

Oct. 30, 1956

H. S. BLACK ET AL  
LAMINATED CONDUCTOR

2,769,150

Filed Nov. 14, 1952

2 Sheets-Sheet 1

FIG. 1

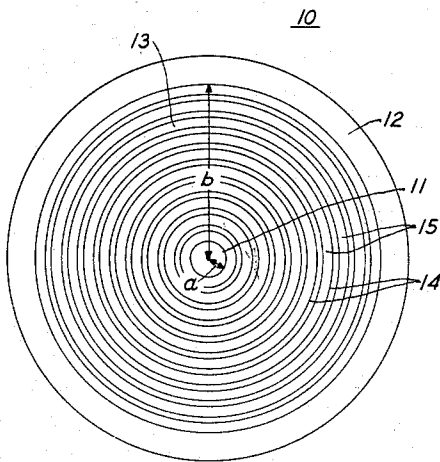


FIG. 2

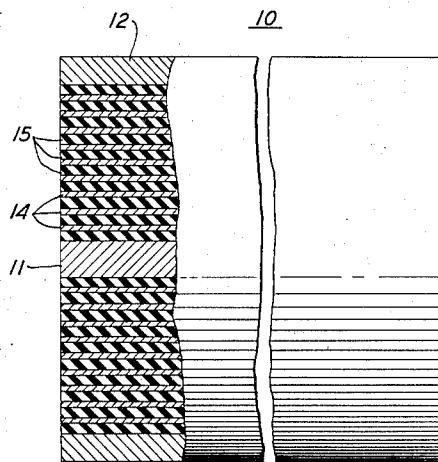
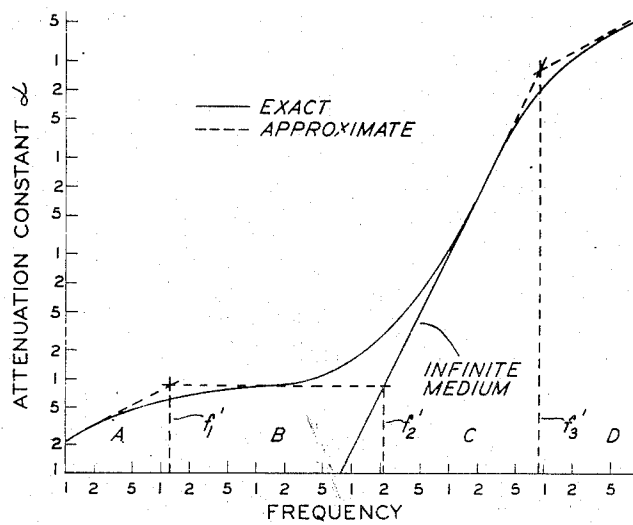


