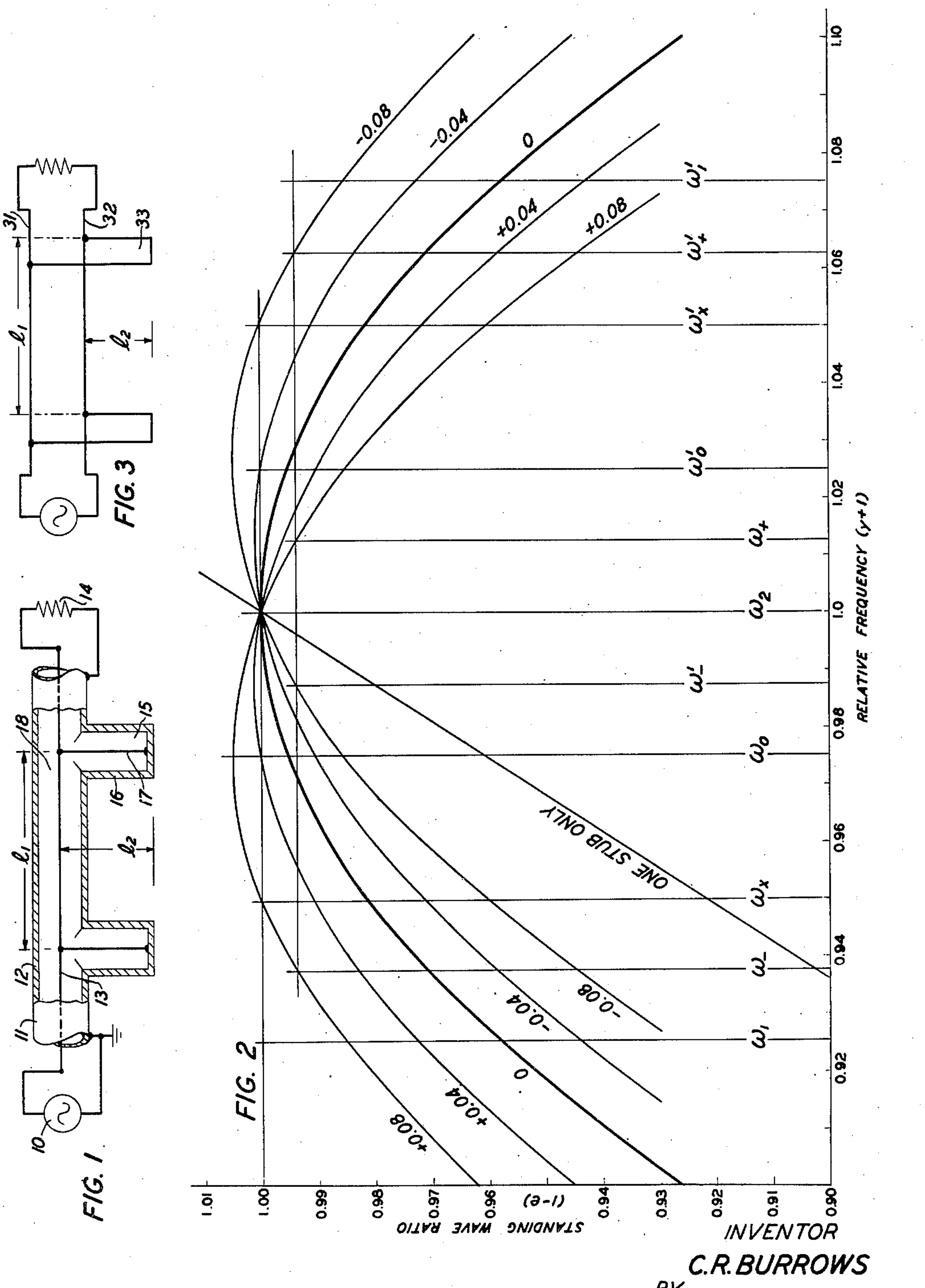
SUPPORT FOR CONDUCTORS OF SIGNAL TRANSMISSION LINES

Filed Feb. 29, 1944



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Patented Apr. 12, 1949

2,467,292

# UNITED STATES PATENT OFFICE

2,467,292

## SUPPORT FOR CONDUCTORS OF SIGNAL TRANSMISSION LINES

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Application February 29, 1944, Serial No. 524,346

7 Claims. (Cl. 178—44)



This invention relates to a line for transmitting signal waves extending over a band of relatively high frequencies, and more particularly to an arrangement for supporting the conductors of such line.

