

Sept. 11, 1956

S. P. MORGAN, JR

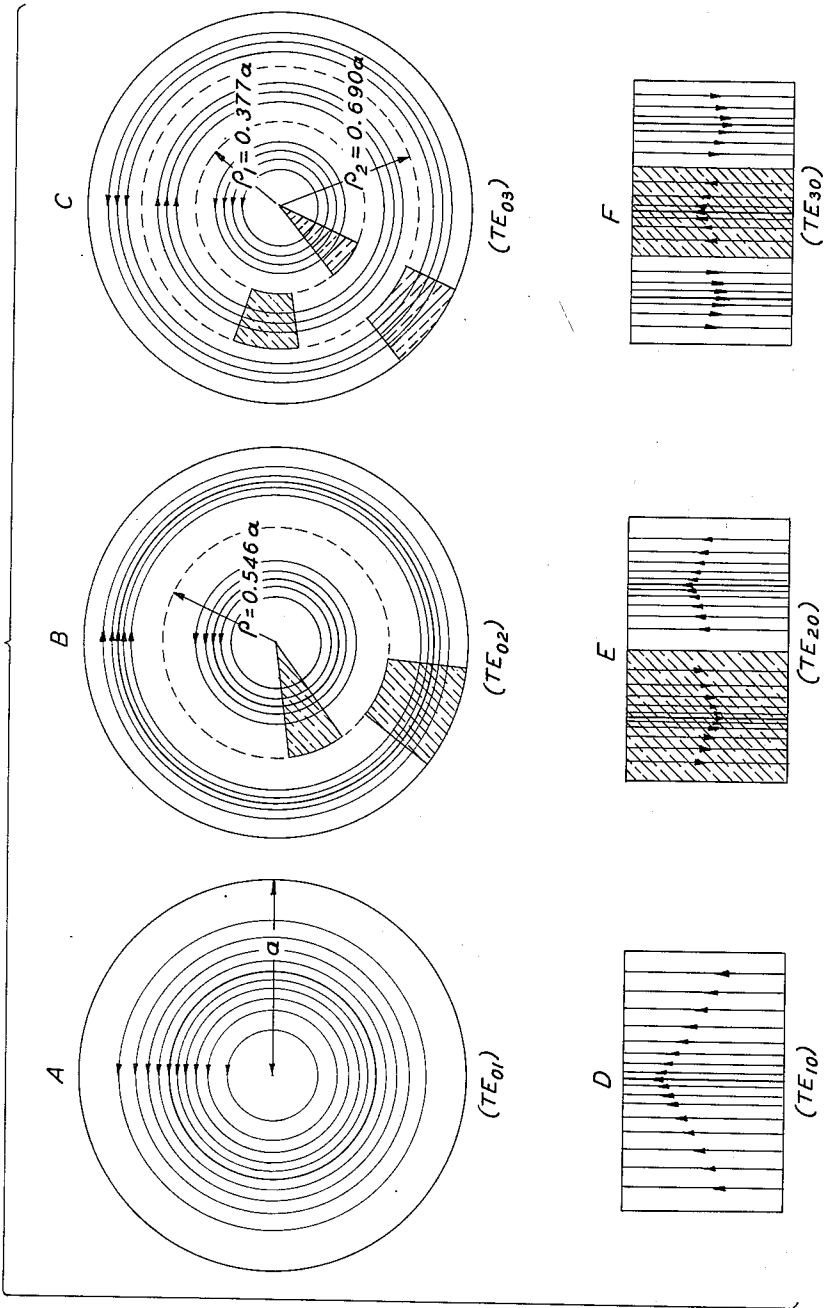
2,762,982

MODE CONVERSION IN WAVE GUIDES

Filed May 17, 1951

3 Sheets-Sheet 1

FIG. 1



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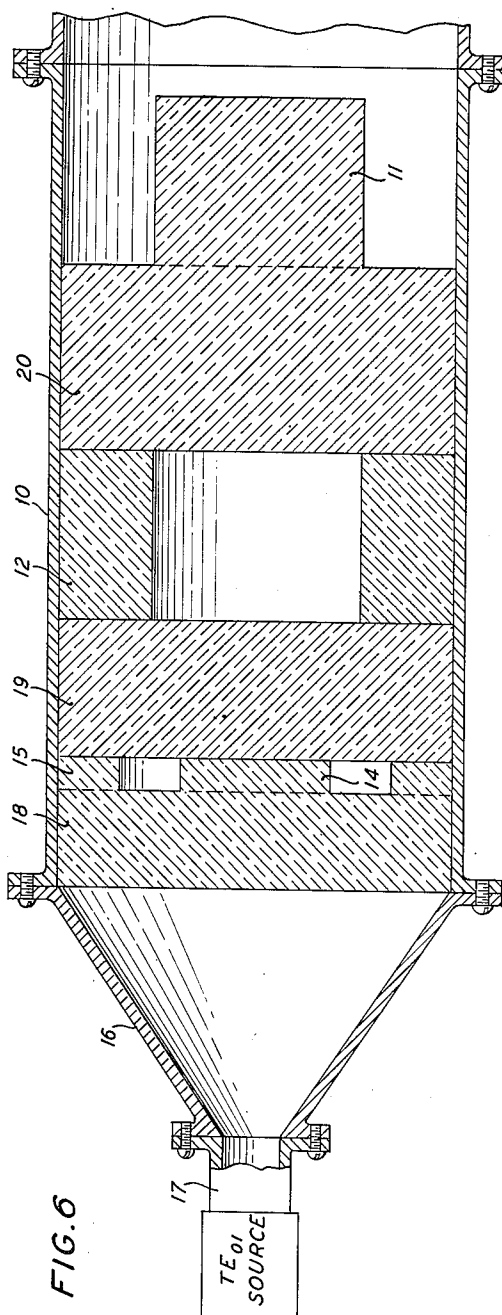
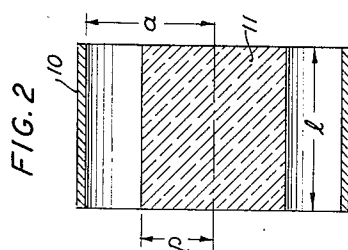
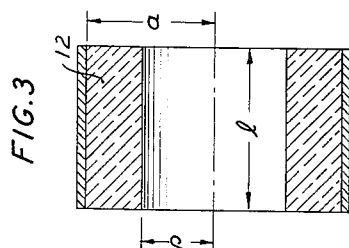
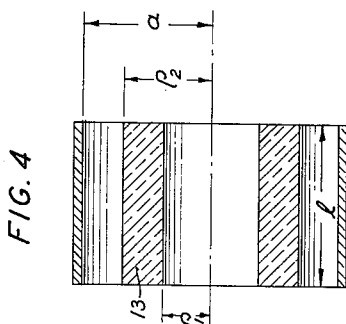
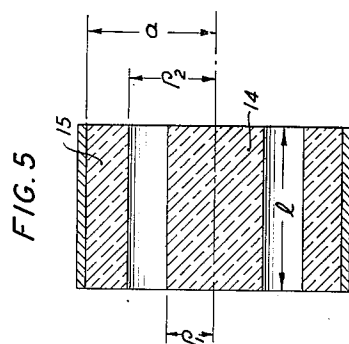
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3 Sheets-Sheet 2



