

April 9, 1957

J. G. LINVILL

2,788,496

ACTIVE TRANSDUCER

Filed June 8, 1953

2 Sheets-Sheet 1

FIG. 1

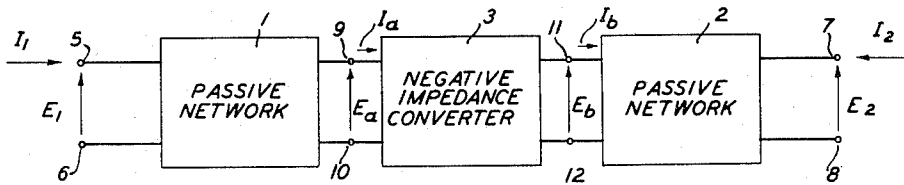


FIG. 2

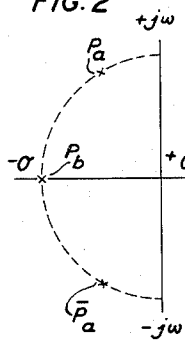


FIG. 3

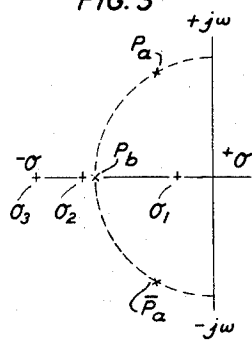


FIG. 5

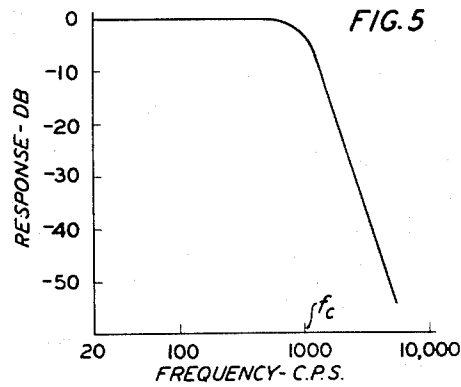


FIG. 4

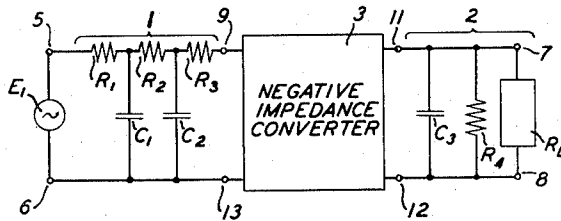


FIG. 7

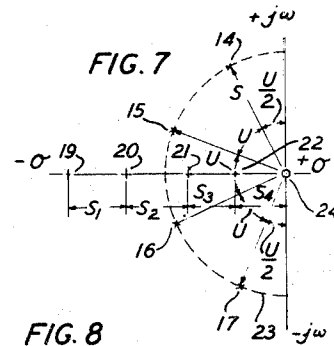


FIG. 8

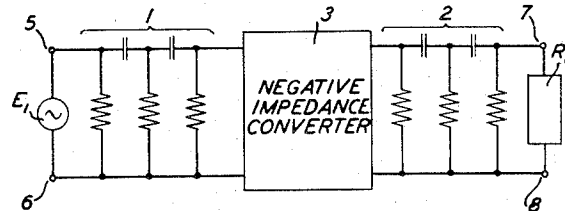


FIG. 12

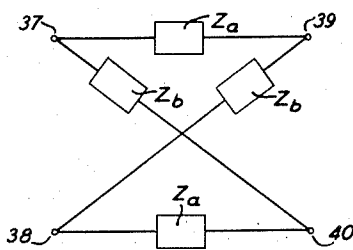
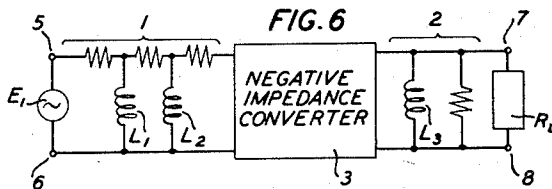


