

April 17, 1951

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2,549,065

FREQUENCY DISCRIMINATIVE ELECTRIC TRANSDUCER

Filed Nov. 2, 1948

3 Sheets-Sheet 1

FIG. 1

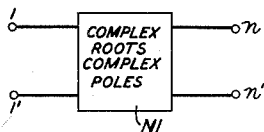


FIG. 2

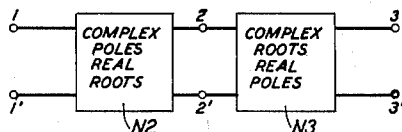


FIG. 3

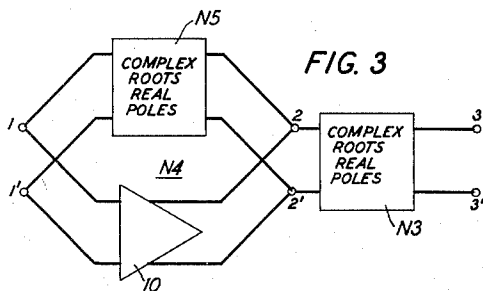


FIG. 4

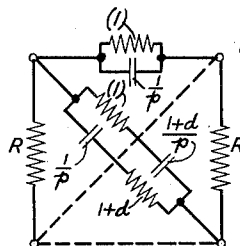


FIG. 5

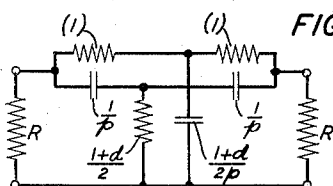


FIG. 6

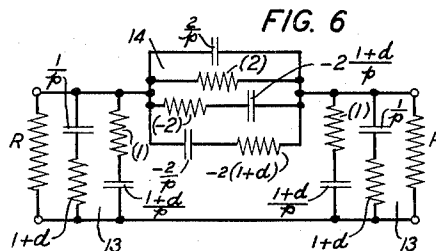


FIG. 7

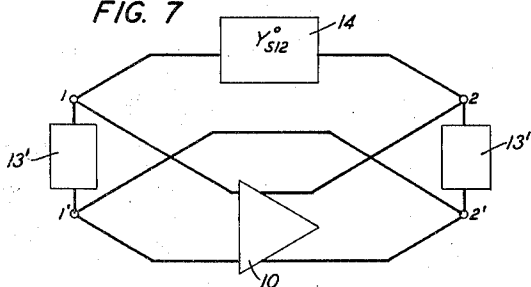


FIG. 8

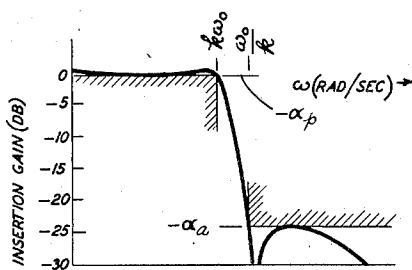
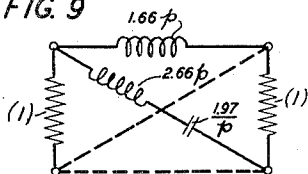


