

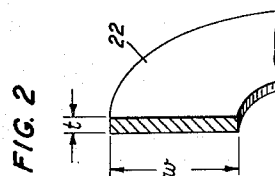
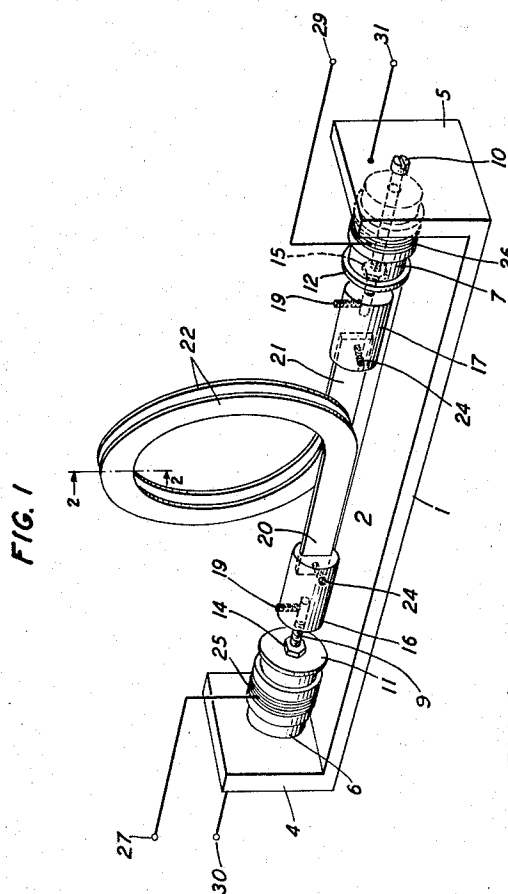
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TORSIONAL HIGH-PASS MECHANICAL FILTER

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TORSIONAL HIGH-PASS MECHANICAL
FILTER

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This invention relates to systems for the transmission of mechanical vibrations for such purposes as selecting waves in a given frequency range, or effecting desired delays in transmission.

It is particularly concerned with lowering the cut-off frequency and reducing the amount of space required for a high-pass filter of the type disclosed by E. L. Norton in United States Patent No. 1,799,634, issued April 7, 1931, and entitled "Wave Transmission." The Norton invention utilized a piezoelectric crystal to set up torsional vibrations in one end of a rod of uniform circular cross-section bent naturally into helical form. The vibrations were selectively propagated along the rod by virtue of its mechanical characteristics, and applied at the opposite end of the helix to a second crystal. The stressed crystal produced electrical impulses delayed in time and corresponding to those components of the vibrations applied to the first crystal which were within the selected range of frequencies.

The mechanical transmission element afforded an extremely sharp discrimination at the lower limit of frequencies permitted to pass, which was highly desirable. Its use for the transmission of audio frequencies of high quality was limited, however, by the size of the apparatus. To obtain a lower cut-off at 1000 cycles per second, for example, would have required a helix substantially four feet in diameter, while for a 100-cycle cut-off the helix required would have a diameter of forty feet.

I have discovered that this disadvantage may be overcome by forming the helix from a strip of rectangular cross-section with a high ratio of width to thickness, wound with its longer cross-sectional dimension extending along a diameter of the helix, and normal to the axis thereof.

This reduces the cut-off frequency so greatly that the filter may be of a size usable in conventional central office installations on audio frequency circuits. It makes possible the convenient use of such equipment in the delay circuit of radar moving target indicator systems. Other examples of the uses to which a solid delay line of convenient size may be applied are found in test equipment for studying fading and other transmission problems on long distance radio circuits, and in equipment for giving a more natural tone to studio broadcasts, where delay circuits are used to simulate echoes.

Other applications will occur to those skilled in the art for a unit providing a compact, simple, high-pass filter with an extremely sharp cut-off

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characteristic at substantially reduced frequencies, or for a delay line which will pass relatively low frequencies while providing substantial delay periods with a minimum of maintenance.

The invention may be better understood by reference to the drawings, in which:

Fig. 1 represents a unit incorporating the principles of the invention; and

Fig. 2 is a fragmentary view partially in section, taken as indicated by line 22 in Fig. 1.

It will be understood that the construction shown is illustrative only of the principles of the invention, and that any equivalent structure may be used within the scope of the appended claims.