FIG. 6



INVENTOR
J. G. LINVILL
BY
Ralph T. Holcomb
ATTORNEY

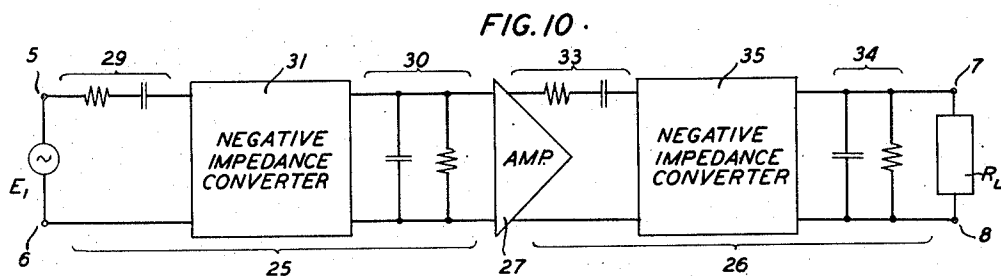
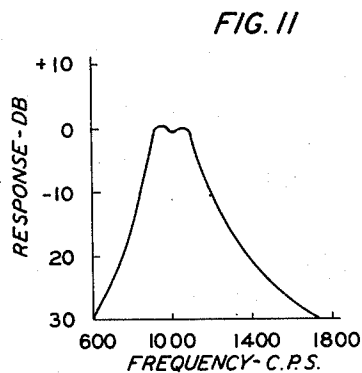
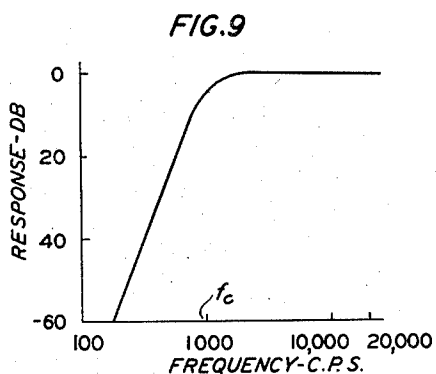
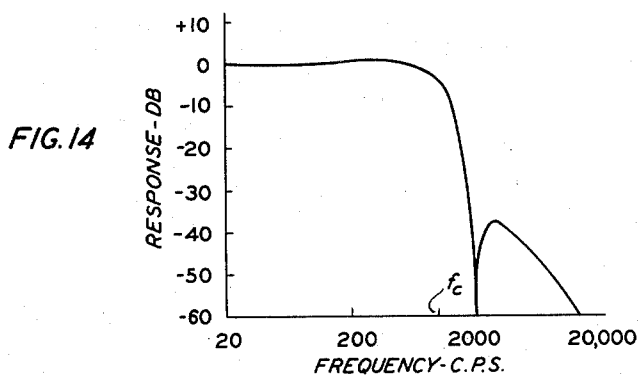
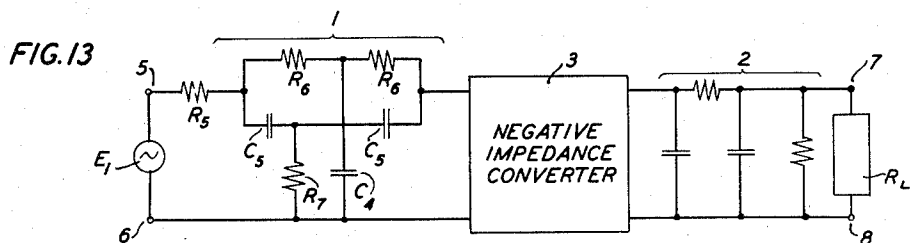
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2 Sheets-Sheet 2



INVENTOR
J. G. LINVILL
BY
Ralph T. Holcomb
ATTORNEY

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2,788,496

ACTIVE TRANSDUCER

John G. Linvill, Whippany, N. J., assignor to Bell Telephone Laboratories, Incorporated, New York, N. Y., a corporation of New York

Application June 8, 1953, Serial No. 360,301

39 Claims. (Cl. 333—80)

This invention relates to wave transmission networks and more particularly to an active transducer which may be designed to have any physically realizable transmission characteristic.

An object of the invention is to simplify the design procedure for an active transducer having an unrestricted transmission characteristic. Other objects are to reduce the loss, or even provide a gain, in the transmission band of an active wave filter. Another object is to eliminate one type of reactor in a transducer without restricting its transmission characteristic. Another object is to reduce the number of component reactors required. Further objects are to decrease the size and cost of a transducer which must meet particularly severe transmission requirements.

The network designer often encounters transmission requirements which he cannot meet economically in a passive transducer. For example, a wave filter which will transmit very low frequencies usually requires very large and expensive inductors. For this case, a passive transducer made up of only resistors and capacitors, usually called an R-C network, is attractive. However, for a given characteristic, an R-C filter introduces excessive loss in the transmission band and requires many more elements than does a filter which includes both inductors and capacitors. These defects may be overcome by including an active element. One active transducer of this type is disclosed in United States Patent 2,549,065, issued April 17, 1951, to R. L. Dietzold. The active element is a stabilized feedback amplifier. The passive elements are resistors and only one type of reactor, either capacitive or inductive.

The present invention is directed to another type of active transducer with unrestricted transmission characteristic. The circuit comprises two passive networks and a negative impedance converter connected in tandem between them. The negative impedance converter, hereafter called simply a converter, has an impedance conversion ratio, designated M , which is negative. Thus, it presents at one pair of its terminals an impedance which is M times the impedance connected to its other terminal pair. Each of the passive networks is made up of one or more resistors and one or more reactors. The reactors may include both inductors and capacitors, or they may be all of the same type. Usually, it is preferred to use only capacitors for the reactors. Resistors and capacitors are, in general, cheaper, smaller, and more nearly ideal elements than are inductors. The passive networks may be simple structures, either balanced or unbalanced. They may, for example, be lattice, ladder, bridged-T, or twin-T networks. The total number of reactors required in the networks is no greater than that required in a passive transducer having a comparable transmission characteristic.

At their ends facing the converter, the passive networks have driving-point impedances one of which is equal to $-M$ times the other at one or more preselected

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natural frequencies of the transducer. The natural frequencies are the complex frequencies at which the transfer impedance of the transducer is infinite. The transfer impedance of the transducer is unrestricted as to roots and poles in that any physically realizable transmission characteristics may be provided. This is true even when the passive networks are restricted to reactors of only one type. As examples, low-pass, high-pass, and band-pass wave filters are disclosed. The loss in the transmission band may be reduced by the active portion of the transducer. Also, a gain in the band may be provided by proper design of the converter or by inserting an amplifier in tandem between two active transducers.