FIG. 9



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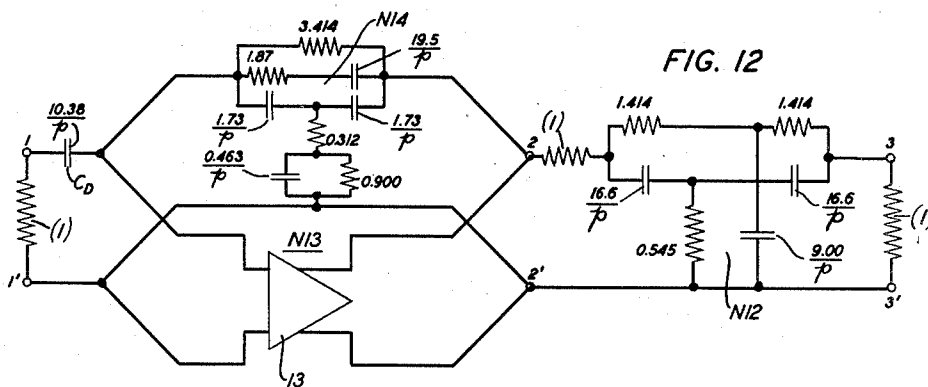
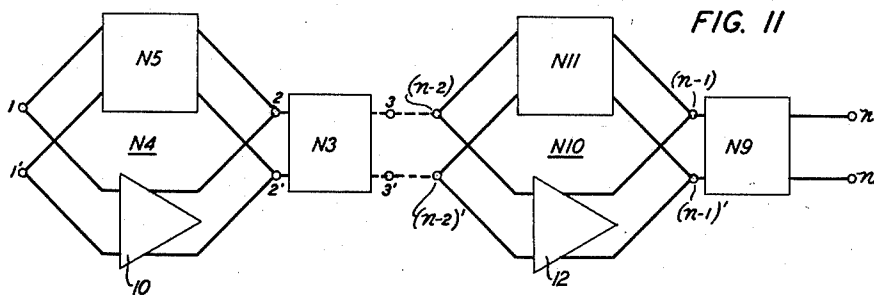
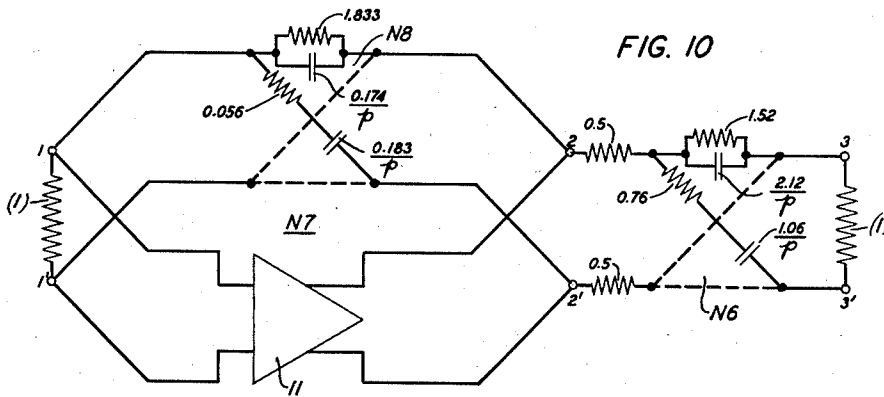
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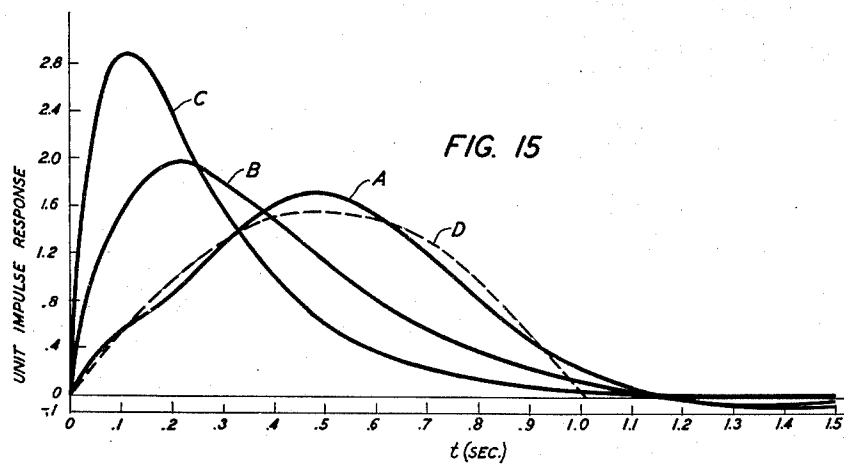
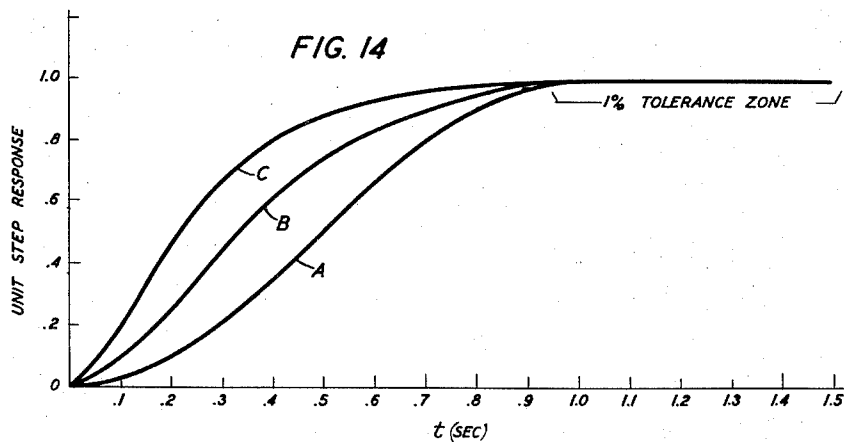
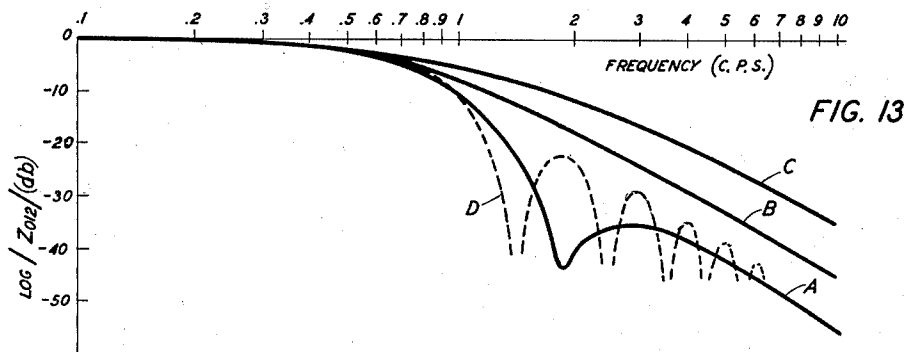
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FREQUENCY DISCRIMINATIVE ELECTRIC TRANSDUCER

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3 Sheets-Sheet 3



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UNITED STATES PATENT OFFICE

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FREQUENCY DISCRIMINATIVE ELECTRIC
TRANSDUCER

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Application November 2, 1948, Serial No. 57,879

20 Claims. (Cl. 178-44)

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This invention relates to wave transmission networks and more particularly to a transducer which requires only one type of reactor but may be designed to have any physically realizable transmission characteristic.

An object of the invention is to eliminate one type of reactor in a transducer without restricting the transmission characteristic obtainable. Another object is to extend the range of physically realizable transmission characteristics obtainable in a transducer. A further object is to reduce the size and cost of transducer which must meet particularly severe transmission requirements.

There are often encountered transmission requirements of such a character that they cannot be met in a passive transducer because of the uneconomic size or cost of one type of component reactor. For example, selective properties may be required at very low frequencies. For this case resistance-capacitance networks are practical but inductors must be excluded. The most general transmission characteristic requires the presence of both complex roots and complex poles in the transfer impedance of the transducer. Passive networks which are restricted to only one type of reactor may be designed to provide complex roots but will not provide complex poles.

In accordance with the present invention this deficiency is overcome by including in the transducer an active section comprising a stable feedback amplifier having a network in the feedback path. This feedback network may comprise only one type of reactor and so will have only complex roots and real poles, but due to the inverting property of the amplifier the active section will provide complex poles in the transfer impedance of the transducer. The required complex roots are furnished by a tandem-connected passive network which also requires only one type of reactor. There is thus provided a transducer which may be designed to have any physically realizable transmission characteristic without requiring the use of a type of reactor which may be too large or too expensive to build practically.