The main object of the invention is to provide a support for the inner conductor of a concentric conductor line transmitting signal waves extending over a preselected transmission band.

Another object is to decrease loss in the transmission band.

A further object is to minimize reflection effects in the transmission band.

In a specific embodiment of the present invention, the outer and inner conductors of a concentric conductor line transmitting signal waves extending over a preselected band of relatively high frequencies are concentrically spaced by a support comprising a series section of the line, and a short-circuited stub connected to each of the two opposite ends of the series section. The length of the stubs may be one quarter wavelength at a frequency near one end of the transmission band but inside thereof, and the length of the series section may be one quarter wavelength at a frequency near the opposite end of the transmission band but lying outside thereof. Thus the length of the stubs may be one quarter wavelength at a frequency equivalent to

$$\left(1+\frac{\Delta}{2\sqrt{2}}\right) \text{ or } \left(1-\frac{\Delta}{2\sqrt{2}}\right)$$

times the mid-frequency of the transmission band; and the length of the series section may be

$$\left(1+\frac{3\Delta}{2\sqrt{2}}\right) \text{ or } \left(1-\frac{3\Delta}{2\sqrt{2}}\right)$$

times the length of the stubs, where $\Delta$ is the fractional transmission band. In these expressions, a (+) sign for the subs requires a (−) sign for the series section, and vice versa. Thus, the lengths of the series section and stubs may be different at a preselected frequency in the transmission band, but it is immaterial whether the series section or stubs is the longer.

The invention is also applicable to a transmission line including two parallel conductors such that the support comprises a series section of the line, and a shunt section of the line connected to each of the two opposite ends of the series section.



The invention will be readily understood from the following description taken together with the accompanying drawing in which:

Fig. 1 shows a specific embodiment of the invention included in a concentric conductor transmission line;

Fig. 2 is a family of curves illustrating certain action obtainable in Figs. 1 and 3; and

Fig. 3 shows an alternate embodiment of the invention included in a transmission line embodying two parallel conductors.

Referring to Fig. 1, a source of signal waves extending over a certain frequency band is connected to the input end of a concentric conductor transmission line 11 comprising an outer conductor 12 and an inner conductor 13 arranged concentrically therewith, and the output of the line is applied to a load 14. The inner conductor 13 is supported by a pair of stubs 15, each of which includes an outer conductor 16 and an inner conductor 17 arranged concentrically therewith. Corresponding ends of the stubs are connected to the opposite ends of a series section 18 of the line, while the opposite ends of the stubs are short-circuited. Thus, the effective support for the inner conductor 13 comprises the two stubs 15 joined together by the series section 18. Assuming the impedance of the wave source, transmission line and load are so matched that substantially no standing wave is produced on the line, the only other factor tending to produce standing waves on the line is the above-mentioned support.

In accordance with the present invention, the design of this support is such as to produce substantially no standing wave on the line, or substantially the minimum magnitude of such wave, and is determined as follows:

The ratio of the input admittance to the characteristic admittance of a pair of short-circuited stubs of length $l_2$ shunted across a transmission line at two points separated by a length $l_1$ is given by

$$\frac{Yin}{Y_0}=\frac{1-j\left(\frac{Y_r}{Y_0}-j\cot\alpha_2\right)\cot\alpha_1}{\frac{Y_r}{Y_0}-j\cot\alpha_2-j\cot\alpha_1}-j\cot\alpha_2 \quad (1)$$