The nature of the invention and its various objects, features, and advantages will appear more fully in the following detailed description of preferred embodiments illustrated in the accompanying drawing, of which—

Fig. 1 is a block diagram of an active transducer in accordance with the invention;

Fig. 2 represents the complex frequency plane on which are plotted the poles of the transfer impedance of a typical low-pass filter in accordance with the invention;

Fig. 3 shows, on the complex frequency plane, the distribution of the zeroes and the poles of the difference between the driving-point impedances of the passive networks at their ends facing the converter, for the low-pass filter example;

Fig. 4 shows the circuit of the low-pass filter when the passive networks are unbalanced ladder structures made up of resistors and shunt capacitors;

Fig. 5 is a typical relative response versus frequency characteristic obtainable with the low-pass filter of Fig. 4;

Fig. 6 shows the circuit of a high-pass filter in accordance with the invention, in which the passive networks are constituted by resistors and shunt inductors;

Figs. 7, 8, and 9 show, respectively, the pole-zero distribution, the network configuration, and a typical characteristic of a second high-pass filter in accordance with the invention;

Fig. 10 shows two active band-pass filters in accordance with the invention connected in tandem by an amplifier;

Fig. 11 shows the relative response characteristic of the filter of Fig. 10;

Fig. 12 shows a generalized lattice structure which may be used for the passive networks in Fig. 1; and

Figs. 13 and 14 show, respectively, the network configuration and the characteristic of a low-pass filter in accordance with the invention in which a twin-T network provides a peak of attenuation at a finite frequency.

Fig. 1 shows in block diagram an embodiment of an active transducer in accordance with the invention. The transducer comprises two four-terminal passive networks 1 and 2 and an interposed negative impedance converter 3 connected in tandem between a pair of input terminals 5, 6 and a pair of output terminals 7, 8. A suitable source of signals, not shown, may be connected to the input terminals and a suitable load, not shown, may be connected to the output terminals.

The converter 3 is an active four-pole network which presents at its input terminals 9, 10 an impedance which, over the frequency range of interest, is equal to M times the impedance connected to its output terminals 11, 12. The converter 3 may be of the vacuum-tube type, examples of which are disclosed in the paper by J. L. Merrill, Jr., entitled, Theory of the negative impedance converter, in the Bell System Technical Journal, vol. XXX, No. 1, January 1951, pages 88 to 109. Preferably, however, the converter 3 is of the transistor type, suitable circuits of which are disclosed in the copending United States patent application of R. L. Wallace, Jr.,

and the present applicant, Serial No. 310,084, filed September 17, 1952, now Patent No. 2,726,370 issued December 6, 1955. A converter using one or more transistors is preferred because it may be designed to have more nearly ideal characteristics. Therefore, a transducer employing a converter of this type may be designed to meet a prescribed transmission characteristic within closer limits, and the characteristic will be more stable with time.

The converter 3 has a current transfer ratio M_i , which is the ratio of the input current I_a to the output I_b , and a voltage transfer ratio M_e , given by the ratio of the input voltage E_a to the output voltage E_b . One of these ratios is always negative. The ratio M_i is substantially unity if junction transistors are used in the converter. Its value may be approximately doubled by using point contact transistors, but at a sacrifice in the stability of the converter. This ratio may, of course, be extended by associating a transformer with the converter. As explained in the above-mentioned patent application, the magnitude of M_e may be selected within wide limits.

The impedance conversion ratio M of the converter, which is the ratio of M_e to M_i , may be given any negative value within a wide range by choosing appropriate values of M_e and M_i . A judicious choice of M may help in obtaining convenient values for the component impedance elements in the passive networks 1 and 2.

It is obvious that a given impedance conversion ratio M may be obtained with an infinite number of combinations of the ratios M_i and M_e . However, their choice determines the gain factor G , which is the product of M_i and M_e . If the networks 1 and 2 and the ratio M all remain fixed, the power gain of the transducer from input to output is inversely proportional to the magnitude of G .

The theory of the design of a transducer of the type shown in Fig. 1 using such a converter will now be presented. The transfer functions of a four-terminal transducer made up of lumped impedance elements are rotational functions of the frequency f . The analysis is simplified by the introduction of the parameter p , called the complex frequency and defined as

$$p = \sigma + j\omega \quad (1)$$

where σ is the real part, $j\omega$ is the imaginary part, and ω is the radian frequency $2\pi f$. A fuller discussion of the concept of complex frequency and the complex frequency plane may be found, for example, in chapter II of the book "Network Analysis and Feedback Amplifier Design," by H. W. Bode, published by D. Van Nostrand Company, New York, 1945.

The transfer impedance Z_T of any four-terminal network may be expressed as the ratio of two polynomials in p , giving

$$Z_T(p) = \frac{N(p)}{D(p)} \quad (2)$$

The transfer impedance becomes infinite at the complex frequencies at which the denominator $D(p)$ is zero. Therefore, these complex frequencies are the natural frequencies of the network. In passive R-C networks, or R-L networks (comprising only resistors and inductors), these zeroes are restricted to the negative real axis of the complex frequency plane. This constraint seriously limits the quality of approximation to an ideal filter characteristic obtainable if $N(p)$ and $D(p)$ are polynomials of a limited degree. Active R-C or R-L networks, however, can have natural frequencies anywhere in the left-half plane, the same as passive networks comprising resistors, inductors, and capacitors.

In the ideal case, if the current transfer ratio M_i is unity, the current I_a entering the converter 3 is equal to the current I_b leaving it, that is,

$$I_a = I_b \quad (3)$$

Assuming a voltage transfer ratio M_e of -1 , the input voltage E_a is the negative of the output voltage, that is,

$$E_a = -E_b \quad (4)$$

With these relationships in mind, the transfer and driving-point impedances of the transducer shown in Fig. 1 may be expressed in terms of the four-pole parameters of the passive networks 1 and 2. The open-circuit transfer impedance Z_{21} from the input terminals 5, 6 to the output terminals 7, 8, which is the ratio of the output voltage E_2 to the input current I_1 when the output current I_2 is zero, is given by

$$Z_{21}(p) = \frac{N(p)}{D(p)} = \frac{E_2}{I_1} = \frac{Z_{12a}Z_{12b}}{Z_{22a} - Z_{11b}} \quad (5)$$