The nature of the invention will be more fully understood from the following detailed description and by reference to the accompanying drawing in which like reference characters are used to designate similar or corresponding parts and of which:

Fig. 1 is a block diagram of a transducer N1 indicating that, for the most general type of transmission characteristic, both complex roots and complex poles are required;

Fig. 2 is another block diagram indicating that the transmission characteristics of the transducer N1 may be achieved by the tandem com-

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bination of a network N2 having complex poles and real roots and a network N3 having complex roots and real poles;

Fig. 3 shows how, in accordance with the invention, the network N2 of Fig. 2 may be realized in an active section N4 comprising an amplifier which has in its feedback path a network N5 having complex roots and real poles;

Figs. 4 and 5 are schematic circuits of a typical lattice and the equivalent parallel-T network, respectively, comprising only resistors and capacitors, which may be used for the networks N3 and N5 shown in Fig. 3;

Fig. 6 is a non-physically realizable π equivalent of the networks of Figs. 4 and 5;

Fig. 7 is a schematic diagram of the active network N4 of Fig. 3, used in the analysis;

Fig. 8 is a typical insertion gain-frequency characteristic of a sharply selective filter of which the conventional lattice embodiment comprising inductors and capacitors is shown in Fig. 9 and the embodiment according to the invention, requiring no inductors, in Fig. 10;

Fig. 11 is a block diagram showing, in accordance with the invention, a plurality of tandem-connected transducers of the type shown in Fig. 3;

Fig. 12 is a schematic circuit of a linear-phase transducer in accordance with the invention comprising a bridged-T feedback network N14 and a parallel-T tandem network N12;

Fig. 13 compares the insertion gain-frequency characteristic, curve A, of the transducer of Fig. 12 with curves B and C obtainable from two passive transducers and with an ideal characteristic, curve D;

Fig. 14 compares the square-wave response of the three transducers; and

Fig. 15 compares the weighting functions, or unit impulse responses, of the transducers with the ideal parabolic response, curve D.

For networks according to the invention, the transmission is in general most conveniently specified by the open-circuit transfer impedance, Z_{o1n} . This is defined for the transducer N1 of Fig. 1 as the voltage appearing across the output terminals $n-n'$ in response to unit current driven through the input terminals $1-1'$. When the network is energized by a source of finite impedance and delivers signal to a load of finite impedance, these terminating impedances are understood to be included in the network. In the special case of operation between terminations of zero impedance, the transmission is measured by the short-circuit transfer admittance, Y_{s1n} . This is defined as the current flowing through the closed terminals $n-n'$ in response to unit voltage established across the terminals $1-1'$ of the transducer N1.

These two transmission functions are simply

related to the branches of certain common configurations. For example, it is seen from the definition that Z_{01n} is the impedance of the shunt arm of the equivalent T of the network. Likewise, Y_{S1n} is the admittance of the series arm of the equivalent π of the network. For networks in the configuration of a symmetrical lattice having arms A and B,

$$Z_{01n} = \frac{Z_A - Z_B}{2} \quad (1)$$

and

$$Y_{S1n} = \frac{Y_A - Y_B}{2} \quad (2)$$

where Z_A and Z_B are the impedances of the arms A and B, respectively, and Y_A and Y_B are their admittances.

If meshes are chosen for a general network so that the terminals 1-1' are part of the first mesh only, and terminals $n-n'$ part of the n th mesh only, then these functions can be computed from the determinant of mesh impedances as

$$Z_{01n} = \frac{\Delta_{1n}}{\Delta_{11nn}} \quad (3)$$

$$Y_{S1n} = \frac{\Delta_{1n}}{\Delta} \quad (4)$$

The determinant and its minors are real functions of the complex frequency variable,

$$p = \sigma + i\omega$$

where σ is the real part and where the imaginary part, ω , is the real angular frequency. Therefore, the roots are either real, or occur in conjugate pairs. As is well known, the roots of the network determinant and its principal minors have non-positive real parts, but in general are otherwise unrestricted. In the special case of networks containing resistances and but one type of reactance, the roots are further restricted to non-positive real values of p . Upon the roots of Δ_{1n} , which is not a principal minor, there is no special restriction.

Therefore, the transmission of a transducer as measured either by Z_{01n} or Y_{S1n} is characterized generally by complex roots and also complex poles, as indicated in Fig. 1, but in case the transducer contains only one kind of reactive element, the poles must be real. At roots of the network function, the transmission is extinguished; by the use of bridge balance these infinite loss points may be located on or near the axis of physical frequencies whether or not the network is restricted to one type of reactive element. The poles of the network function are the frequencies of free response; in the unrestricted network these infinite gain points may be located close to the axis of physical frequencies by the use of resonance effects. The restriction to one kind of reactive element excludes this possibility and locates the infinite gain points remote from physical frequencies, that is, on the real p -axis.

This behavior is illustrated by the symmetrical lattice network of Fig. 4, which is of a type utilized in one embodiment of the invention. Each series branch of the network comprises a capacitor and a resistor in parallel and each diagonal branch comprises two parallel arms, each constituted by a capacitor and a resistor in series. The broken lines in this and subsequent lattice diagrams indicate series and diagonal branches identical with those shown explicitly. In Fig. 4 and other similar network representations the impedance of each element is given by the expression associated therewith. In case this is a

whole number it is enclosed in parentheses to avoid confusion with like reference numerals used elsewhere. The representation assumes a unit impedance level for the network, with the impedance level R for the terminations. Specification of the design constant d determines the performance of the network for this terminating condition.

