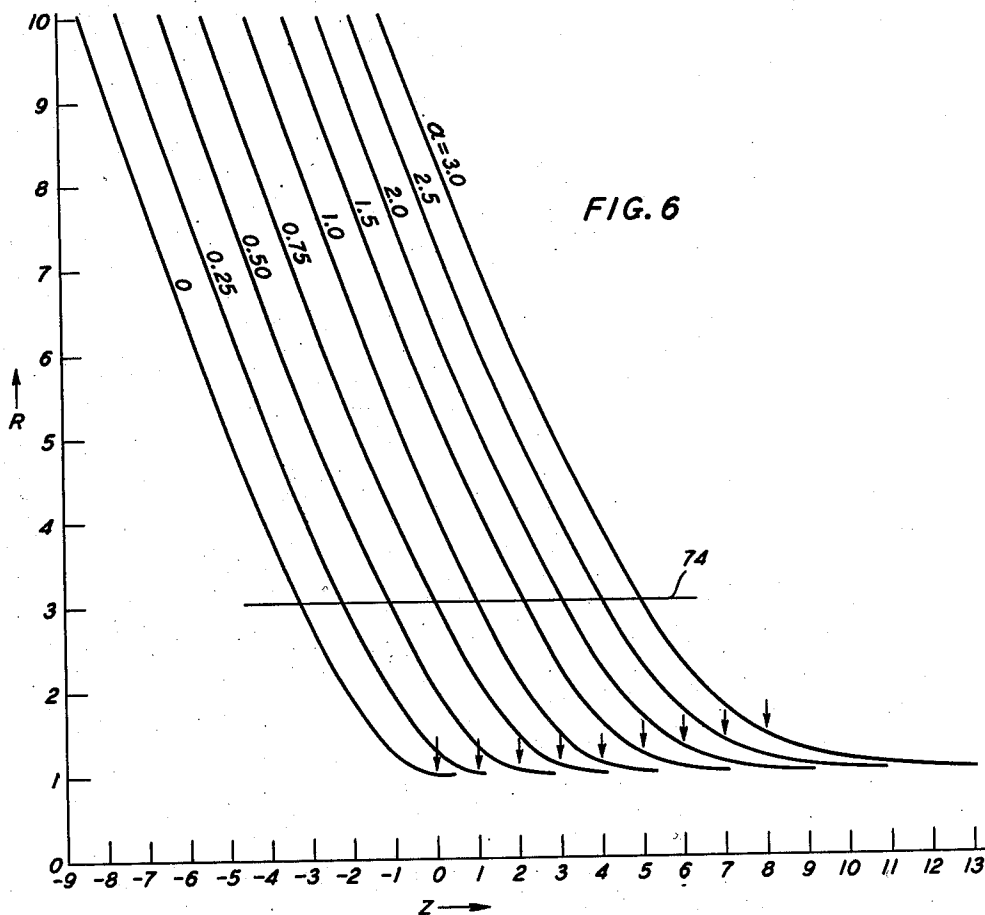
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3 Sheets-Sheet 3



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TRAVELING WAVE TYPE ELECTRON DISCHARGE DEVICES

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1 Claim. (Cl. 315—3.5)

This invention relates to electron discharge devices and more particularly to such devices known as traveling wave tubes wherein an electron beam is projected through an elongated helical conductor.

In such traveling wave tubes it is desired to produce and utilize long electron beams of small diameter. Since it is well known that in the absence of applied field, electron beams will spread radially because of space charge effects, a magnetic field is applied parallel to the axis of the beam to counteract the space charge effects. It is highly desirable, however, that the beam be in a perfectly circular cylindrical form with the constant diameter electron beam rotating about its axis, as a solid, while traveling in the axial direction at a constant velocity. For each electron the radially outward forces due to space charge and centrifugal force are exactly balanced by the radially inward force due to the magnetic field. Thus the electron beam is confined by the axially symmetrical magnetic field which, over a portion of the length of the beam, has a substantially constant strength, and the strength of which varies with distance outside of this region. Particularly, the strength of the field in the region of the electron gun is advantageously zero.

