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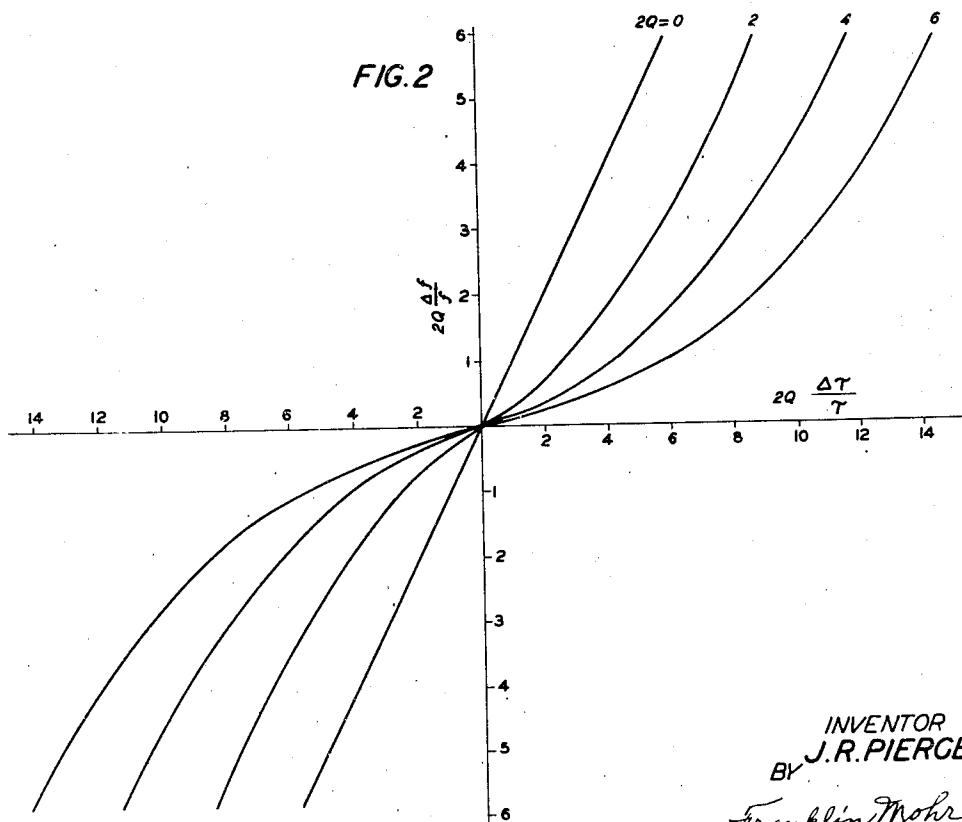
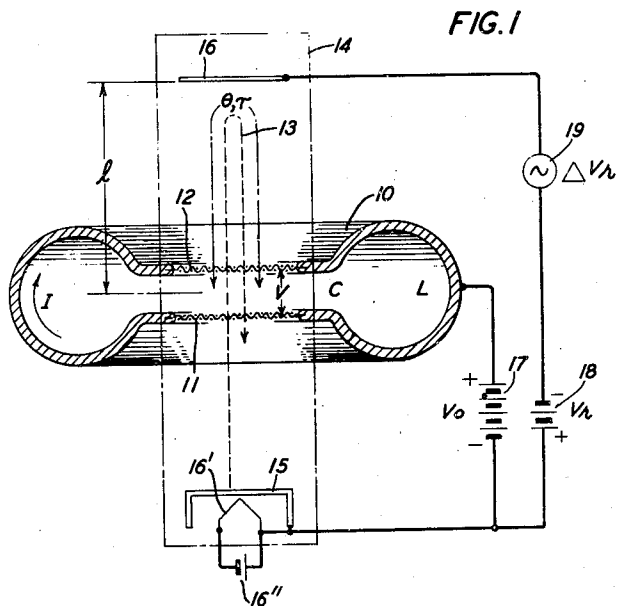
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2,476,765