In Equation 5, Z_{12a} is the transfer impedance of the network 1, Z_{12b} is the transfer impedance of the network 2, Z_{22a} is the driving-point impedance of the network 1 at the terminals 9, 10, and Z_{11b} is the driving-point impedance of the network 2 at the terminals 11, 12. The derivation of Equation 5 involves, as an intermediate step, the evaluation of the input current I_a to the converter 3. The load at the terminals 9, 10 of the network 1 is the input impedance Z_i seen at the terminals 9, 10 of the converter 3, given by

$$Z_i = MZ_{11b} \quad (6)$$

where M is the impedance conversion ratio of the converter. When M is -1 , as assumed here,

$$Z_i = -Z_{11b} \quad (7)$$

Therefore, in accordance with network theory,

$$I_a = \frac{I_1 Z_{12a}}{Z_{22a} - Z_{11b}} \quad (8)$$

But since the current I_b flowing into the network 2 is equal to I_a , the output voltage E_2 is

$$E_2 = I_a Z_{12b} \quad (9)$$

Substituting in Equation 9 the value of I_a from Equation 8 gives

$$E_2 = \frac{I_1 Z_{12a} Z_{12b}}{Z_{22a} - Z_{11b}} \quad (10)$$

Dividing each side of Equation 10 by I_1 gives Equation 5.

The transfer impedance Z_{12} from output to input, with I_1 equal to zero, is

$$Z_{12} = \frac{E_1}{I_2} = \frac{-Z_{12a}Z_{12b}}{Z_{22a} - Z_{11b}} = -Z_{21} \quad (11)$$

It will be observed from Equation 11 that, in the present case where M_e and M_i are each of unit magnitude, the transducer obeys the law of reciprocity only in magnitude. The transfer impedance changes sign when the input and output terminal pairs are interchanged.

The driving-point impedance Z_{11} of the transducer at the input end is

$$Z_{11} = \frac{E_1}{I_1} = Z_{11a} - \frac{Z_{12a}^2}{Z_{22a} - Z_{11b}} \quad (12)$$

where E_1 is the input voltage and Z_{11a} is the driving-point impedance of the network 1 at the terminals 5, 6. The driving-point impedance Z_{22} at the output end of the transducer is

$$Z_{22} = \frac{E_2}{I_2} = Z_{22b} - \frac{Z_{12b}^2}{Z_{11b} - Z_{22a}} \quad (13)$$

where Z_{22b} is the driving-point impedance of the network 2 at the terminals 7, 8.

In Equation 2, the zeroes of the numerator $N(p)$ are associated with the structure of the network, not with its natural frequencies. For instance, ladder networks have zeroes of transfer impedance at frequencies where shunt elements become short circuits or series elements become open circuits. Thus, an R-C ladder-type net-

work made up of resistors and three shunt capacitors will have three zeroes of transfer impedance at infinite frequency, where the capacitors are short circuits, irrespective of the values of the capacitors or the natural frequencies of the network. Lattice, bridged-T, and twin-T networks will have zeroes of transmission at frequencies where a bridge-like balance occurs, regardless of the location of the natural frequencies of the complete network.

A suggested procedure for designing an active transducer in accordance with the invention comprises three steps. First, the designer prescribes the desired transfer impedance, within a constant multiplier, in the form of Equation 2. Next, he selects the zeroes of the denominator $D(p)$ as the natural frequencies of an active network of the type shown in Fig. 1. Then, by a method to be described below, he obtains for the networks 1 and 2 a pair of driving-point impedances Z_{22a} and Z_{11b} which are consistent with these natural frequencies. Finally, he synthesizes networks which have the driving-point impedances Z_{22a} and Z_{11b} . The networks chosen must also be of a form which will provide the desired zeroes of transmission at the zeroes of the numerator $N(p)$.

To illustrate the application of the procedure outlined above, the design of several active wave filters of the form shown in Fig. 1 will now be described. The first example is a low-pass filter which has a Butterworth characteristic, a cut-off frequency f_c of 1000 cycles per second, and an attenuation rising at the rate of 18 decibels per octave of frequency. In the form of Equation 2, the transfer impedance of such a filter may be written as

$$Z_{21} = \frac{N(p)}{D(p)} = \frac{K}{\left(\frac{p}{2\pi f_c}\right)^3 + 2\left(\frac{p}{2\pi f_c}\right)^2 + \frac{2p}{2\pi f_c} + 1} \quad (14)$$

where K is a numerical constant which determines the impedance level of the filter. The transfer impedance will have three poles, corresponding to the zeroes of $D(p)$, and three zeroes. The zeroes all fall at infinite frequency. The poles are known to fall on a semicircle in the left half of the complex frequency plane. Fig. 2 shows the poles P_a , $\bar{P}_a$, and P_b plotted on a complex frequency plane in which the real (or σ) axis is horizontal and the imaginary (or $j\omega$) axis is vertical. The coordinates of these poles are

$$P_a = -2\pi 500 + j2\pi 866 \quad (15)$$

$$\bar{P}_a = -2\pi 500 - j2\pi 866 \quad (16)$$

$$P_b = -2\pi 1000 + j0 \quad (17)$$

It is seen that the poles P_a and $\bar{P}_a$ are conjugates and the pole P_b falls on the negative real axis.

The natural frequencies of the filter must occur at these complex frequencies. The circuit shown in Fig. 1 will have natural frequencies when the driving-point impedance Z_{22a} of the network 1 and the driving-point impedance Z_{11b} of the network 2 are equal. At each of these frequencies, the sum Z of the impedance $-Z_{11b}$ seen looking into the converter 3 at its input terminals 9, 10 and the impedance Z_{22a} is zero, that is, when

$$Z = Z_{22a} - Z_{11b} = 0 \quad (18)$$

The zeroes of Z will, therefore, be at the complex frequencies P_a , $\bar{P}_a$, and P_b .