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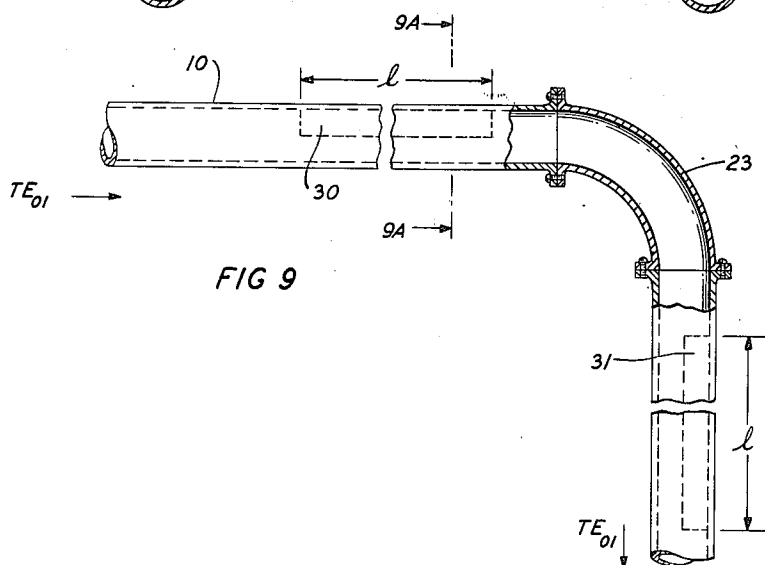
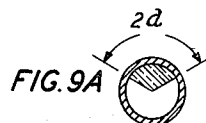
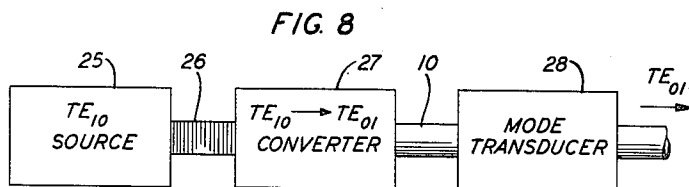
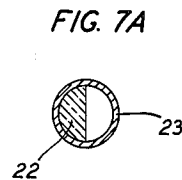
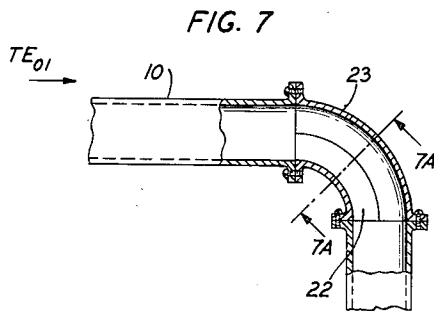
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MODE CONVERSION IN WAVE GUIDES

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3 Sheets-Sheet 3



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2,762,982

MODE CONVERSION IN WAVE GUIDES

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Application May 17, 1951, Serial No. 226,869

7 Claims. (Cl. 333—21)

This invention relates to the transmission of electromagnetic wave energy in the microwave frequency range through wave guides. It is concerned primarily with the spurious wave modes that may tend to appear in a wave guide in addition to the desired or operating wave mode, and with the transformation of guided waves from one mode to another.

In the transmission of electromagnetic energy through a hollow conductive pipe or other wave guide it is well-known that the energy can propagate in one or more transmission modes, or characteristic field configurations, depending on the cross-sectional size and shape of the particular guide and the operating frequency, and that the larger the cross-section of the guide is made the greater is the number of modes in which the energy can propagate at a given operating frequency. Very generally it is desired to confine propagation of the energy to one particular mode, chosen because its propagation characteristics are favorable for the particular application involved. If the desired mode happens to be the so-called dominant mode it is feasible to restrict the cross-sectional dimensions of the guide so that no modes other than the dominant mode can be sustained therein. This expedient is not available, however, if the desired mode is not the dominant mode or if a guide of large cross section is prescribed in order, for example, that advantage may be taken of its relatively low attenuation.

For practical example, it may be desired to transmit circular electric waves of lowest order viz., TE_{01} , through a long-distance wave guide of circular cross section having a diameter several times the operating wavelength. Both the choice of mode and large diameter in this case tend to reduce the attenuation, or power loss, suffered by the waves as they propagate through the guide. In such case, however, the guide can sustain numerous other modes including circular electric modes of higher order such as TE_{02} and TE_{03} , and to the extent that the wave power is actually transmitted in these other modes the power loss is substantially increased and other ill effects may occur.

The unwanted modes may appear because of some lack of symmetry in the mechanism for launching the operating mode into the guide, or irregularities in the guide may result in the conversion of power from the principal, or operating mode into the subordinate, spurious mode. Bends, elbows and taper sections of guide, for example, tend to generate additional modes and these will be propagated, if the guide be large enough, and thereby introduce deleterious effects.

Heretofore mode-selective absorbers or attenuators have been proposed for use in wave guides to substantially prevent the transmission of unwanted modes. These, however, are generally limited in respect of the modes they can accommodate, or tend to be resonant or otherwise to be effective only over a narrow range of frequencies, or tend to present an impedance mis-match to waves of the desired mode and thereby to cause un-

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wanted reflection of the latter, and they depend on absorption of wave power.

The present invention has as one of its general objectives the reduction or suppression of unwanted modes in a wave guide. More particular objectives are to effect such reduction or suppression with minimum loss of wave power, over relatively broad frequency bands, and/or without substantial reflection of wave power in the operating mode or other ill effect.

Another objective of the invention is to counteract the tendency of wave-guide bends, taper sections and the like to generate unwanted modes.

The present invention involves in one aspect the controlled progressive transformation of energy from the operating mode in a wave guide into one or more other modes, including more particularly a transformation into one or more other modes in such relative phase and amplitude as to oppose or neutralize any spurious mode that accompanies the operating mode.

In accordance with embodiments of the invention to be described hereinafter in detail, the conversion of wave power from the operating mode to another mode is effected by a dielectric transducer comprising dielectric material that is disposed within the guide in predetermined relation to the characteristic electric field pattern of the wave mode to be excited, and that may extend a substantial distance longitudinally through the guide. In some embodiments the transducer serves to neutralize a spurious mode that is introduced elsewhere into the guide, whereas in others the transducer is disposed in a section of guide that itself tends to introduce a spurious mode.

I have found that while each elemental length of the dielectric transducer tends to convert wave energy from the principal mode into the secondary mode, in general there is effectively a progressive shift in phase in the respective contributions from successive elemental lengths, and that the net amount of conversion would oscillate through maximum and zero values if the length of the transducer were to be increased indefinitely. In accordance with a feature of the invention, the dielectric transducer may be divided into longitudinally separated dielectric transducer elements, either to obtain a greater transformation than is possible with a single transducer element or to facilitate adjustment of the vector-resultant amplitude of the secondary mode. In accordance with a related feature complementary dielectric transducers may be used in succession with or without a space interval between them.

In accordance with a further feature the transducers may be constructed of a material having a dielectric constant that is only slightly higher, for example less than ten per cent higher and preferably in some cases only of the order of two per cent higher, than the dielectric constant of the medium (air or other gas, e. g.) filling the rest of the guiding structure.