It has been indicated by theory that a perfectly cylindrical electron flow in the region of constant magnetic field might be obtained when a parallel beam of electrons enters a completely abruptly starting magnetic field. Such a field could be provided theoretically by an imperforate magnetic shield of infinite area. But it is obvious that this is a theoretical ideal incapable of attainment, as an aperture of finite size must be provided in the shield to allow for passage of the electron beam from the electron gun to the magnetic field. The field therefore cannot be infinitely abrupt, but instead has a definite buildup characteristic.

The electrons in leaving the gun, where there are no magnetic fields, will follow a path whose contour is known in the art as the universal beam spread curve. In the absence of any applied fields the beam contour reaches a minimum value, whose radius is r_0 , and then diverges. However, in accordance with this invention, before the beam minimum is reached, but while the electrons are following the universal beam spread curve, the electrons enter the region of gradually increasing magnetic field, and enter in a specific manner so that when the beam is in the region of substantially constant magnetic field strength the beam is of substantially constant diameter. Priorly the electron gun and magnetic shield were so assembled and positioned, with the magnetic shield in the vicinity of the beam minimum, that the electron beam in the uniform magnetic field had a scalloped or wavy outline, which deviations from a constant diameter beam are known as perturbations. But, as explained above, it is highly desirable in such devices, in order to obtain the smallest possible electron beam for a given current, voltage and magnetic field strength in the region of uniform magnetic field, that the electrons travel in this region with a con-

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stant radius from the axis of symmetry. We have found that this can be attained by making the parameters of the electron gun and its spacing from the region of commencement of magnetic field assume certain specific values which are definable in terms of and are calculable from the beam size, current, voltage and the magnetic field in the region of uniform magnetic field, and the specific form of variation of the magnetic field with distance along the axis outside the region of uniform magnetic field.

In one specific embodiment of this invention, we have found that the transitional section of the beam between the portion following the universal beam spread curve and the portion of constant diameter in the constant magnetic field is under the control of three parameters which are the magnetic field strength, the position of the magnetic shield relative to the anode of the electron gun, and the size of the aperture in this shield through which the electron beam passes. When these parameters are related to each other in a particular manner, the electrons will go from the universal beam spread curve to the desired constant diameter.

When these parameters are not so related, which relationship is fully set forth below, perturbations or sinusoidal variations in the beam diameter occur in the electron flow, which perturbations can be so severe as to cause electrons to impinge on the rods supporting the helix of the traveling wave tube, so that the period of the sinusoidal variations can be visually observed by the spots of fluorescence which appear on the rods when the electrons strike them. The amplitude of these variations can be decreased by increasing the magnetic field strength, but these perturbations cannot be removed from the beam.

The conditions for an electron beam of constant diameter in an axial magnetic field of constant strength are known. Thus if the beam is to be of constant diameter in the region of constant magnetic field, then the current I , the accelerating voltage V_0 , the beam radius r_m and magnetic field strength in the region of constant field, B_0 , all measured in MKS units, must satisfy the relation

$$I = \frac{\pi \epsilon \eta^{3/2}}{\sqrt{2}} B_0^2 V_0^{1/2} r_m^2 = 1.45 \times 10^6 B_0^2 V_0^{1/2} r_m^2 \quad (1)$$

This is Equation 9.29 of the book "Theory and Design of Electron Beams" by J. R. Pierce, Van Nostrand, 1949, but with a different symbol for the beam radius in the region of the constant field. But even though these conditions are met, if the electron gun and magnetic shield parameters are not related so as properly to introduce the electron beam into the region of constant field, a constant diameter beam cannot be obtained, as perturbations will be present. Priorly, attempts were made to reduce these perturbations by employing an external shield whose position was varied until experimental evidence was obtained indicating that the electron beam appeared to approach a cylindrical form, which indication was given by minimum electron current in the helix of the tube. It is obvious, however, that such a technique, while appropriate to laboratory experimentation, would greatly hinder the commercial production of traveling wave tubes. It is therefore advantageous to position the magnetic shield, with appropriate size aperture, within the tube envelope so that the tube may be fabricated as a single complete unit capable of immediate application for the purpose intended.

