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Oct. 19, 1948.

H. L. BARNEY
TRANSVERSAL FILTER

2,451,465

Filed Feb. 27, 1947

3 Sheets-Sheet 1

FIG. 1A

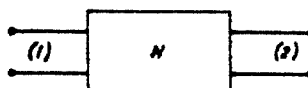


FIG. 1B

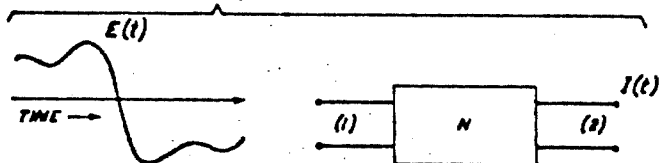


FIG. 1C

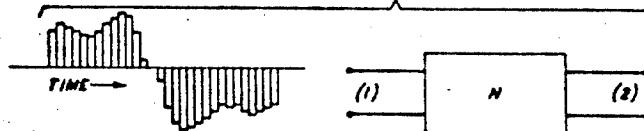


FIG. 1D

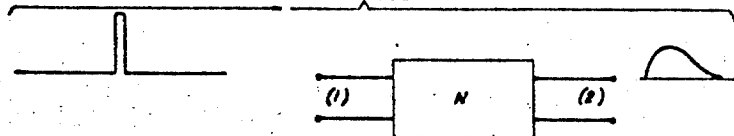
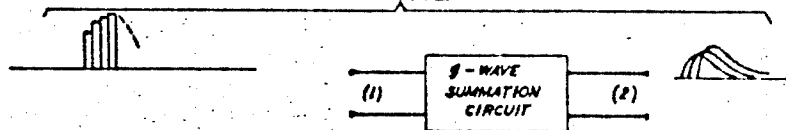


FIG. 1E



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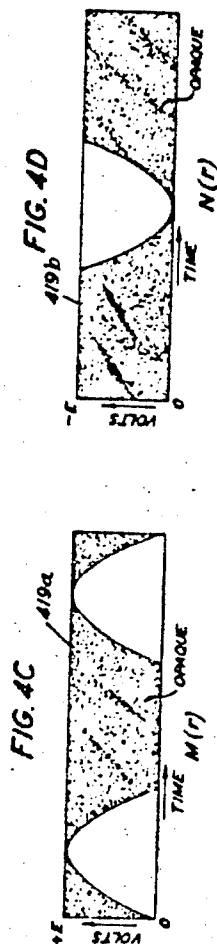
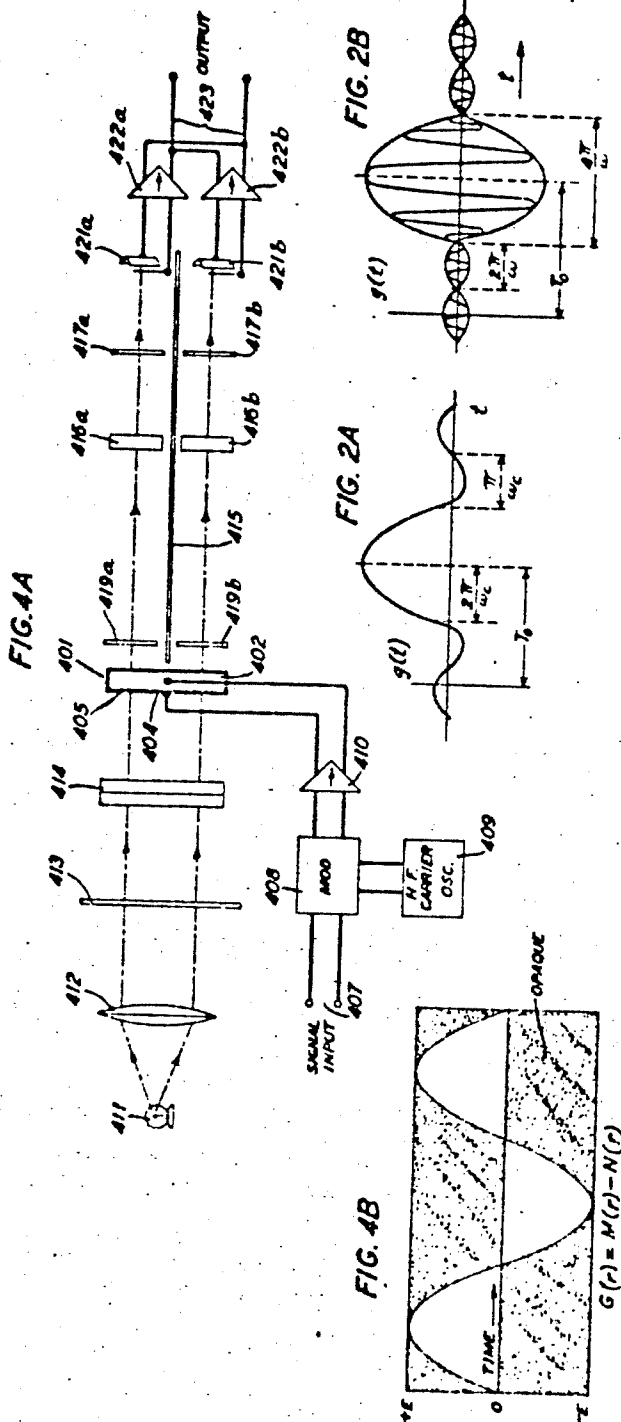
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3 Sheets-Sheet 2