In accordance with still another feature of the invention a bend or other wave guide element that has a natural propagation mode other than the operating mode, is preceded by a first mode transducer such that the wave power passes through the element without further mode conversion, and a second mode transducer follows the element to restore the wave power to the operating mode. This subject matter is further disclosed and claimed in my application Serial No. 255,836 filed November 10, 1951.

The nature of the present invention and its various objectives, features and advantages will appear more fully on consideration of the embodiments illustrated in the accompanying drawings and hereinafter described.

In the drawings:

Fig. 1 comprises a set of diagrams that, in part, illus-

trate the field configurations of certain wave modes;

Figs. 2 to 5 illustrate elementary mode transducers in accordance with the invention;

Fig. 6 illustrates a more complex mode transducer associated with a taper section of wave guide; and

Figs. 7, 7a, 8, 8a, 9 and 9a illustrate further embodiments of the invention.

Referring now to Fig. 1, there is represented diagrammatically at A the electric field configuration characteristic of the TE_{01} mode in a circular guide, while B and C are applicable similarly to the TE_{02} and TE_{03} modes, respectively. In each case the electric field is symmetrical about the axis and varies in intensity from center to periphery of the guide in known manner. In the TE_{01} mode the phase is the same from axis to periphery, as indicated by the arrows. In the TE_{02} mode the phase is the same from the axis out to a cylindrical node of radius ρ_1 equal to $0.546a$ where a is the radius of the guide, and the opposite relative phase prevails in the tubular zone extending from that node to the periphery. In the TE_{03} mode there are three such "cophasal" zones, viz., a central zone extending from the axis to a node of radius ρ_1 equal to $0.377a$; an outer cophasal zone, of the same phase as the first, extending from a node of radius ρ_2 equal to $0.690a$ to the periphery; and a third cophasal zone, opposite in phase relative to the other two and lying between them.

I have found that if in a wave guide carrying the TE_{01} mode, as in A, dielectric material be placed in one or the other of the two cophasal zones of B, some of the energy of the operating mode will be converted into the TE_{02} mode, and further that any tendency to generate non-circular modes can be reduced by disposing the dielectric material symmetrically about the axis of the guide, i. e., in the form of a (solid or hollow) cylinder coaxial with the guide. The maximum generation of the TE_{02} mode occurs if the dielectric just fills the inner cophasal zone, i. e., the space from the axis of the guide to the first node of the electric field, as illustrated in Fig. 2, or if it just fills the outer cophasal zone, i. e., the space between the first node and the metal shell 10 of the guide, as illustrated in Fig. 3. The two dielectric elements 11, 12, of Figs. 2 and 3, assuming them to be of the same length, are complementary in that the TE_{02} waves they generate are equal in amplitude and opposite in phase. This can be readily appreciated on observing that the two (generalized) cylinders together would form a solid dielectric plug filling the guide and that such a plug would generate no modes other than TE_{01} under the influence of a pure TE_{01} wave.

Similarly, the most efficient disposition of dielectric material for the generation of the TE_{03} mode is in the form of a hollow cylinder or tube 13 just filling the intermediate cophasal zone, as illustrated in Fig. 4, or in the form of a pair of cylinders, 14, 15, one just filling the inner cophasal zone and the other a tube just filling the outer one, as illustrated in Fig. 5. The dielectric structures in Figs. 4 and 5, respectively, are complementary to each other with respect to the generation of the TE_{03} mode or any other mode.

The same principles are applicable to other guide shapes and modes generally, excepting only unusual cases where the electric field vector of the mode to be generated is perpendicular at every point to the electric field vector of the operating mode. The cophasal zones can be identified readily by superposing the electric field diagrams of the operating mode and the mode to be derived, and marking the boundaries between regions in which the electric vector of the operating mode (or, more generally, the projection thereof) coincides in direction and phase with the electric vector of the other mode, and similar regions in which the relative phases are reversed. Diagram D in Fig. 1, for example, illustrates the dominant mode TE_{10} in a guide of rectangular cross section, while at E are illustrated the TE_{20} mode in the same guide, the two equal cophasal zones that appear, and the disposition of dielec-

tric material in one of the two cophasal zones for maximum conversion of wave power from the first of these modes to the second. For maximum generation of the TE_{30} mode the dielectric material may fill the central one of the three equal cophasal zones shown at F in Fig. 1 or it may fill both of the adjoining zones which together constitute a cophasal region of the opposite relative phase.

The specific dimensions that have been assigned to the dielectric mode transducers of Figs. 2 to 5 are, as indicated, those that, with a given dielectric material, yield maximum generation of a prescribed secondary mode for a given over-all length of transducer, and it has been pointed out that complementary transducers are equally effective. In various practical applications of the invention, however, other factors than maximum generation are significant or even controlling and they may dictate the choice of one transducer over another or call for modification of the dimensions given. One such factor is ease of fabrication, or mechanical support for the dielectric members. The tubular dielectric member in Fig. 3, e. g., is self-supporting within the guide; the complementary dielectric member in Fig. 2 is not. The latter can be supported by a contiguous dielectric plug (as in Fig. 6); or if the transducer is interposed in a wave guide that is filled with solid dielectric material it can be supported by a surrounding tube of that same material. In some cases another factor, the total volume of dielectric material required, may favor the choice of one design instead of another.

Another factor in the design of a dielectric mode transducer that may be significant in certain circumstances is the tendency of the transducer to generate modes other than the prescribed secondary mode or modes. The transducers of Figs. 2 and 3, for example, generate the TE_{03} mode in some slight degree, because the dielectric fills one of the like-phased cophasal zones of this mode (Fig. 1C), and thus tends to generate the TE_{03} mode but it does not extend quite the proper distance into the adjacent oppositely-phased zone to effect cancellation. A secondary mode so generated will be suppressed, however, if its cut-off frequency is greater than the operating frequency, i. e., if the guide be of sufficiently small size. If not so suppressed it may happen to be of such amplitude and phase as to oppose and reduce a wave component of the same mode originating elsewhere in the guide. The generation of any particular secondary mode can be obviated, in any event, by so disposing the dielectric material of the mode transducer that any dielectric material in one cophasal zone of that mode is balanced by dielectric material in a cophasal zone of the opposite phase. Applying this principle to the transducers of Figs. 2 and 3, for example, it will be found that if the indicated radius is changed from $0.546a$ to $0.529a$ none of the TE_{03} mode will be generated.

Still another factor is the relation between the dielectric constants of the several dielectric media, that is, the two dielectric media in the transducer section and the dielectric medium in the adjoining sections of guide. The more closely the average or effective dielectric constant in the transducer section approximates the dielectric constant in the adjoining sections, the more completely is the transducer reflectionless. The more nearly alike the dielectric constants of the media in the transducer section the lower are internal reflections and reflection losses in composite transducers such as the one illustrated in Fig. 6. The greater, too, is the length of transducer required, and in some cases this in itself is advantageous. It will be understood, then, that as a special case the adjoining guide may contain a solid dielectric medium and the transducer may comprise a first dielectric of the same or lower dielectric constant and a second dielectric (a gas, for example) of still lower dielectric constant.

Although a mode transducer in accordance with the invention may be designed with the aid of mathematical formulas given hereinafter to meet the requirements of a

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particular problem in practice, the invention is especially well adapted to an empirical method of design and fabrication. This approach will be treated first and applied to a specific example of practice before the more general and elaborate mathematical approach is considered.