In the network of Fig. 4 an infinite loss point may be brought close to the physical frequency, $\omega=1$, by assigning a small value to d , the damping constant, in which case the transfer impedance of the network terminated at each end in a resistance R , as shown, is given very nearly by

$$Z_{01n} = \frac{1}{2} \frac{R^2}{1+R} \frac{1+dp+p^2}{\left(1-\frac{p}{\omega_1}\right)\left(1-\frac{p}{\omega_2}\right)} \quad (5)$$

where the infinite gain points

$$\omega_1 = -\frac{1}{1+(2-d)R}, \quad \omega_2 = -\left(1+\frac{1}{R}\right) \quad (6)$$

are both negative reals. For a source and load of vanishing impedance, the short-circuit transfer admittance is computed as

$$Y_{S1n} = \frac{1}{2} \frac{(1+dp+p^2)(1+p)}{\left[1+\frac{p}{\left(1+\frac{1}{R}\right)}\right]\left[1+\frac{p}{(1+p)}\right]} \quad (7)$$

Unbalanced equivalents to lattice networks can sometimes be exhibited according to methods familiar in the art. The network of Fig. 4 has the well-known equivalent parallel- T form shown in Fig. 5. In this connection reference is made to United States Patent No. 2,058,210, issued October 20, 1936, to H. W. Bode. It cannot have a physical embodiment in an ordinary ladder section, since bridge action is required to provide the complex roots. In fact, the π -section equivalent of Fig. 6, comprising two equal shunt branches 13, 13 and an interposed series branch 14, is seen to include negative elements. This non-physical representation is nevertheless convenient in analysis, since it exhibits the short-circuit transfer admittance explicitly as the series branch 14 of the π .

The π -representation of Fig. 6 facilitates the calculation of the open-circuit transfer impedance for an active section, such as N4 of Fig. 3, comprising an amplifier 10 having a network N5 in the feedback path. As shown schematically in Fig. 7, the network N5 is replaced by the equivalent π , the input and output impedances of the amplifier being absorbed in the terminations, R , of the network, forming part of the shunt impedances 13', 13' of Fig. 7. The gain of the amplifier 10 is measured by the trans-admittance, g , defined as the ratio of the current discharged through the output terminals 2-2' to the voltage existing across the input terminals 1-1', the sense being taken so that g is positive when the amplifier comprises an odd number of resistively coupled stages. If this is done, an elementary calculation shows that

$$Z_{012} = -\left(\frac{1}{Y_{S12}^0} - \frac{1}{g}\right) \frac{gZ_{012}^0}{1+gZ_{012}^0} \quad (8)$$

where

$$Z_{012}^0$$

is the passive transfer impedance of the network, that is, the value of Z_{012} with $g=0$, and where

$$Y_{S12}^0$$

is the admittance of the series arm 14 of the equivalent π of the passive network. Over the

useful frequency range, the gain of the amplifier 10 is understood to be very great, so that $g \rightarrow \infty$, and

$$Z_{012} \rightarrow \frac{1}{Y_{012}^0} \quad (9)$$

except for a trivial crossing of terminals. Thus the active open-circuit transfer impedance of the active section N4 is reciprocal to the passive short-circuit transfer admittance of the network N5 in the feedback path, and the complex factors which occur in the numerator of

$$Y_{012}^0$$

appear in the denominator of Z_{012} . Therefore, if a network whose passive transmission is characterized by complex roots and real poles is used as the feedback circuit for a high-gain amplifier, the resulting active transmission will be characterized by real roots and complex poles. If such active transmission characteristics can be combined with passive transmission characteristics in such a way that all the wanted factors are preserved while any unwanted real factors are cancelled out, then the combination of active and passive transmissions is an unrestricted network function.

The law of combination is very simple. High gain is required of the amplifier 10 to reciprocate the passive transmission of the feedback network N5. The same requirement of high gain in the amplifier insures that the active impedance presented at the amplifier terminals is vanishingly small. Another elementary calculation shows that Z_1 , the active impedance at terminals 1-1' of Fig. 7, is related to

$$Z_1^0$$

the passive impedance at terminals 1-1', by the formula

$$Z_1 = \frac{Z_1^0}{1 + gZ_{012}^0} \quad (10)$$

The same relation obtains for Z_2 and

$$Z_2^0$$

the active and passive impedances, respectively, at terminals 2-2' of Fig. 7. Thus high feedback insures substantially zero input and output impedances for the active sections in a tandem combination. This property permits active sections, such as N4 and N10, and passive sections, such as N3 and N9, to be joined alternately in tandem sequences, as shown in Fig. 11, without interaction. The active section N10 comprises an amplifier 12 with a feedback network N11.

It is necessary to distinguish between the case of an active section driving a passive section and the case of a passive section driving an active section. Consider first the case of an active section driving a passive section, as occurs at the junctions 2-2', or $(n-1)-(n-1)'$, of Fig. 11. The voltage, V_2 , across the terminals 2-2' is $I_1 Z_{012}$, where I_1 is the current supplied by an external source to the terminals 1-1', and Z_{012} is the transfer impedance of the active network N4. Now let the output terminals of N4 be introduced in series with the input terminating resistance, R_2 , of the subsequent passive section N3. Then N3 is actually excited by a zero impedance voltage generator of strength V_2 acting through the terminating impedance. This is exactly equivalent to exciting N3 by an infinite impedance current

generator acting across the impedance R_2 , provided the current source is of strength V_2/R_2 . Thus the passive section can be regarded as driven by the current V_2/R_2 with the terminating resistance included in the network. This is the manner of excitation which was assumed in the calculation of the open-circuit transfer impedance of the section.

At such junctions as 3-3' or $(n-2)-(n-2)'$ in Fig. 11, the case of a passive section driving an active section is encountered. If the zero-impedance input terminals of the active section are introduced in series with the output terminating resistance of the passive section, the delivery of current to the terminating resistance will be in no wise affected. The current drawn by the output resistance of the passive section then becomes the exciting current for the subsequent active section. This current is calculated in the following way. The voltage established across the output terminating resistance of the passive section (for example, across the output resistance R_3 of network N3) is $I_2 Z_{023}$ where I_2 is the strength of the equivalent current generator driving the terminals 2-2' (that is, V_2/R_2), and Z_{023} is the open-circuit transfer impedance of N3. Then the current I_3 which flows through the terminals 3-3' is $I_2 Z_{023}/R_3$.