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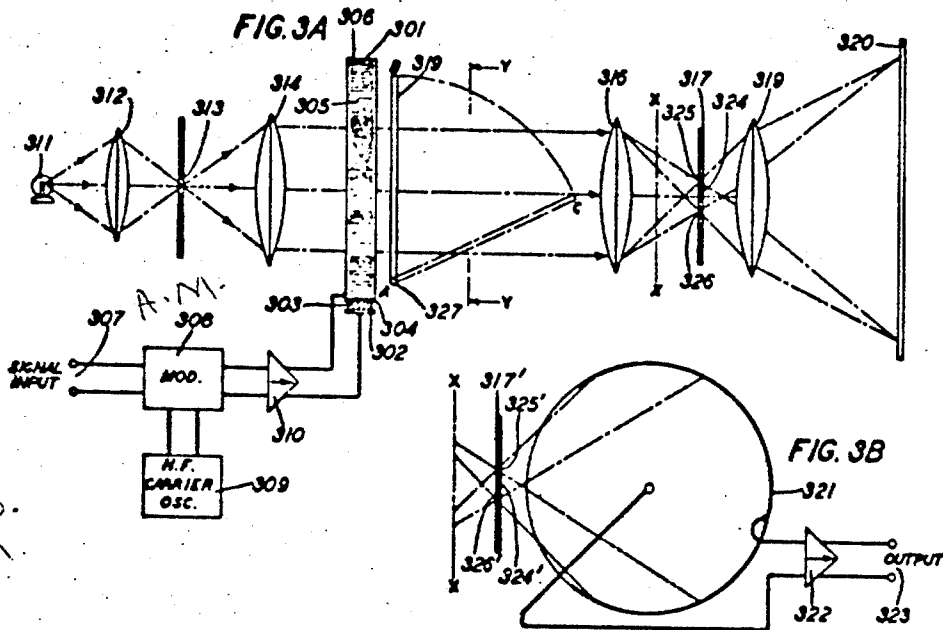
Oct. 19, 1948.

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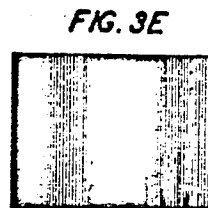
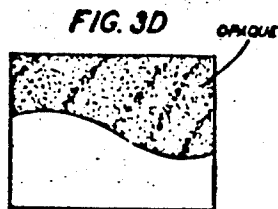
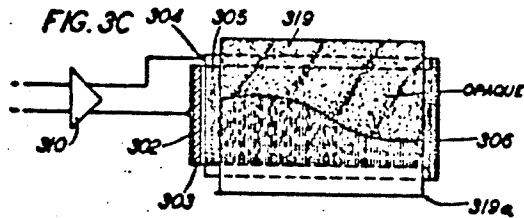
2,451,465

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3 Sheets-Sheet 3



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UNITED STATES PATENT OFFICE

2,451,465

TRANSVERSAL FILTER

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Application February 27, 1947, Serial No. 731,343

13 Claims. (Cl. 250-41.5)

1 This invention relates to the modification of time-varying functions in accordance with preselected patterns; more particularly, it relates to electrical transducers known in the art as transversal filters.

The behavior of electrical networks can be specified in two ways representing two different physical points of view. Ordinarily, one thinks first of the well-known steady state point of view which describes the network performance in terms of the concepts of amplitude and phase response versus frequency. In addition to this more conventional view point there is the time function concept in which the network is described in terms of its amplitude-time response at the receiving end resulting from the application of an impulse of infinitesimal duration at the sending end. Network response may thus be considered either in terms of frequency or time functions. The bridge between these two avenues of approach is the Fourier Integral which may be thought of as a mathematical device for expressing a time function in terms of steady state phenomena.

For the most part prior art practice has been to base the design of communication networks upon the steady state frequency-amplitude characteristics; and an elaborate theory has been worked out for such design procedures. The networks thus obtained contain as elements resistances, inductances and capacitances, the frequency and/or phase selective effects of which are used in various combinations to secure desired response characteristics.

On the other hand when network design is considered from time function point of view, that is, when time rather than frequency is taken as the independent variable, one is led to a broad group of selective circuits whose principle of operation does not depend upon resonant combinations of network elements. Thus, selective circuits embodying the time function concept have been disclosed in Patents 2,024,900, December 17, 1935; 2,124,599, July 26, 1938; and 2,128,257, August 30, 1938, to N. Wiener and Y. Lee, and elsewhere in the art.

A principal object of this invention is to provide certain improvements and simplifications in the art of modifying functions of time in accordance with preselected patterns of amplitude, frequency, and phase variation.

A more specific object of this invention is to provide simplified technique and apparatus for modifying an impressed electrical input in accordance with a preselected admittance function

2 which may be varied at will without substantial changes in the applied techniques or apparatus.

A certain class of devices, known in the art as transversal filters, substitute the time-function approach for the conventional steady-state approach in the simulation of network response, operating through a series of steps which include the following:

- (1) Transitorily recording or storing the input signal;
- (2) Weighting the stored record in accordance with a predetermined multiplying function; and
- (3) Integrating the weighted increments of record to produce a modified output.

15 In accordance with a preferred embodiment of the present invention, a supersonic light valve, such as disclosed in Patent 2,287,587 to G. W. Willard and elsewhere in the art, is utilized in a transversal filter as an element with functions to store progressive portions of an impressed input signal in the form of modulated supersonic waves passing longitudinally through the cell. A light beam directed through the cell transversely to the direction of travel of the modulated supersonic waves is diffracted in accordance with the modulations. Weighting of the stored record in accordance with a predetermined multiplying function is provided by a mask interposed in the path of the modulated light beam emerging from the cell, the interposed mask having a light transmission characteristic which varies in a plane normal to the direction of travel of the beam. The light emergent therefrom is continuously collected and integrated by means of a photoelectric cell, in the output of which is produced the desired modified response.

A modified form of the present invention is adapted to utilize multiplying functions having both positive and negative components. In accordance with the modified embodiment, the modulated light output of the supersonic cell is divided, half passing through a mask representing the positive component of a multiplying function, and the other half passing through a mask representing the negative component of the multiplying function. The light from each of the two masks is collected separately on two different photoelectric cells, whose respective outputs are combined in reversed phase to produce the desired modified output response.