LINEAR FREQUENCY MODULATION SYSTEM

Filed May 7, 1943

2 Sheets-Sheet 1



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2 Sheets-Sheet 2

FIG. 3

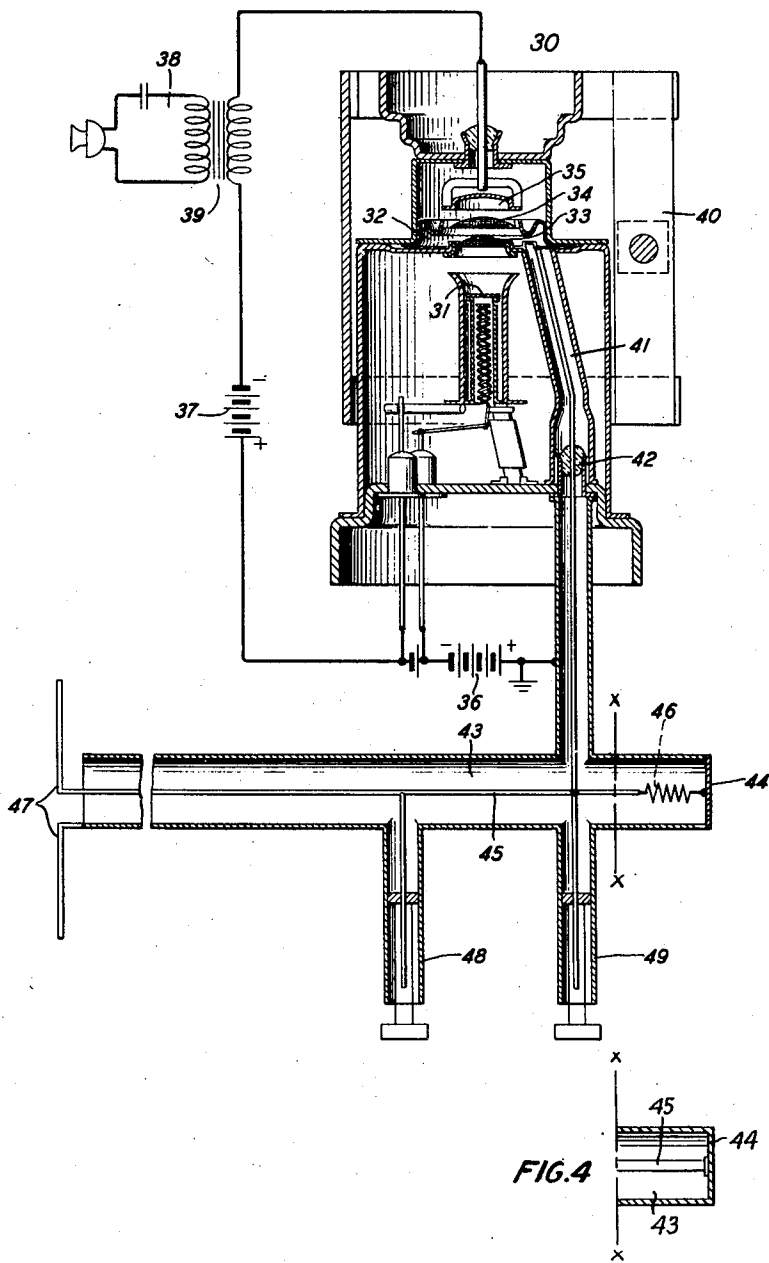


FIG. 4

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LINEAR FREQUENCY MODULATION SYSTEM

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6 Claims. (Cl. 332-25)

1

This invention relates to frequency modulation systems, particularly for use with an ultra-high carrier frequency.

The principal object of the invention is to secure in a frequency modulation system, a high degree of linearity of frequency as a function of modulating potential. In other words, the object is to improve the fidelity with which the frequency changes follow the wave form of the modulating potential, which latter may be controlled by voice waves, radio program material, television signals or the like.

The main feature of the invention is the use of increased damping, which preferably takes the form of a close degree of coupling between the oscillating circuit that produces the carrier waves and the load to which the waves are to be supplied. By a close coupling is meant a greater degree of coupling than the one which accompanies maximum power transfer, i. e., than the one which corresponds to an exact impedance match between the source and the load. The closeness of the coupling may be adjusted by means of an impedance transforming device between the source and the load. Alternatively, the damping may be increased by inserting resistance in the circuit between the source and the load.

In frequency ranges below the ultra-high, it has been found necessary in many cases to insert resistance to increase the damping of the oscillation circuit of a frequency modulator and thus to reduce the frequency stability of the circuit to a point which will enable frequency changes to be effected that are sufficiently rapid to follow the modulation potentials. In the ultra-high frequency or microwave range, however, especially when cavity resonators or the like are employed in the oscillation generating system, the frequency transmission band of the typical resonator is so very wide compared to the highest frequency of the modulating potential that the band width is ample to permit frequency changes as rapid as are required for the frequency modulation without the deliberate introduction of any additional damping. It is found, however, that in many cases the system if utilized without any change, does not give sufficient linearity of response to meet exacting requirements. Applicant has discovered that the linearity of the response in such a system may be improved by increasing the damping connected with the oscillating system, as by adding resistance or by using a close coupling to the load.