It is also highly advantageous to be able to manufacture traveling wave tubes wherein the parameters of the transitional beam section necessary for a constant diameter electron beam have been incorporated into the tube in the proper relationship when it is intended to employ permanent magnets, rather than coil structures, to provide the magnetic field. The elimination of power requirements for the magnetic field makes it advan-

tageous to employ permanent magnets, but it is apparent that their use is incompatible with variations in the magnetic field strength to decrease the effect of perturbations or experimental variations in an attempt to minimize perturbations or achieve a constant diameter beam.

It is one object of this invention to enable the attainment of a constant diameter flow of electrons in electron discharge devices employing small diameter electron beams in magnetic fields of constant strength.

It is a further object of the invention to expedite the fabrication of such devices wherein the relationship between the parameters necessary for the attainment of constant diameter electron flow is incorporated into the structure of the device.

It is another object of this invention to facilitate the manufacture of traveling wave tubes.

It is a still further object of this invention to provide an improved traveling wave tube.

These and other objects of this invention are achieved in accordance with one specific embodiment of this invention by positioning the magnetic shield within the envelope of the device so that the position of the magnetic shield, the size of the aperture therein, and the strength of the magnetic field are related to each other in accordance with a specific relationship, specifically by one solution of the equation

$$\frac{d^2R}{dZ^2} = \frac{1}{2R} - \frac{[F(Z/a)]^2 R}{2} \quad (2)$$

wherein R and Z are coordinates of the edge of the electron beam expressed in particularly weighted variables, and $F(Z/a)$ is a function which describes the variation in the magnetic field with distance where a is the size of the aperture in the shield and with the initial conditions that $R=0$ and $dR/dZ=0$ when $F(Z/a) \cong 1$. This equation is a particular form of the paraxial ray equation. These terms are explained more fully below.

In one specific embodiment of this invention, the magnetic shield is a cup completely encompassing the electron gun, the base of the cup having an aperture therein for passage of the electron beam. The electron gun and cup-shaped magnetic shield comprise a unitary structure located within the envelope of a traveling wave tube. Pole pieces and associated field structure or permanent magnets are positioned adjacent the electron gun external to the tube envelope and the electron collector. The magnetic shield shields the electron gun from the influence of the magnetic field. For the desired perveance and radius of the electron beam, the position of the shield, size of the aperture at the base of the cup, and the strength of the magnetic field are related to each other by the above equation.

More generally, in accordance with our invention, the specific relationships between the electron gun and the magnetic shield can be ascertained for any form of magnetic shield and magnetic field variation in the transitional beam section.

A complete understanding of this invention and of the various features thereof may be gained from consideration of the following detailed description and the accompanying drawing, in which:

Fig. 1 is a side elevational view of a traveling wave tube embodying this invention, a portion of the envelope being broken away and the field structure and wave guides being shown in section;

Fig. 2 is a sectional view of the electron gun of the embodiment of Fig. 1;

Fig. 3 is a diagrammatic representation of the electron gun of Fig. 2;

Fig. 4 is a plot of the magnetic field variation showing the buildup of the magnetic field B_z along the axis through the aperture in the magnetic shield;

Fig. 5 is a plot of dR/dZ as a function of Z for various apertures, being the family of curves of the first integral of Equation 2, supra;

Fig. 6 is a plot of R as a function of Z for various apertures, being the family of curves of the second integral of Equation 2; and

Fig. 7 is a graph representing the solution of the equation for the specific illustrative embodiment of the invention disclosed.