Transversal filters designed in accordance with the present invention have certain advantages over devices of the prior art.

In accordance with one feature of the invention, summation of increments representing the

product of the signal amplitude and the characteristic time function is continuously performed by an optical system utilizing the progressive signal storage of the supersonic cell, thus eliminating the necessity for multiple pick-up points.

In accordance with another feature of the invention, the over-all loss-frequency characteristic of the system may be changed by merely varying the position of the impulse response mask thereby permitting controllable cut-off frequencies of low and high-pass filters, and proportional variations in the band widths and mid-band frequencies of band-pass filters. This feature provides a new means for constructing tracking filters which supplements the techniques now known in the art.

A further advantage inherent in systems of the present invention is their physical simplicity and lack of bulk as compared to transversal filter systems of the types disclosed heretofore in the art.

Other objects, features, and advantages of the present invention will be apparent from a study of the detailed description hereinafter and the attached drawings in which:

Figs. 1(A)-1(E) are a series of diagrams illustrating a discussion of the theory of the invention;

Figs. 2(A) and 2(B) are graphical representations of specific filter characteristics which may be simulated in accordance with the theory of the present invention;

Fig. 3(A) shows a plan view of a preferred embodiment of a transversal filter utilizing a supersonic light cell in accordance with the present invention, which disclosed for the purposes of illustrating the theory an assemblage including the screen 320 shown as connected across the line X-X' to receive the light output of the system, in place of the assemblage shown in Fig. 3(B) which includes the photoelectric collecting device 321;

Fig. 3(C) shows a cross-sectional view of the system of Fig. 3(A) in a plane represented by the line Y-Y' of Fig. 3(A);

Figs. 3(D) and 3(E) show alternative forms of the impulse response mask 318 of Fig. 3(A);

Fig. 4(A) shows a modified form of a transversal filter in accordance with the present invention which is adapted to utilize impulse response functions having both positive and negative components;

Fig. 4(B) is a graphical illustration of an impulse response function having positive and negative components;

Figs. 4(C) and 4(D) are respective illustrations of the positive and negative impulse response masks representing the function shown in Fig. 4(B).

Reference will be made hereinafter to the transfer indicial admittance of a system. This quantity is defined by J. R. Carson in *Electric Circuit Theory and the Operational Calculus*, McGraw-Hill, 1926, page 14, as the ratio of the output current of the system, expressed as a time function, to the magnitude of the steady electromotive force suddenly inserted at the input of the system at time $t=0$.

The time rate-of-change of the transfer indicial admittance defined above is a function of time designated $g(t)$. The function $g(t)$ is variously referred to in the specification and claims hereinafter as the "impulse response" or merely the " g -function" of a system.

Further discussion and definition of certain mathematical concepts, such as that of the "unit

impulse," which will be relied on in the detailed description hereinafter will be found in volume I of *Transients in Linear Systems*, by Gardner and Barnes, John Wiley and Sons, 1942, pages 255-263.

The broad principles upon which the time function point of view are based are illustrated in Figs. 1(A) to 1(E), to which reference is now made. Consider a frequency selective network such as is illustrated schematically by N in Fig. 1(A). Let us assume that the complex voltage wave $E(t)$ shown in Fig. 1(B) which is any continuous function of voltage versus time is impressed upon the input (1) of the network. At the output (2) there will then appear a current wave which we designate as $I(t)$. Now let it be supposed that the voltage wave $E(t)$ is split up into a series of narrow pulses as shown in Fig. 1(C). With this pulsed wave impressed upon the input terminals (1) one should expect to obtain at the output terminals (2) very nearly the same current wave $I(t)$ obtained before.

Now referring to Fig. 1(D) assume that there is impressed upon the network a single pulse of the sort into which the voltage wave $E(t)$ has been subdivided. At the network output terminals (2) there now appears a function which as the pulse-width approaches zero is proportional to the g -function of the network as defined above. It should be noted that any other pulse of different amplitude would result in the same approximate g -function except that its amplitude would vary in proportion to the applied pulse amplitude and that moreover its time of occurrence would depend on the time of pulse application. Thus it follows as shown in Fig. 1(E) that in the limit the current wave $I(t)$ which appears at the output terminals (2) as a result of the application of the voltage wave $E(t)$ at the input terminals (1) is the sum of a number of overlapping g -functions whose relative strengths or amplitudes vary in accordance with the impressed voltage wave $E(t)$. Using a somewhat more precise language one can say that if the network is subjected at the input (1) to an initial pulse at some arbitrary time which for convenience may be called zero, and if this pulse is followed by others at specified values of time, the total response at the output terminals (2) at any later time will be the sum of the responses which have occurred up to that time.

Thus two important principles applicable to this approach to network theory may be derived from the above. First, the network response to a unit impulse of infinitesimal duration completely determines the response to any other input wave. Second, the response at any time depends upon the history of the applied input wave previous to the time in question so that the past history must be available at least over a time interval within which the g -function is of appreciable magnitude.

Therefore, the network can be looked upon as a circuit for effecting the summation of a series of time displaced g -functions in which the individual amplitude of each of the respective g -functions is proportional to the corresponding time-displaced instantaneous value of the impressed voltage wave $E(t)$. This process is schematically indicated in Fig. 1(E).

Adopting a slightly different point of view one can also look upon the output wave as representing at any time a weighted history or record of the input wave where the g -function has acted as the weighting factor.

The foregoing statements may be summarized by deriving a mathematical expression for the

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network response to an arbitrary driving force from the assumption that the net behavior of a linear system at any instant is a function of the linear superposition of all the responses which have occurred up to that time counting from some arbitrary starting point. Assume, for example, that a network is subjected to an initial voltage pulse $E(t)$ at the time $t=0$ and that this pulse is followed by others at specified values of time, then the total response at any later time will be the sum of the responses which have occurred up to that time, due allowance being made for the time at which each pulse was applied.

