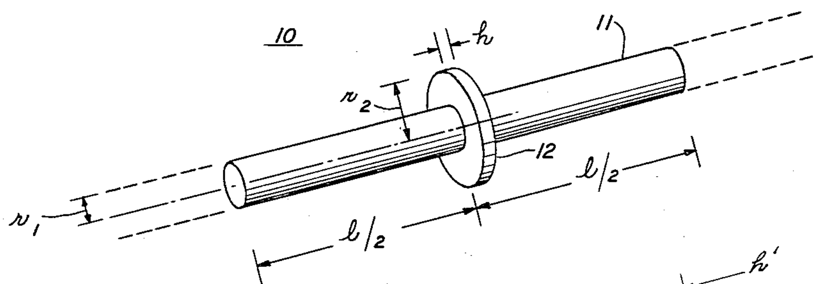
Dec. 11, 1956 W. P. MASON ET AL 2,774,042
ELECTROMECHANICAL WAVE FILTER
Filed April 29, 1953

FIG. 1



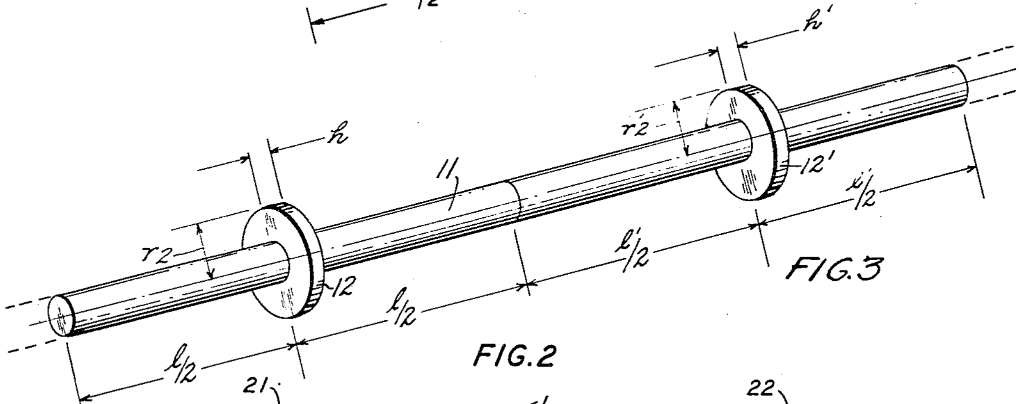
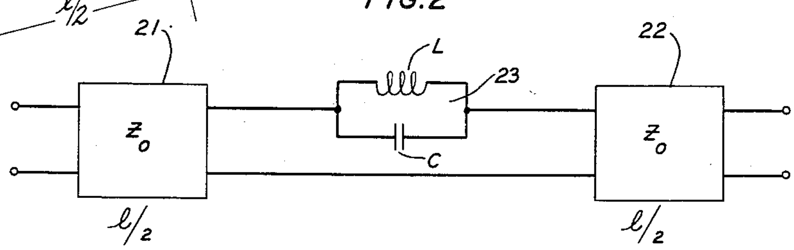


FIG. 3

FIG. 2



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ELECTROMECHANICAL WAVE FILTER

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7 Claims. (Cl. 333—71)

This invention relates to mechanical wave filters and more particularly to those filters which are torsionally excited.

The principal object of this invention is to improve the frequency filtering characteristics of a mechanical wave filter.

One common form of mechanical wave filter consists of a relatively long, small-diameter rod of acoustic material and one or more members of greater diameter centrally connected thereto and adapted to be set into flexural vibration when longitudinal vibrations are impressed upon one end of the rod. Peaks of attenuation occur at the antiresonant frequencies of the transverse members and these peaks are usually located close to the frequency pass band limits to obtain sharp cut-off characteristics. In such a filter, however, when certain signals are applied, spurious resonances resulting from mechanical coupling to other than the longitudinal mode of vibration appear and, as a consequence, the output signals are undesirably distorted in these instances. Previous to this invention, various ways, including operating the filter at low signal levels or making the frequency cut-off characteristics less sharp, have been tried for the purpose of eliminating this difficulty but so far as is known none has proved as satisfactory as might be wished. The present invention is directed to a simple and inexpensive solution of this problem by means which also permit very sharp frequency cut-offs and substantial signal levels.

In accordance with the present invention in one embodiment thereof a small-diameter rod of acoustic material of length l has connected to its midpoint a larger diameter thin disk or plate of a similar material which vibrates in a torsional mode when an angular rotational torque is applied to one end of the rod. By properly dimensioning this centrally located disk, as, for example, in accordance with the formulas set forth herein, a peak of attenuation can be placed either above or below the frequency pass band at a chosen frequency, thereby facilitating almost any desired degree of sharpness in the cut-off characteristics.