Referring now to the drawing, the illustrative embodiment of this invention shown in Fig. 1 comprises a traveling wave tube 10 having a glass envelope with an enlarged portion 11 to which is attached a base 12 with suitable leads 13 extending therethrough. The envelope includes also an elongated portion 15 wherein is situated the helix 16 and support rods 17 therefor. The electron gun 18, shown more clearly in Fig. 2, is positioned within the envelope portion 11 adjacent the elongated envelope portion 15; an electron collector 19 is positioned at the opposite end of the elongated portion 15. A suitable heat radiator 20 is provided for the collector 19.

The base portion 11 of traveling wave tube 10 rests in a cartridge 23 which is attached, as by being screwed thereon, to a ring member 24 screwed to the wall of the input wave guide 25 at one end of the helix 16. An output wave guide 27 encompasses the traveling wave tube 10 adjacent the other end of the helix 16. The input and output wave guides 25 and 27 each have a cylindrical portion 28 and 29, respectively, extending towards the other and being contiguous for a short portion, as at 30, thereby defining a non-magnetic tubular member surrounding the helix 16 and extending between the two wave guides 25 and 27.

A first tubular pole piece 32 encompasses the cartridge 23 and the base portion 11 of the traveling wave tube and a second tubular pole piece 33 encompasses the radiator 20, the pole pieces being part of the magnetic circuit including the cylindrical field structure 34, which is substantially of E shape section with the middle arm 35 of the E terminating adjacent the helix 16. Although a field structure 34 utilizing coils has been shown, it is to be understood that permanent magnets may be utilized to provide the axial magnetic field for the electron beam.

Referring now to Fig. 2, the electron gun 18 for this specific illustrative embodiment of the invention comprises a cathode 38 having a coated surface 39 which defines a segment of a sphere. The cathode 38 is supported in a cathode cup member 42 by strut members 43. Also positioned within the cathode cup member 42 is a cathode heater 44, which is positioned above a heat reflector 45 secured to the cathode 38. The cathode heater leads extend through insulating eyelets 46 in the base of the cathode cup member 42. A beam focusing electrode 49 has an inner surface 50 contiguous to and forming a continuation of cathode surface 39. The beam focusing electrode 49 is secured to the cathode cup 42 by pins 51 but separated therefrom by a washer 52 interposed between the cathode cup 42 and the electrode 49. The electrode 49 is thus closely adjacent the cathode surface 39 but thermally insulated from it by the long heat path through the strut members 43, the cathode cup 52, and the pins 51 and washer 52.

The anode 54 is separated from the focusing electrode 49 by an insulating, as ceramic, ring 55. The anode 54 has a flat central portion having an aperture 56 therein, and the position of the anode with respect to the other elements of the gun and to the magnetic shield is determined with respect to the front plane of this aperture. A shield member 58 is secured to the anode 54 and extends adjacent the insulating ring 55 thereby shielding the ring from any barium that might be evolved from the coated cathode surface 39 during operation of the device.

The cathode cup member 42, cathode 38, focusing electrode 49 and an anode 54 are all mounted within a non-magnetic housing 60, which is directly secured to the anode 54 and thus is at anode potential. The focus-

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ing electrode 49, and thus the cathode 38 and cathode cup member 42, are also supported from the housing 60 by a support ring 61 from which they are insulated by the ceramic ring 62. The housing 60 has a long tubular nose portion 63 integral therewith which, being itself at anode potential, provides a field free space for the electrons in the transitioned space between the electron gun and the magnetic field portion of the traveling wave tube. Additionally, the end of the nose 63 provides a mounting for the helix structure, including the rods 17 and the helix 16 of the traveling wave tube.

The gun housing 60 is in turn surrounded entirely by a cup-shaped magnetic shield 65 which shields the gun from the influence of the magnetic fields. The cup-shaped magnetic shield 65 has a base portion 66 having an aperture 67 therein through which the end of the nose portion 63 of the gun housing 60 protrudes. This base portion 66 is considered as the magnetic shield defining the beginning of the axial magnetic field and is so referred to in portions of this specification where the position of the magnetic shield is being discussed. Also from a theoretical standpoint, this base portion 66 which defines the magnetic shield may be considered as of infinite extent but no substantial error is introduced by a finite cup base with the sides of the cup encompassing the gun housing 60.