Let the time axis then be divided into short intervals $\Delta\tau$ of equal width, the electromotive force $E(t)$ being approximated by a series of rectangular pulses applied for the duration of each time interval $\Delta\tau$. The total response at a specified time t is then approximately the sum at that instant of all the elementary responses started previous to that instant. If the interval $\Delta\tau$ is very small, that is, approaches zero as a limit the response at time t to the first pulse is $\Delta\tau E(0)g(t)$, where $g(t)$ is the response to unit pulse or g -function as defined hereinbefore, and where $E(0)$ is the amplitude of the voltage wave $E(t)$ at time $t=0$. Consider now the $(n+1)$ th pulse. The response at the time t is $E(n\Delta\tau) \Delta\tau g(t-n\Delta\tau)$. In this expression it should be noted that $n\Delta\tau$ is the time of pulse application. The reason that the argument of the g -function in the latter expression is $t-n\Delta\tau$ and not t is that this pulse does not come into existence until the time $n\Delta\tau$ and the expression is only valid for time equal to or greater than $n\Delta\tau$. Assume an arbitrary point on the time scale to be denoted by τ , thus $\tau=n\Delta\tau$. The current response $I(t)$ at the instant t is the sum at time t of all elementary responses which have occurred between time equal to 0 when the first pulse was started and time t as the length of time interval $\Delta\tau$ approaches zero. Hence

$$I(t) = \lim_{\Delta\tau \rightarrow 0} \sum_{r=0}^{t/\Delta\tau} E(t) \Delta\tau g(t-r) \quad (1)$$

By definition of an integral this may also be written

$$I(t) = \int_0^t E(\tau) g(t-\tau) d\tau \quad (2)$$

or

$$I(t) = \int_0^t E(t-\tau) g(\tau) d\tau \quad (3')$$

This equation thus expresses the system response to an arbitrary driving force in terms of the response to a unit impulse of infinitesimal duration. Thus it follows that a knowledge of the response $g(t)$ to a unit impulse, that is, an applied pulse which in the limit approaches unit area and infinitesimal duration is sufficient to specify completely the system performance. This implies in particular that the steady state performance of a particular network may also be determined from a knowledge of $g(t)$ for that network. Suppose for example, that the network is a filter passing a certain band of frequencies. This is then merely a reflection of the fact that $g(t)$ behaves in a very definite manner. To illustrate this in a general way, assume that a sinusoidal voltage $E(t) = E \sin \omega t$, where E represents the steady-state amplitude and ω the angular frequency, has been applied to the network at $t=0$ and that all transients have died out. The steady state current can then be written as

$$I(t) = E |Y_{11}(j\omega)| \sin[\omega t - \varphi(\omega)] = E a_{11}(\omega) \sin \omega t + E b_{11} \cos \omega t \quad (3)$$

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where $Y_{11}(j\omega)$ is the transfer admittance between input and output terminals of the selected network and $\varphi(\omega)$ its phase angle. By writing $Y_{11}(j\omega) = a_{11}(j\omega) + j b_{11}(j\omega)$ in which a_{11} and b_{11} are constants, the second of expressions (3) is obtained. An equivalent expression may also be obtained from (2) or (3'), for by inserting $E(t) = E \sin \omega t$ and extending the range of integration to infinity (which means that transient distortion has died out) one obtains,

$$I(t) = E \sin \omega t \int_0^\infty \cos \omega g(\tau) d\tau - E \cos \omega t \int_0^\infty \sin \omega g(\tau) d\tau \quad (4)$$

By comparison of (3) and (4) it follows that

$$\left. \begin{aligned} a_{11}(\omega) &= \int_0^\infty \cos \omega g(\tau) d\tau \\ b_{11}(\omega) &= - \int_0^\infty \sin \omega g(\tau) d\tau \end{aligned} \right\} \quad (5)$$

The points to be emphasized in general are, first, that the expressions (5) establish quantitative relations between the frequency selection properties of the network and the response to unit impulse excitation, and second, that both the real and imaginary components of the transfer admittance can be calculated from a knowledge of the response to unit impulse. This relationship may also be further illustrated as follows: Multiply the second of equations (5) with j , the imaginary unit, and add to the first; then use Euler's formula. The result is

$$Y_{11}(j\omega) = \int_0^\infty g(\tau) e^{-j\omega\tau} d\tau \quad (6)$$

from which it follows that the steady state transfer admittance is the Fourier transform of the unit impulse response. From a principal point of view, it is thus irrelevant whether frequency selective properties of a network are stated in terms of steady state frequency response to sinusoidal driving forces or whether they are given as the time response to a unit impulse. The frequency response is merely the spectral analysis of the time response to a unit impulse. It also follows from (6) that

$$g(t) = \frac{1}{2\pi} \int_0^\infty |Y_{11}(j\omega)| \cos(\omega t + \theta(\omega)) d\omega \quad (7)$$

where $Y_{11}(j\omega)$ denotes the amplitude and $\theta(\omega)$ the phase of the steady state transfer admittance. Equation (7) in principle allows $g(t)$ to be calculated from a knowledge of the frequency spectrum of the steady state transfer admittance, i. e. from the amplitude frequency characteristic of the network. Moreover we have from (5)

$$\left. \begin{aligned} g(t) &= \frac{2}{\pi} \int_0^\infty a_{11}(\omega) \cos \omega t d\omega \\ g(t) &= \frac{2}{\pi} \int_0^\infty b_{11}(\omega) \sin \omega t d\omega \end{aligned} \right\} \quad (8)$$

Hence it follows that the time response $g(t)$ to unit impulse is completely determined when either the real or the imaginary component of the steady state transfer impedance is specified over the entire frequency range.