$$=\frac{1-jx+j^2xy}{1-j(x+y)}-jy$$

where $Y_r$=load admittance

$Y_0$=characteristic admittance

$$\alpha_1=\frac{2\pi l_1}{\lambda}=\frac{\pi}{2}\frac{\omega}{\omega_1}$$

$$\alpha_2=\frac{2\pi l_2}{\lambda}=\frac{\pi}{2}\frac{\omega}{\omega_2}$$

$l_1$=length of line between stubs

$l_2$=length of stubs

$x=\cot\alpha_1$

$y=\cot\alpha_2$

Since we are interested in evaluating the input admittance at frequencies for which $l_1$ and $l_2$ are approximately a quarter wavelength and hence the cotangent functions are small quantities, it is convenient to perform the indicated division, which gives:

$$\frac{Yin}{Y_0}=1-y(y+2x)-jy(y+2x)(y+x)-\ldots \quad (2)$$

This may be written in terms of a parameter that depends upon frequency and another that depends upon the ratio of $l_1$ to $l_2$ both of which are small quantities, by defining

$$\delta=\frac{\omega_2}{\omega_1}-1=\frac{l_1}{l_2}-1$$

$$\gamma=\frac{\omega}{\omega_2}-1$$

In terms of these, $x$ and $y$ are given by

$$y=-\tan\frac{\pi}{2}\gamma\sim-\frac{\pi}{2}\gamma$$

$$x=-\tan\frac{\pi}{2}(\gamma\delta+\gamma+\delta)\sim\frac{\pi}{2}(\gamma\delta+\gamma+\delta)$$

Neglecting the third and higher order terms in expression (2) for the input admittance, it may be written

$$\frac{Yin}{Y_0}\sim1-2\frac{\pi^2}{4}\gamma\delta-\gamma^2\frac{\pi^2}{4}(3+2\delta) \quad (3)$$

This is the equation of a parabola which, when put in its normal form, becomes

$$\left(\gamma+\frac{\delta}{3+2\delta}\right)^2=\frac{4}{\pi^2(3+2\delta)}\left(\epsilon+\frac{\pi^2\delta^2}{4(3+2\delta)}\right) \quad (4)$$

where now the ratio of the input admittance to the characteristic admittance of the line is

$$\frac{Yin}{Y_0}=1-\epsilon$$

From Equation 4 is it evident that the vertex of the parabola occurs at the frequency for which

$$\gamma=-\frac{\delta}{3+2\delta}$$

and the ratio of the input admittance to the characteristic admittance differs from unity by

$$\frac{\pi^2\delta^2}{4(3+2\delta)}$$

at this frequency.

As the frequency is varied the ratio of the input admittance to the characteristic admittance increases from a small quantity through unity to a quantity somewhat greater than unity and then decreases through unity to a small quantity. Accordingly, to obtain the maximum band of frequencies for a given standing wave ratio, we will make the ratio of the input admittance to the characteristic admittance at the edges of the band differ from unity by the same amount that it does in the middle of the band. When this is done, the edges of the band are given by



$$\gamma_1=-\frac{\delta}{3+2\delta}(1+\sqrt{2}) \quad (5)$$

$$\gamma_2=-\frac{\delta}{3+2\delta}(1-\sqrt{2})$$

If we call the fractional transmission band width

$$\gamma_1-\gamma_2=\Delta$$

then

$$\epsilon=\frac{3\pi^2}{32}\Delta^2\sim\Delta^2 \quad (6)$$

where $(1+\epsilon)$ is the maximum standing wave ratio within the fractional transmission band $\Delta$. This small amount of standing waves is made possible by using a support comprising a series section of the line, and a short-circuted stub connected to each of the two opposte ends thereof, and arranged such that the length of the stubs is one quarter wavelength at a frequency equivalent to

$$\left(1+\frac{\Delta}{2\sqrt{2}}\right)\text{ or }\left(1-\frac{\Delta}{2\sqrt{2}}\right)$$

times the mid-frequency of the transmission band $\delta$ and such that the length of the series section is

$$\left(1+\frac{3\Delta}{2\sqrt{2}}\right)\text{ or }\left(1-\frac{3\Delta}{2\sqrt{2}}\right)$$

times the length of the stubs. In these expressions, a (+) sign for the stubs requires a (−) sign for the series section, and vice versa.

