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W. C. JAKES, JR
MICROWAVE ANTENNA SYSTEM

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2 Sheets-Sheet 1

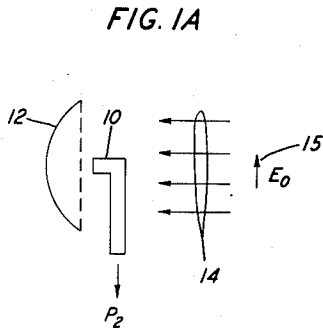
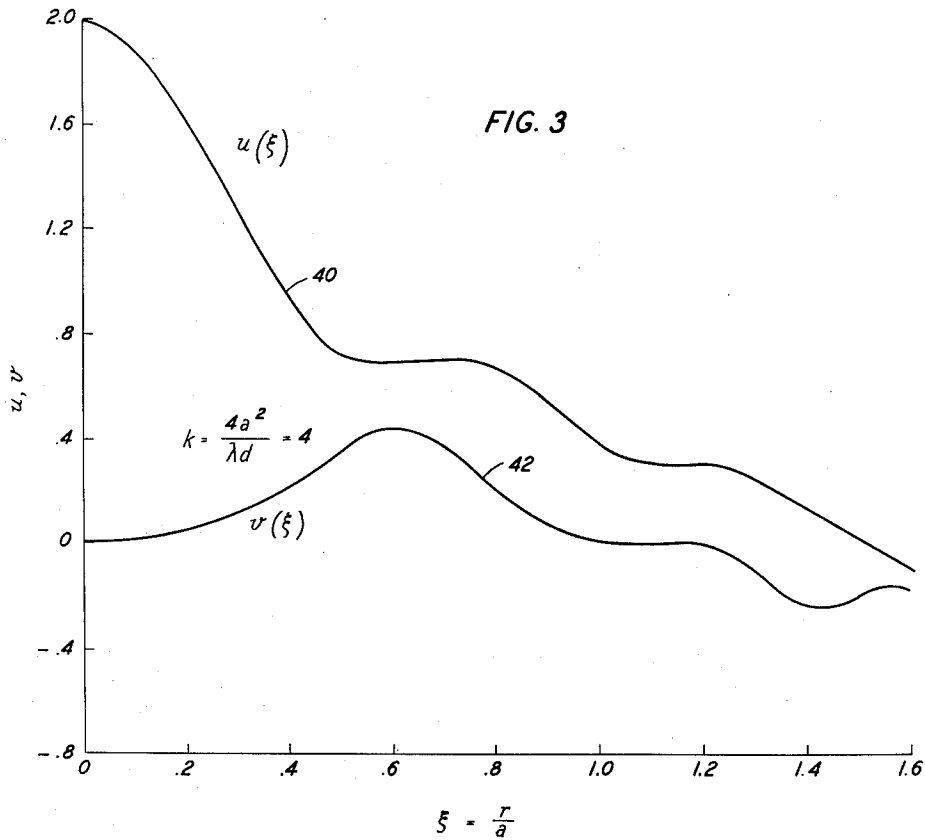
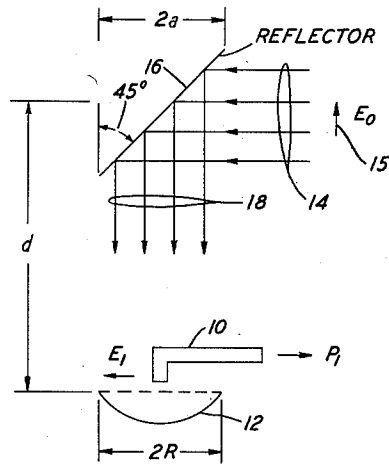