Stressing the physical interpretation of the facts presented rather than the mathematical analysis, three steps are suggested by means of which a given input function may be modified in accordance with certain admittance characteristics to produce a desired output response without resort to the conventional concepts of selective networks. They are:

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1. Recording or storage of the input wave.
2. Weighting of the stored record by means of selected g -functions; and
3. Summation of the weighted record.

These are fundamental steps which can serve as building blocks in frequency selective devices operating on a time function basis. It should be noted that in arriving at these steps no reference has been made to vibrating systems such as coil and condenser combinations nor has any use been made of the concepts of amplitude and phase versus frequency response. These concepts have now been replaced by the single concept of the g -function. In other words the physical phenomena conventionally described by the amplitude and phase versus frequency functions are now described by the single function $g(t)$.

As concrete examples of g -functions consider two cases of functions in which the positive and negative values are symmetrical with respect to a certain value of time, say time T_0 , where $T_0 > 0$. Consider first a low pass filter having a uniform transfer impedance equal to K from frequency zero to a cut-off frequency angular ω_c . Outside this range it is assumed that no transmission occurs. As a consequence of the stipulation of even time response the phase shift $\theta(\omega)$ is linear and is given by the following equation:

$$\theta(\omega) = -\omega T_0 + \pi/2 \quad (9)$$

For the low pass filter under consideration there is obtained a particular g -function which will be designated $g_1(t)$ by substituting the above in Equation 7 which relates to the generalized function $g(t)$ and integrating the expression over the chosen range of angular frequencies from zero to ω_c .

$$g_1(t) = \frac{1}{\pi} \int_0^{\omega_c} \cos(\omega t - \omega T_0) K d\omega$$

or

$$g_1(t) = \frac{\omega_c K}{\pi} \left[\frac{\sin \omega_c(t - T_0)}{\omega_c(t - T_0)} \right] \quad (10)$$

This function is shown as Fig. 2(A). It may be demonstrated that the width of the main oscillatory lobe is inversely proportional to the band width f_c . It is also seen that the received signal reaches its maximum at the time $t = T_0$ and that the maximum response is proportional to the area $\omega_c K$ under the amplitude characteristic.

As a second example we consider an idealized band-pass filter of even time response and with a flat amplitude characteristic between the cut-off frequencies ω_1 and ω_2 ($\omega_2 > \omega_1$). Outside this frequency range it is assumed that no transmission can take place. From Equation 7 substituting the conditions imposed by Equation 9, and integrating over the angular frequency range from ω_1 to ω_2 , there is obtained a function which will be designated $g_2(t)$, which represents the particular case of the generalized g -function $g(t)$ as applied to an idealized band-pass filter having the above characteristics.

$$g_2(t) = \frac{K}{\pi} \int_{\omega_1}^{\omega_2} \cos(\omega t - \omega T_0) d\omega$$

which may be reduced to

$$g_2(t) = \frac{\omega K}{\pi} \frac{\sin \frac{\omega(t - T_0)}{2}}{\frac{\omega(t - T_0)}{2}} \cos \omega_m(t - T_0) \quad (11)$$

Here ω represents the band width $\omega_2 - \omega_1$, and ω_m the arithmetic mean of the two cut-off frequencies ω_1 and ω_2 , and may thus be considered to

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coincide with the mid-band frequency. Equation 11 which is roughly plotted on Fig. 2(B) represents an amplitude modulated carrier wave with a carrier frequency equal to that of mid-band. The maximum response occurs at $t = T_0$ and is proportional to ωK which is the area under the amplitude response characteristic and the length of the main oscillatory lobe is

$$\frac{4\pi}{\omega}$$

which is inversely proportional to the band width.

The examples selected show that several important properties of the steady state characteristics can be obtained directly from an inspection of the plots of the g -functions. It must be emphasized, however, that too much significance cannot be attached to the calculated g -functions since they are based upon assumptions which cannot be realized. On the other hand, the general qualitative and quantitative properties of the g -functions for the filters in question are believed to have been preserved, although one is not justified in attaching very much significance to any of the finer details.

The computation of the g -function, as discussed hereinbefore, has necessarily been in broad general terms with several specific applications by way of illustration. From the previous discussion, the procedure will be apparent to those skilled in the art for uniquely computing g -functions to comply with specific sets of conditions imposed in other particular cases.

In accordance with an embodiment of the invention shown in Figs. 3(A), 3(B), and 3(C) of the drawings, portions of the impressed signal are progressively stored in the form of modulated supersonic waves passing longitudinally through a supersonic light cell. The impressed function is continuously multiplied by a predetermined weighting function through the agency of a light beam passing transversely through the supersonic cell and an impulse response mask positioned adjacent thereto. The increments of emerging light, which are proportioned to the weighted product, are focused on the input of a photoelectric cell where they are progressively integrated to produce a desired filtered output current.

The system shown in Fig. 3(A) comprises in combination a supersonic light cell 301 of a type such as disclosed, for example, in Patent 2,287,587 to G. W. Willard, June 23, 1942. For present purposes, the cell 301 is constructed substantially as shown in Figs. 1 and 2 of the above-cited patent, but with the separate outer electrode elements 24 and 25 as shown in those figures replaced by a single outer electrode 302 as shown in Fig. 3(A) of the present drawings. The electrode 302 is attached to the outer face of the piezoelectric crystal element 303 which comprises a section of X-cut quartz disposed at one end of the cell 301. The electrode 304 is attached to the inner face of the crystal element 303.

The input system connected across the electrodes 302 and 304 includes the signal input terminals 307 adapted for connection to the source of the signal to be filtered. The terminals 307 feed the impressed input signal into the modulating circuit 308 where it is combined with the output of the carrier frequency oscillator 309. The double side-band amplitude modulated carrier output from the modulator 308 is then amplified in the circuit 310 and impressed across the electrodes 302 and 304 to drive the piezoelectric element 303 in compressional vibration.