Negative feedback for distortion correction has been used in various forms in frequency modula-

2

tors to secure the over-all effect of linearity of frequency as a function of the modulating potential. The present invention has advantages over feedback systems in simplicity and considerable saving in the amount of apparatus required to secure the desired result.

The invention contemplates the use of any suitable ultra-high frequency oscillator which is capable of frequency control in response to variation of some electrical or mechanical parameter, for example, by application of modulating potentials between two electrodes of a vacuum tube. The theory of the invention will be developed hereinafter with reference specifically to an oscillator of what is commonly referred to as the electron velocity variation type and more particularly to one in which the electron stream is reflected or reversed at some point by suitable means so that the stream twice traverses a single gap in a resonating chamber or cavity resonator. At the first passage of the electron stream across the gap, the successive electrons receive velocity variations in accordance with the instantaneous value of the potential produced across the gap by electrical oscillations within the cavity resonator. After an interval, referred to as the transit time or transit angle, the electron stream has acquired electron density variations in well-known manner which may be described briefly as being caused by electrons accelerated in the gap overtaking other electrons which have been slowed down. The electron grouping or density variations will be periodic with a frequency determined by the frequency of oscillation in the cavity resonator. The oscillations will be self-sustained if the transit angle is such that each group or concentration of electrons returns to the gap at the proper instant to be decelerated by the electric field then existing across the gap. The theory may be developed equally well for the type of electron variation oscillator in which separate resonators or separate gaps in a single resonator are provided for the first and second passages of the electron stream, respectively, as described above, in which case the electron stream passes the gaps in succession without turning about. The results of the theory will be readily recognized as applicable also to other types of oscillator mechanisms which are capable of frequency variations. In fact, the invention is applicable to any oscillating system in which frequency variation is obtained by a variation in the phase of the feed-back regeneration which is a substantially linear function of the modulating voltage or other modulating means.

In the drawings:

Fig. 1 is a diagram useful in explaining the theory of the invention with reference to a reflection type velocity variation oscillator;

Fig. 2 is a graph of the relation between frequency change and transit time change in an oscillator under various conditions of damping;

Fig. 3 is a schematic representation of a preferred embodiment of the invention; and

Fig. 4 shows a modification of a portion of the system of Fig. 3.

Referring to Fig. 1, there is represented in cross-section in that figure a cavity resonator 10 of generally toroidal shape, the central portion of which is perforated to form grids or screens 11 and 12. The envelope of a vacuum tube enclosing the grids 11 and 12 is represented schematically by a broken line 14. The grids 11 and 12 serve to define a gap across which the alternating potentials developed by electromagnetic waves in the resonator 10 can act upon an electron beam contained within the envelope 14. A cathode 15 is shown as a source of electrons and may be heated by filament 16' and a source 16'' of electromotive force. On the opposite side of the grids from the filament 15 is a repeller electrode 16 at a distance l from the gap. Steady energizing potentials V_0 and V_r are impressed upon the resonator 10 and the repeller electrode 16 by suitable sources 17 and 18, respectively, which are illustrated as batteries. In series with the battery 18 there is represented a generator 19 which superimposes a modulating potential ΔV_r upon the voltage V_r . The path of a typical electron is represented schematically by a broken line 13 which passes upward to the gap between the grids 11 and 12, is turned back and again traverses the gap, this time in the downward direction. The transit angle θ and the transit time τ are represented schematically and are measured between the first and second passages of the gap. The transit angle is understood to be measured in terms of an integral number of cycles or fraction thereof, 360 electrical degrees corresponding to a complete cycle of oscillations of electromagnetic waves in the cavity resonator. The resonator, for purposes of this analysis, is contemplated as having a lumped capacity C concentrated at the gap and a lumped inductance comprising a single turn and residing in the toroidal surface of the resonator. For purposes of the discussion, the instantaneous voltage across the gap will be designated V and the instantaneous current on the inner surface of the resonator will be presented by I .

The ratio of I to V constitutes an admittance. It is the admittance of the resonator acting as an absorber of energy from the electron stream and is conventionally to be regarded as positive. In the case of sustained oscillations, the electron stream serves as a mechanism for delivering energy to the resonator through the process of velocity variation followed by charge density variation. The electron stream as a source of alternating current energy may be regarded as having an admittance which is negative and which in order to sustain the oscillations must exactly neutralize the positive admittance of the resonator.