FIG. 1B



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FIG. 2

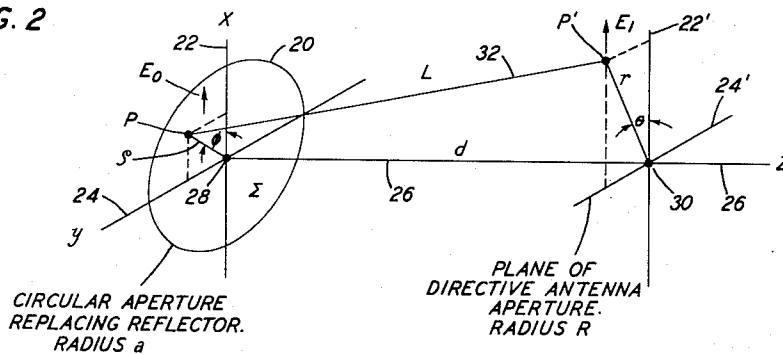
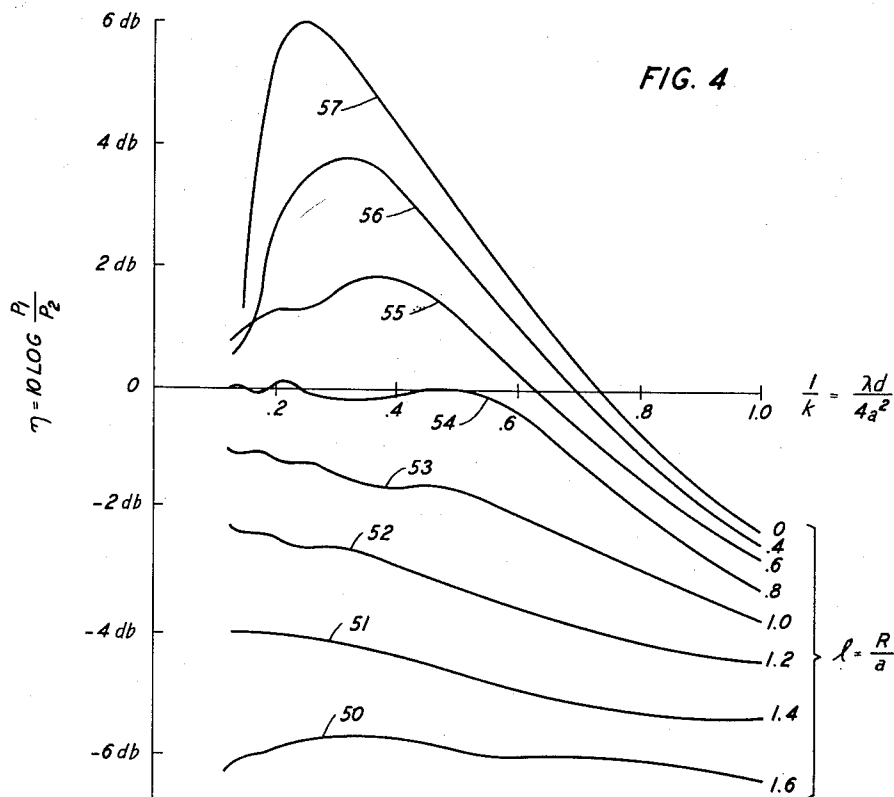


FIG. 4



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MICROWAVE ANTENNA SYSTEM

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5 Claims. (Cl. 343—834)

This invention relates to the transmission and reception of very high frequency, or microwave, electromagnetic wave energy. More particularly, it relates to the use of a plane reflector in conjunction with a highly directive microwave antenna, the relative proportions of the plane reflector and the antenna and the distance relation between them being, in one form of the invention, such that the gain of the combination is greater than that realizable by use of the antenna alone. In a second form of the invention, the relative proportions of the plane reflector and the antenna are such that the combination has a substantially uniform gain over an extremely broad range or band of frequencies.

One principal object of the invention is, therefore, to provide a more efficient microwave transmitting and receiving antenna system.

Another principal object of the invention is to provide an antenna system having substantially uniform gain over a very broad range or band of frequencies.

Other and further objects will become apparent during the course of the following description of illustrative embodiments of the principles of the invention.

The nature and principles of the invention will become apparent from the detailed analysis given hereinafter of specific illustrative arrangements embodying said principles and from the accompanying drawings in which:

Figs. 1A and 1B illustrate one form of highly directive microwave antenna employed without and with the reflector of the invention, respectively;

Fig. 2 is a diagram employed in connection with the mathematical analysis of structures of the invention;

Fig. 3 shows curves of the real and imaginary parts, respectively, of a function employed in the mathematical analysis; and

Fig. 4 shows curves from which the gain or loss, and the variations thereof with frequency, resulting from any of numerous relative values of parameters for a system of the invention can be determined.