The compressional vibrations generated by the piezoelectric element 303 set up waves in the liquid medium 305, such as water, which fills the cell 301, the resulting condensations and rarefactions progressing longitudinally therethrough to the far end of the cell, where they are absorbed by several layers of wire mesh 306 and other devices designed to prevent reflection, such as more fully described in the patent to Willard supra.

Positioned along a line approximately perpendicular to the center point of one of the sides of the cell 301 and at a short space therefrom, is a light source 311 which is focussed on the vertical slit 312 by means of the lens 312. The emergent face of the slit 312 is positioned at the line of focus of the cylindrical lens 314, whereby a uniform light beam is produced having the shape of a parallelepiped which has a depth perpendicular to the plane of Fig. 3(A) which is substantially coextensive with the depth of the cell 301, and enough breadth in a horizontal direction to include a large portion of the cell 301 in its path. The beam is directed to pass through the cell 301 transversely to the direction of travel of the compressional waves from the driving element 303.

The beam emerging from the opposite side of the cell 301 falls on a second cylindrical lens 316, so positioned that the central undiffracted portion of the beam is focussed on the vertical bar 324 which is formed in the central portion of the baffle 317, which is disposed in a plane perpendicular to the path of the beam. Adjacent to the bar 324, on each side, are disposed vertical slits 325 and 326 perpendicular to the plane of Fig. 3(A), which are coextensive, and so positioned that only diffracted portions of the beam pass therethrough. For the purposes of illustration, the light output of the slits 325 and 326 is shown as passing through the lens 318 which serves to focus the diffracted light images on the screen 320.

In the absence of the impulse response mask 318, which will be fully discussed hereinafter, the system operates as follows. Assume, for example, that a carrier frequency of 10 megacycles modulated by a frequency of 100 kilocycles is impressed across the electrodes 302 and 304. Bands of high amplitude modulated waves alternating at space intervals of about one-half inch with bands of low amplitude modulated waves are produced in the medium 305, traveling longitudinally down the cell 301 at a speed of about 4000 feet per second. The light rays passing through the cell from the source 311 are diffracted as though passing through a diffraction grating having a line spacing in accordance with the periodic rarefactions and compressions of the carrier wave. However, whereas an ordinary diffraction grating allocates a fixed amount of light to each order of diffracted beam including the zero or undiffracted order, the supersonic wave grating gives an amount of light in the different order beams which varies with the changes in amplitude of the waves. Thus, when the waves are zero amplitude, all of the light falls in the principal or undiffracted beam; whereas, as the amplitude of the waves is increased, the amount of light in the principal or undiffracted beam is decreased, and that in the higher orders proportionately increased. Thus, as the rays are diffracted at any given spot in the cell, a smaller proportion of the light therefrom is focussed in the undiffracted beam on the bar 324, and proportionately more light is focussed in the dif-

fracted beams directed on the slits 325 and 326, the light passing therethrough being therefor a function of the amplitude modulation of the supersonic waves passing through the cell 301.

The images cast on the screen 320 thus take the form of alternate light and dark bands traveling across its surface at a high rate of speed in the direction of travel of the supersonic waves through the cell 301. The carrier wave, in effect, provides a bias on the modulating signal, such that with no signal modulation, the amount of light transmitted is a median value. The maxima and minima in the envelope of the signal modulated carrier wave then correspond to less and more light, respectively, than the amount transmitted when there is no signal modulation.

Assume now, that the supersonic light cell 301 is to be used to simulate a given network response or admittance characteristic, as taught by the present invention. To adapt the system described hereinbefore for this purpose, an impulse response mask 318 is interposed in the path of the diffracted beam emerging from the cell 301. The mask 318 takes the form of a rectangular plate having a light transmission characteristic which varies in accordance with a chosen multiplying function in a direction transverse to the direction of travel of the light beam. Fig. 3(C) which represents a cross-sectional view through the line Y-Y in the plan view of Fig. 3(A), shows a preferred position of the mask 318 adjacent to and substantially parallel to the light-emerging side of the cell 301, the plate 318 being preferably coextensive with the cross-sectional dimension of the diffracted light beam in the direction of travel of the supersonic waves through the cell 301.

The impulse response mask 318 may alternatively assume different forms, two of which are shown in Figs. 3(D) and 3(E). A variable area mask, such as shown in Fig. 3(D), can be simply made by merely plotting a large-scale graph of the weighting function, blacking in the area above the curve so that the transmitting portion of the mask represents the weighting function, and reducing the picture to the desired size by a photographic process. A variable density mask of the type shown in Fig. 3(E) can be made by focussing a vertical ribbon-shaped light beam which is intensity varied in accordance with a desired weighting function to shine on a light sensitive film which is moved transversely across the path of the beam at a uniform rate, and then making a positive reproduction of the negative film so that the light ultimately passed by the variable density mask will be proportioned to the desired function, rather than its inverse.

Preferably, the weighting function utilized in the preparation of the mask 318 is the chosen g -function which is calculated as discussed hereinbefore in accordance with a desired output function and which represents the time rate-of-change of the indicial transfer admittance to be simulated. Assume the calculated curve in this case to be a function of time, $g_1(t)$, which will be translated into a function of space $g_1(x)$. As will be apparent from the discussion hereinafter, it may be desirable to introduce an additional exponential factor, $e^{-\alpha x}$, in combination with the calculated curve for the impulse response mask, in order to compensate for attenuation sustained by the modulated supersonic waves as they move down the cell 301. Therefore, the impulse response mask will be prepared in accordance with a function which takes the form $g_1(x)e^{-\alpha x}$, where $g_1(x)$ is the g -function discussed above, α is the

distance from some reference point measured in the direction of travel of the supersonic wave along the cell 301, and a is an attenuation constant.