The outline of the electron beam is shown by the dotted lines 69. Also the distance z_s of the magnetic shield, i. e., the base portion 66, from the anode 54 of the electron gun is indicated on Fig. 2 to facilitate the subsequent discussion, as are the radius a of the aperture 67 in the magnetic shield and the radius r_m of the electron beam in the region of constant magnetic field.

We have discovered that a constant diameter electron beam flow is entered into by the electron beam on passing through the shield 66 when the cup-shaped magnetic shield is positioned so that the distance z_s from the anode 54 to the base 66 of the cup is related to the radius a of the aperture 67 in the base 66 and to the field strength B_z by a particular solution of the equation

$$\frac{d^2 R}{dZ^2} = \frac{1}{2R} - \frac{[F(Z/a)]^2 R}{2} \quad (3)$$

for an electron beam of particular perveance and radius r_m . To arrive at a particular solution of a second order differential equation, two initial conditions must be specified. In solving Equation 3, these initial conditions are: $R=1$ (i. e., $r=r_m$), when $F(Z/a) \approx 1$ and $dR/dZ=0$ when $F(Z/a) \approx 1$. When $F(Z/a) \approx 1$, the magnetic field is substantially constant. These initial conditions specify the perfect cylindricity of the beam when in the region of constant magnetic field strength. Each of the terms of this equation is itself a function of other terms and is defined below. By referring to Equation 3 one is enabled to incorporate the magnetic shield within the envelope of the device and to determine during the fabricating of the device all of the parameters necessary for entrance of the electron beam into a constant diameter flow. It is believed that this will be more clearly understood by employing the specific embodiment shown as an example, which is done below after defining the terms involved.

Z and R are normalized variables, as defined below, and $F(Z/a)$ is a function representing the axial field strength. Thus Z is defined as

$$Z = \left[\frac{I}{2\pi\epsilon\sqrt{2\eta V_0}} \right]^{1/2} \frac{z}{r_m} \quad (4)$$

where

$$\eta = \frac{e}{m}$$

the ratio of charge to the mass of the electron V_0 is the axial potential relative to the cathode, ϵ the dielectric constant of free space, I the current enclosed by the radius r , z the distance along the axial direction of the

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beam, and r_m the radius of the beam in the region of constant field. r_m is uniquely defined in terms of beam voltage and current and axial magnetic field strength by Equation 1. Gathering together the constants and simplifying

$$Z = 174\sqrt{Pz}/r_m \quad (5)$$

where P is the perveance of the electron beam. R is defined by

$$R = \frac{r}{r_m} \quad (6)$$

where r is the radius of the envelope of the electron beam at any point z . $F(Z/a)$, which is indicative of the axial field strength, is the ratio of the axial field strength at any point z to the maximum field strength. For the axial variation of the magnetic field through a circular aperture in a thin shield, which is the structure of the magnetic shield in these traveling wave tubes, the normalized magnetic field strength $F(Z/a)$ is known to vary with Z as follows:

$$\text{For } Z/a > 0, F(Z/a) = 1 - 1/\pi \left[\arctan(a/Z) - \frac{Z/a}{1+(Z/a)^2} \right] \quad (7a)$$

$$\text{For } Z/a < 0, F(Z/a) = 1/\pi \left[\arctan(-a/Z) + \frac{Z/a}{1+(Z/a)^2} \right] \quad (7b)$$

where a is the size of the aperture 67 in units of Z .

Fig. 4 is a plot of the value of the axial field B_z as it builds up from zero to the maximum value. The physical length of this buildup can be appreciated from Fig. 4 in conjunction with Fig. 3 which is drawn to the same scale along the z -axis.