The filter assembly is indicated generally as 1 in Fig. 1 of the drawings. The unit has a heavy mounting base 2 with upturned ends 4 and 5, and may be formed of a good conductor, such as brass. Crystal units 6 and 7 are mounted on ends 4 and 5 respectively. Each unit consists of a sodium potassium tartrate or composite Rochelle salt crystal cemented to its supporting end and centrally bored to allow a bolt to pass therethrough. Bolts 9 and 10 act as the internal electrodes, providing electrical connection to the crystal interior, in which they are fixed in accordance with the principles set forth in A. M. Nicolson Patents 1,562,578 and 1,574,302.

Bolts 9 and 10 are threaded so that they may be secured to their respective supporting ends 4 and 5. Plates 11 and 12, bored to fit over bolts 9 and 10, are held against the free ends of the crystals by lock nuts 14 and 15, respectively. If desired, the crystals may be placed under an initial compression by tightening these nuts. The bolts also provide rigid mechanical support for the crystals relative to their supporting ends 4 and 5. The resilience of the bolts 9 and 10 permits them to transfer torsional movements from the crystal to the helix and vice versa, through the coupling arrangements described hereafter.

The bolts 9 and 10 are secured at their inner ends to coupling sleeves 16 and 17 by set screws 19. The adjacent ends of these sleeves are formed to receive the ends 20 and 21 of the helix 22; set screws 24 maintain a rigid connection between the helix ends and the sleeves.

The helix 22 is shown as having two turns, but the number may be varied as desired to produce the requisite characteristics. These design principles were set forth in the Norton patent referred to above. It may be of any readily procurable metal, such as steel, brass, copper, aluminum or the like. For each material the proper values of the constants μ , ρ , and E must, of course, be

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used in computing the performance characteristics.

One electrical connection to each crystal is made by means of conventional girdle electrodes 25 and 26, to which are attached leads 27 and 30. The other electrical connection, to the interior of each crystal, is made through leads 30 and 31, ends 4 and 5, and bolts 9 and 10. An electrical impulse applied through leads 27 and 30 will produce a mechanical torsion in the crystal 6, which is transmitted through rod 9, coupling 16, helix 22, coupling 17 and rod 10 to crystal 7. It there produces mechanical stress in the crystal 7 which results in a potential across leads 29 and 31 exactly duplicating that applied to leads 27 and 30, but delayed and attenuated by the desired amount.

Comparing the results of this construction with that of Norton's round rod helix referred to above, having a cut-off at 3500 cycles and a helix diameter of 13.1 inches, a tape helix with a ratio of thickness to width of 1:100 would have its cut-off frequency reduced by the factor

$$.01 \times \sqrt{2}$$

so that it would be 49.5 cycles per second instead of 3500. Still greater reductions would be obtainable by reducing the thickness to width ratio further.

Thus a unit may be designed having a diameter sufficiently small to fit within any space conveniently available, and yet having a desired cut-off frequency. When it is to be used as a delay line, additional turns may be added to the helix to secure the requisite transmission time through the system.

The way in which the invention makes possible a much lower cut-off frequency for the same radius of curvature may be better understood by considering the mathematical analysis of the effects of torsional vibration of a rod of round cross-section bent into a circle. Such analysis shows that the cut-off frequency may be expressed by the equation

$$f_c = \frac{1}{2\pi r} \sqrt{\frac{E}{2\rho}} \quad (1)$$

where

f_c = the cut-off frequency;
 r = the radius of the circle into which the wire is bent;
 E = Young's modulus of elasticity; and
 ρ = the density of the rod material.

It will be seen from Equation 1 that for a given material the cut-off frequency depends only on the radius of the coils, and varies inversely with that radius. The Norton patent gives as a practical result a cut-off frequency of 3500 cycles for a steel wire coil of 13.1 inches in diameter. To obtain a cut-off frequency nearer the lower limit of the frequency band which must be transmitted for reasonably good quality of speech, for example, 100 cycles per second, would, therefore, require a diameter of about 40 feet. Hence, the equipment required to practice the teachings of the Norton patent at voice frequency ranges is not suited for use in conventional central office installations.

Equation 1 was derived from a consideration of the propagation constant for the filter, which involved equating the potential and kinetic reactions to obtain the wave propagation equation. Assuming as a starting point that the rod is straight and of length S , that μ is the coefficient

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of rigidity, b the angular displacement in the plane of the cross-section of the rod, and I_0 the moment of inertia of the section about the central axis of the rod normal to the section, it will be seen that

$$\mu I_0 \frac{\partial^2 b}{\partial S^2} = \rho I_0 \frac{\partial^2 b}{\partial t^2} \quad (2)$$