If the admittance of the resonator is designated by Y_c and the admittance of the electron stream by Y , then the condition for sustained oscillations is

$$Y_c + Y = 0 \quad (1)$$

Regarding the resonator as equivalent to a sim-

ple tuned circuit, and representing 2π times the operating frequency by ω , the admittance of the resonator is

$$Y_c = i\omega C - \frac{i}{\omega L} + G_c \quad (2)$$

where G_c stands for the conductance. Equation (2) indicates that Y_c is a function of ω hence can vary to accommodate a suitable value of Y with the result that sustained oscillations can occur for a particular value of ω which will satisfy Equation (1). The operating frequency may of course differ from the resonant frequency of the resonator. The value of ω at resonance may be designated ω_0 and since at resonance the reactances of the resonator are neutralized, the condition of resonance gives rise to the relation

$$Y_c(\omega_0) = G_c \quad (3)$$

By a well-known method of approximation employing Taylor's Series, the value of Y_c in the neighborhood of resonance may be expressed as

$$Y_c = G_c + i2C\Delta\omega \quad (4)$$

Considering now the electron stream, the current induced in the resonator by the stream is assumed to be proportional to the transit angle θ , since the electron grouping may be considered to proceed at a uniform time rate. It is further assumed that the faster electrons do not pass by the slower ones. The phase relation between I and V will also depend upon θ and the conductance may be expressed conveniently as an exponential function

$$Y = K\theta e^{-i\theta} = K\theta(\cos \theta - i \sin \theta) \quad (5)$$

where K is a constant of proportionality. A relationship may be found between Y and $\Delta\theta$ by assuming Y to take a new value $Y + \Delta Y$ corresponding to a new value $\theta + \Delta\theta$. By conventional methods of approximation it may be shown that

$$Y + \Delta Y = Y e^{-i\Delta\theta} \quad (6)$$

the approximation involving the assumption that $\Delta\theta$ is a small fraction of θ .

Suppose that Y_0 is the value of Y corresponding to ΔY and $\Delta\theta$ both equal to zero. The magnitude of Y_0 will be a function of the amplitude of oscillation. Then any particular value of Y in the neighborhood of Y_0 may be expressed as

$$Y = Y_0 e^{-i\Delta\theta} = Y_0 (\cos \Delta\theta - i \sin \Delta\theta) \quad (7)$$

Substituting the approximate values of Y_c and Y from Equations (4) and (7) respectively into Equation (1) gives

$$G_c + 2iC\Delta\omega + Y_0 \cos \Delta\theta - iY_0 \sin \Delta\theta = 0 \quad (8)$$

Equating the real and imaginary parts of Equation (8) gives two real equations

$$\begin{aligned} G_c + Y_0 \cos \Delta\theta &= 0 \\ 2C\Delta\omega - Y_0 \sin \Delta\theta &= 0 \end{aligned} \quad (9)$$

In the establishment of equilibrium, the amplitude and hence Y_0 , and the frequency will adjust themselves so that Equations (9) are satisfied.

Equations (9) may be solved to obtain

$$\Delta\omega = -\frac{G_c}{2C} \tan \Delta\theta \quad (10)$$

Equation (10) actually applies to any oscillator whose resonant circuit may be treated as a single tuned circuit, according to (4), $\Delta\theta$ being merely the change in phase angle of the feedback. Substituting the commonly used symbol Q for the ratio of reactance to resistance in the resonating

system, or what is the same thing the ratio of conductance to susceptance, after dividing Equation (10) through by ω , the result is

$$\frac{\Delta\omega}{\omega} = \frac{\Delta f}{f} = \frac{1}{2Q} \tan \Delta\theta \quad (11)$$

In order to relate the frequency change to a voltage variation upon the repeller electrode, Equation (11) is best modified to secure a corresponding relationship in terms of the transit time τ required for the typical electron between the first and second passages of the gap. The transit time is related to θ and ω by

$$\theta = \omega\tau \quad (12)$$

from which it follows that

$$\Delta\theta = \omega\Delta\tau + \tau\Delta\omega \quad (13)$$

and

$$\frac{\Delta\tau}{\tau} = -\frac{\Delta\omega}{\omega} + \frac{\Delta\theta}{\theta} \quad (14)$$

Using Equations (11) and (14)

$$\frac{\Delta\tau}{\tau} = \frac{1}{2Q} \tan \Delta\theta + \frac{\Delta\theta}{\theta} \quad (15)$$

A relationship between the transit time τ and the potential V_r of the repeller electrode may be worked out with reference to Fig. 1 and the fundamental principles of electron motion.