FIG. 3



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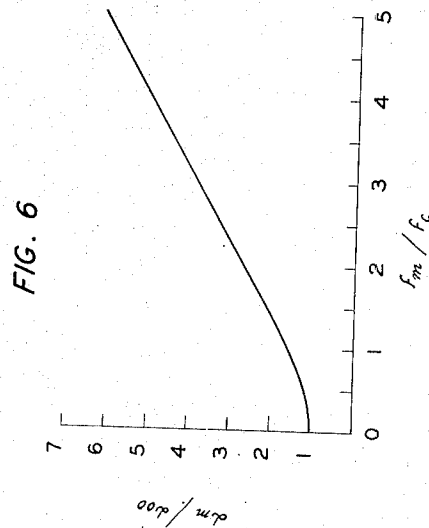
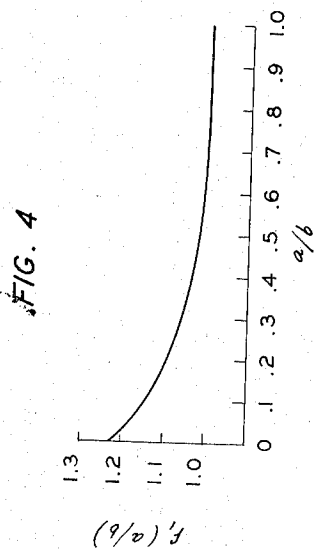
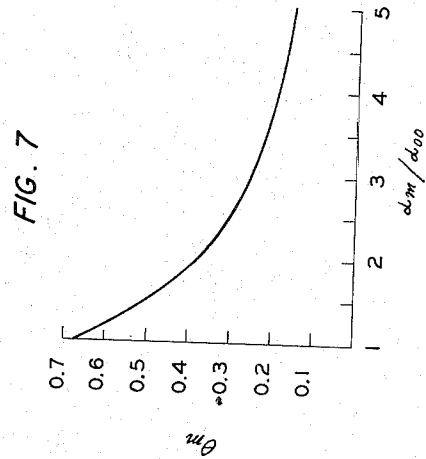
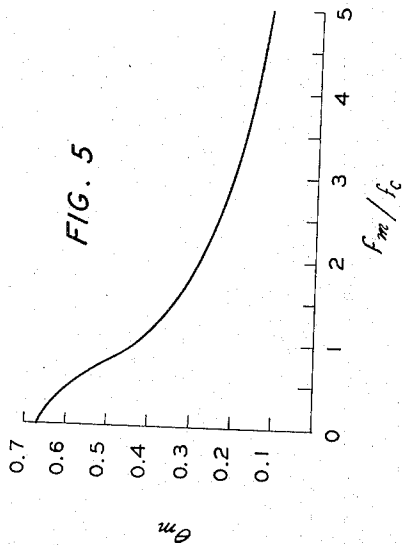
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2 Sheets-Sheet 2



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2,769,150

## LAMINATED CONDUCTOR

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Application November 14, 1952, Serial No. 320,468

4 Claims. (Cl. 333—96)

This invention relates to electrical conductors and more particularly to composite conductors formed of a multiplicity of insulating conducting portions and which have come to be known as "Clogston conductors." Specifically, the invention relates to a particular type of Clogston conductor which comprises a single coaxial stack of insulated metallic cylinders and which is called a "Clogston 2" conductor.

It is an object of this invention to improve the current distribution in "Clogston 2" conductors, and particularly to effect such improvement by the proportioning of the insulating and metallic material in the stack.

In a copending application of A. M. Clogston, Serial No. 214,393, filed March 7, 1951, there are disclosed a number of composite conductors each of which comprises a multiplicity of insulated conducting elements of such number, dimensions, and disposition relative to each other and to the orientation of the electromagnetic wave being propagated therein as to effect a more favorable distribution of current and field within the conducting material. In one specific embodiment shown in Fig. 17A of the Clogston application, a coaxial transmission line has the entire region between an outer sheath and an inner core filled with alternate thin cylinders or laminae of metal and dielectric material, respectively. The metal cylinders are made as thin as possible compared with the skin depth  $\delta$ , where  $\delta$  is given by the expression

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \quad (1)$$

where  $\delta$  is expressed in meters,  $f$  is the frequency in cycles per second,  $\mu$  is the permeability of the metal in henries per meter, and  $\sigma$  is the conductivity of the metal in mhos per meter. The factor  $\delta$  measures the distance in which the current or field penetrating into a slab of the material many times  $\delta$  in thickness will decrease by one neper, i. e., their amplitude will become equal to  $1/e=0.3679$  . . . times their amplitude at the surface of the slab.

This factor  $\delta$ , called one skin thickness or one skin depth, is many times, for example, 10, 100, or even 1000 times larger than the thickness of each metal or insulating lamina. It has been found that when the composite conductor has such a laminated structure, it will propagate a wave at a certain critical velocity which is determined by the geometry of the structure and this wave will penetrate further into the conductor and under proper conditions will have lower attenuation than the transmission mode of a conventional coaxial cable of the same size. The critical velocity mentioned above is determined by the thickness of the metal and insulating laminae, the dielectric constant of the insulating laminae and the permeabilities of the conducting and insulating laminae. It is further shown in the aforementioned Clogston application that the critical velocity is achieved by equating the product of the average dielectric constant of the laminated stack and the average

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permeability thereof to the product of the dielectric constant and permeability of the remaining portion of the cable through which the wave propagates. When these products are equal, the velocity of propagation of the waves is substantially uniform throughout the cross-sectional area of the cable.