The nature of this invention will be more fully understood from the following detailed description given in connection with the accompanying drawings, in which:

Fig. 1 is a perspective view of a mechanical wave filter in accordance with the invention employing a centrally positioned disk;

Fig. 2 is a diagram of the electrical equivalent of the mechanical filter shown in Fig. 1; and

Fig. 3 is a perspective view of a cascaded mechanical wave filter in accordance with the invention.

Referring now particularly to the drawings, Fig. 1 shows, for the purposes of illustration, as an embodiment of the invention mechanical filter **10** which consists of a small-diameter rod **11** of a suitable acoustic material to which is connected disk or plate **12** of a similar or like material. This disk is fastened to rod **11** substantially midway between the ends thereof and is preferably, though not necessarily, formed as an integral part. Since disk **12** is the principal resonating element of the filter, its mechanical Q should be as high as possible in order to minimize loss and also to minimize loss distortion in the pass band and consequently to improve the cut-off characteristics of the filter. Accordingly it should be chosen from a low loss material such as a nickel-iron alloy although in many, if not all, applications certain ceramics or crystals are well suited.

The dimensions of rod **11** and disk **12** will most conveniently be set forth in connection with the formulas characterizing the electrical behavior of the equivalent circuit shown in Fig. 2. It should be noted here, however, that the dimensions of filter **10** for a given frequency of operation in the torsional mode are entirely different from those of a filter having the same operating frequency but excited in a longitudinal mode. One noticeable difference is that the length of filter **10** is approximately only one-half that of a comparable longitudinal mode filter.

Fig. 2 shows the electrical analogue of the mechanical filter shown in Fig. 1. This electrical equivalent circuit consists of two identical wave transmission lines **21** and **22**, each having a length $l/2$ which is approximately a quarter wavelength long at the frequency of operation, whose adjacent ends are joined together by a serially connected parallel tuned circuit **23**. The image propagation constant θ and image impedance Z_i for the filter are given by the following

$$\operatorname{Cosh}\theta=\cos\left(\frac{\omega l}{v}\right)+\frac{jZ_s}{2Z_0}\sin\left(\frac{\omega l}{v}\right) \tag{1}$$

$$Z_i=Z_0\left[\frac{\sin\frac{\omega l}{v}-\frac{jZ_s}{Z_0}\cos^2\frac{\omega l}{2v}}{\sin\frac{\omega l}{v}+\frac{jZ_s}{Z_0}\sin^2\frac{\omega l}{2v}}\right]^{1/2} \tag{2}$$

where v is the velocity of propagation along the line, l is the total length of both sections of line, Z_0 is the characteristic impedance of each line, ω is angular frequency, Z_s is the impedance of the resonant circuit **23** given by

$$\frac{j\omega L}{1-\left(\frac{\omega}{\omega_r}\right)^2}$$

ω_r is the angular frequency at resonance, and L is equivalent to the moment of inertia of disk **12**.

Expressed in mechanical quantities, the characteristic impedance of lines **21** and **22** is

$$Z_0=\frac{\pi}{2}r_1{}^4\rho v \tag{3}$$

where r_1 is the radius of rod **11** and v the velocity of propagation equals

$$\left(\frac{\mu}{\rho}\right)^{1/2}$$

μ is the shear modulus in the acoustic material and ρ is the density.

The series impedance Z_s for disk **12** near its antiresonant frequency vibrating in a torsional mode is

$$Z_s=\frac{j\omega I_1}{1-\frac{f^2}{f_r{}^2}} \tag{4}$$

where f is frequency

$$f_r=\frac{.4636r_1v}{r_2{}^2-r_1{}^2}$$

r_1 is the inner radius of disk **12** and r_2 is the outer radius, $I_1=1.559\rho(r_2{}^4-r_1{}^4)h$, and h is the thickness of disk **12**, these distances in centimeters.