It will be assumed that, in this example, each of the passive networks 1 and 2 will be an R-C structure. Therefore, the poles of Z will fall singly on the negative real axis of the complex frequency plane. Fig. 3 shows three suitable points σ_1 , σ_2 , and σ_3 whose abscissas are $-2\pi 300$, $-2\pi 1050$, and $-2\pi 1500$, respectively. Since the impedance Z will thus have zeroes at the points P_a , $\bar{P}_a$, and P_b , also shown in Fig. 3, and at σ_1 , σ_2 , and σ_3 , we may write

$$Z = \frac{D(p)}{(p - \sigma_1)(p - \sigma_2)(p - \sigma_3)} \quad (19)$$

where $D(p)$ is the denominator in Equation 14. The selection of the points σ_1 , σ_2 , and σ_3 is arbitrary as far as the transfer impedance Z_{21} of the filter is concerned. Of course, none should coincide with the pole P_b . However, as explained below, the driving-point impedances and other characteristics of the filter are influenced by the selection.

There remains, now, only the synthesis of the passive R-C networks 1 and 2. A suggested method involves, first, the expansion of Equation 19 in partial fractions. It is found that the residues are always real but may be positive or negative. The terms are divided into a first group with positive residues and a second group with negative residues. It is known that any function with simple poles on the negative real axis and positive real residues in those poles is the driving-point impedance of an R-C network. Therefore, the first group of terms is associated with the impedance Z_{22a} of the network 1. The second group of terms is associated with the impedance Z_{11b} of the network 2. Although the network 2 can only provide positive residues in the poles of Z_{11b} , they will appear as negative residues when viewed from the input terminals 9, 10 of the converter 3. It is now assumed that the networks 1 and 2 will be ladder-type structures comprising resistors and shunt capacitors. The required values of the component elements are found by making a Cauer synthesis.

For the distributions of the critical frequencies P_a , $\bar{P}_a$, P_b , σ_1 , σ_2 , and σ_3 shown in Fig. 3, the networks 1 and 2 will have the configurations shown in Fig. 4. The network 1 comprises three series resistors R_1 , R_2 , and R_3 and two shunt capacitors C_1 and C_2 . The network 2 consists of the parallel combination of a resistor R_4 and a capacitor C_3 . The fact that the forms shown for the networks 1 and 2 are correct can be understood from the following analysis. From the distribution of the poles of Z shown in Fig. 3, it can be observed that the residues in the poles at σ_1 and σ_2 are positive while the residue in the pole at σ_3 is negative. Therefore, the constant K , appearing in Equation 14, and the partial fractions associated with the poles at σ_1 and σ_2 are identified with the impedance Z_{22a} of the network 1 and the partial fraction associated with the pole σ_3 is identified with the impedance Z_{11b} as seen through the converter 3. Accordingly, the network 1 will include two capacitors and the network 2 only one capacitor. The required values of the resistors R_1 , R_2 , R_3 , and R_4 , in ohms, are 600, 2040, 1200, and 1300, respectively, and the values of the capacitors C_1 , C_2 , and C_3 , in microfarads, are 0.384, 0.172, and 0.0820, respectively. It is assumed that the impedance conversion ratio M of the converter 3 is -1 .

Fig. 5 shows a typical relative response characteristic obtainable with the low-pass filter of Fig. 4. It is assumed that the signal source of voltage E_1 connected to the terminals 5, 6 has zero internal impedance. If the source has an equivalent series impedance R_0 , the series resistor R_1 is replaced by a resistor of smaller value R_1' given by

$$R_1' = R_1 - R_0 \quad (20)$$

It is also assumed that the load impedance R_L connected to the output terminals 7, 8 is infinite in impedance. If R_L has a finite impedance greater than R_4 , the shunt impedance R_4 is preferably replaced by a resistance of larger value R_4' so chosen that the impedance of R_L and R_4' in parallel is equal to R_4 . This value of R_4' is given by

$$R_4' = \frac{R_4 R_L}{R_L - R_4} \quad (21)$$

In Fig. 5, the ratio of the output voltage E_2 to the input voltage E_1 , expressed in decibels, is plotted against the frequency in cycles per second, on a logarithmic frequency scale. The response relative to that at zero frequency is shown. By properly choosing the gain

factor G of the converter 3, there may be provided any gain which is consistent with the stability of the converter and the networks 1 and 2.

Fig. 6 shows the circuit of a second wave filter in accordance with the invention. This is a high-pass filter having a Butterworth characteristic and an attenuation which rises at the rate of 18 decibels per octave. The passive networks 1 and 2 are ladder-type R-L structures, comprising resistors and shunt inductors. Their configurations are the same as those shown for the low-pass filter of Fig. 4 except that the three shunt capacitors C_1 , C_2 , and C_3 are replaced, respectively, by the three shunt inductors L_1 , L_2 , and L_3 . Otherwise, the circuit of Fig. 6 is similar to that of Fig. 4. The component resistors and inductors required in the filter of Fig. 6 may be evaluated by the same procedure described above in connection with the filter of Fig. 4.