It has been discovered that in "Clogston 2" conductors of the type just described, the attenuation constant increases gradually from very low frequencies up to a first certain frequency  $f_1'$ , in practice a few kilocycles, and is then substantially uniform up to a second certain frequency  $f_2'$ , which may in practice be many megacycles. The values of these two certain frequencies  $f_1'$  and  $f_2'$  are dependent on various factors among which are the composition, thickness, and proportioning of the various laminae in the composite conductors. Above this certain second frequency  $f_2'$  the attenuation constant rises, in many cases quite steeply, to a third certain frequency  $f_3'$ , which may be hundreds of times  $f_2'$ , at which frequency  $f_3'$  the rate of increase of attenuation constant decreases somewhat. In the description which follows, all frequencies above  $f_2'$  are called "elevated frequencies." The present invention is concerned with the proportioning of "Clogston 2" conductors to give optimum performance in this elevated frequency area.

In the above-identified Clogston application, it is pointed out that for certain conditions the thickness  $W$  of each metal lamina should be twice the thickness  $t$  of each insulating lamina in the stack. In other words,  $\theta$ , the fraction of the stack filled with conducting material is equal to  $\frac{2}{3}$ . This  $\theta = \frac{2}{3}$  relationship is particularly applicable in cables having substantially infinitely thin insulating and metal layers and for frequencies below  $f_2'$ . However, for finite thicknesses of metal and insulating laminae and especially for frequencies in the "elevated" range, that is, above  $f_2'$ , it has been discovered that other values of  $\theta$  produce lower attenuation.

It is accordingly another object of this invention to produce as low a loss as possible at elevated frequencies in "Clogston 2" cables having conducting and insulating layers of finite thicknesses.

The present invention is based on the discovery that the attenuation can be minimized in the elevated frequency region, that is, for frequencies above  $f_2'$ , for various assigned conditions by fixing the value of  $\theta$  at something other than this previously prescribed value, that is, something other than  $\frac{2}{3}$ , the specific value of  $\theta$  chosen being dependent upon the particular conditions. For example, it is possible to select a value of  $\theta$  so that the attenuation of the cable is minimized for a particular frequency. Also, it is possible to pick a value of  $\theta$  so that the attenuation of the cable is maintained below a predetermined value for the widest possible frequency range.

The invention will be more readily understood by referring to the following description taken in connection with the accompanying drawings forming a part thereof in which:

Fig. 1 is an end view of a coaxial composite conductor of the "Clogston 2" type used in the practice of the invention;

Fig. 2 is a longitudinal view, with portions broken away, of the composite conductor of Fig. 1;

Fig. 3 is a graph of attenuation constant versus frequency for a representative composite conductor of the type shown in Figs. 1 and 2; and

Figs. 4 through 7 are other curves used in explaining the invention.

Referring more particularly to the drawings, Figs. 1 and 2 show, by way of example for purposes of illustration, a conductor 10 in accordance with the invention, Fig. 1 being an end view and Fig. 2 being a longi-

tudinal view. The conductor 10 comprises a central core 11 which may be either of metal or of dielectric material, an outer sheath 12 of metal or other suitable shielding material, and a composite conductor or stack 13 filling the entire space between the members 11 and 12 and comprising many laminations of metal 14 spaced by insulating material 15. As disclosed in the above-mentioned Clogston application, with particular reference to Fig. 17A which shows a composite conductor similar to the composite conductor 10, each of the metal layers 14 is made very thin compared to the skin depth of the conductor being used, which for example, can be copper, silver, or aluminum. The insulating layers 15 are also made very thin and may be of any suitable material. Examples of satisfactory insulating materials are: polyethylene, polystyrene quartz, and polyfoam. The stack 13 has, for example, ten or a hundred or more metal and insulating layers 14 and 15 and since there are a large number of insulating and metallic layers, it makes no difference whether the first or the last layer of the stack 13 is of metal or insulation.