In both cases, active and passive sections are joined by inserting the terminals of the active sections in series with the terminating resistances of the passive sections. Then the combined transfer impedance Z_{01n} of an alternating sequence of active and passive sections, such as N4, N3, . . . , N10, N9 of Fig. 11, is seen to be

$$Z_{01n} = Z_{012} \times \frac{Z_{023}}{R_2} \times \frac{Z_{034}}{R_3} \times \dots \quad (11)$$

where Z_{012} , Z_{023} , . . . are the open-circuit transfer impedances of the component sections, N4, N3, . . . , and R_2 , R_3 , . . . are the passive network terminations in series with which the terminals of the active sections are connected.

There is much freedom in the assignment of impedance levels in the successive sections, on account of the isolation of effects provided by high feedback in the active sections N4 and N10. The only precaution to be observed is that the impedances of the passive sections N3 and N9 be not so low as to destroy the feedback in a connected active section. The impedance of adjoining passive sections must be reckoned in the terminating impedances of the active sections, not for calculating the transmission, but only for estimating the feedback,

$$gZ_{012}^0$$

It is unnecessary to know this exactly, but it is necessary to insure that it be large.

Flexibility in the choice of impedance levels is used to control the element sizes and to adjust the general level of the response. Commonly modest losses in the passive sections are compensated by modest gains in the active sections. In the usual applications, the amplifiers 10 and 12 are direct-current amplifiers. So long as the gain is mostly consumed in feedback, the problem of drift is not troublesome; high over-all gains are rendered impracticable by drift, and are not contemplated by the invention.

The analysis so far presented demonstrates that the transmission between input terminals 1-1' and output terminals 3-3' of the tandem com-

combination of an active section N4 and a passive section N3, as shown in Fig. 3, may have the distribution of poles and zeros shown in Fig. 2, in which the section N2 represents the active section N4 of Fig. 3, even though the networks include only one type of reactive element. As indicated in Figs. 2 and 3 and illustrated by the example of Fig. 4, the complex roots introduced by the passive network N3 are accompanied by real poles, which may not be desired in the composite transfer impedance. In order that the tandem combination of Fig. 2 be equivalent to the unrestricted network N1 of Fig. 1, it is necessary that unwanted real poles in the passive section N3 be removed by identical real roots in the active section N2. It is always possible to accomplish this, as is shown by the following examples.

The proportioning of networks in accordance with the invention will be illustrated by two examples. The examples also illustrate that useful network functions are in fact characterized by complex roots as well as complex poles. It further appears from the examples that the physical embodiment utilizing capacitors and inductors, such as is shown in Fig. 9, is impracticable when selective properties are required at very low frequencies.

The first example, shown schematically in Fig. 10, is a sharply selective low-pass filter. A typical requirement is that the insertion loss shall not be greater than a tolerance, α_p , up to some fraction, $k < 1$, of the angular cut-off frequency ω_0 , while beyond the frequency ω_0/k the loss shall not be less than the tolerance, α_a . These requirements are indicated by the partly cross-hatched areas of Fig. 8, in which insertion gain, the negative of insertion loss, is plotted against the angular frequency ω .

Darlington has shown (Jour. Math. Phys., vol. XVIII, No. 4, pp. 257-353, September 1939) that this requirement is most efficiently met by an appropriate assignment of values to the angular frequencies $\omega_1, \omega_2, \dots, \omega_N$ in the expression

$$e^{2\alpha} = 1 + \sqrt{(e^{2\alpha_p} - 1)(e^{2\alpha_a} - 1)} \frac{\omega^2(\omega_1^2 - \omega^2)^2(\omega_2^2 - \omega^2)^2 \dots (\omega_N^2 - \omega^2)^2}{(1 - \omega_1^2\omega^2)^2(1 - \omega_2^2\omega^2)^2 \dots (1 - \omega_N^2\omega^2)^2} \quad (12)$$

where N is the number of sections, and where the unit of frequency has been adjusted to place the cut-off at $\omega_0 = 1$. Darlington finds the least value of N consistent with a given k , and so determines $\omega_1, \dots, \omega_N$ in (12). From the real function (12) he generates the complex transfer impedance

$$Z_{012} = \frac{1}{2} \frac{(1 + \omega_1^2 p^2) \dots (1 + \omega_N^2 p^2)}{\left[1 + \frac{p}{\rho_0}\right] \left[1 + d_1 \left(\frac{p}{\rho_1}\right) + \left(\frac{p}{\rho_1}\right)^2\right] \dots \left[1 + d_N \left(\frac{p}{\rho_N}\right) + \left(\frac{p}{\rho_N}\right)^2\right]} \quad (13)$$

Except for the single real pole at $p = -\rho_0$, the poles are in every case complex, that is, the value of each damping constant d_i is less than two.

If α_p corresponds to 0.5 decibel and α_a corresponds to 24 decibels then $N=1$ for $k=0.8$ and Darlington's theory gives

$$e^{2\alpha} = 1 + 5.16 \frac{\omega^2(.512 - \omega^2)^2}{(1 - .512\omega^2)^2} \quad (12')$$

for (12), and

$$Z_{012} = \frac{1}{2} \frac{1 + \left(\frac{p}{1.40}\right)^2}{\left(1 + \frac{p}{.604}\right) \left[1 + .437 \left(\frac{p}{.860}\right) + \left(\frac{p}{.860}\right)^2\right]}$$

for (13). The theoretical insertion gain characteristic of this filter is shown in Fig. 8. The conventional embodiment in a symmetrical lattice network utilizing inductors and capacitors is shown by Fig. 9.