Figs. 7, 8, and 9 relate to another high-pass wave filter in accordance with the invention. The filter has a Butterworth characteristic with an attenuation rising at the rate of 24 decibels per octave. On the complex frequency plane of Fig. 7, the points 14, 15, 16, and 17 marked by x 's are poles of the transfer impedance Z_{21} and zeroes of the impedance Z . These points all fall on a semicircle 23 in the left half of the plane and have a uniform spacing S from the origin. They also have a uniform angular spacing U of 45 degrees on the semicircle 23, and the points 14 and 17 have equal angular spacings of $U/2$ from the $j\omega$ axis. Thus, the points 14 and 17 are conjugate, as are also the points 15 and 16. The points 19, 20, 21, and 22 marked by $+$'s are poles of Z . These poles all fall on the negative real axis and have spacings of S_1 , S_2 , and S_3 , as shown. The displacement of the nearest pole 22 from the origin is S_4 . The impedance Z_{21} has a fourth-order zero at the origin, as shown by the circle 24.

Fig. 8 is a schematic circuit of a filter having the distribution of poles and zeroes shown in Fig. 7. Each of the passive networks 1 and 2 is an R-C ladder structure comprising two series capacitors and three shunt resistors. The procedure described in connection with Fig. 4 may be used to find the required values of these elements. Fig. 9 shows a relative response characteristic obtainable with the filter of Fig. 8 when the source E_1 and the load R_L each have a high impedance compared to the resistance of the end resistor connected in parallel therewith.

In accordance with the invention, two or more active transducers may be connected in tandem and isolated from each other by one or more amplifiers. As an example, Fig. 10 shows two band-pass wave filters 25 and 26 connected in tandem between input terminals 5, 6 and output terminals 7, 8 and an interposed amplifier 27, preferably of the transistor type. The filter 25 comprises two passive networks 29 and 30 connected through a converter 31. In the filter 26, the corresponding units are designated 33, 34, and 35. Each of the networks 29 and 33 is constituted by the series combination of a resistor and a capacitor in a series branch. Each of the networks 30 and 34 is made up of the parallel combination of a resistor and a capacitor in a shunt branch.

Fig. 11 shows a typical relative response characteristic obtainable with the filter of Fig. 10. The mid-band frequency is located at 1000 cycles per second and the band width is approximately 200 cycles. Zeroes of transmission occur at zero and infinite frequencies. By properly designing and adjusting the amplifier 27, considerable gain in the transmission band may be achieved, if desired.

In the filter circuits shown in Figs. 4, 6, 8, and 10, the zeroes of transmission occur only at zero or infinite frequency. With the R-C and L-C ladder-type passive networks used in these filters, it is impossible to obtain a zero of transmission at a real frequency between zero and infinity. However, in accordance with the inven-

tion, intermediate zeroes of transmission (peaks of attenuation) may be provided by using R-C or L-C lattice networks, or unbalanced equivalents thereof such as bridged-T or twin-T structures.

Fig. 12 shows the generalized circuit of a lattice structure which may be used for either or both of the passive networks 1 and 2 in Fig. 1. The lattice comprises two equal series impedances Z_a , Z_a and two equal diagonal impedances Z_b , Z_b connected between a pair of input terminals 37, 38 and a pair of output terminals 39, 40. The synthesis of an active filter of the type shown in Fig. 1 using two such lattice networks to provide intermediate attenuation peaks proceeds as explained above in connection with Fig. 4 through the specification of the desired transfer impedance Z_{21} , the selection of the impedance Z , and the evaluation of the driving-point impedances Z_{22a} and Z_{11b} . At this point, one has the driving-point impedances of the networks 1 and 2 and knows the frequencies at which these networks should introduce peaks of attenuation.

The driving-point impedance Z_D at either end of the lattice of Fig. 12 is

$$Z_D = \frac{Z_a + Z_b}{2} = \frac{A(p)}{B(p)} \quad (22)$$

and the transfer impedance Z_T in either direction is

$$Z_T = \frac{Z_b - Z_a}{2} = \frac{T(p)}{B(p)} \quad (23)$$

It is already known that the driving-point impedance of the network 1 will be Z_{22a} and the driving-point impedance of the network 2 will be Z_{11b} . When one substitutes expressions for the driving-point and the transfer impedances for the lattices, as given in Equations 22 and 23, into an equation of the form of 5, the numerators of the expressions for Z_{12a} and Z_{12b} are found to include constant multipliers and all of the factors of $N(p)$ but no others. This is so because the driving-point and the transfer impedances of a lattice have the same denominator $B(p)$. These factors of $N(p)$ are divided in conjugate pairs between the numerators of Z_{12a} and Z_{12b} in any way which makes each numerator of no higher degree than the associated denominator. One thus obtains an expression for the transfer impedance of each of the lattices 1 and 2 in the form of Equation 23 in which the factors of the numerator $T(p)$ are specified but a constant multiplier is yet to be determined. Expressions for the impedances Z_a and Z_b of each of the lattices 1 and 2 may now be found by adding Z_D to Z_T to obtain Z_b and subtracting Z_T from Z_D to get Z_a . Now, if the impedances Z_a and Z_b are to be R-C structures, one selects the largest permissible values for the constant multipliers. Finally, he determines the configurations of these branches and the required values of the component resistors and capacitors to provide the impedances Z_a and Z_b . The synthesis of an R-C lattice network from a specified driving-point impedance and a transfer impedance specified within a constant multiplier is described in greater detail, for example, in the paper by J. L. Bower and P. F. Ordung entitled, The synthesis of resistor-capacitor networks, in the Proceedings of the I. R. E., vol. 38, No. 3, March 1950, pages 263 to 269.

It is often desirable to convert the lattices thus found for the networks 1 and 2 into equivalent unbalanced structures. However, it is usually difficult, if not impossible, to predict the configuration of the equivalent unbalanced network from the form of the lattice. When it is desired to obtain, for one of the networks 1 and 2, a particular type of unbalanced structure, one may use a modified method which avoids first finding a lattice and then seeking the unbalanced equivalent. This method, which is applicable to either R-C or R-L networks, involves exchanging part of the inherent latitude in the choice of poles of the impedance Z for a constraint in the form of one of the networks 1 and 2.