Fig. 3 shows a curve, drawn on a log-log scale, of attenuation constant  $\alpha$  versus frequency for a particular cable of the general type shown in Figs. 1 and 2. It will be appreciated that this curve is merely representative and that a different curve can be drawn for a Clogston 2 conductor of different size and proportioning. Hence exact values taken from this curve are of no significance. In general, the shapes of all curves of this kind, however, are similar to one another. This curve can be considered as being in four parts. The low frequency region A extends from very low frequencies up to a frequency designated in Fig. 3 as  $f_1'$ , a second region B extends from frequency  $f_1'$  to a frequency designated  $f_2'$ , a third region C extends from  $f_2'$  to a higher frequency  $f_3'$ , and a fourth region D comprises all frequencies above  $f_3'$ . The frequencies of points  $f_1'$ ,  $f_2'$  and  $f_3'$  are determined by the points at which the dotted line curve (approximate curve shown in Fig. 3) changes slope. Referring now to this approximate dotted curve in Fig. 3, it will be noted that in the region A, the attenuation constant increases substantially in proportion to the square root of the frequency to approximately the frequency of  $f_1'$ , which in practice may be of the order of a few kilocycles. Between  $f_1'$  and  $f_2'$  the attenuation constant is substantially independent of frequency and  $f_2'$  may be as high as a few megacycles. Between  $f_2'$  and  $f_3'$  the attenuation constant is substantially proportional to the square of the frequency, while in the region D above  $f_3'$ , which may be of the order of a hundred megacycles, the attenuation constant is substantially constant to the square root of the frequency. As pointed out above, the frequencies above  $f_2'$  are designated "elevated" frequencies, and it is within this region that the improvement in attenuation constant is most marked. Prior to the present invention, the emphasis on minimizing the attenuation constant has been primarily in the region below  $f_2'$ .

In accordance with the present invention the value of  $\theta$  is selected to produce desirable operating characteristics for various assigned conditions in the elevated frequency area. Before specifying these conditions it seems desirable to present a mathematical analysis of the cable 10 shown in Figs. 1 and 2 in order to provide a proper background for the discussion of the present invention.

In this analysis:

- $a$ =inner radius of stack
- $b$ =outer radius of stack
- $t_1$ =thickness of each conducting layer (called  $W$  in the above-identified Clogston application)
- $t_2$ =thickness of each insulating layer (called  $t$  in the Clogston application)

- $\theta=t_1/(t_1+t_2)$ ; fraction of stack filled with conducting material
- $\theta_m$ =optimum value of  $\theta$  as defined herein
- $2T_1=\theta(b-a)$ ; total thickness of conducting material in stack
- $\epsilon_2$ =dielectric constant of the insulating layers
- $\bar{\epsilon}=\epsilon_2/(1-\theta)$ ; average dielectric constant of stack
- $\mu_1$ =permeability of conducting layers
- $\mu_2$ =permeability of insulating layers
- $\bar{\mu}=\theta\mu_1+(1-\theta)\mu_2$ ; average permeability of stack
- $\sigma_1$ =conductivity of conducting layers
- $\bar{\sigma}=\theta\sigma_1$ ; average conductivity of stack
- $\delta_1$ =skin depth in conducting layers
- $\alpha$ =attenuation constant
- $\alpha_0$ =low-frequency ("flat") attenuation constant of any Clogston 2 conductor
- $\alpha_{00}$ =minimum value of  $\alpha_0$  (obtained with  $\theta=2/3$  for non-magnetic lines)
- $f$ =frequency
- $f_c$ =a characteristic frequency associated with a Clogston 2 cable and defined by Equation 9 below
- $f_m$ =highest frequency in the operating band
- $\chi$ =a root of Equation 3 below
- $p$ =a mode number; the principal mode corresponds to  $p=1$
- $f_p(a/b)$ =a numerical function; for  $p=1$ ,  $f_1(a/b)$  is plotted against  $a/b$  in Fig. 4.