In more detail in Fig. 1A, a wave-guide feed 10, is positioned with its feed aperture at the focal point of the parabolic reflector 12. A plane wavefront, linearly polarized, very high frequency wave, represented by arrows 14 and having a polarization direction as shown by arrow 15 and a uniform value E_0 , is shown impinging perpendicularly upon reflector 12. The resultant output power, from the lower end of wave-guide feed 10, is designated P_2 .

In Fig. 1B, a plane reflector 16, placed at a 45 degree angle with respect to both the direction of propagation of wave 14 and reflector 12, is added to reflect the incoming wave 14, as wave 18 having a value E_1 , to the reflector 12, the resulting output of wave guide 10, at its right end, being P_1 . The distance between the front of reflector 12 and the center of reflector 16 is designated d . The diameter of reflector 12 is designated $2R$. The periphery of the plane reflector 16 is elliptical, the minor axis of the periphery being $2a$ and the major axis being

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$$\frac{2a}{\sin 45^\circ} = 2a\sqrt{2}$$

both horizontal and vertical projections of reflector 16 being, therefore, circles of diameter $2a$.

In analyzing the arrangement illustrated by Fig. 1B, the problem is, of course, to determine the ratio

$$\frac{P_1}{P_2}$$

i. e., the ratio of the power received with the arrangement of Fig. 1B to that received with the arrangement of Fig. 1A, for various values of the parameters a , R , λ and d , where λ is the wavelength of the microwave energy being received.

The reflector 16 can be considered, for the purpose of mathematical analysis, as being replaced by a circular aperture parallel to the face of reflector 12 and having a diameter $2a$, with a uniform plane wavefront field of value E_0 , polarized in the x -direction (vertically), impinging upon it. The diffracted field (18) can also be considered as having a plane wavefront polarized in the x -direction and a value E_1 .

The solution of the above-mentioned problem can be conveniently divided into two steps. First, the distribution of the reflected field over the directive antenna aperture (aperture of 12) must be calculated. Then this field is integrated over the antenna aperture, it being assumed, by way of a typical specific example, that the aperture field of the directive antenna when used as a transmitter has a parabolic amplitude distribution and is tapered from the center toward its periphery by 10 decibels.

1. Reflected field

The reflector may be replaced by a circular aperture 20, of radius a , as shown in Fig. 2.

In Fig. 2, the Z axis 26 constitutes a longitudinal axis, in the direction of propagation, which includes the center points 28 and 30, of aperture 20 and of the aperture of the highly directive antenna (such, for example, as the aperture of antenna 12 of Fig. 1B) respectively. The vertical arrows labeled E_0 and E_1 represent the incident and diffracted fields impinging upon aperture 20 and the directive antenna aperture, respectively. The distance between center points 28 and 30 along the axis 26 is designated by the letter d . The X and Y axes of the plane of aperture 20 are designated 22 and 24, respectively, and those of the plane of the directive antenna aperture are designated 22' and 24', respectively, as shown. The symbol Σ designates the entire area of the aperture 20. Energy passing through any point P in the aperture 20, will impinge upon a point P' of the directive antenna aperture, the path of the energy being along line 32, designated L . The polar coordinates of the point P with respect to the origin 28, in the plane of the aperture 20 are the radius ρ and the angle φ . Similarly, the polar coordinates of the point P' with respect to the origin 30, in the plane of the directive antenna aperture are the radius r and the angle θ .

The incident field is assumed to be E_0 and the diffracted field, E_1 , as stated above. By Huygens' principle

$$E_1 = \frac{iE_0}{\lambda} \int_{\Sigma} \frac{e^{-i\beta L}}{L} ds \quad (1)$$

where

$$\beta = \frac{2\pi}{\lambda} \text{ and } L = \text{the distance } \overline{PP'}$$

i. e., line 32. The symbol Σ under the integral sign signifies that the quantity following the integral sign is to be integrated over the entire area Σ of the aperture 20.