Assume for specific example that a transducer of the form shown in Fig. 3 is called upon to generate TE₀₂ waves of predetermined amplitude and phase in a guide carrying pure TE₀₁ waves. Since the amplitude of the generated TE₀₂ waves depends on the length of the dielectric member the latter can be fabricated in the form of washers which are assembled in the guide one after another until the generated TE₀₂ wave is increased to the desired amplitude. Then the dielectric assemblage is shifted longitudinally in the guide to a position such that the generated waves have the desired phase at the prescribed point. This method of fabrication by parts is applicable also to other forms of dielectric transducer although the mechanical problem is greater in some cases than in others.

In carrying out the foregoing procedure a point may be reached where additional washers contribute progressively less to the amplitude of the generated waves and, in fact, a point may be reached at which additional washers reduce the amplitude. If a greater amplitude be required in such case additional washers may be assembled in the same manner to form another dielectric tube at a distance along the guide where tests show they increase the amplitude further. Alternatively in such case the first transducer element may be followed directly by one of the complementary form shown in Fig. 2. Mode transducer members conforming with Fig. 4 and/or Fig. 5 may be provided additionally if the TE₀₃ mode is to be generated at the same time.

Where two or more dielectric transducer members appropriate to the same secondary mode are arranged in tandem for one reason or another, the amplitude of the resultant generated wave can be adjusted precisely by adjusting the distance between members, for this distance determines the relative phase in which the contribution from each member to the desired secondary mode combines with the contributions from the other members. The assemblage of dielectric members may then be shifted, as a unit, to adjust the phase of the resultant generated wave as desired. The contributions from transducer members of the same form are directly additive if, in the interval between members, the number of wavelengths of the operating mode differs by an integer (e. g., 1, 2, 3 . . .) from the number of wavelengths of the secondary mode being generated.

There is little tendency for any of the generated secondary modes to give rise in any significant degree to still other modes by interaction with the dielectric transducers, for the secondary modes will ordinarily be many times smaller in amplitude than the operating mode.

Fig. 6 illustrates a composite mode transducer that is disposed adjacent the larger end of a conical wave guide adapter, or taper section, 16 in a TE₀₁ system and that is designed to substantially neutralize TE₀₂ and TE₀₃ modes generated by the adapter. The dielectric structure may be divided into parts, as shown, for ease of manufacture. The lefthand dielectric element may be resolved into a cylindrical plug 18 and a TE₀₁-TE₀₃ transducer 15 conforming with Fig. 5, the plug serving to fix accurately the distance from the conical adapter to the transducer. This is followed by another spacing plug 19, a TE₀₁-TE₀₂ transducer 12 conforming with Fig. 3, and an element comprising another spacer 20 and a second TE₀₁-TE₀₂ transducer 11 complementary to the first.

In one instance in practice the conical adapter 16 was twelve inches long and joined two air-filled metal guides having inside diameters of two inches and 4.732 inches, respectively. The smaller guide 17 was supplied with pure TE₀₁ waves having a free-space wavelength of 3.33

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centimeters. The dielectric structure was constructed of polystyrene foam having a relative dielectric constant of 1.0235 and a low power factor. The lengths of the spacers were 2.92, 4.19 and 5.64 centimeters, respectively, taking them in order from left to right as they appear in Fig. 6. In the same order, the lengths of the transducer sections were 1.10, 5.07 and 5.07 centimeters, respectively, and the radii were scaled as in Figs. 2, 3 and 5 for maximum generation of the two secondary modes. (Fig. 6 shows the structure substantially to scale except for shortening of the adapter 16.)

It will be noted that the operation of the dielectric mode transducer does not depend on power absorption. The transducer tends instead to restore the power received in the form of spurious modes into the original mode. Although the function of the dielectric transducer in Fig. 6 is analogous to that of an optical lens in so far as it tends to restore the curved wave front incident upon it to planar form, the principles of operation are distinctly different. Since the space dimensions involved are comparable with the wave length, geometric optics cannot be applied.

The design of dielectric mode transducers in accordance with the invention may be treated from a theoretical or mathematical standpoint as follows.

Any steady-state harmonically varying electromagnetic field in a perfectly conducting wave guide filled with a homogeneous dielectric but containing no charge and current distributions in its interior, may be regarded as a superposition of one or more of the transmission modes characteristic of the cross-sectional shape (round, rectangular, or other) of that particular guide. If an obstacle of different dielectric constant is now introduced into the guide, the amplitudes of these modes and in general the number of modes present will be altered, in order that the total fields may satisfy appropriate boundary conditions at the surface of the obstacle. An exact calculation of the amplitudes and phases of all the modes induced by the presence of the obstacle would be an exceedingly difficult undertaking in any but the simplest special cases; but it happens that if the relative dielectric constant of the obstacle differs only slightly from unity, we can calculate its effect on the wave guide fields to first approximation by a perturbation method. This method we now proceed to describe.

It is shown in works on electromagnetic theory (S. A. Schelkunoff, *Electromagnetic Waves*, D. van Nostrand Co., Inc., New York, 1943, pp. 90-94) that if a homogeneous dielectric "island" is placed in a harmonically varying electromagnetic field in another dielectric medium, the effect of the island on the field may be calculated by replacing it with fictitious polarization currents and charges.

If the relative dielectric constant ϵ_r of the island differs but little from unity, i. e., if

$$(\epsilon_r - 1) \ll 1, \epsilon_r = \epsilon / \epsilon_0 \quad (1)$$

where ϵ is the dielectric constant of the obstacle and ϵ_0 the dielectric constant of the surrounding material, then we may obtain a first approximation to the effect of the dielectric on the fields by assuming the polarization currents $\vec{J}$ and charges q_s to be given by

$$\vec{J} = i\omega\epsilon_0(\epsilon_r - 1)\vec{E}_0 \quad (2)$$

$$q_s = \epsilon_0(\epsilon_r - 1)E_{0n} \quad (3)$$

where $\vec{E}_0$ is the electric field which would have existed at the point in question in the absence of the obstacle, E_{0n} is

the normal component of $\vec{E}_0$ at the surface of the obstacle, and ω is the angular frequency. Since there are known expressions for the radiation fields of current and charge elements, either in a wave guide or in an unbounded space (see for example W. R. Smythe, *Static and Dynamic Electricity*, 2nd ed., McGraw-Hill, New York, 1950, pp.

521-523, 551, 552), one has only to integrate these expressions, suitably weighted, over the entire volume occupied by the dielectric in order to get the first approximation to the change in the original field due to the presence of the dielectric. Presumably this first approximation might be used in turn to calculate a second approximation, and so forth.