To this approximation, the input admittance is real so that the ratio of the admittance at the point along the line where it has its maximum value to the admittance at the point along the line where it has its minimum value is equal to the square of the ratio of the input admittance to the characteristic admittance, or the square of its reciprocal. Since the voltage and current standing wave ratios are equal to the square root of the impedance or admittance standing wave ratios, the voltage and current standing wave ratios are also equal to the ratio of the input admittance to the characteristic admittance for the case under consideration. These standing wave ratios are given as a function of frequencies in Fig. 2. Referring to the latter figure, standing wave ratios are given for $\delta=0$, $\pm0.04$ and $\pm0.08$ where the curve "0" indicates that $\delta$, the ratio between the lengths of the series section 18 and stubs 15, is unity, that is, the length of the series section 18 is substantially equal to the length of each stub 15, each of these lengths being one quarter wavelength at the relative frequency "1.0" which is effectively the mid-frequency of the transmitted band. The curve "+0.04" indicates that $\delta$, the ratio between lengths of the series section 18 and stubs 15 is different from unity by "+0.04," that is, the length of the series section 18 is longer than the length of the stubs 15 which are one quarter wavelength at the relative frequency "1.0"; and the curve "−0.04" indicates that $\delta$, the ratio between the lengths of the series section 18 and stubs 15 is different from unity by "−0.04," that is, the length of the series section 18 is less than the length of the stubs 15 which are one quarter wavelength at the relative frequency "1.0." The curves "±0.08" indicate that the ratio between the lengths of the series section 18 and stubs 15 differ from unity in the manner similar to the "±0.04" curves.

Referring again to Fig. 2, the curve "0" indicates, for example, that for an 0.07 fractional transmission band width say, from frequencies 0.965 to 1.035 times the frequency represented by the relative frequency "1.0," the most unfavorable standing wave ratio is approximately 0.991; that for an 0.07 fractional transmission band width, say from frequencies 0.95 to 1.02 times the frequency represented by the relative frequency "1.0," the most unfavorable standing wave ratio is approximately 0.991 or 1.001; and that for an 0.07 fractional transmission band width, say from frequencies 0.94 to 1.01 times the frequency represented by the relative frequency "1.0," the most unfavorable standing wave ratio is approximately 0.995 or 1.005. In connection with the "0" and "±0.08" curves, it is apparent that the deviation of the standing wave from unity has been decreased approximately by 50 per cent. For fractional frequency transmission bands $\Delta$ wider than 0.07, it can be similarly demonstrated that the improvement of the deviation of the standing wave from unity becomes substantially greater. In other words, as the transmission band is made wider, the improvement of the deviation of the standing wave from unity becomes greater. For the purpose of simplicity such curves are omitted from Fig. 2.

In Fig. 2, with reference to the "+0.08" curve, the line $\omega_-$ is the frequency at the lower limit of the transmission band; the line $\omega_+$ is the frequency at the upper limit of the transmission band; the line $\omega_0$ is the frequency at the middle of the transmission band; the line $\omega_2$ is the frequency at which the length of the stubs is one quarter wavelength; and the line $\omega_1$ is the frequency at which the length of the series section is one quarter wavelength. In Fig. 2, the reference to the "−0.08" curve, the line $\omega'_-$ is the frequency at the lower limit of the transmission band; the line $\omega'_+$ is the frequency at the upper limit of the transmission band; the line $\omega'_0$ is the frequency at the middle of the transmission band; the line $\omega_2$ is the frequency at which the length of the stubs is one quarter wavelength; and the line $\omega'_1$ is the frequency at which the length of the series section is one quarter wavelength. With reference to the "+0.08" curve, no standing waves are produced on the transmission line at the frequencies represented by the lines $\omega_x$ and $\omega_2$; and with reference to the "−0.08" curve, the same condition obtains at the frequencies represented by the lines $\omega'_x$ and $\omega_2$. Also, with reference to the "+0.08" curve, the maximum deviation from unity within the transmission band occurs at the three frequencies represented by the lines $\omega_-$, $\omega_0$ and $\omega_+$; and with reference to the "−0.08" curve, the maximum deviation within the transmission band occurs at the three frequencies represented by the lines $\omega'_-$, $\omega'_0$ and $\omega'_+$. In like manner, the lengths of the stubs and series section may be demonstrated with reference to the "±0.04" curves, and other curves not shown in Fig. 2 but contemplated therefor as mentioned above.