For a cut-off frequency of 1000 cycles per second or greater, the section might actually be constructed in this way. If the selective properties are needed at very low frequencies, however, the required element values become very great. The size of the reactive elements of one type only can be controlled by adjusting the impedance level. The capacitors can be made small by raising the impedance level sufficiently, in which case the required inductors are enormous; alternatively the inductors may be kept small by lowering the impedance level, in which case the required capacity is enormous. When reactive elements of both types are present in the network, the impedance level must be adjusted to a compromise value, equalizing the difficulty of constructing the two types of reactive element. If the conventional section shown in Fig. 9 were to be built for a cut-off of one cycle per second, the required capacity could be limited to 100 microfarads by fixing the impedance level at 1600 ohms. The required inductance is then more than 2000 henries. For coils of this size the impedance at one cycle per second is usually more resistive than reactive, so that the theoretical behavior shown in Fig. 8 would be completely obscured by dissipative effects. Thus, even though the cost and bulk of the elements were acceptable, the desired performance could not be obtained.

The specified transfer impedance (13') can be realized practically in the circuit shown in block in Fig. 3. For the passive section N3, the network of Fig. 4 is suitable, if the damping constant d is taken equal to zero and the frequency is transformed to place the roots at $\omega = \pm 1.4$. The location of the real poles depends upon the ratio of the impedance of the network to the impedance of the terminations R. This ratio is adjusted so that a pole of the network is the real pole required by (13'). The smaller pole may be so identified. From the first of Equations 6

$$\frac{.604}{1.40} = \frac{1}{1 + 2R}, R = 66$$

The second of Equations 6 locates the other pole at $\omega = 3.52$. If the whole network is now multiplied by $1/R$, the terminations become the unit impedance, and the network is as shown in the passive section N6 of Fig. 10. Note that the re-

sistor at the input end of the network N6 is divided into equal parts, since this is a balanced circuit, and one part is inserted in each side of the line connecting the networks N6 and N7. The corresponding transmission Z_{023} of the network N6 is

$$Z_{023} = \frac{1}{2} \frac{1}{1.66} \frac{1 + \left(\frac{p}{1.4}\right)^2}{\left(1 + \frac{p}{.604}\right) \left(1 + \frac{p}{3.52}\right)}$$

When this partial transmission is divided out of Equation 13', the requirement on the feed-

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back network N8 of the active section N7 is seen to be

$$Y_{s12}^0 = \frac{1 + .437\left(\frac{p}{.860}\right) + \left(\frac{p}{.860}\right)^2}{1.66\left(1 + \frac{p}{3.52}\right)}$$

which has the partial fraction expansion,

$$.60 + 2.86p - \frac{1}{.112 + \frac{.366}{p}}$$

This short-circuit transfer admittance has a realization in a symmetrical lattice by Equation 2, if

$$\frac{Y_A}{2} = .60 + 286p$$

$$\frac{Y_B}{2} = \frac{1}{.112 + \frac{.366}{p}}$$

This lattice is shown as the feedback network N8 in the active section N7 of Fig. 10. The composite transfer impedance is exactly 13', since the unwanted pole at $p = -3.52$ which is introduced by the passive section N6 is cancelled by a similar root introduced by the active section N7. The networks may be conveniently constructed with the unit impedance taken as one megohm. At this level, and with the cut-off at one cycle per second, the total capacity required is 4 microfarads. Unbalanced equivalents of the two networks can be found by well-known methods. In the unbalanced form, the capacity requirement is nearly halved.

Higher discrimination or sharper selectivity in the specified behavior implies the presence of more sections in the network representation. A multi-section filter may be assembled in the usual way from component sections according to the invention, as shown by Fig. 11, active sections N4, N10 and passive sections N3, N9 alternating in the tandem combination. The networks are individually of minimum complexity when each contributes a single complex factor to the composite transfer impedance. It is always possible to assemble a representation for any physically realizable transmission function in this way. However, economy in the employment of amplifiers can be achieved by combining sections. How this is done is illustrated by the next example.

The second example is a linear phase shift filter. As shown schematically in Fig. 12, the filter comprises a passive section N12 and an active section N13 connected in tandem. The active section includes an amplifier 13 with a network N14 in the feedback path.

Filtering to protect the signal against noise is particularly important when it is desired to obtain by electrical differentiation the time rate of change of an input upon which random perturbations are superposed. Since the output of a filter is the average over past time of the input, weighted according to the impulse response of the filter, it is simplest to specify the optimum filtering in terms of the weighting function. If T is the time over which the average may be extended, the ideal weighting function is known to be

$$\begin{aligned} A'(t) &= 0 & t < 0 \\ &= 6\frac{t}{T}\left(1 - \frac{t}{T}\right), & 0 < t < T \\ &= 0 & T < t \end{aligned} \quad (14)$$

The second example is a smoothing filter approxi-

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mating to the parabolic weighting function shown by the broken-line curve D of Fig. 15, in which T is equal to one second.

The frequency transform of this impulse response is the linear phase shift characteristic,

$$\begin{aligned} Z_{012} &= 6 \frac{(p+2)e^{-p} + (p-2)}{p^3} \\ &= 3 \frac{\sin \frac{\omega}{2} - \frac{\omega}{2} \cos \frac{\omega}{2}}{\left(\frac{\omega}{2}\right)^3} e^{-\frac{i\omega}{2}}, \quad (p = i\omega) \end{aligned}$$

A rational approximation of the fourth degree is found by the method of Padé (Perron, Lehre v. d. Kettenbrüchen, Ch. 10, to be

$$Z_{012} = \frac{\left(1 + .090 \frac{p}{11.75} + \frac{p^2}{11.75^2}\right)}{\left(1 + 1.90 \frac{p}{5.80} + \frac{p^2}{5.80^2}\right) \left(1 + 1.19 \frac{p}{6.68} + \frac{p^2}{6.68^2}\right)}$$

There is an easy embodiment of this function in two active sections separated by one passive section. An alternative embodiment which combines the active sections will be found, illustrating this feature of the invention. To illustrate a further feature of the invention, the special requirement will be imposed that the networks be unbalanced to ground.