The typical electrons leaving the cathode 15 arrive at the gap 11, 12 with a velocity

$$v_o = \sqrt{2\eta V_o} \quad (16)$$

where η is the ratio of charge to mass for an electron. Taking V_r as a positive quantity when the repeller 16 is negative with respect to the cathode 15, the transit time τ is readily found to be

$$\tau = \frac{2lV_o}{v_o(V_o + V_r)} \quad (17)$$

The variation of τ corresponding to a variation ΔV_r may be expressed as

$$\tau + \Delta\tau = \frac{2lV_o}{v_o(V_o + V_r + \Delta V_r)} \quad (18)$$

From Equation (18) by simple algebraic manipulation, it may be shown that

$$\frac{\Delta\tau}{\tau} = \frac{-\frac{\Delta V_r}{V_o + V_r}}{1 + \frac{V_r}{V_o + V_r}} \quad (19)$$

It will be noted that for small voltage variations such as are encountered in practice, this equation becomes

$$\frac{\Delta\tau}{\tau} = \frac{\Delta V_r}{V_o + V_r} \quad (20)$$

Equation (20) shows that for variations in repeller voltage of the magnitude used in frequency modulation $\Delta\tau/\tau$ is nearly linearly proportional to ΔV_r . Thus, an idea of the shape of the frequency variation curve may be obtained by plotting $\Delta f/f$ against $\Delta\tau/\tau$ by means of Equations (11) and (15). This will likewise be true in the case of other oscillators for which the variation in phase of the feedback is a substantially linear function of the control means. Fig. 2 shows curves plotted in this manner to several values of the parameter

$$\frac{2Q}{\theta}$$

It is apparent from Fig. 2 and Equation (23) that the lower the value of Q , that is, the more

tightly the load is coupled to the oscillator, or the greater the damping of the oscillator, the more nearly linear the variation of frequency with repeller voltage will be. Consequently when a very nearly linear relation of frequency to voltage is desired, as in the use of a reflection type oscillator to produce frequency modulated waves in a high fidelity system, the linearity can be improved by coupling the load to the oscillator more tightly than for optimum power output. Added damping may, of course, be obtained by the insertion of a resistance in the load circuit in any suitable manner.

Fig. 3 shows an embodiment of the invention as applied to a reflection type oscillator disclosed in a copending application of R. L. Vance, Serial No. 439,375, filed April 17, 1942, now Patent Number 2,411,912, issued December 3, 1946. A cross-sectional view of the tube is shown at 30. The tube has a cathode 31, resonating chamber 32, grids 33 and 34 and a repeller electrode 35 corresponding respectively to elements 15, 10, 11, 12, and 16 of Fig. 1. Batteries 36 and 37 are provided for acceleration and retardation, respectively, corresponding to 17 and 18 in Fig. 1. A microphone circuit 38 and a transformer 39 take the place of the generator 19. Mechanical tuning means including a toggle arrangement 40 may be used to vary the spacing between the grids 33 and 34 for initial adjustment of the resonant frequency. A coaxial lead 41 is coupled to the resonating chamber for extracting useful output. A seal 42 prevents loss of vacuum through the lead 41.

The output lead 41 is connected as a side branch of a coaxial line 43 which is short-circuited at one end by an end plate 44 near the junction with the lead 41. The line 43 has an inner conductor 45 which may be interrupted near the end plate 44 where a series resistor 46 may be inserted. Opposite the end plate 44, the line 43 may extend to any suitable load, for example, a doublet antenna 47 as illustrated. Tuning stubs 48 and 49 are preferably supplied for adjusting the wave conditions in the line 43.