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$$E_1 = \frac{iE_0}{\lambda d} e^{-i\beta} \left(\frac{d+r^2}{2d} \right) \int_0^a e^{-\frac{i\beta r^2}{2d}} \rho d\rho \int_0^{2\pi} e^{\frac{i\beta}{d} r \cos(\xi-\varphi)} d\varphi \quad (2)$$

where

r = the radial distance from point 30 to P'

ρ = the radial distance from point 28 to P

ξ = the angle made by r with axis 22'

φ = the angle made by ρ with axis 22

using the approximate formula for L:

$$L = d + \frac{\rho^2 + r^2 - 2\rho r \cos(\xi - \varphi)}{2d} \quad (3)$$

The integral on φ in Equation 2 may be evaluated directly, giving

$$E_1 = imE_0 e^{-i\beta d - i\frac{m^2}{2}} \int_0^1 J_0(m\xi t) e^{-i\frac{m}{2}t^2} t dt$$

$$= E_0 e^{-i\beta d} g(\xi) \quad (4)$$

where

$$t = \rho/a, \quad \xi = r/a, \quad m = \frac{\beta a^2}{d}, \quad J_0 = \text{zero order Bessel function} \quad (5)$$

Note that E_1 is independent of ξ , i. e., has circular symmetry.

Equation 4 defines the function $g(\xi)$ which is by rearrangement of Equation 4, found to be

$$g(\xi) = ime^{-i\frac{m}{2}\xi^2} \int_0^1 J_0(m\xi t) e^{-i\frac{m}{2}t^2} t dt \quad (4')$$

By taking

$$\frac{d}{d\xi} g(\xi)$$

and comparing this to the integral by parts of $g(\xi)$ one gets a differential equation for $g(\xi)$:

$$g'(\xi) e^{i\frac{m}{2}\xi^2} - mJ_1(m\xi) e^{-i\frac{m}{2}} = 0 \quad (5)$$

where J_1 = first order Bessel function with initial condition

$$g(0) = 1 - e^{-i\frac{m}{2}} \quad (6)$$

$g(0)$ gives the field variation along the z -axis, and $g(\xi)$ gives the field variation in the x - y plane for a given value of d .

Equation 5 was solved by the Bell Telephone Laboratories, Incorporated differential analyzer for values of

$$m = k\pi/2, \quad k = 1, 2, 3-8$$

giving both real and imaginary parts of

$$g(\xi) = u(\xi) + iv(\xi)$$

which latter are shown as curves 40 and 42, respectively, in Fig. 3, for a value of the dimensionless parameter

$$k = \frac{4a^2}{\lambda d} = 4$$

II. Power relations

Suppose the aperture field of the antenna when used for transmitting is

$$E_{\text{TRANS}} = E_T h(\xi), \quad h(\xi) \text{ real and } |h(\xi)| \leq 1 \quad (7)$$

where h gives the variation with ξ of E_{TRANS} .

The ratio of the power which would be received by the directive antenna (area = s) when subjected to an incident field E_1 to that transmitted is given by the following equation:

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$$\frac{P_1}{P_T} = \frac{\left| \int_s E_0 e^{-i\beta d} g(\xi) E_T h(\xi) ds \right|^2}{\left[\int_s E_T^2 h^2(\xi) ds \right]^2} \quad (8)$$

where, as given in Equation 4, above, $E_1 = E_0 e^{-i\beta d} g(\xi)$.

The ratio of the power which would be received by the antenna when subjected to an incident field E_0 to that transmitted is

$$\frac{P_2}{P_T} = \frac{\left| \int_s E_0 E_T h(\xi) ds \right|^2}{\left[\int_s E_T^2 h^2(\xi) ds \right]^2} \quad (9)$$

Then P_1/P_2 (see Figs. 1A and 1B) is given by

$$\frac{P_1/P_T}{P_2/P_T} = \frac{\left| \int_0^l g(\xi) h(\xi) \xi d\xi \right|^2}{\left[\int_0^l h(\xi) \xi d\xi \right]^2} = \frac{P_1}{P_2} \quad (10)$$

where $l = R/a$. Since the area of a circle is π times the square of its radius the factor l is the square root of the ratio of the area of the antenna aperture to the projected area of the reflector.