For the passive section N12 the parallel-T network of Fig. 5 is suitable, with $d = 0.09$ and the frequency scaled for a peak at $\omega = 11.75$. The impedance level of the network relative to the terminations may be selected at pleasure. The choice somewhat affects the spread of element values in the feedback network N14 of the active section N13, and the ease with which the network N14 may be unbalanced. If subsequent analysis proves an arbitrary choice to be unfortunate, the design may be repeated with the choice improved. The frequency characteristic of the passive section N12 is nearly symmetrical in respect to the peak frequency if the impedance level corresponds to $R = \sqrt{2}/2$. If this value is selected arbitrarily, and the network scaled to unit terminating resistance, there results

$$Z_{023} = \frac{1}{2} \frac{1}{1.707} \frac{1 + 0.09 \frac{p}{11.75} + \frac{p^2}{11.75^2}}{\left(1 + \frac{p}{5.00}\right) \left(1 + \frac{p}{28.38}\right)}$$

The physical embodiment is the passive section N12 of Fig. 12.

The two real poles in the passive section N12 must be removed in the active section N13. In determining the active section, precaution should be taken not to upset the stability of the amplifier 13. If the transmission of the feedback path were allowed to change very rapidly with frequency, the amplifier 13 could be caused to sing unless compensatory equalization were introduced in the forward circuit. This could always be done, but might often be inconvenient. No difficulty with the stability should be encountered if the networks in the feedback path for combined active sections are restricted to have the same asymptotic performance as the simple networks representing single sections. For these, the passive, or feedback, transmission is flat at high frequencies, and the short-circuit transfer admittance changes at the rate of 6 decibels per octave.

This stability condition can be met in the present instance by associating a third real pole with

the short-circuit transfer admittance required of the feedback network N14, which becomes

$$Y_{s12}^0 = \frac{\left(1 + 1.90 \frac{p}{5.80} + \frac{p^2}{5.80^2}\right) \left(1 + 1.19 \frac{p}{6.68} + \frac{p^2}{6.68^2}\right)}{3.414 \left(1 + \frac{p}{5.00}\right) \left(1 + \frac{p}{\omega_2}\right) \left(1 + \frac{p}{28.38}\right)} \quad 5$$

Any positive value may be assigned to ω_2 , and a physical lattice representation obtained. For certain values, however, the unbalanced equivalent of the lattice does not exist. The value assigned to ω_2 is therefore adjusted to insure an equivalent unbalanced form.

Such a value for ω_2 may be ascertained by expanding Y_{s12} in partial fractions and noting the variation of the lattice elements with changing ω_2 . It can turn out that there is no real value of ω_2 which satisfies the unbalance condition. In this event, the passive section N12 should be re-adjusted to obtain more comfortable values for the other two poles. In the present instance, the process goes through as follows.

The expansion by partial fractions of

$$Y_{s12} = \frac{1 + a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4}{\left(1 + \frac{p}{\omega_1}\right) \left(1 + \frac{p}{\omega_2}\right) \left(1 + \frac{p}{\omega_3}\right)} = \frac{N(p)}{D(p)} \quad 15 \quad 25$$

is arranged as

$$Y_{s12} = 1 - \frac{C_1 p}{1 + \frac{p}{\omega_1}} + \frac{C_2 p}{1 + \frac{p}{\omega_2}} - \frac{C_3 p}{1 + \frac{p}{\omega_3}} + C_{\infty} p \quad 30$$

where the constants C_1 , C_2 , etc. are all positive if $N(p)$ has complex roots and $\omega_1 < \omega_2 < \omega_3$. In fact,

$$\begin{aligned} C_1 &= \frac{1}{1 - \frac{\omega_1}{\omega_2}} \left[\frac{1}{\omega_1} \frac{N(-\omega_1)}{1 - \frac{\omega_1}{\omega_3}} \right] \\ C_2 &= \frac{1}{\omega_2} \frac{N(-\omega_2)}{\left(\frac{\omega_2}{\omega_1} - 1\right) \left(1 - \frac{\omega_2}{\omega_3}\right)} \\ C_3 &= \frac{1}{\frac{\omega_3}{\omega_2} - 1} \left[\frac{1}{\omega_3} \frac{N(-\omega_3)}{\frac{\omega_3}{\omega_1} - 1} \right] \\ C_{\infty} &= \omega_2 [\omega_1 \omega_3 a_4] \end{aligned} \quad 35 \quad 40 \quad 45 \quad (17)$$

The bracketed factors are independent of ω_2 . The positive terms in the expansion are identified with one arm of the lattice, and the negative terms with the other. The elements of the physical network are determined by the constants of the partial fraction expansion. Thus,

$$\frac{Y_A}{2} = 1 + \frac{C_2 p}{1 + \frac{p}{\omega_2}} + C_{\infty} p$$

is the admittance of a network having three branches in parallel, of which one is a unit conductance, one is a capacitance C , and one is a series combination of a capacitance, C_2 , with a resistance, R_2 . The admittance of this combination is

$$\frac{C_2 p}{1 + R_2 C_2 p}$$

so that the constant C_2 of the partial fraction expansion is the capacitance C_2 of the network, and $R_2 = 1/\omega_2 C_2$.

The other lattice arm has the admittance given by

$$\frac{Y_B}{2} = \frac{C_1}{1 + \frac{p}{\omega_1}} + \frac{C_3}{1 + \frac{p}{\omega_3}}$$

and is seen to consist of two branches in parallel, each being a series combination of resistance and

capacitance. A more useful equivalent form is found by expanding the reciprocal of this admittance (that is, the impedance) in partial fractions. This leads to a determination of the elements for the equivalent configuration comprising the resistance $1/(\omega_1 C_1 + \omega_3 C_3)$ and the capacitance $(C_1 + C_3)$ in series with a shunt combination of resistance and capacitance.