Consider specifically, by way of example, the circular electric (TE_{0m}) modes characteristic of a round wave guide. There are an infinite number of these modes, of which however at most a finite number will propagate freely in a pipe of given diameter at a particular operating frequency. If we take a cylindrical coordinate system (ρ, φ, z), where $\rho=a$ represents the inner surface of the guide and z is measured along the guide axis, the electric vector of these modes has only a φ -component, which is independent of the angle φ . The electric field of the TE_{0m} mode is given, up to an arbitrary amplitude factor by (S. A. Schelkunoff, *Electromagnetic Waves*, pp. 380-381, 389-390)

$$(E_{0m})_{\varphi} = J_1(X_{0m}\rho) e^{-\Gamma_{0m}z}, \quad m=1, 2, \dots \quad (4)$$

where

$$\Gamma_{0m} = (X_{0m}^2 - \beta^2)^{1/2}, \quad \beta = 2\pi/\lambda \quad (5)$$

Here λ is the wavelength, at the operating frequency, of a free wave in an unbounded medium of dielectric constant ϵ_0 , and $X_{0m}a = k_{0m}$ is the m th positive root of the Bessel function $J_1(x)=0$. For the modes which are above cut-off (i. e., freely propagated) at the operating frequency we may write

$$\Gamma_{0m} = i\lambda_{0m} = i(\beta^2 - X_{0m}^2)^{1/2} = 2\pi i/\lambda_{0m} \quad (6)$$

where λ_{0m} is the wavelength of the TE_{0m} wave in the guide. The field pattern of the circular electric modes in any plane normal to the axis of the guide is quite simple, the electric lines of force being merely concentric circles. The electric field vanishes on the axis of the guide and at the perfectly conducting outer wall; in addition there are, for the TE_{0m} mode, $(m-1)$ nulls of electric field between the axis and the guide wall.

If in a round wave guide the electric field vector has only a φ -component, the field may always be regarded as a super-position of one or more TE_{0m} modes with appropriate amplitudes and phases. If the wave guide extends indefinitely, in the positive z -direction and there is no generation, reflection, or absorption of modes to the right of some reference plane $z=z_0$, then for $z \geq z_0$ we may write

$$E_{\varphi} = \sum_{m=1}^{\infty} A_m J_1(X_{0m}\rho) e^{-\Gamma_{0m}z} \quad (7)$$

where the A_m 's are complex constants (of which all but a finite number may be zero). We shall call A_m the amplitude of the TE_{0m} component of the field whose electric vector is given by (7). The choice of how we define unit amplitude of a given mode is of course arbitrary, the definition adopted here being simplest for our present purpose.

Suppose that we have a wave guide propagating an initially pure TE₀₁ wave in the positive z -direction, the electric field of this wave being given by

$$E_{0\varphi} = A J_1(X_{01}\rho) e^{-\Gamma_{01}z} \quad (8)$$

If we insert a rotationally symmetric dielectric obstacle coaxially into the guide then, unless the dielectric forms a solid cylinder completely filling a section of guide between two transverse planes, in addition to altering the amount of TE₀₁ mode present it will generate TE₀₂, TE₀₃, . . . modes in amounts depending on the dimensions and shape of the obstacle. The total electric field in the guide may then be written as

$$E_{\varphi} = E_{0\varphi} + (\epsilon_r - 1)E_{1\varphi} \quad (9)$$

where $(\epsilon_r - 1)E_{1\varphi}$ is the field induced by the presence

of the obstacle of relative dielectric constant ϵ_r . The factor $(\epsilon_r - 1)$ is inserted explicitly since if ϵ_r differs but slightly from unity the quantity $E_{1\varphi}$ can be calculated as a function of the geometry of the obstacle, the radius of the guide, and the operating wavelength, independent of ϵ_r to first approximation.

To the right of the obstacle we evidently have

$$(\epsilon_r - 1)E_{1\varphi} = A \sum_{m=1}^{\infty} (\epsilon_r - 1)B_m J_1(X_{0m}\rho) e^{-\Gamma_{0m}z} \quad (10)$$

where $(\epsilon_r - 1)B_m$ is the relative amplitude of the TE_{0m} mode generated by the interaction between the dielectric and the original TE₀₁ field. If $(\epsilon_r - 1) \ll 1$, the B_m 's may be calculated as described above, by replacing the dielectric by elementary rings of polarization current with current density given by (2) and (8). Since the electric field is tangential to the surface of the obstacle there will be no surface polarization charges.

Calculation shows that the relative amplitudes of TE₀₂ and TE₀₃ waves generated by the dielectric structures of Figs. 2 to 5, dimensioned for maximum mode generation as described, are as follows:

Fig. 2:

$$B_2 = -i0.2824 \frac{2\pi a \lambda_{02}}{\lambda^2} \frac{\sin \frac{1}{2}(h_{01} - h_{02})l}{\frac{1}{2}(h_{01} - h_{02})a} e^{-i(h_{01} - h_{02})(c + \frac{1}{2}l)} \quad (11)$$

$$B_3 = +i0.0289 \frac{2\pi a \lambda_{03}}{\lambda^2} \frac{\sin \frac{1}{2}(h_{01} - h_{03})l}{\frac{1}{2}(h_{01} - h_{03})a} e^{-i(h_{01} - h_{03})(c + \frac{1}{2}l)} \quad (12)$$

Fig. 3:

$$B_2 = +i0.2824 \frac{2\pi a \lambda_{02}}{\lambda^2} \frac{\sin \frac{1}{2}(h_{01} - h_{02})l}{\frac{1}{2}(h_{01} - h_{02})a} e^{-i(h_{01} - h_{02})(c + \frac{1}{2}l)} \quad (13)$$

$$B_3 = -i0.0289 \frac{2\pi a \lambda_{03}}{\lambda^2} \frac{\sin \frac{1}{2}(h_{01} - h_{03})l}{\frac{1}{2}(h_{01} - h_{03})a} e^{-i(h_{01} - h_{03})(c + \frac{1}{2}l)} \quad (14)$$

Fig. 4:

$$B_3 = +i0.3275 \frac{2\pi a \lambda_{03}}{\lambda^2} \frac{\sin \frac{1}{2}(h_{01} - h_{03})l}{\frac{1}{2}(h_{01} - h_{03})a} e^{-i(h_{01} - h_{03})(c + \frac{1}{2}l)} \quad (15)$$

Fig. 5:

$$B_3 = -i0.3275 \frac{2\pi a \lambda_{03}}{\lambda^2} \frac{\sin \frac{1}{2}(h_{01} - h_{03})l}{\frac{1}{2}(h_{01} - h_{03})a} e^{-i(h_{01} - h_{03})(c + \frac{1}{2}l)} \quad (16)$$

Here B_2 and B_3 have the meaning as in Equation 10, and as before

$$h_{01} = 2\pi/\lambda_{01}, \quad h_{02} = 2\pi/\lambda_{02}, \quad h_{03} = 2\pi/\lambda_{03} \quad (17)$$

Inspection of Equations 11-16 shows that on account of the sine factors the B_m 's do not increase in magnitude indefinitely as l is increased but are oscillatory functions of the lengths of the dielectric cylinders. (This is in contrast to the predictions of a too naive wavefront argument, which might lead one to imagine that the distortion of the wavefront could be increased indefinitely by "slowing down" certain portions of the wavefront with a dielectric cylinder of arbitrarily great length.)