There will now be presented, as an example of this modified method, an active low-pass wave filter of the type shown in Fig. 1 in which an unbalanced, twin-T, R-C network is used to provide a peak of attenuation at a finite frequency. As shown in Fig. 13, the passive network 1 comprises a resistor R_5 in series with a parallel twin-T structure. One of the T's is constituted by the two equal series resistors R_6 , R_8 and an interposed shunt capacitor C_4 . The other is made up of the equal series capacitors C_5 , C_6 and the interposed shunt resistor R_7 . The network 2 is an R-C ladder structure with a series resistor, a shunt resistor, and two shunt capacitors which will provide a 12-decibel per octave attenuation rate at the higher frequencies. Fig. 14 gives the relative response characteristic of the filter. The band cuts off at 1000 cycles per second. The attenuation peak occurs at 2000 cycles.

As in the previous examples, the first step in designing the circuit of Fig. 13 is to select the poles and zeroes of the transfer impedance Z_{21} of the filter to provide an acceptable characteristic, that is, one having a reasonably flat pass band from zero to 1000 cycles, an attenuation peak at 2000 cycles, and a high-frequency attenuation rate of 12 decibels per octave. The method described in my paper entitled, The approximation with rational functions of prescribed magnitude and phase characteristics, in the Proceedings of the I. R. E., vol. 40, No. 6, June 1952, pages 711 to 721, may be followed in determining the locations of the poles and zeroes.

In the network 1, the initial resistor R_5 is selected in accordance with the impedance of the voltage source E_1 . The twin-T structure is designed to have reasonable values for the component elements R_6 , R_8 , R_7 , C_4 , C_5 , and C_6 and to provide the attenuation peak at 2000 cycles. A method of designing such a twin-T network is presented, for example, in United States Patent 2,106,785, to H. W. Augustadt, issued February 1, 1938.

Next, the driving-point impedance Z_{22a} of the network 1 at its end facing the converter 3 is expressed in the form of a partial fraction expansion. Then, the driving-point impedance Z_{11b} of the network 2 at its end facing the converter 3 is determined. This can be done through a consideration of the expression for Z , which, as given by Equation 18, is the difference between Z_{22a} and Z_{11b} . One knows certain factors of Z at this point. Its numerator has a constant multiplier K_1 and includes the factors of $D(p)$. Its denominator includes as factors the denominator of Z_{22a} and the denominator of Z_{11b} , as yet undetermined but known to be of the form $(p+a)(p+b)$, where a and b are as yet unknown. One writes an expression for Z , employing the known factors and putting the unknown factors in literal form, and expands this expression in partial fractions. By known mathematical methods, the unknown constants K_1 , a , and b are selected consistent with the requirements that the residues in the poles of Z_{22a} must be positive and of the values already known and that the residues in the remaining poles, associated with Z_{11b} , must be negative. With the constants K_1 , a , and b determined, one may write an explicit expression for Z . The impedance Z_{11b} is found by subtracting Z from Z_{22a} . With Z_{11b} thus determined, the final step is to synthesize an R-C ladder structure with shunt capacitors to obtain the network 2 shown in Fig. 13.

The synthesis procedure described above in connection with Fig. 4 results in a filter having a desired prescribed transmission characteristic. However, it does not lead to unique structures for the passive networks 1 and 2 because of the permissible latitude in the choice of the poles of the impedance Z . An infinite number of embodiments of the networks 1 and 2 may be found which will provide the desired transmission characteristic for the filter as a whole. The different filters will, however, differ in their driving-point impedances and also in the

effects on the transmission characteristic of imperfections in the converter 3.

In some applications, it is important to be able to prescribe the driving-point impedance of the filter at one or both of the pairs of terminals 5-6 and 7-8. Part of the latitude in the choice of the poles of Z may be used to obtain a more desirable driving-point impedance at one or both of the ends of the filter. Of course, any desired impedance transformation within the transducer may be provided by properly choosing the impedance conversion ratio M of the converter 3.

As seen in Equation 6, the input impedance Z_i of the converter 3 depends upon the impedance conversion ratio M . In a practical converter, the ratio M changes with time, temperature, or other environmental conditions, causing a corresponding change in Z_i . Equation 5 shows that a change in Z_i from the value $-Z_{11b}$ will not affect the zeroes of the transfer impedance Z_{21} of the filter but will change the poles. A change in the locations of these poles will cause a change in the transmission characteristic of the filter. For example, moving a pole such as 14 in Fig. 7 toward the $j\omega$ axis to a new position which is a fractional part H of its original distance from this axis will cause a maximum change T in the response characteristic of the filter given in decibels by

$$T = 20 \log \frac{1}{H} \quad (26)$$

A mathematical analysis shows that, for a given change in the ratio M , the resulting shift in pole location and consequent change T in the filter characteristic are dependent upon the distribution of the poles of the impedance Z on the negative real complex frequency axis — σ . From the standpoint of drift of the filter characteristic, a good practical rule to apply is as follows: Assuming that Z has W poles, one is spaced from the origin by a distance Q which is between half and triple the average displacement of the zeroes of Z from the origin, another is located not more than Q/W from the origin, and the remaining poles are spaced approximately uniformly between these two. A somewhat involved mathematical analysis, here omitted, shows that in general the drift will be less with such a distribution of poles than if either of the end poles falls outside of the limits given, even though the spacing between poles is kept uniform. The analysis further shows that a non-uniform spacing, especially one in which adjacent poles have a spacing much less than the average, aggravates the drift problem. In Fig. 7, it is seen that the distribution of the four poles 19, 20, 21, and 22 of the impedance Z falls within the rule. The distance Q from the origin of the most remote pole 19 is greater than $S/2$ but not more than $3S$, where S is the radius of the semicircle 23 on which all of the zeroes 14, 15, 16, and 17 of Z fall. Also, the pole spacings S_1 , S_2 , and S_3 are approximately equal.