The lattice has a simple unbalanced equivalent if either the series resistance of arm B exceeds the shunt resistance of arm A, or the shunt capacitance of arm A exceeds the series capacitance of arm B. These two relations are

$$\omega_1 C_1 + \omega_3 C_3 \leq 1 \quad (18)$$

and

$$C_1 + C_3 \leq C_{\infty} \quad (19)$$

With N , ω_1 , ω_3 given, and the C 's evaluated by Equation 17, the equality signs lead to quadratic equations for the determination of ω_2 . With the numerical values of the example, the relation 19 is satisfied for

$$5.74 < \omega_2 < 10.38$$

If ω_2 is taken as an end point of the interval, an element disappears from the balanced form. The value 10.38 is assigned to ω_2 . The lattice elements are now determined by Equations 16 and 17, and the unbalanced bridge-T equivalent exhibited by conventional methods. After multiplication by the indicated scale factor, the network appears as the feedback network N14 of Fig. 12.

The combined transfer impedance of the active and passive sections N13 and N12 contains an unwanted real root at $p = -10.38$. This root is removed by building up the impedance of the source which supplies signal to the differentiating condenser C_D . This circuit is shown on Fig. 12 scaled for unit source impedance. On account of the isolating effect of the active section N13, the networks may be easily adapted to variations in generator and load impedances, and to adjustment of the general level of transmission, simply by scaling the impedance levels.

The network representation admitting complex roots and poles in the transmission function permits a very great improvement in the quality of the smoothing. Although the characteristic ideal for smoothing is not very sharply selective, it is not well approximated by purely passive networks comprising only resistance and capacitance. With actual networks, the time over which the averaging process may be extended is limited by the time required for the output voltage to become a good approximation to its final value, subsequent to a sudden change in the input. Since a useful approximation is always fairly close, say to within one per cent of the change, this settling time may considerably exceed the period during which the input signal is heavily weighted in forming the average. The settling time for any network may be ascertained by inspecting its unit step response, $A(t)$, of which the derivative, $A'(t)$, is the weighting function.

Fig. 14 compares the unit step response, as given by curve A, of the transducer of Fig. 12 in accordance with the invention with that of two passive resistance-capacitance networks of refined design, as given by curves B and C. All the networks are scaled for one per cent settling time of one second. The network for curve B is a bridge network having complex roots but real

poles. The network for curve C is a ladder network having no complex factors in its transmission function. The ideal response corresponding to the parabolic function cannot be distinguished from curve A, and the three curves cannot be distinguished over the one per cent tolerance zone.

The weighting functions, or unit impulse responses, are compared by the correspondingly designated curves of Fig. 15. It is observed that for equal settling times the network in accordance with the invention, curve A, has much the longest memory, avoiding the temptation to favor recent data at the expense of old. The broken-line curve D gives the ideal parabolic response.

In Fig. 13 the steady-state transmission of the three filters, given by curves A, B and C, may be compared with the ideal shown by the broken-line curve D. To avoid complicating the drawing, only the real part is given. The phase shift of the transducer of Fig. 12, which is the Padé approximation, is, of course, closely linear with frequency up to the vicinity of the peak of loss.

What is claimed is:

1. A transducer with a transfer impedance unrestricted as to roots and poles comprising a stable feedback amplifier, a network in tandem therewith, and a network in the feedback path thereof, each of said networks comprising resistors and only a single type of reactor, said tandem-connected network introducing a complex root into said impedance, and the combination of said feedback network and said amplifier introducing a complex pole into said impedance through the reciprocation of said feedback network's transmission function by said amplifier.

2. A transducer in accordance with claim 1 in which said single type of reactor is a capacitor.

3. A transducer in accordance with claim 1 in which said feedback network is of the bridge type.

4. A transducer in accordance with claim 1 in which said tandem-connected network is of the bridge type.

5. A transducer in accordance with claim 1 in which said feedback network is of the bridged-T type.

6. A transducer in accordance with claim 1 in which said tandem-connected network is of the parallel-T type.

7. A transducer in accordance with claim 1 which has an unbalanced circuit.

8. A transducer in accordance with claim 1 in which the short-circuit transfer admittance of said feedback network has a real pole so located as to remove, by virtue of reciprocation through the action of said amplifier, an unwanted real pole introduced into said transfer impedance by said tandem-connected network.

9. A transducer in accordance with claim 1 having a sharply selective, low-pass transmission characteristic with cut-off below two cycles per second.

10. A transducer in accordance with claim 1 in which the unit impulse response is substantially parabolic.

11. In combination, a plurality of transducers in accordance with claim 1 connected in tandem.

12. The combination in accordance with claim 11 in which the short circuit transfer admittance of one of said feedback networks has a real pole so located as to remove, by virtue of reciprocation through the action of the associated amplifier, an unwanted real pole introduced into the transfer impedance of said combination by one of said tandem-connected networks.

13. A transducer in accordance with claim 1 which includes a resistor connected in series between said amplifier and said tandem-connected network.

14. A transducer comprising an active section and a passive section connected in tandem, said active section comprising an amplifier having a feedback path and a network in said feedback path, said passive section and said network each comprising resistors and only a single type of reactor and said passive section introducing a complex root and said active section introducing a complex pole into the transfer impedance of said transducer.

15. A transducer in accordance with claim 14 in which said single type of reactor is a capacitor.

16. A transducer in accordance with claim 14 in which said feedback network is of the bridge type.

17. A transducer in accordance with claim 14 which has an unbalanced circuit.

18. A transducer in accordance with claim 14 in which the transfer impedance of said active section has a real root so located as to remove an unwanted real pole introduced into said transfer impedance of said transducer by said passive section.

19. A transducer in accordance with claim 14 having a sharply selective, low-pass transmission characteristic with cut-off not exceeding a few cycles per second.

20. A transducer in accordance with claim 14 which includes a resistor connected in series between said active section and said passive section.

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