The integral

$$\int_0^l g(\xi) h(\xi) \xi d\xi \quad (11)$$

was evaluated by the above-mentioned differential analyzer for values of $l = .4, .5, .6-1.6$. The transmitted aperture field was taken as

$$h(\xi) = 1 - .684 \frac{\xi^2}{l^2} \quad (10 \text{ decibel parabolic taper}) \quad (12)$$

The gain or loss afforded by the combination of the reflector and the directive antenna is the function

$$\eta(k, l) = 10 \log \frac{P_1}{P_2}$$

and is plotted in the curves 50 to 57, inclusive, of Fig. 4. Since the projected area of the reflector is πa^2 , the factor

$$\frac{1}{k}$$

is substantially the ratio of the product of the wavelength and the separation between the antenna and the reflector to the projected area of the reflector.

As is obvious from curves 50 to 57, inclusive, of Fig. 4, where parameter

$$l = \frac{R}{a}$$

is smaller than 0.8 and

$$\frac{1}{k}$$

is smaller than 0.6, the power received by the directive antenna in combination with the plane reflector of elliptical contour exceeds that which the directive antenna would receive if it were subject to the field E_0 incident on the reflector. As a practical example, if d (the distance or elevation of the reflector with respect to the directive antenna) is made 140 feet, the projected diameter $2a$ of the reflector is made 10 feet and the directive antenna aperture diameter is made 5 feet, a gain in the order of $2\frac{1}{2}$ decibels will be realized at a frequency of 3950 megacycles by the combined use of the reflector and directive antenna as compared with the use of the antenna alone in the position of the reflector (i. e., at an elevation of 140 feet).

In addition to the improved efficiency, the combination also effects a saving of 140 feet of wave-guide, or other type of appropriate, very high frequency (microwave), transmission line, which would be required to connect the directive antenna to apparatus on the ground were the directive antenna to be mounted in the position of the reflector. Furthermore, a somewhat lighter, simpler

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and less expensive tower structure would suffice to support the reflector, and if changes in the direction of transmission are desired, the problem of rotating the reflector is appreciably more simple than that of rotating the directive antenna if the latter were used in place of the reflector.

Since the parameter

$$\frac{1}{k}$$

is a function of λ (wavelength or frequency) it is also apparent from the curves 50 to 57, inclusive, of Fig. 4 that as l is increased within the limits illustrated, the transmission characteristic of the over-all combination of the invention, represented for various values of l by the curves 50 to 57, inclusive, becomes substantially uniform over relatively broad portions or ranges of frequency. For example, for $l=0.8$ the curve 54 is flat within a few tenths of a decibel between the values of .1 to .6 for the parameter.

$$\frac{1}{k}$$

This is significant because it means that the combination can be proportioned to have substantially uniform gain over a very broad frequency range or band. In many practical applications such a "broad band" characteristic is of considerably more importance than a transmission gain of several decibels over a much narrower band of frequencies. Accordingly, a system of the invention proportioned so that the parameter l has a value near 0.8 represents an extremely "broad band" antenna system. Where even a broader frequency range of uniform response may be of paramount importance, it is not inconceivable that a loss as great as 6 decibels could be accepted in which case a value of l near 1.6 could be employed to provide a response uniform within $\frac{1}{2}$ decibel over the range of values for

$$\frac{1}{k}$$

between 0.125 to 1.00, inclusive.

By a similar line of reasoning, it becomes obvious that, since the parameter

$$\frac{1}{k}$$

is also a function of d and a , at any given frequency either or both of these last-mentioned parameters can be varied over substantial ranges without adversely affecting the over-all operation of the system to any considerable degree, where values of $l=0.8$ or greater are employed.

While in the above example the angles of incidence and reflection (diffraction) from the reflector were taken as 45 degrees each, structures embodying the principles of the invention are obviously not limited to the use of this specific value. Angles near 45 degrees are, however, usually more convenient where the desired direction of propagation is substantially horizontal and it is desired to mount the directive antenna on the ground at or near the base of the tower structure employed to support the reflector. If an angle other than 45 degrees is employed, the shape of the reflector is, of course, modified so that its projection at the chosen angle is a circle of the proper radius.