It is to be understood that the above-described arrangements are illustrative of the application of the principles of the invention. Numerous other arrangements may be devised by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. An active transducer comprising two passive networks and a negative impedance converter connected in tandem therebetween, said networks having at their ends facing said converter driving-point impedances which, at a prescribed natural frequency of said transducer, are related to each other by a numerical factor equal in magnitude to the impedance conversion ratio of said converter.

2. A transducer in accordance with claim 1 having a transfer impedance with unrestricted zeroes and poles, said networks comprising resistors and only a single type of reactor.

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3. A transducer in accordance with claim 2 in which said single type of reactor is capacitive.
4. A transducer in accordance with claim 2 in which said single type of reactor is inductive.
5. A transducer in accordance with claim 1 in which said impedance conversion ratio is approximately -1 .
6. A transducer in accordance with claim 1 in which said impedance conversion ratio has a magnitude greater than unity.
7. A transducer in accordance with claim 1 in which said converter is of the vacuum-tube type.
8. A transducer in accordance with claim 1 in which said converter is of the transistor type.
9. A transducer in accordance with claim 1 having the transmission characteristic of a low-pass filter.
10. A transducer in accordance with claim 9 in which said transmission characteristic has a peak of attenuation at a finite frequency other than zero.
11. A transducer in accordance with claim 1 having the transmission characteristic of a band-pass filter.
12. A transducer in accordance with claim 1 having the transmission characteristic of a high-pass filter.
13. A transducer in accordance with claim 1 having the transmission characteristic of a wave filter with a peak of attenuation at a finite frequency other than zero.
14. A transducer in accordance with claim 1 in which one of said passive networks is a twin-T structure.
15. A transducer in accordance with claim 14 in which said twin-T structure has a peak of attenuation at a finite frequency other than zero.
16. A transducer in accordance with claim 14 in which said twin-T structure comprises only resistors and capacitors.
17. A transducer in accordance with claim 14 in which said twin-T structure comprises two T-networks connected in parallel, one of said T-networks including two series resistors and an interposed shunt capacitor and the other of said T-networks including two series capacitors and an interposed shunt resistor.
18. A transducer in accordance with claim 1 in which one of said passive networks is a ladder-type structure.
19. A transducer in accordance with claim 18 in which said ladder-type structure comprises only resistors and capacitors.
20. A transducer in accordance with claim 18 in which said ladder-type structure comprises only resistors and inductors.
21. A transducer in accordance with claim 18 in which said ladder-type structure comprises a series resistor and a shunt capacitor.
22. A transducer in accordance with claim 18 in which said ladder-type structure comprises a series resistor and a shunt inductor.
23. A transducer in accordance with claim 18 in which said ladder-type structure comprises a series capacitor and a shunt resistor.
24. A transducer in accordance with claim 1 in which each of said passive networks is a ladder-type structure.
25. In combination, two transducers in accordance with claim 1 and an amplifier connected in tandem therebetween.
26. A transducer in accordance with claim 1 in which the difference between said driving-point impedances is an impedance having a plurality of zeroes and a plurality of poles, said poles being W in number, all of said poles being located on the negative real axis of the complex

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- frequency plane, one of said poles being spaced from the origin by a distance Q which is between half and triple the average displacement of said zeroes from said origin, a second of said poles being located not more than Q/W from said origin, and the remaining poles being spaced approximately uniformly between said first and said second poles.
27. A transducer in accordance with claim 26 in which said zeroes fall approximately on a semicircle in the left half of said plane.
28. A transducer in accordance with claim 27 in which said zeroes are approximately equally spaced on said semicircle.
29. A transducer in accordance with claim 28 in which two of said zeroes are conjugate.
30. A transducer in accordance with claim 1 in which said impedances are also related to each other by said ratio at a second prescribed natural frequency of the transducer.
31. A transducer in accordance with claim 1 in which said impedances are also related to each other by said ratio at a plurality of other prescribed natural frequencies of the transducer.
32. A transducer comprising two passive networks and an interposed negative impedance converter connected in tandem, said converter having an impedance conversion ratio approximately equal to -1 , said networks comprising resistors and only a single type of reactor, and said networks having at their ends facing said converter driving-point impedances which are substantially equal at a preselected natural frequency of the transducer.
33. A transducer in accordance with claim 32 in which said single type of reactor is capacitive.
34. A transducer in accordance with claim 32 in which said single type of reactor is inductive.
35. A transducer in accordance with claim 32 in which one of said networks is of the ladder type.
36. A transducer in accordance with claim 32 in which each of said networks is of the ladder type.
37. A transducer in accordance with claim 32 in which said impedances are also substantially equal at a second preselected natural frequency of the transducer.
38. A transducer in accordance with claim 32 in which said impedances are also substantially equal at a plurality of other preselected natural frequencies of the transducer.
39. A transducer in accordance with claim 32 in which the difference between said driving-point impedances is an impedance having a plurality of zeroes and a plurality of poles, said poles being W in number, all of said poles being located on the negative real axis of the complex frequency plane, one of said poles being spaced from the origin by a distance Q which is between half and triple the average displacement of said zeroes from said origin, a second of said poles being located not more than Q/W from said origin, and the remaining poles being spaced approximately uniformly between said first and said second poles.

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