If the rod is now bent into a circle or helix, the potential energy factor represented by the left side of Equation 2 must be changed, since half the rod is put in compression and half in tension throughout its length when torsion is applied, and the restoring force is a function of b alone. Letting I_1 represent the moment of inertia of a cross-section of the rod about a diameter coinciding with a diameter of the helix, the potential energy P of the material is

$$P = \frac{1}{2} E I_1 \left(\frac{b}{r} \right)^2 \quad (3)$$

which gives as the restoring force F , when differentiated with respect to b

$$F = \frac{E I_1}{r^2} b \quad (4)$$

Equation 2 then becomes

$$\mu I_0 \frac{\partial^2 b}{\partial S^2} = \frac{E I_1}{r^2} b = \rho I_0 \frac{\partial^2 b}{\partial t^2} \quad (5)$$

For the case of a rod of circular cross-section,

$$I_1 = \frac{1}{2} I_0$$

and since b is a harmonic function of time, Equation 5 may be written thus

$$\frac{\partial^2 b}{\partial S^2} = \left(\frac{E}{2r^2 \mu} - \frac{\omega^2 \rho}{\mu} \right) b \quad (6)$$

where $\omega = 2\pi f$. The solution of Equation 6 is the sum of two exponential terms representing the original and the reflected waves. The latter may be neglected in considering the propagation constant, and the solution is

$$b = B e^{-\theta s} \quad (7)$$

where B is the value of b when $s=0$, and

$$\theta = s \sqrt{\frac{E}{2r^2 \mu} - \frac{\omega^2 \rho}{\mu}} \quad (8)$$

which may be readily reduced to the form

$$\theta = \frac{s}{r} \sqrt{\frac{1}{2} \frac{E}{\mu} \left[1 - \left(\frac{f}{f_c} \right)^2 \right]} \quad (9)$$

where f is the applied frequency, and the cut-off frequency f_c is defined as in Equation 1.

The equation of motion (5) involved the moments of inertia I_0 and I_1 . For a round rod,

$$I_1 = \frac{1}{2} I_0, \text{ or } \frac{I_1}{I_0} = \frac{1}{2}$$

Substituting this ratio in Equation 9 gives for the propagation constant

$$\theta = \frac{s}{r} \sqrt{\frac{I_1 E}{I_0 \mu} \left[1 - \left(\frac{f}{f_c} \right)^2 \right]} \quad (10)$$

and the cut-off frequency becomes, for the round rod,

$$f_c = \frac{1}{2\pi r} \sqrt{\frac{I_1 E}{I_0 \rho}} \quad (11)$$

The cut-off frequency may then be reduced by replacing the round rod by one of cross-section such that the ratio I_1/I_0 is small compared with $\frac{1}{2}$. Using an edgewise-wound helix, having the longer dimension of the cross-section, width w , normal to the axis of revolution of the helix, and

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the shorter dimension, or thickness t , of the cross-section parallel to that axis, the ratio becomes

$$\frac{I_1}{I_0} = \frac{t^2}{w^2 + t^2} \quad (12)$$

When the thickness t is small compared with the width w of the strip or tape, Equation 12 closely approximates

$$\frac{t}{w} = \sqrt{\frac{I_1}{I_0}} \quad (13)$$

Substituting this equivalent expression in Equation 11, the cut-off frequency f_c' of the edgewise wound strip becomes

$$f_c' = \frac{1}{2\pi r} \cdot \frac{t}{w} \sqrt{\frac{E}{\rho}} \quad (14)$$

The ratio of the cut-off frequency of the edgewise wound strip to that of the round rod (Equations 1 and 14) becomes

$$\frac{f_c'}{f_c} = \frac{\frac{t}{w} \sqrt{\frac{E}{\rho}}}{\sqrt{\frac{E}{2\rho}}} = \frac{t}{w} \sqrt{2} \quad (15)$$

That is, the cut-off frequency is reduced by using an edgewise-wound strip in the ratio of the thickness to the width of the tape times the square root of two.

The propagation constant is reduced in the same ratio, so that the distance along the helix at which a given attenuation or phase shift occurs is increased by the reciprocal of this ratio.

It should be observed that the use of tape wound with the longer cross-sectional dimension parallel to the helix axis would not produce a comparable result. This would make t large compared with w , and Equation 12 would approximate unity, and the ratio of Equation 15 would become, letting $f_{cs'}$ represent the cut-off frequency of this helix

$$\frac{f_{cs'}}{f_c} = \frac{\sqrt{\frac{E}{\rho}}}{\sqrt{\frac{E}{2\rho}}} = \sqrt{2} \quad (16)$$

That is, the cut-off frequency would then actually be increased over that of the round rod, by the factor

$$\sqrt{2}$$

It will be seen from the foregoing description that I have made a new and useful contribution to the field of mechanical wave transmission systems, and have thereby increased the field of application of such devices.