In Fig. 2 the length of the stubs 15 is one quarter wavelength at a frequency near the upper edge of the transmission band for the "+0.08" curve, and near the lower edge of the transmission band for the "−0.08" curve. These curves show that it is desirable to have the lengths of the stubs and series section slightly different in accordance with the above equations, but that it



is immaterial which length is the longer. If the length of the series section is greater than the length of the stubs, most of the useful frequency band lie below the frequency for which the stubs are a quarter wavelength long. If the length of the series section is less than the length of the stubs, most of the useful frequency band lie above the frequency for which the stubs are a quarter wavelength long. For purposes of comparison, the standing wave ratio for a single stub is also shown in Fig. 2. If the stubs are approximately three quarters of a wavelength long instead of one quarter wavelength long, the fractional transmission band $\Delta$ is one third as great. These same curves give the standing wave ratios for this case if the frequency scale is expanded by this factor of three.

The above results may be generalized to apply to two stubs that are any odd quarter wavelength long separated by a length of transmission line that is also an odd quarter wavelength long not necessarily the same number of quarter wavelengths. To do this, we will write

$$\alpha_2=\frac{n\pi}{2}(\gamma+1)$$

where $n$ is an odd number equal to the length of the stub in quarter wavelengths at the frequency for which $\gamma$ is unity. The ratio of the length of transmission line between stubs to the length of the stubs is

$$\frac{l_1}{l_2}=\frac{m}{n}(\delta+1)$$

where $m$ is an odd integer approximately equal to the length of the transmission line between the stubs in quarter wavelengths. This gives

$$\alpha_1=\frac{m\pi}{n}(\gamma+1)(\delta+1)$$

Proceeding as before, the equation corresponding to Equation 4 is

$$\left(\gamma+\frac{m\delta}{n+2m(1+\delta)}\right)^2=\frac{4}{\pi^2 n(n+2m)(1+\delta)} \text{ times } \left(\epsilon+\frac{m^2 n\pi^2\delta^2}{4(n+2m)(1+\delta)}\right)$$

The edges of the band are now

$$\gamma_1=-\frac{m\delta}{n+2m(1+\delta)}(1+\sqrt{2})$$

$$\gamma_2=-\frac{m\delta}{n+2m(1+\delta)}(1+\sqrt{2})$$

As before, if we call the fractional band width $\gamma_2-\gamma_1=\Delta$ we obtain expression (7).

If, for mechanical or other reasons, it is desirable to increase either the length of the stubs or their spacing, the band width is decreased to that given by

$$\epsilon=\frac{\pi^2}{32}n^2\left(1+\frac{2m}{n}\right)\Delta^2 \qquad (7)$$

where $m$ and $n$ are odd numbers giving the numbers of quarter wavelengths in the length of line separating the stubs and the length of the stubs, respectively, and $\Delta$ is the fractional transmission band. This is possible by making the ratio of the length of line separating the stubs, to the length of the stubs

$$\frac{l_1}{l_2}=\frac{m}{n}+\frac{1+2\frac{m}{n}}{2\sqrt{2}}\cdot\Delta \text{ or } \frac{l_1}{l_2}=\frac{m}{n}-\frac{1+2\frac{m}{n}}{2\sqrt{2}}\cdot\Delta$$

and making the stubs $n$ quarter wavelengths long at a frequency equivalent to

$$\left(1+\frac{\Delta}{2\sqrt{2}}\right) \text{ or } \left(1-\frac{\Delta}{2\sqrt{2}}\right)$$

times the mid-frequency of the fractional transmission band $\Delta$. In these expressions, a (+) sign for the ratio requires a (−) sign for the stub length, and vice versa.