Assume then, that the impulse response mask 319 is placed in the system in the path of the light beam, as described, and that an infinitesimally short pulse is sent down the cell on the carrier, modulated so as to cause the carrier to approach zero amplitude momentarily. Neglecting the effect of attenuation, the total light passing through the slits 325 and 326 and falling on the screen 320 will trace out a pattern of intensity variations with time which is a replica of the space variations in the light transmission characteristic of the impulse response mask 319. Now if a complex signal is sent down the cell, it may be considered as divided into an infinite number of very short pulses in succession, each one of which will trace out the impulse response function. The total output of light from the slits 325 and 326 will accordingly be the summation of all of the increments of signal multiplied by the impulse response function.

This operation may be expressed mathematically as follows. Assume that a signal $E(t)$ is impressed across the input terminals, and that the signal $E(t)$ travels down the cell 301 without attenuation, and is completely absorbed in the characteristic impedance 306 at the far end of the cell, so that no reflections are produced. At any distance x , which is measured in a horizontal direction from the edge of the beam nearest the input, the instantaneous amplitude of a wave passing down the cell is $E(t-x/u)$, where u is the speed of wave propagation. Assume that the scale of the impulse response mask has been adjusted by a constant factor $1/u$ to make it correspond to the velocity of travel of the compressional waves down the cell. Then the g -function value measured at any point x in a horizontal direction along the mask 319 equals $g_1(x/u)$. Thus, the light passing through an infinitesimal vertical segment of the cell 301 having a width dx and through a corresponding portion of the mask 319, is proportioned to the quantity

$$E(t-x/u)g_1(x/u)dx$$

Replacing x/u by τ , the delay, and letting $t=0$ when $\tau=0$, it is apparent that the total light, L , falling on the screen 320 from the cell 301 and interposed mask 319 is proportioned to the following quantity.

$$L \int_{-\infty}^0 E(t-\tau)g_1(\tau)d\tau \quad (12)$$

The above is seen to be the equivalent of the response integral derived in the early part of the specification as Equation 2'. Hence, the action of a network is simulated without conventional network elements.

Equation 12 above, specifies the operation of the system under ideal conditions which are not actually attained in practice. In order to more closely approximate actual operating conditions, it must be assumed that the supersonic waves traveling down the cell 301 undergo a constant decrease in amplitude because of attenuation, so that the actual amplitude of the waves at any point x in the horizontal direction of travel along the cell 301 is represented by the quantity

$$E(t-\tau)e^{-a\tau}$$

where a is the attenuation coefficient of the medium. In order to compensate for the factor

$e^{-a\tau}$ so that the product as given in Equation 12 is truly representative of the response integral, an additional factor, $e^{+a\tau}$ must be introduced into the calculations for constructing response mask 319, so that the actual mask function plotted is $g_1(\tau)e^{+a\tau}$, as pointed out hereinbefore.

In the foregoing description for the purposes of illustration, it has been assumed that the light output of the system of Fig. 3(A) traces out patterns on the screen 320. In accordance with the present invention, the photoelectric collecting system shown in Fig. 3(B) is substituted to the right of the line X—X for the screen 320 and associated elements shown in Fig. 3(A). The diffracted light beams passing through the slits 324' and 325' falls on the light sensitive surface of the conventional photoelectric cell 321, producing an electrical output current therefrom which is proportional to the response integral described above. The output current from the cell 321, which is continuously modified in accordance with a preselected admittance characteristic as described, passes through the amplifying circuit 322 and is impressed on the output terminals 323.

As pointed out in the introductory portion of the specification, a particular novel feature of systems, in accordance with the present invention is that their over-all frequency characteristics may be altered in each case by simply rotating the impulse response mask about an axis in such a way as to change the proportion of light from the supersonic cell which is intercepted by the mask. Assume, for example, that the impulse response mask 319 is rotated about an axis 327 perpendicular to the plane of the paper, and aligned with the bounding edge of the mask 319 nearest the input end of the cell 301. If the mask 319 is rotated from the position AB to the position AC, only half of the light beam emerging from the cell 301 is intercepted by the mask. Therefore, the impulse response function occupies only half as long a time period as were the entire beam intercepted by the mask, thereby doubling the frequency scale of the over-all loss characteristic of the system. For example, if the system were set up to simulate the operation of a low-pass filter, the cut-off frequency could be shifted over a wide range of values by simply rotating the mask at will. Similarly, in a system for simulating the operation of a band-pass filter, the band-width could be made to change in the same ratio as the mid-band frequency by a similar operation.

Another problem which arises in a system of the type described is the necessity for taking into account impulse response functions having both positive and negative components. A system modified to perform this function is shown in Fig. 4(A) of the drawings.

Fig. 4(A) shows a side elevation view of a system in accordance with the present invention which is in general similar in structure and operation to the system shown in the plan view of Figs. 3(A) and 3(B), like elements being correspondingly numbered in the two systems. However, the system of Fig. 4(A) differs from the system of Figs. 3(A) and 3(B) in that the light output from the cell 401 is separated horizontally by the opaque barrier 410 into two equal components, each of which passes through a separate system including a respectively different response mask, lens system, and receiving photocell, the separate photocell output currents being combined in reverse phase.

Assume that the impulse response function

comprises both positive and negative components, and takes the form

$$G(\tau) = M(\tau) - N(\tau) \quad (13)$$

In accordance with the presently described systems, the positive and negative components of the curve $G(\tau)$ are recorded separately on two different masks, $M(\tau)$, the positive component, on mask 418a, as shown in Fig. 4(C), and $N(\tau)$, the negative component, on mask 418b, as shown in Fig. 4(D).

The portion of the diffracted beam emerging from the upper portion of the supersonic cell 401 above the barrier 416 passes through the mask 418a, and is thereby multiplied by the function $M(\tau)$. The weighted increments of light from the mask 418a are then focussed by the cylindrical lens 416a on the slits in the baffle 417a, which are similar in form, the slits 325 and 326 described with reference to Fig. 3(A), passing therethrough onto the light sensitive surface of the photoelectric cell 421a, and thence into the output through the amplifier 422a. Light passing through the mask 418b is weighted in accordance with the negative response function $N(\tau)$, and arrives by a similar series of steps at the output of the amplifier 422b. The photoelectric cells 421a and 421b and their respective amplifying circuits are connected in what is known as "push-pull" relation, so that their respective output currents are combined 180 degrees out of phase.