The attenuation constant of the  $p$ th mode in a coaxial Clogston 2 cable is given approximately by

$$\alpha = \frac{\chi_p^2}{2\sqrt{\bar{\mu}/\bar{\epsilon}}\bar{\sigma}} \left[ 1 + \frac{\theta^2 t_1^2}{3\chi_p^2 \delta_1^4} \right] = \frac{\chi_p^2}{2\sqrt{\bar{\mu}/\bar{\epsilon}}\bar{\sigma}} + \frac{\theta^2 t_1^2 \pi^2 \mu_1^2 \sigma_1^2 f^2}{6\sqrt{\bar{\mu}/\bar{\epsilon}}\bar{\sigma}} \quad (2)$$

where  $\chi_p$  is the  $p$ th root of the equation

$$J_1(\chi a)N_1(\chi b) - J_1(\chi b)N_1(\chi a) = 0 \quad (3)$$

To a rough approximation  $\chi_p$  is equal to  $p\pi/(b-a)$ ; accurate values can be found from curves or tables. In general one can write

$$\chi_p = \frac{p\pi f_p(a/b)}{b-a} \quad (4)$$

where  $f_p(a/b)$  is a numerical factor of the order of unity. For the principal mode ( $p=1$ ),  $f_1(a/b)$  is plotted against  $a/b$  in Fig. 4. Equation 2 for the attenuation constant is valid from frequencies where  $\delta_1 \approx T_1$  (or  $T_1/p$  for the higher modes) to frequencies where  $\delta_1 \approx t_1$ , so that it may be applied over the whole range of engineering interest.

The problem of minimizing the attenuation constant of a Clogston cable of given size, made of given materials, at a preassigned top frequency  $f_m$ , by proper choice of the value of  $\theta$  will first be considered. For definiteness it can be supposed that the thickness  $t_1$  of the conducting layers is given, and that  $\theta$  is varied by varying the thickness of the insulating layers.

The attenuation constant  $\alpha_m$  of the cable at the frequency  $f_m$  is given by

$$\alpha_m = \frac{\sqrt{\epsilon_2}[3\chi_p^2 + \theta^2 t_1^2 \pi^2 \mu_1^2 \sigma_1^2 f_m^2]}{(1-\theta)^{1/2}[\theta\mu_1 + (1-\theta)\mu_2]^{1/2}\bar{\sigma}_1} \quad (5)$$

By logarithmic differentiation it is found that the right-hand side of (4) is a minimum when  $\theta$  satisfies the expression

$$\frac{1}{\theta} + \frac{1}{2(1-\theta)} \frac{(\mu_1 - \mu_2)}{2[\theta\mu_1 + (1-\theta)\mu_2]} + \frac{2\theta t_1^2 \pi^2 \mu_1^2 \sigma_1^2 f_m^2}{3\chi_p^2 + \theta^2 t_1^2 \pi^2 \mu_1^2 \sigma_1^2 f_m^2} = 0 \quad (6)$$

When cleared of fractions, Equation 6 reduces to a quartic equation in  $\theta$ , which may be solved either by formulas or by successive approximations in any particular case. The optimum value of  $\theta$  is the root  $\theta_m$  which lies between zero and one, and the minimum value of  $\alpha_m$  at  $f_m$  is obtained by substituting  $\theta_m$  for  $\theta$  in Equation 5.