Numerous and varied additional applications of the principles of the invention will readily occur to those skilled in the art. For example, one or more reflectors could obviously be employed to direct the beam of a highly directive antenna located at the base of a cliff to a reflector on the summit of the cliff from which the last-mentioned reflector beam would be launched horizontally toward a distant receiving antenna system. By proportioning and positioning each of said reflectors and the directive antenna in accordance with the principles of

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this invention, a gain could be realized at each reflector, in addition to obtaining transmission or reception from an inaccessible but advantageously located point at high elevation, without requiring that the directive antenna and the associated transmitting and receiving apparatus be located inconveniently near the inaccessible point. Obviously, also, any of the numerous types of highly directive antennas well known to those skilled in the art can be employed in place of the type illustrated in the drawings.

What is claimed is:

1. In combination, a highly directive microwave radio antenna and a plane reflector of elliptical contour adapted and positioned to reflect radio waves transmitted along a predetermined path from and into said antenna, the dimensions of the reflector with respect to those of the antenna, and the spacing between said reflector and antenna being proportioned in accordance with the relations

$$l = \frac{R}{a} \text{ and } \frac{1}{k} = \frac{\lambda d}{4a^2}$$

where R is the radius of the aperture of the directive antenna, a is the radius of the projected aperture of the plane reflector, d is the distance between the directive antenna aperture and the center point of the plane reflector and λ is the wavelength of the radio waves to be transmitted and received by said combination, the said parameters being such that l is less than 0.8 and

$$\frac{1}{k}$$

is greater than 0.1 and less than 0.6 whereby the effect of the combination is to increase the efficiency or gain of said combination over that realizable by use of said antenna alone.

2. In combination, a highly directive microwave radio antenna and a plane reflector of elliptical contour adapted and positioned to reflect radio waves transmitted along a predetermined path from and into said antenna, the dimensions of the plane reflector with respect to those of the antenna and the spacing between said antenna and said reflector being proportioned in accordance with the relations

$$l = \frac{R}{a} \text{ and } \frac{1}{k} = \frac{\lambda d}{4a^2}$$

where R is the radius of the aperture of the directive antenna, a is the radius of the projected aperture of the plane reflector, d is the distance between the directive antenna aperture and the center point of the plane reflector and λ is the wavelength of the radio waves to be transmitted and received by said combination, the said parameters being such that l is greater than 0.6 and

$$\frac{1}{k}$$

is greater than 0.1 and less than 1.0 whereby the frequency band width over which a substantially uniform gain of the over-all combination can be obtained is of a width to permit wide latitude in variations of the various operating parameters.

3. In combination, a highly directive microwave radio antenna and a plane reflector adapted and positioned to reflect radio waves transmitted along a predetermined path from and into said antenna, the projected area toward said antenna of said reflector being related to the area of the aperture of said antenna by the ratio l , where l is the square root of the ratio of the area of the antenna aperture to the area of the reflector as projected toward the antenna, the value of l being greater than 1.0 and no greater than 1.6, the spacing between said reflector and said antenna being at least several times the maximum dimension of said antenna aperture, whereby said combination will have transmitting and receiving characteristics which are substantially uniform over an extremely wide band of frequencies.

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4. In combination, a highly directive microwave radio antenna and a plane reflector adapted and positioned to reflect radio waves transmitted along a predetermined path from and into said antenna, the distance d between said antenna and the midpoint of said reflector being several times the largest dimension of said antenna and of said reflector, the square root of the ratio of the area of the aperture of said antenna to the projected area toward said antenna of said reflector being less than 0.8, the ratio of the product of the wavelength of the wave energy being transmitted by said combination and said distance d to the projected area toward said antenna of said reflector being greater than 0.1 and less than 0.7 whereby said combination will have an effective overall transmission gain exceeding that which can be obtained by employing said antenna alone to directly receive and transmit said wave energy.

5. In combination, a highly directive microwave radio antenna and a plane reflector adapted and positioned to reflect radio waves transmitted along a predetermined

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path from and into said antenna, the distance d between said antenna and the midpoint of said reflector being several times the largest dimension of said antenna and of said reflector, the ratio of the product of the wavelength of the wave energy being transmitted by said combination and said distance d to the projected area toward said reflector being greater than 0.1 and the square root of the ratio of the area of the antenna aperture to the projected area of the reflector being no greater than 1.6, whereby either increased gain or extremely broad ranges of tolerable operating parameters may be obtained.

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