The total output current may be expressed as follows.

$$I(t) = K \int_0^t E(t-\tau) M(\tau) d\tau - K \int_0^t E(t-\tau) N(\tau) d\tau \\ = K \int_0^t E(t-\tau) [M(\tau) - N(\tau)] d\tau \quad (14)$$

Substituting Equation 13 above, the expression becomes

$$I(t) = K \int_0^t E(t-\tau) G(\tau) d\tau \quad (15)$$

which is seen to be the response integral, as derived hereinbefore in Equation 2'.

It will be apparent, that for best performance of the system of Fig. 4(A) the same attenuation correction should be applied to $G(\tau)$ of Equation 15 as was discussed with reference to Equation 12 above.

The supersonic light cell, which is utilized as an element of the combinations disclosed in both Figs. 3(A) and 3(B) and Fig. 4, may take many other forms than the particular one shown. For example, the lateral slits 325 and 326 which are disclosed in Fig. 3(A) as separated by the bar 325, may be replaced by a single vertical slit, in which case the output of the photocell 321 is merely reversed in phase.

Many modifications of the apparatus comprising the illustrative embodiments disclosed herein which are within the scope of the present invention will be apparent to those skilled in the art.

What is claimed is:

1. The method which comprises translating applied waves into elastic waves progressive across a predetermined space interval, directing electromagnetic energy through said space interval in a direction transverse to the direction of progression of said elastic waves, receiving and integrating components of said electromagnetic energy that pass through said progressive elastic waves, and weighting various of said energy components that pass through respectively different parts of

said space interval in accordance with a predetermined weighting function.

2. The method which comprises generating compressional waves in a medium, said waves being modulated in accordance with an impressed function, directing a beam of electromagnetic radiant energy to pass through said medium transversely to the direction of propagation of said compressional waves therethrough, whereby components of said beam are differently diffracted, varying the intensity of the respective components of said beam in accordance with a predetermined weighting function, and collecting and summing said components thereby to produce a modified output function.

3. The method of modifying a given input function which comprises storing portions of said function as supersonic waves in a supersonic light cell, producing variations in the light output of said cell in accordance with a predetermined weighting function, and summing increments of said light output to produce a modified output function.

4. The method of modifying a given input function in accordance with a predetermined admittance characteristic which comprises storing portions of said function as supersonic waves in a supersonic light cell, producing space variations in the intensity of said light output of said cell in accordance with a function of said admittance characteristic and summing increments of said light output to produce a modified output function.

5. The method which comprises storing portions of an impressed function as supersonic waves in a supersonic light cell, producing variations in the light output of said cell in accordance with a predetermined weighting function, and directing components of said light output to fall on the light sensitive surface of a photoelectric cell, whereby to produce a modified output function in the electrical output of said cell.

6. The method of modifying a given input function in accordance with a predetermined admittance characteristic which comprises storing portions of said function as supersonic waves in a supersonic cell, interposing a mask in the path of the light output of said cell whereby said light output is space varied in accordance with a function of said admittance characteristic, and directing components of said light output to fall on the light sensitive surface of a photoelectric cell, whereby the desired modified function is produced in the electrical output of said photoelectric cell.

7. The method in accordance with claim 6 in which said predetermined admittance characteristic is varied by variations in the position of said mask with respect to the light output of said cell.

8. A system which comprises in combination means for propagating elastic waves across a given space interval, a source of electromagnetic energy, means for directing said electromagnetic energy through said space interval in a direction transverse to the direction of propagation of said elastic waves, means for receiving and integrating components of said electromagnetic energy that pass through said elastic waves, and means for weighting various of said energy components that pass through respectively different parts of said space interval in accordance with a predetermined weighting function.

9. A system adapted to modify an applied input function in accordance with a predetermined

admittance characteristic which comprises in combination means for propagating waves across a given space interval, a source of a beam of radiant energy, means for modifying components of said beam in accordance with certain variations in said waves, means for receiving and integrating various components of said beam that pass through said space interval, and means for weighting respective ones of said components that pass through different parts of said space interval which comprises a mask interposed in the path of said beam of radiant energy, said mask having a transmission characteristic for the energy components of said beam which varies in space in accordance with a function of said predetermined admittance characteristic.

10. A system in accordance with claim 9 which includes means for changing the position of said mask with respect to said beam, whereby to vary said predetermined admittance characteristic.

11. A system in accordance with claim 9 in which said certain variations in said waves comprise variations in the amplitude of said waves.

12. A system comprising in combination means for generating elastic waves in a given medium, means for modulating said elastic waves in accordance with a given impressed function, a source of a beam of electromagnetic radiation, means for directing said beam to pass through said medium transversely to the direction of

travel of said elastic waves through said medium, means interposed in the path of said beam for varying the intensity of said beam in accordance with a predetermined weighting function, and means for collecting and summing components of said beam.

13. In combination with a system comprising a supersonic light valve, said valve including a vessel containing a medium adapted for the propagation of elastic waves therethrough, a piezoelectric means disposed at one end of said vessel for generating elastic waves in said medium, means comprising an input circuit connected to said piezoelectric means for impressing modulations on said elastic waves in accordance with an applied input function, and a source of a beam of light disposed relative to said vessel whereby said beam is directed to pass through said medium transversely to the direction of travel of said elastic waves through said medium, means comprising at least one mask positioned in the path of said beam and having a light transmission characteristic which is space varied in accordance with a predetermined weighting function, and means comprising a photoelectric responsive device positioned to receive and integrate weighted components of said beam.

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No references cited.