Suppose then that in a wave guide (say for $z \geq z_0$, where z_0 is some reference plane) the electric field is given by

$$E_{\varphi} = A \left[J_1(X_{01}\rho) e^{-\Gamma_{01}z} + \sum_{m=2}^{\infty} C_m J_1(X_{0m}\rho) e^{-\Gamma_{0m}z} \right] \quad (18)$$

where

$$|C_m| \ll 1, \quad m=2, 3, 4, \dots$$

The field represented by (18) consists predominantly of the TE₀₁ mode, with small amounts of TE₀₂, TE₀₃, etc.,

present as impurities. We assume that the modes above TE_{03} are beyond cut-off at the operating frequency, although in principle we could deal with any finite number of propagating modes. We further suppose that the complex relative amplitudes C_2 and C_3 are known, either by calculation or by experimental measurement. Then with the help of Equations 11-16 we can determine proper dimensions and spacing for an array of hollow dielectric cylinders whose interactions with the TE_{01} component of the original field (18) will generate higher mode fields of amplitude and phase just sufficient to cancel out the TE_{02} and TE_{03} modes occurring in (18). Comparing (9) and (10) with (18) we see that the conditions to be met are

$$(er-1) \Sigma B_2 = -C_2 \quad (20)$$

$$(er-1) \Sigma B_3 = -C_3 \quad (21)$$

where the summation signs merely indicate that we are to add the contributions from all the elementary cylinders of which the lens will be made. If (20) and (21) are satisfied simultaneously, then at any point beyond the lens we should find only the TE_{01} mode plus modes which are beyond cut-off at the given frequency. The latter modes represent local fields which vanish within a few wavelengths from the obstacle, leaving a pure TE_{01} wave.

In applying the invention to the compensation of a conical adapter, as in Fig. 6, some assistance may be derived from the following approximate theoretical formulae expressing the relative amplitude and phase of certain modes generated in such an adapter.

The field in the guide at the larger end of the conical section is given approximately by

$$E_\phi = A \left[J_1(X_{01}\rho) e^{-\Gamma_{01}z} + \sum_{m=2}^{\infty} C_m J_1(X_{0m}\rho) e^{-\Gamma_{0m}z} \right] \quad (22)$$

where the complex coefficients C_m are functions only of the dimensionless parameter

$$\alpha = \pi a^2 / \lambda_{01} L \quad (23)$$

and L is the perpendicular distance from the vertex of the cone to the mouth of the large cylindrical guide of radius a . The quantity α depends on the geometry of the taper and the operating wavelength. One can derive explicit formulas for the C_m 's in terms of definite integrals which can be evaluated numerically for any desired choices of m and α . The values of $C_2(\alpha)$ and $C_3(\alpha)$, for a ranging from 0.0 to 1.2, are given in the following table:

α	$C_2(\alpha)$	$C_3(\alpha)$
0.0	0.0000 $e^{+i1.571}$	0.0000 $e^{-i1.571}$
0.2	0.0434 $e^{+i1.566}$	0.0127 $e^{-i1.665}$
0.4	0.0969 $e^{+i1.562}$	0.0257 $e^{-i1.762}$
0.6	0.1454 $e^{+i1.557}$	0.0394 $e^{-i1.855}$
0.8	0.1941 $e^{+i1.552}$	0.0539 $e^{-i1.943}$
1.0	0.2430 $e^{+i1.547}$	0.0696 $e^{-i2.028}$
1.2	0.2921 $e^{+i1.542}$	0.0867 $e^{-i2.108}$

Since the Expression 22 for the fields in the guide which follows the tapered section is of exactly the same form as (18), with known values of the coefficients C_m , we can apply the preceding theory to design a transducer which will eliminate the undesired higher modes appearing in it.

As a numerical example consider a structure to compensate for a tapered section 12 inches long, which joins a guide of inside diameter 2 inches to a guide of inside diameter 4.732 inches. At the design wavelength of 3.33 centimeters the only circular electric mode which

will propagate in the small guide is TE_{01} , while the large guide will support TE_{01} , TE_{02} , and TE_{03} . We set

$$\begin{aligned} a &= 6.01 \text{ cm.} \\ \lambda &= 3.33 \text{ cm.} \\ L &= 52.79 \text{ cm.} \end{aligned} \quad (24)$$

where L is the perpendicular distance from the vertex of the conical section to the mouth of the large guide. The guide wavelengths of the propagating circular electric modes are:

$$\lambda_{01} = 3.54 \text{ cm., } \lambda_{02} = 4.24 \text{ cm., } \lambda_{03} = 7.54 \text{ cm.} \quad (25)$$

For the relative dielectric constant of the transducer material we shall take

$$\epsilon_r = 1.0235 \quad (26)$$

this being the value measured for a particular sample of polystyrene foam, with an uncertainty of perhaps two or three units in the last figure.

The first step in designing the transducer is to obtain the coefficients C_2 and C_3 for the particular taper employed. We find from (23), (24), and (25) that

$$\alpha = 0.6074 \quad (27)$$

then by interpolation in the foregoing table,

$$C_2 = 0.1472 e^{+i1.557} \quad (28)$$

$$C_3 = 0.399 e^{-i1.858} \quad (29)$$

To eliminate the TE_{02} and TE_{03} mode fields in the region beyond the transducer we must satisfy (20) and (21). We shall satisfy these conditions by successive approximations. Since according to (28) and (29) the TE_{02} mode predominates over TE_{03} in the original field, we first design a cylinder which eliminates TE_{02} , i. e., which satisfies (20) without regard to (21). We then design, using (21), a second cylinder to compensate for the total TE_{03} amplitude generated by the tapered section plus the first cylinder. Since the amount of TE_{02} mode reintroduced by the second cylinder is negligible, the transducer is then complete.

In order to satisfy (20) we take a hollow cylinder of the form shown in Fig. 3 whose internal and external radii are $0.546a = 3.28$ centimeters and $a = 6.01$ centimeters, and whose length and position are to be determined. Substituting for B_2 from (13), for ϵ_r from (26), and for C_2 from (28), we find that (20) becomes

$$i0.1086 \sin \frac{0.882l}{a} e^{-i1.764(c+\frac{1}{2}l)/a} = -0.1472 e^{+i1.577} \quad (30)$$

Inasmuch as the absolute value of the sine factor cannot exceed unity, we see that with a transducer of such low dielectric constant it is necessary to correct the TE_{02} mode with two cylinders, the second one being the complement of the first and being placed at such a point in the guide that beyond the transducer the contributions of the two cylinders add in phase. If we require the first cylinder to cancel half of the total TE_{02} field, we get

$$-i \sin \frac{0.882l}{a} e^{-i1.764(c+\frac{1}{2}l)/a} = 0.678 e^{+i1.557} \quad (31)$$

On equating the moduli of the complex members on both sides of (31), we get

$$\sin \frac{0.882l}{a} = 0.678 \quad (32)$$

$$l = 5.07 \text{ cm.}$$

while from equality of the phases,

$$\begin{aligned} 3\pi/2 - 1.764(c+\frac{1}{2}l)/a &= 1.557 \\ c &= 8.21 \text{ cm.} \end{aligned} \quad (33)$$

The complementary cylinder which cancels the other half of the TE_{02} field has the form of Fig. 2, with a radius of

3.28 centimeters and a length of 5.07 centimeters. Its distance c' from the mouth of the guide is determined by

$$\begin{aligned} 1.764c'/a &= 1.764c/a + \pi \\ c' &= 18.92 \text{ cm.} \end{aligned} \quad (34)$$