What is claimed is:

1. A mechanical wave transmission system, including a helix formed of metal tape having the longer cross-sectional dimension thereof disposed normal to the axis of said helix and the shorter cross-sectional dimension disposed parallel to the axis of said helix, means at one end of said helix for setting up torsional vibration in said tape, and means at the other end of said helix for utilizing torsional vibrations transmitted therethrough.

2. In a mechanical wave transmission system, a high-pass filter element comprising a helix formed of a metal tape having its longer cross-sectional dimension disposed normal to the axis of said helix and its shorter cross-sectional dimension extending parallel to the axis of said helix, means at one end of said tape for intro-

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ducing vibration torsionally about the longitudinal axis thereof, and means at the opposite end of said tape for utilizing torsional vibrations transmitted therethrough.

3. In a mechanical wave transmission system, a high-pass filter having a reduced cut-off frequency comprising means for vibrating a member torsionally, a helically formed member adapted to be vibrated in torsion and arranged to have a ratio for a cross-section of said member of substantially less than one-half between I_1 and I_0 , where I_1 is the moment of inertia of said cross-section about a diameter of said helix passing through said cross-section, and I_0 is the moment of inertia of said cross-section about the central axis thereof extending at right angles to said cross-section, and means for receiving torsional vibrations transmitted through said member.

4. A high-pass mechanical filter having a lowered cut-off frequency, comprising means for converting electrical impulses into torsional impulses, a solid element arranged to receive said impulses, and means for reconvertng said impulses into electrical impulses, wherein said solid element consists of a helix of flat metal tape wound edgewise, and so shaped that a ratio of substantially less than one-half is obtained between I_1 and I_0 for a cross-section of said tape where I_1 is the moment of inertia of said cross-section about a diameter of said helix passing through said cross-section, and I_0 is the moment of inertia of said cross-section about the central axis of said cross-section extending normally thereto.

5. In a mechanical wave transmission system, a mechanical high-pass filter comprising a helix formed by a bar having its longer cross-sectional dimension disposed normal to the axis of said helix and its shorter cross-sectional dimension parallel to said axis, wherein the cross-section of said bar has a ratio of substantially less than one-half between the moment of inertia about a line coinciding with a diameter of said helix and the moment of inertia about the central axis of said bar normal to said section, means for applying torsion to one end of said helix, and means for utilizing torsional effects therein at the opposite end of said rod.

6. In a wave transmission system, the combination of a piezoelectric crystal arranged for the production of torsional impulses on the application of electrical potentials thereto; a mechanical element comprising a helix of resilient material joined at one end to said crystal and formed of material having a cross-sectional shape such that the ratio of the moment of inertia of that cross-sectional shape about a diameter of said helix to the moment of inertia of said cross-sectional shape about a central axis normal thereto is substantially less than one-half; and a second piezoelectric crystal joined to the other end of said helix and arranged to produce electrical impulses on the application thereto of torsional impulses through said helix.

7. In a mechanical wave transmission system, a delay line comprising a torsion-producing member effective on application thereto of electric potentials, a helix formed of a strip of resilient material connected to said member and having a ratio of substantially less than one-half between the moment of inertia of the cross-section of said strip about a line therein coinciding with a diameter of said helix and the moment of inertia of said cross-section about the central axis of said strip normal to said section, and a

member arranged to convert torsional impulses in said helix into electric potentials.

8. A delay line, comprising a piezoelectric crystal arranged for the production of torsional impulses on the application of potentials thereto, a helix coupled at one end to said crystal, a similar crystal coupled to the opposite end of said helix and arranged to produce electrical potentials on the application of torsional impulses thereto, wherein said helix is formed of a strip of resilient material edgewise wound to reduce the ratio of I_1 to I_2 for the cross-section of said strip to a minimum, where I_1 is the moment of inertia of said cross-section about a diameter of said helix passing therethrough, and I_2 is the moment of inertia of such cross-section about the central axis thereof extending normally thereto.

9. In a mechanical wave transmission system, the combination of a piezoelectric crystal arranged for the production of torsional impulses on the application of electric potentials there-

to; a mechanical element joined at one end thereof to said crystal and comprising a helix of resilient material of rectangular cross-section having the longer dimension thereof disposed normal to the axis of said helix and the shorter dimension thereof disposed parallel to the axis of said helix; and a crystal connected to the other end of said helix and arranged for the production of electrical impulses corresponding to torsional impulses transmitted thereto through said helix.

RALPH V. L. HARTLEY.

REFERENCES CITED

15 The following references are of record in the file of this patent.

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Number	Name	Date
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