For the case where f_r, the anti-resonant frequency of disk **12**, lies below the frequency pass band, it can be shown from the above equations that the image impedance at mid-band is

$$Z_i=\frac{\pi^2}{8}\left(\frac{f_U-f_L}{f_m}\right)\sqrt{\rho\mu}\,r_1^4\left(\frac{f_U}{b^2f_L+\left(\frac{\pi}{4}\,\frac{f_U-f_L}{f_m}\right)^2(f_U+b^2f_L)}\right)^{1/2} \quad (5)$$

where f_U is the upper cut-off frequency corresponding to $\cosh\theta=-1$ in Equation 1, f_L is the lower cut-off frequency corresponding to $\cosh\theta=+1$ in Equation 1, $f_m=\sqrt{f_Uf_L}$, and

$$b^2=\frac{1-\left(\frac{f_r}{f_L}\right)^2}{1-\left(\frac{f_r}{f_U}\right)^2}$$

Similarly it can be shown that where f_r lies above the upper cut-off frequency

$$Z_i=\frac{\pi^2}{8}\left(\frac{f_U-f_L}{f_m}\right)\sqrt{\rho\mu}\,r_1^4\left(\frac{f_Lb^2}{f_U+\left(\frac{\pi}{4}\,\frac{f_U-f_L}{f_m}\right)^2(f_Lb^2+f_U)}\right)^{1/2} \quad (6)$$

By using Equations 5 and 6, the dimensions of a mechanical filter, such as filter **10**, may be readily determined once the two cut-off frequencies f_L and f_U, the frequency of infinite attenuation f_r and the image impedance Z_i, which should be made equal to the impedance of the driving source, have been chosen.

In order to obtain a peak of attenuation both above and below the pass band, a structure similar to the one shown in Fig. 3 may be used. As shown, two filters, one having a peak above and the other below the band, may be connected in series. As discussed above, the dimensions of each section of the filter may be determined from Equations 5 and 6. By thus cascading filter sections of the type shown in Fig. 1, increasingly sharp cut-off characteristics may be obtained. Although filter **10** will have transmission bands in addition to the principal one considered here, these may, if objectionable, be eliminated by attenuation provided by the torsional input and output transducers used in conjunction with the filter.

It should be understood that the invention described herein is not limited to wholly mechanical systems but will be very useful in electromechanical filters such as telephone channel frequency-separating filters and superheterodyne radio receiver intermediate-frequency filters. These electromechanical devices of course require transducers, such as described in Electro-Mechanical Transducers and Wave Filters by W. P. Mason, Van Nostrand Company, 1948, to convert the electrical signals into the vibrations applied to the mechanical filters.

Changes and modifications in the illustrative embodiment described herein will occur to those skilled in the art and may be made without departing from the spirit or scope of the invention.

What is claimed is:

1. A mechanical wave filter for transmitting a band of frequencies, said filter having a lower cut-off frequency f_L, and an upper cut-off frequency f_U, comprising a small-diameter cylindrical rod of acoustic material of radius r_1 having a length l that is approximately a half wavelength long in the torsional mode of vibration at a frequency within said band, and an annular disk of acoustic material of distributed mass having an inner radius r_1, an outer radius r_2, and a thickness h that is less than one-half wavelength in the torsional mode of vibration at a frequency within said band connected to said rod and dimensioned to resonate in a single torsional mode at a frequency f_r when said rod is torsionally excited by mechanical vibrations applied to an end thereof, whereby a peak of attenuation is provided at a frequency f_r at a desired point outside the pass band of said filter.

2. The combination of elements as in claim 1 in which said disk is an integral part of said rod.

3. The combination of elements in claim 1 in which said disk is positioned symmetrically on said rod midway between the ends thereof.

4. A cascaded mechanical wave filter comprising a plurality of mechanical filters connected in series, each filter like the filter in claim 1, at least one of said filters having its annular disk resonant at a frequency differing from the resonant frequencies of the corresponding disks of the other filters to give a peak of attenuation outside the pass band whereby there is a peak of attenuation above and a peak below the band cut-off frequencies.

5. A mechanical wave filter comprising a wave filter like the filter in claim 1, means for impressing torsional vibrations on one end of said rod, and means for receiving torsional vibrations at the other end of said rod.

6. A mechanical wave filter for transmitting a preassigned band of frequencies comprising means including a rod of acoustic material having a length approximately equal to a half wavelength in the torsional mode of vibration at a frequency within said preassigned band for transmitting a band of frequencies including said preassigned band, and means for producing a peak of attenuation at a frequency close to one limit of said preassigned band, said last-mentioned means including a transverse thin continuous annular member made of acoustic material of distributed mass centrally mounted near the center of said rod and dimensioned to resonate in a torsional mode at the frequency of said peak of attenuation when said rod is torsionally excited by mechanical vibrations applied to an end thereof.

7. A mechanical wave filter as in claim 1 coupled to a torsional electromechanical transducer, in which the image impedance Z_i in the torsional mode of vibration of said mechanical wave filter is matched to the output mechanical impedance of said torsional mechanical transducer.

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