If the conducting and insulating layers have equal permeabilities, that is, if  $\mu_1 = \mu_2$ , and in particular if all

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materials used are non-magnetic, the formal results simplify a good deal. Equation 5 then becomes

$$\alpha_m = \frac{\chi_p^2}{2\theta(1-\theta)^{1/2}\sqrt{\mu_2/\epsilon_2\sigma_1}} \left[ 1 + \frac{\theta^2 t_1^2 \pi^2 \mu_1^2 \sigma_1^2 f_m^2}{3\chi_p^2} \right] \quad (7)$$

which may be reduced to dimensionless form by introduction of the characteristic attenuation constant  $\alpha_{00}$  and the characteristic frequency  $f_c$  defined as follows:

$$\alpha_{00} = \frac{3\sqrt{3}\chi_p^2}{4\sqrt{\mu_2/\epsilon_2\sigma_1}} \quad (8)$$

$$f_c = \frac{3\sqrt{3}}{2} \frac{\chi_p}{t_1 \pi \mu_1 \sigma_1} = \frac{3\sqrt{3}}{2} \frac{p f_p (a/b)}{\mu_1 \sigma_1 t_1 (b-a)} \quad (9)$$

Physically  $\alpha_{00}$  represents the minimum low-frequency attenuation constant obtainable with the given cable, by choosing  $\theta = 2/3$ , and  $f_c$  is the frequency at which the attenuation constant of a cable with  $\theta = 2/3$  would be double its low-frequency value  $\alpha_{00}$ . The frequency  $f_c$  is the same as what was called  $f_2'$  before. In terms of these quantities, Equation 7 may be written

$$\frac{\alpha_m}{\alpha_{00}} = \frac{2}{3\sqrt{3}\theta(1-\theta)^{1/2}} \left[ 1 + \frac{9\theta^2 f_m^2}{4 f_c^2} \right] \quad (10)$$

The relation satisfied by the optimum value of  $\theta$ , namely Equation 6, may be reduced when  $\mu_1 = \mu_2$  to the cubic equation

$$9(f_m/f_c)^2[\theta^3 - 2\theta^2] - 12\theta + 8 = 0 \quad (11)$$

The root  $\theta_m$  of Equation 11 is plotted against the ratio  $f_m/f_c$  in Fig. 5. It is worth noting that  $\theta_m$  approaches  $2/3$  as  $f_m/f_c$  approaches zero, and  $\theta_m$  is approximately equal to  $2f_c/3f_m$  when  $f_m \gg f_c$ . The value of  $\alpha_m/\alpha_{00}$  corresponding to  $\theta_m$  is given by

$$\frac{\alpha_m}{\alpha_{00}} = \frac{2}{3\sqrt{3}\theta_m(1-\theta_m)^{1/2}} \left[ 1 + \frac{9\theta_m^2 f_m^2}{4 f_c^2} \right] \quad (12)$$

and is plotted against  $f_m/f_c$  in Fig. 6. The ratio  $\alpha_m/\alpha_{00}$  approaches unity as  $f_m/f_c$  approaches zero, and is approximately equal to  $2f_m/\sqrt{3}f_c$  when  $f_m \gg f_c$ .

Thus, given a particular top frequency  $f_m$ ,  $f_c$  can be calculated from Equation 9, with reference also to Fig. 4 for the principal mode or to a similar curve for other modes. One then calculates the value of the ratio  $f_m/f_c$  and reads off the value of  $\theta_m$  corresponding to this ratio from the curve of Fig. 5.

A closely related problem will now be considered. Given a certain maximum attenuation constant  $\alpha_m$ , the value of  $\theta$  to maximize the frequency band over which the attenuation constant of the cable does not exceed the preassigned value  $\alpha_m$  will be determined.

In the general case where the permeabilities  $\mu_1$  and  $\mu_2$  of the conducting and insulating laminae may be different, the frequency  $f_m$  at which  $\alpha$  is equal to  $\alpha_m$  is given by

$$f_m = \frac{\sqrt{3}}{\pi \mu_1 \sigma_1 t_1} \left[ \frac{2\alpha_m \sigma_1 \sqrt{[\theta \mu_1 + (1-\theta)\mu_2](1-\theta)/\epsilon_2}}{\theta} \chi_p^2 \right]^{1/2} \quad (13)$$

It is assumed throughout that  $\alpha_m$  is at least as large as the minimum attenuation which can be achieved with this cable, so that  $f_m$  as given by Equation 13 is real.