Thus in the general differential equation

$$\frac{d^2 R}{dZ^2} = \frac{1}{2R} - \frac{[F(Z/a)]^2 R}{2} \quad (8)$$

R and Z , as defined above, represent the beam radius and axial position along the beam respectively in normalized coordinates, and $F(Z/a)$ is a function representing the form of buildup of the magnetic field from zero to its maximum value and is a function of Z/a . The constant a , in fact, is a measure of the range of Z over which this buildup occurs and is equal, in units of Z , to the radius of the aperture in a magnetic shield which produces a buildup given by $F(Z/a)$.

In one specific embodiment of the invention, as shown in the drawings, the electron beam which is to have a constant diameter in traversing through the helix 16 of the traveling wave tube had the characteristics of a perveance of 0.654×10^{-6} amperes/volts^{3/2} and a constant radius, r_m , of 0.0225 inch, for which a maximum magnetic field strength of 454 gauss was required at a beam potential of 1600 volts. The specific electron gun employed had a radius of curvature of the cathode, $\bar{r}_c$, of 0.500 inch, a cathode radius of 0.150 inch, and a radius of curvature of the anode, $\bar{r}$, of 0.227 inch. The beam radius at the anode, r_{anode} , is determined by the cathode radius and the perveance of the beam and in this instance was 0.068 inch. It is known that dR/dZ at the anode is only a function of $\bar{r}_c/\bar{r}$ or in this instance of .500/.227 and can be shown to be equal to 1.0 for this specific gun. Also by Equation 4 above R at the anode is equal to r_{anode}/r_m or 3.02 in this instance.

From these known conditions of the electron beam at the anode, the necessary position of the magnetic shield from the anode and the size of the aperture in the magnetic shield for the embodiment shown can be determined from the solution of Equation 8 given by Figs. 5 and 6. Specifically, one procedure is to assume various values of a , the aperture radius, and ascertain for each value of a the values of Z for $dR/dZ=1$ from Fig. 5 and for $R=3.02$ from Fig. 6. The values of Z and a

are then plotted, as in Fig. 7, giving two lines 70 and 71, the intersection of which at 72 is the particular solution desired. Line 70 represents a curve of Z against a for $dR/dZ=1$ and line 71 a curve of Z against a for $R=3.02$. A line 73 has been drawn on the curves of Fig. 5 at $dR/dZ=1$ and a line 74 on Fig. 6 at $R=3.02$ to facilitate this particular solution. It should be noted that only the $a=0$ curves of Figs. 5 and 6 are properly oriented with respect to the origin. The other curves have been moved along the Z axis to facilitate reading and the origin of each is indicated by the small arrow.

The point 72, which is the particular solution desired, has the coordinates on Fig. 7 of $Z=3.01$ and $a=0.83$. These are of course normalized variables and must be converted to the actual distance between the magnetic shield and the anode and the actual radius of the aperture 67. As $Z=0$ represents the position of the magnetic shield, the distance, z_0 , therefrom of the anode is given by

$$z_0 = Z \frac{r_m}{174\sqrt{P}} \quad (9)$$

which in this instance equals 0.480 inch. Similarly, the actual aperture radius, a^* , is given by

$$a^* = a \frac{r_m}{174\sqrt{P}} \quad (10)$$

which in this instance equals 0.132 inch.

Thus in this specific embodiment for an electron beam of a permeance of 0.654 ampere/volts^{3/2} and constant radius of 0.0225 inch adjacent the helix 16, the cup-shaped magnetic shield 65 has a base portion 66 .480 inch from the anode 54 of the gun and has an aperture 67 therein having a radius of 0.132 inch. It is of course to be realized that other solutions of the Equation 1 will yield different parameters for different traveling wave tubes.