The invention may also be employed in a transmission line embodying two parallel conductors **31** and **32**, Fig. 3, for which the support comprises a series section of both these conductors, and the two shunt stubs **33**. The dimensions of this support may be obtained substantially in the manner hereinbefore described in connection with Figs. 1 and 2.

What is claimed is:

1. A line for transmitting signal waves extending over a certain range of frequencies including two spaced conductors, and means for supporting said conductors such that reflection effects tend to reduce as the width of said certain frequency range tends to increase, comprising two conductor stubs spaced from each other and short-circuited at one end, a section of said line connecting the opposite ends of said two stubs, each of said two stubs having a length equivalent to a preselected odd number of quarter wavelengths at a frequency determined by the product of a first predetermined fraction and the mid-frequency of said certain frequency range, and said section having a length equivalent to the product of a second predetermined fraction and the length of said stubs, said fractions being correlated with a predetermined fractional width of said certain frequency range, one of said fractions having a numerical value greater than unity and the other fraction a numerical value less than unity.

2. A transmission line for signal waves extending over a certain band of frequencies including a pair of conductors, and means for supporting said conductors in spaced relation for substantially reducing the deviation of the effective standing wave on said line from unity, comprising two stubs of said conductors arranged such that corresponding ends are joined to spaced points on said line and such that corresponding ends are short-circuited, and a series section of said line comprising the portion intervening between said points to which said two stubs are joined, said two stubs being one quarter wavelength long at a frequency equivalent to

$$\left(1+\frac{\Delta}{2\sqrt{2}}\right)$$

times the mid-frequency of said certain band, and said series section having a length equivalent to

$$\left(1-\frac{3\Delta}{2\sqrt{2}}\right)$$

times the length of said stubs, $\Delta$ being a fractional width of said certain band.

3. The transmission line according to claim 2 in which said stubs are one quarter wavelength long at a frequency equivalent to

$$\left(1-\frac{\Delta}{2\sqrt{2}}\right)$$

times the mid-frequency of said certain band, and said series section has a length equivalent to



$$\left(1+\frac{3\Delta}{2\sqrt{2}}\right)$$

times the length of said stubs.

4. A transmission line for signal waves extending over a certain band of frequencies including a pair of conductors, and means for supporting said conductors in spaced relation to reduce substantially the tendency of said supporting means to introduce reflection effects over said certain band, comprising two conductor stubs having their corresponding ends connected to two spaced points on said line and their opposite ends short-circuited and a series section of said line intervening between said two spaced points, the optimum ratio of the length $l_1$ of said series line section to the length $l_2$ of said two stubs being

$$\frac{l_1}{l_2}=\frac{m}{n}+\left(\frac{1+2\left(\frac{m}{n}\right)}{2\sqrt{2}}\right)\Delta$$

where ($m$) and ($n$) are integers of the number of quarter wavelengths of said stubs and said series line section respectively, $\Delta$ is a fractional width of said certain frequency band, and the ($n$) quarter wavelengths of said stubs are at a frequency which is

$$\left(1-\frac{\Delta}{2\sqrt{2}}\right)$$

times the mid-frequency of said certain frequency band.

5. The transmission line according to claim 4 in which the optimum ratio of the length $l_1$ of said series line section to the length $l_2$ of said two stubs is

$$\frac{l_1}{l_2}=\frac{m}{n}-\left(\frac{1+2\left(\frac{m}{n}\right)}{2\sqrt{2}}\right)\Delta$$

where ($m$) and ($n$) are integers of the number of quarter wavelengths of said stubs and said series line section respectively, $\Delta$ is a fractional width of said certain frequency band, and ($n$) quarter wavelengths of said stubs are at a frequency which is

$$\left(1+\frac{\Delta}{2\sqrt{2}}\right)$$

times the mid-frequency of said certain frequency band.

6. A transmission line according to claim 1 in which the numerical values of said first and second fractions are preselected to render the length of said stubs longer than the length of said section.

7. A transmission line according to claim 1 in which the numerical values of said first and second fractions are preselected to render the length of said stubs shorter than the length of said section.

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