It is desired to maximize  $f_m$  considered as a function of  $\theta$ , that is, to choose the conductor-to-insulator ratio, which is equal to  $\theta/(1-\theta)$ , so as to maximize the frequency band over which  $\alpha$  does not exceed  $\alpha_m$ . Differentiation shows that  $\theta$  must satisfy the relation

$$\frac{\alpha_m \sigma_1 \theta}{\sqrt{\epsilon_2}} \left[ \frac{\theta \mu_1 + 2(1-\theta)\mu_2}{(1-\theta)^{1/2}[\theta \mu_1 + (1-\theta)\mu_2]^{1/2}} \right] = 2\chi_p^2 \quad (14)$$

If both sides are squared and cleared of fractions, Equation 14 reduces to a quartic equation in  $\theta$ , which may be solved either by formulas or by successive approximations in any particular case. Denoting by  $\theta_m$  the root of Equation 14 which lies between zero and one, the maxi-

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imum value of  $f_m$  is found by substituting  $\theta_m$  for  $\theta$  in Equation 13.

If the conducting and insulating layers have equal permeabilities ( $\mu_1 = \mu_2$ ), then Equation 5 becomes

$$\frac{f_m}{f_c} = \frac{2}{3} \left[ \frac{3\sqrt{3}(1-\theta)^{1/2}}{2\theta} \frac{\alpha_m}{\alpha_{00}} - \frac{1}{\theta^2} \right]^{1/2} \quad (15)$$

in terms of the characteristic attenuation constant  $\alpha_{00}$  and the characteristic frequency  $f_c$  defined by Equations 8 and 9 respectively. The equation satisfied by the optimum value of  $\theta$  takes the form

$$\frac{(2\theta - \theta^2)}{\sqrt{1-\theta}} = \frac{8}{3\sqrt{3}} \frac{\alpha_{00}}{\alpha_m} \quad (16)$$

and if  $\theta_m$  is the root of this equation between zero and one, the maximum value of  $f_m$  is given by

$$\frac{f_m}{f_c} = \frac{2}{3\theta_m} \left[ \frac{2-3\theta_m}{2-\theta_m} \right]^{1/2} \quad (17)$$

The ratio  $\alpha_m/\alpha_{00}$  is just the ratio of the maximum permissible attenuation constant to the minimum low-frequency attenuation constant attainable in a Clogston cable of the same diameter. The value of  $\theta_m$  obtained from Equation 16 is plotted against  $\alpha_m/\alpha_{00}$  in Fig. 7;  $\theta_m$  is equal to  $2/3$

when  $\alpha_m/\alpha_{00}$  is equal to unity, and for large values of the attenuation ratio  $\theta_m \approx 4\alpha_{00}/3\sqrt{3}\alpha_m$ . The relationship between  $f_m/f_c$  and  $\alpha_m/\alpha_{00}$  for the optimum cable is the same as that shown in Fig. 6. For  $f_m/\alpha_{00} = 1$ ,  $f_m/f_c = 0$  to the present approximation; and for large values of  $\alpha_m/\alpha_{00}$ ,  $f_m/f_c \approx \sqrt{3}\alpha_m/2\alpha_{00}$ .

Thus, given a particular value of  $\alpha_m$ ,  $\alpha_{00}$  can be calculated from Equation 8, with reference also to Fig. 4 for the principal mode or to a similar curve for other modes. One then calculates the value of the ratio  $\alpha_m/\alpha_{00}$  and reads off the value of  $\theta_m$  corresponding to this ratio from the curves of Fig. 7.

It is obvious that in a (non-magnetic) Clogston cable with  $\theta_m \neq 2/3$ , the low-frequency attenuation constant  $\alpha_0$  will be greater than  $\alpha_{00}$ . In fact,  $1/2\alpha_m < \alpha_0 < \alpha_m$ , which means that in choosing the optimum value of  $\theta$  as has been done above, not only has the top frequency  $f_m$  been increased but also the attenuation vs. frequency characteristic is flattened by raising the low-frequency end. This result reduces the problem of equalizing the Clogston cable over an extended frequency band.