Next we have to calculate the total amplitude of TE_{03} mode introduced by the tapered section plus the two cylinders already designed. Let this amplitude be C_3' ; then from (29), (14) and (12),

$$\begin{aligned} C_3' &= 0.0399e^{-i1.858} \\ -i0.0062 \sin \frac{2.832l}{a} &\left[e^{-i5.665(c+\frac{1}{2}l)/a} - e^{-i5.665(c'+\frac{1}{2}l)/a} \right] \\ &= -0.0346e^{i1.440} \end{aligned} \quad (35)$$

To cancel this TE_{03} field we introduce the pair of coaxial cylinders shown in Fig. 5, the radii of the inner cylinder being

$$0 \text{ and } 0.377a = 2.26 \text{ cm.} \quad (36)$$

while the radii of the outer cylinder are

$$0.690a = 4.14 \text{ cm. and } a = 6.01 \text{ cm.} \quad (37)$$

The length l'' and position c'' of these cylinders are determined by

$$(e_r - 1) B_3 = -C_3' \quad (38)$$

which becomes, if we refer to (16) and substitute numerical values,

$$-i0.0698 \sin \frac{2.832l''}{a} e^{-i5.665(c''+\frac{1}{2}l'')/a} = +0.0346e^{i1.440} \quad (39)$$

By equating moduli we find

$$\begin{aligned} \sin \frac{2.832l''}{a} &= 0.496 \\ l'' &= 1.10 \text{ cm.} \end{aligned} \quad (40)$$

while by equating phases,

$$\begin{aligned} 3\pi/2 - 5.665(c''+\frac{1}{2}l'')/a &= 1.440 \\ c'' &= 2.92 \text{ cm.} \end{aligned} \quad (41)$$

An axial section of the complete transducer is shown in Fig. 6.

Referring now to Fig. 7, there is illustrated an embodiment of the present invention in which a dielectric mode transducer is incorporated in a microwave device that tends to generate a spurious mode. The particular device shown is a 90-degree arcuate bend 23 in a round wave guide in which the energy is supplied in the TE_{01} mode. The theory of propagation of TE_{01} waves around a curved bend has been extensively treated in the literature (see for example M. Jouguet, *Cables et Transmission* 1, 133-153 (1947)), and it is there shown that when such a bend is encountered the applied TE_{01} waves are progressively converted into the TM_{11} mode. It is also known that the two modes have exactly the same phase velocity in a straight pipe, a circumstance which leads to close coupling between these two modes in a curved guide. In the bend the conversion into the TM_{11} mode may be partial or complete, depending on the dimensions, and in fact there may be several complete conversions from one mode to the other in alternation.

In Fig. 7 the dielectric member 22 is disposed uniformly throughout the arcuate length of the bend 23, in accordance with the design principles set forth hereinbefore, to generate from the operating mode the same mode that the bend itself tends to generate, viz., the TM_{11} mode. More particularly, the member 22 is proportioned to generate this mode with the same amplitude as that generated by the bend and with the same orientation. The relative phases are opposite so that the spurious mode generated in the bend is cancelled as rapidly as it is formed. In general the centroid of the dielectric-filled area is displaced somewhat from the geometrical center of the guide cross section toward the center of curvature

of the bend. The optimum position of the dielectric and the necessary cross-sectional area can be calculated fairly readily since the literature teaches what amount of the TM_{11} mode must be cancelled per unit length of curved guide. In the specific example illustrated in Fig. 7 the dielectric member appears in cross section as a 180-degree segment the chord of which is perpendicular to the plane of the bend. In a relatively sharp bend it may be necessary to use material having a dielectric constant substantially higher than the values hereinbefore suggested if there is to be complete cancellation of all of the TM_{11} mode. In any such case the extremities of the dielectric member may be tapered to reduce the tendency toward impedance mismatch.

The detailed design of the dielectric member 22 in Fig. 7 may be arrived at empirically. If, for example, the 180-degree segment shown in Fig. 7 is found to overcompensate the bend, it may be removed and material shaved from its flat face until test shows that the emerging wave is substantially free of the TM_{11} mode. Alternatively the dielectric member may be in the form of a flexible strip which can be progressively reduced in width until the desired condition is obtained. The member may be cemented in place, or supporting plugs may be used at its extremities, or other devices for holding it in position may be employed as will be evident to those skilled in the art. The subject matter of Fig. 7 is further disclosed and claimed in my application Serial No. 580,914, filed April 26, 1956.

The technique described with reference to Fig. 7 can be applied also to other operating modes and cross sections such as, for example, the dominant mode in an oversized, rectangular guide, the dielectric member being disposed with reference to the cophasal zones involved in accordance with the principles set forth hereinbefore. It will be understood, too, that the mode transducer may alternatively precede and/or follow the bend, in which case the spurious mode appears in the bend although not in the output section of guide.

Another practical application of the dielectric mode transducer is illustrated in Fig. 8. Here a wave source 25 delivers power in the dominant mode TE_{10} to a rectangular guide 26 which leads to a mode converter 27. The latter more or less completely transforms the wave power into the TE_{01} mode in a circular guide 10 for transmission therethrough. The converter 27 tends to produce also spurious modes and these are cancelled by the mode transducer 28 which follows it. One form of converter, for example, comprises a hollow-pipe guide the cross section of which is deformed progressively along its length from rectangular shape into the sector of a circle supporting the TE_{01} sectoral mode, the sector progressively flaring out in turn until the angle reaches 360 degrees and the sectoral mode converted into the TE_{01} circular electric mode. Another form of converter involves the simultaneous expansion of two sectors, each of which carries half of the final circular electric mode and operates to convert the TE_{20} mode in a rectangular guide into the TE_{01} mode in a round guide. Such transducers, unless made very long compared to the transverse dimensions of the wave guides, introduce spurious modes such as TE_{11} and TE_{21} into the TE_{01} output.

To cancel the TE_{11} mode the mode transducer 28 may comprise a section of round guide in which dielectric material is disposed on one side only of a plane forming a diameter of the guide, as in Fig. 2A or Fig. 9A, for example. To cancel the TE_{21} mode the transducer may comprise also a section in which dielectric material is disposed symmetrically in opposite quadrants, for example as illustrated in Fig. 8A, the rotational position of the dielectric being adjusted, of course, in accordance with the particular orientation of the TE_{21} mode to be cancelled.

In the embodiment of the invention illustrated in

Fig. 9 advantage is taken of the fact that the TE_{01} and TM_{11} modes in a round guide have the same phase velocity, to make the longitudinal position of the dielectric mode transducer uncritical and to secure other advantages as will presently appear.

A curved wave guide has several natural modes, that is, modes which it will propagate without mode conversion. Two of these natural modes in a curved guide of circular cross section are known as the $TE_{01}+TM_{11}'$ and $TE_{01}-TM_{11}''$, each comprising a TE_{01} component and an equal TM_{11} component with a particular orientation relative to the plane of the bend. In Fig. 9 a dielectric mode transducer 30 immediately preceding the bend 23 converts half the power of the incoming TE_{01} waves into the TM_{11} mode oriented to conform with one of these two natural modes of the bend. The total wave power proceeds to and through the bend without further mode conversion to a second dielectric transducer 31, identical with the first, which transforms the wave power back into the TE_{01} mode.