In the specific illustrative embodiment of the invention described above a cup-shaped magnetic shield encompasses the electron gun within the envelope of the traveling wave tube and has an aperture in its base. This structure defines the particular variation of the axial field strength given by Equations 7a and 7b. However, further in accordance with our invention, a constant diameter electron beam can be attained for a wide variety of axially symmetrical magnetic shields and thus many magnetic field variations from substantially zero to a constant value, if the form and position of the electron beam and magnetic shield are related in a specific manner. Thus in regions where the magnetic field is not uniform but in which the axial potential, V_0 with respect to the cathode is constant, the radius of the envelope of the electron beam r , must satisfy the differential equation

$$\frac{d^2r}{dz^2} + \frac{\eta B_z^2 r}{8V_0} - \frac{P}{4\sqrt{2}\eta\epsilon} \frac{1}{r} = 0 \quad (11)$$

This is also the paraxial ray equation but not expressed in special variables. The variation of magnetic field strength, B_z , as a function of distance along the axis, z , is known, either because the functional relationship is specified or because shapes are assigned to the pole pieces or shields 66 and the variation of B_z with z is obtained either by measurement of the magnetic field along the axis or by measurements made on a model in an electrolytic trough.

Likewise, because the beam is to have a constant radius $r=r_m$ in the region of uniform magnetic field strength B_0 , in this region the boundary conditions

$$\begin{aligned} r &= r_m \\ dr/dz &= 0 \end{aligned}$$

must be satisfied. Since these boundary conditions are specified and since I , V_0 , and B as a function of z are specified, the radius of the beam r as a function of distance along the axis z is uniquely specified by Equation 11.

Thus a solution of Equation 11 makes known the r and dr/dz at some point 80, seen in Fig. 3, far enough towards the cathode 39 from the aperture 67 in the magnetic shield or pole pieces 66 that at the point 80 in the magnetic field is substantially zero. Advantageously, point 80 can be considered as being along the end of the anode 54 adjacent the cathode 39, as in the specific embodiment described above, but it need not so be. At the point 80 the solution of Equation 11 gives a particular r and dr/dz for the point, which can be identified as r_f and dr_f/dz respectively. Thus in accordance with our invention if an electron gun has an anode 39 and cathode 54 so formed and positioned in space relative to the shield 66 that it produces an electron beam at the point 80 for which the radius r_0 and dr_0/dz are

$$\begin{aligned} r_0 &= r_f \\ dr_0/dz &= dr_f/dz \end{aligned}$$

and if further the permeance is the same as that already ascribed to the electron beam, then the electron beam will have a constant diameter in the region of constant magnetic field and perturbations will not be present. The design of an electron gun to satisfy these conditions follows known procedures.

It is of course to be understood that the above-described embodiment is illustrative of the application of the principles of the invention and that numerous other arrangements may be devised by those skilled in the art without departing from the spirit and scope of the invention. The various variable and constants recited in the claim are to be understood as defined in the above description.

What is claimed is:

A traveling wave tube comprising an envelope, an axially symmetrical electron gun within said envelope and having an anode and a cathode, a cup-shaped magnetic shield encompassing said gun within said envelope and having a base portion opposite said cathode, said base portion having an aperture therein for passage of the electron beam therethrough, and means creating a magnetic field of substantially constant strength in the region past said base portion removed from said cathode, the ratio of field strength at any point between said cathode and said base portion to said constant field strength being

$$\begin{aligned} F(Z/a) &= 1 - 1/\pi \left[\arctan(a/Z) - \frac{Z/a}{1+(Z/a)^2} \right] \text{ for } Z/a > 0 \\ &= 1/\pi \left[\arctan(-a/Z) + \frac{Z/a}{1+(Z/a)^2} \right] \text{ for } Z/a < 0 \end{aligned}$$

where

$$Z = \left[\frac{I}{2\pi\epsilon\sqrt{2}\eta V_0^{3/2}} \right]^{1/2} z/r_m$$

a is the radius of said aperture in units of Z , r_m is the radius of said electron beam in said region of substantially constant field strength and z is the axial distance, the strength of said magnetic field, the position of said shield portion relative to said anode, and the size of said aperture being related to each other by the equation

$$\frac{d^2R}{dZ^2} = \frac{1}{2R} - \frac{[F(Z/a)]^2 R}{2}$$

where $R=r/r_m$ and the initial conditions are $R=1$, $dZ/dR=0$ at $F(Z/a) \cong 1$.

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