In the operation of the arrangement of Fig. 3, the tube 30 is adjusted to operate at a desired mean frequency and is frequency modulated by signal potentials applied to the repeller electrode 35 by the microphone circuit 38 and transformer 39 in well-known manner. The output of the system is supplied to the antenna 47 through the coaxial lead 41 on the line 43. Reflections from the end plate 44 may be usefully combined with the direct waves supplied by the coaxial lead 41 by adjustment of the tuning stubs 48 and 49. The purpose of the resistor 46 is to absorb energy and thereby increase the damping of the system as a whole. With extra damping of the system provided by the resistor 46, it is found that the linearity of the frequency as a function of the modulating potential is improved. Losses incident to the use of the resistor 46 may be avoided by removing the resistor from the circuit, connecting the conductor 45 directly to the plate 44 as shown in Fig. 4 and changing the adjustment of the tuning stubs 48 and 49 to secure greater than optimum coupling between the oscillator and the load. Increase of the coupling, while causing some decrease in the amount of useful output obtainable, is found to result in increased linearity of the frequency vs. voltage relation.

What is claimed is:

1. A frequency modulator comprising an oscillator, a source of modulating potentials, means

7

to impress potentials from said source upon said oscillator to vary the frequency thereof, said oscillator containing a resonator of low inherent damping and of a resonant frequency very high compared to the highest frequency of said modulating potentials so that in spite of the said low damping the oscillator is capable of changing its frequency in response to said modulating potentials without insertion of additional damping and without introduction of signal distortion due to inertia effects in said resonator, and means to increase the damping of said resonator materially beyond the point of maximum power output from the oscillator to promote increased linearity between the oscillating frequency and the modulating potential at the expense of decreased power output.

2. A frequency modulator comprising an electron beam forming means, a resonating chamber for ultra-high frequency electromagnetic waves, said chamber being arranged in the path of the electron beam and having electron permeable walls permitting said electron beam to pass through and beyond the chamber, an electron-repelling electrode on the opposite side of said resonating chamber from said beam forming means to return electrons to said chamber, thereby to sustain oscillations therein, a source of modulating potentials, means to impress modulating potentials from said source upon said electron repelling electrode to vary the frequency of oscillations in said chamber, and means to increase the damping of said chamber materially beyond the point of maximum power output of oscillations therefrom to promote increased linearity between the oscillating frequency and the modulating potential.

3. A frequency modulator comprising an electron beam forming means, a resonating chamber for ultra-high frequency electromagnetic waves, said chamber being arranged in the path of the electron beam and having an electron permeable wall permitting the passage therethrough of said electron beam, electron repelling means for reversing the direction of travel of electrons to sustain oscillations in said resonating chamber, a source of modulating potentials, means to impress potentials from said source upon said electron repelling means for the purpose of varying the frequency of oscillations in said chamber, and means to increase the damping of said chamber materially beyond the point of maximum power output of oscillations therefrom to promote increased linearity between the oscillation frequency and the modulating potential.

4. A frequency modulator comprising an oscillator, a source of modulating potentials, means to impress potentials from said source upon said

8

oscillator to modulate the frequency thereof, said oscillator containing a resonator of low inherent damping and of a resonant frequency very high compared to the highest frequency of said modulating potentials so that in spite of the said low damping the oscillator is capable of changing its frequency in response to said modulating potentials without introduction of signal distortion due to inertia effects in said resonator, a load circuit, and an impedance transforming means coupling said resonator to said load circuit, said impedance transforming means being adjusted to effect a coupling materially closer than required for an impedance match, whereby the fidelity of the frequency modulation is improved.

5. A frequency modulator comprising a cavity resonator of low inherent damping, a source of an electron beam, means to direct the electron beam from said source through and beyond said cavity resonator, potential actuated means to reflect said beam back to said resonator after first traversing the same, to generate electromagnetic waves therein, means to vary the potential of said reflecting means and thereby correspondingly to vary the frequency of said electromagnetic waves, and means to load the said resonator materially beyond the point of maximum power output to increase the linearity of the relation between the frequency of the generated waves and the actuating potential of said reflecting means.

6. A frequency modulator comprising a cavity resonator of low inherent damping, a source of an electron beam, means to direct the electron beam from said source into said cavity resonator, potential actuated means to reflect said beam after entry into said resonator for the purpose of generating electromagnetic waves therein, means to vary the potential of said reflecting means and thereby correspondingly to vary the frequency of said electromagnetic waves, and means to load the said resonator materially beyond the point of maximum power output to increase the linearity of the relation between the frequency generated and the actuating potential of said reflecting means.

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REFERENCES CITED

The following references are of record in the file of this patent:

UNITED STATES PATENTS

Number	Name	Date
1,595,794	Little	Aug. 10, 1926
2,118,161	Chaffee	May 24, 1938
2,245,627	Varian	June 17, 1941
2,250,511	Varian	July 29, 1941
2,434,704	Kroger	Jan. 20, 1948