The cross section of the dielectric members in Fig. 9 is in the form of a sector of half-angle α disposed with the bisector in the plane of the bend. If the $TE_{01}+TM_{11}'$ mode be chosen, the dielectric is disposed toward the outside of the bend, that is, with the bisector directed away from the center of curvature of the bend. The length l and the half-angle α of each transducer may be calculated as follows:

The half-angle α is a function only of the ratio $\nu=\lambda/\lambda_c$, where λ is the free wavelength corresponding to the operating frequency and $\lambda_c=2\pi a/3.8317$ is the cut-off wavelength for the TE_{01} and TM_{11} modes in a straight guide of the given radius a . α in radians satisfies the equation

$$\cos \alpha = \frac{\sqrt{1-\nu^2}}{0.1957+0.4641\nu^2} \quad (42)$$

where $0<\alpha<\pi$. This equation may be solved readily by cut and try for any particular value of ν .

When α has been determined, the required length l of each mode transducer is given by

$$l = \frac{\lambda}{0.6823(\epsilon_r - 1) \sin \alpha} \quad (43)$$

where ϵ_r is the relative dielectric constant of the transducer material, and it is assumed that ϵ_r does not differ much from unity.

If, for specific example, λ is 1.875 centimeters and a is 1.27 centimeters, ν is calculated to be 0.90 and α , derived from (42), is 40.4 degrees. If ϵ_r is 1.0200, (43) indicates a length l of about 212 centimeters. Increasing ϵ_r to 1.0235 reduces l to about 180 centimeters.

Since the phase velocities of the TE_{01} and TM_{11} modes are identical in a straight guide it is evident that it makes no difference whether or not there is any separation between the ends of the transducers and the ends of the curved section in Fig. 9. Also the transducers may be made in several separated parts if desired so long as the total length of the various parts is equal to the required value of l . The dielectric members may be supported in the guide if necessary by solid dielectric plugs at both ends. The transducer described does not tend to generate modes other than TM_{11} in any substantial degree since all modes other than TM_{11} differ in phase velocity from the TE_{01} mode. Hence over a distance of many wavelengths these other modes will phase in and out so often that the net power transfer will be small.

A third natural mode of the bend is the TM_{11}' mode, which has a different orientation from the TM_{11}'' mode, but has the same velocity as the TE_{01} mode in a straight guide. The TM_{11}' mode could be produced from TE_{01} , if desired alternatively, by a transducer designed according to the general principles described above, and transmitted around the bend in the same way as $TE_{01} \pm TM_{11}''$.

Numerous other embodiments within the spirit and scope of the invention, in addition to those specifically described herein, will be obvious to those skilled in the art.

What is claimed is:

1. In combination, a closed conductive-walled wave guide of circular cross section comprising a flaring section joined at its larger end to a section of uniform size, and a mode transducer in said uniform section adjacent said flaring section, said transducer comprising a generalized cylinder of dielectric material substantially confined to one of the cophasal regions of a circular electric TE_{0n} mode of electromagnetic wave propagation, where n is greater than one.

2. In combination, a closed conductive-walled wave guide of substantially circular cross section comprising a flaring section joined at one end to a section of uniform size, and a mode transducer in said uniform section comprising a generalized cylinder of dielectric material substantially confined to one of the cophasal regions of a circular electric TE_{0n} mode of electromagnetic wave propagation, where n is greater than one.

3. In combination, a hollow circular multiple mode wave guide having a straight section and a transition section and having conducting side walls, said wave guide being substantially filled with a medium having a given dielectric constant, means for applying microwave energy to said wave guide in the TE_{01} electromagnetic wave mode at a frequency sufficiently high that at least four additional modes may be readily propagated through said wave guide, said straight length of said wave guide having a predetermined axis, means comprising said transition section of said wave guide for converting a portion of the microwave energy applied to said wave guide from said TE_{01} mode into a second one of said modes having a given amplitude and phase at one point in said wave guide, said transition section being coupled to said straight section of wave guide and having conducting walls which progressively vary in their spacing from said predetermined axis, said second mode having first electric field regions which are oppositely phased with respect to the electric field of said TE_{01} mode and having second electric field regions having the same phase as the electric field of said TE_{01} mode which exists in the same regions, and means comprising a dielectric mode transducer in said wave guide and extending across only a portion of the cross-section of said guide for converting an additional portion of said TE_{01} mode into wave energy in said second mode, said transducer having such a length that the amplitude of said second mode produced thereby is equal to said given amplitude of said second mode produced by said transition section, said mode transducer being so spaced from said transition section that the phase of the second mode produced by said transducer is opposite to that produced by said transition section at said one point, said mode transducer also being located predominantly in only one of said first and second regions, and the dielectric constant of said transducer differing from that of said medium by not more than approximately ten per cent.

4. In combination, a hollow circular multiple mode wave guide having a straight section and a transition section and having conducting side walls, means for applying microwave energy to said wave guide in the TE_{01} electromagnetic wave mode at a frequency sufficiently high that at least four additional modes may be readily propagated through said wave guide, said straight length of said wave guide having a predetermined axis, means comprising said transition section of said wave guide for converting a portion of the microwave energy applied to said wave guide from said TE_{01} mode into a second one of said modes having a given amplitude and phase at one point in said wave guide, said transition section being coupled to said straight section of wave guide and having conducting walls which progressively vary in their spacing

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from said predetermined axis, said second mode having first electric field regions which are oppositely phased with respect to the electric field of said TE_{01} mode and having second electric field regions having the same phase as the electric field of said TE_{01} mode which exists in the same regions, and means comprising a dielectric mode transducer in said wave guide and extending across only a portion of the cross-section of said guide for converting an additional portion of said TE_{01} mode into wave energy in said second mode, said transducer having such a length that the amplitude of said second mode produced thereby is equal to said given amplitude of said second mode produced by said transition section, said mode transducer being so spaced from said transition section that the phase of the second mode produced by said transducer is opposite to that produced by said transition section at said one point, and said mode transducer being located predominantly in only one of said first and second regions.

5. A combination as defined in claim 4 wherein said transition section is a bend.

6. A combination as defined in claim 4 wherein said transition section is a taper.

7. In combination, a hollow circular multiple mode wave guide having a straight section and a transition section and having conducting side walls, means for applying microwave energy to said wave guide in a preassigned electromagnetic wave mode at a frequency sufficiently high that several additional modes may be readily propagated through said wave guide, said straight length of said wave guide having a predetermined axis, means comprising said transition section of said wave guide for converting a portion of the microwave energy applied to said wave guide from said preassigned mode into a second one of said modes having a given amplitude and phase at one point in said wave guide, said transition section being coupled to said straight section of wave guide and having conducting walls which progressively vary in their

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spacing from said predetermined axis, said second mode having first electric field regions which are oppositely phased with respect to the electric field of said preassigned mode and having second electric field regions having the same phase as the electric field of said preassigned mode which exists in the same regions, and means comprising a dielectric mode transducer in said wave guide and extending across only a portion of the cross-section of said guide for converting an additional portion of said preassigned mode into wave energy in said second mode, said transducer having such a length that the amplitude of said second mode produced thereby is equal to said given amplitude of said second mode produced by said transition section, said mode transducer being so spaced from said transition section that the phase of the second mode produced by said transducer is opposite to that produced by said transition section at said one point, and said mode transducer being located predominantly in only one of said first and second regions.

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