It is to be understood that the above-described embodiments are illustrative of the application of the principles of the invention. Numerous modifications may be devised by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. A cable for the propagation of electromagnetic waves in the elevated frequency range comprising a coaxial stack of conducting members spaced by insulating members, there being a sufficient number of conducting members to carry a substantial portion of the current, the thickness of each conducting member being less than a skin depth at the highest frequency of operation of said cable, the thickness of each conducting member being  $t_1$ , the thickness of each insulating member being  $t_2$ , the ratio  $\theta$  of  $t_1$  to  $(t_1 + t_2)$  having a value between zero and one and satisfying the equation

$$\frac{1}{\theta} + \frac{1}{2(1-\theta)} - \frac{(\mu_1 - \mu_2)}{2[\theta \mu_1 + (1-\theta)\mu_2]} + \frac{2\theta t_1^2 \pi^2 \mu_1^2 \sigma_1^2 f_m^2}{3\chi_p^2 + \theta^2 t_1^2 \pi^2 \mu_1^2 \sigma_1^2 f_m^2} = 0$$

where  $\mu_1$  is the permeability of the conducting members,  $\mu_2$  is the permeability of the insulating layers,  $\sigma_1$  is the conductivity of the conducting members,  $f_m$  is the maximum frequency of operation of the cable, and  $\chi_p$  is the  $p$ th root of the Bessel equations

$$J_1(\chi a)N_1(\chi b) - J_1(\chi b)N_1(\chi a) = 0$$

where  $a$  is the inner radius of the stack and  $b$  is the outer radius of the stack, and  $p$  is equal to the mode

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order of the waves being propagated, the permeability and dielectric constant of the insulating members and the permeability of the conducting members being such that the velocity of propagation is substantially uniform throughout the cross-sectional area of said stack.

2. A cable as claimed in claim 1 wherein  $\theta$  is approximately equal to

$$\frac{2f_c}{3f_m}$$

where

$$f_c = \frac{3\sqrt{3}}{2} \frac{\chi_p}{t_1 \pi \mu_1 \sigma_1}$$

3. A cable for the propagation of electromagnetic waves in the elevated frequency range comprising a coaxial stack of conducting members spaced by insulating members, there being a sufficient number of conducting members to carry a substantial portion of the current, the thickness of each conducting member being less than a skin depth at the highest frequency of operation of said cable, the thickness of each conducting member being  $t_1$ , the thickness of each insulating member being  $t_2$ , the ratio  $\theta$  of  $t_1$  to  $(t_1 + t_2)$  having a value between zero and one and satisfying the equation

$$\frac{\alpha_m \sigma_1 \theta}{\sqrt{\epsilon_2}} \left[ \frac{\theta \mu_1 + 2(1-\theta) \mu_2}{(1-\theta)^{1/2} [\theta \mu_1 + (1-\theta) \mu_2]^{1/2}} \right] = 2\chi_p^2$$

where  $\alpha_m$  is the maximum allowable attenuation,  $\sigma_1$  is the conductivity of the conducting members,  $\epsilon_2$  is the

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dielectric constant of the insulating members,  $\mu_1$  is the permeability of the conducting members,  $\mu_2$  is the permeability of the insulating members, and  $\chi_p$  is the  $p$ th root of the Bessel equations

$$J_1(\chi a) N_1(\chi b) - J_1(\chi b) N_1(\chi a) = 0$$

where  $a$  is the inner radius of the stack and  $b$  is the outer radius of the stack, and  $p$  is equal to the mode order of the waves being propagated, the permeability and dielectric constant of the insulating members and the permeability of the conducting members being such that the velocity of propagation is substantially uniform throughout the cross-sectional area of said stack.

4. A cable as claimed in claim 3 wherein  $\theta$  is approximately equal to

$$\frac{4\alpha_{00}}{3\sqrt{3}\alpha_m}$$

where

$$\alpha_{00} = \frac{3\sqrt{3}\chi_p^2}{4\sqrt{\frac{\mu_2}{\epsilon_2}\sigma_1}}$$

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