

Sept. 28, 1948.

W. P. MASON

2,450,010

PIEZOELECTRIC CRYSTAL APPARATUS

Filed Aug. 9, 1943

3 Sheets-Sheet 1

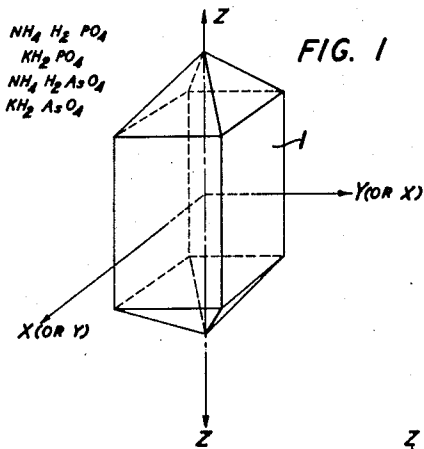


FIG. 1

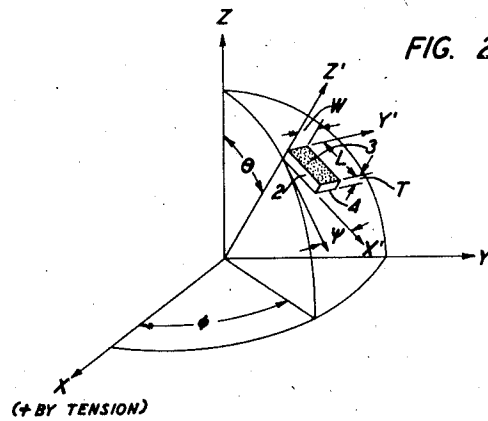


FIG. 2

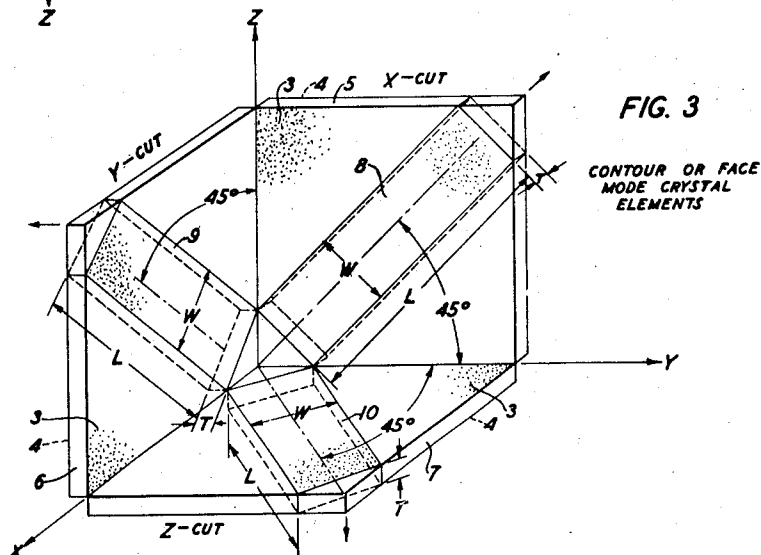


FIG. 3

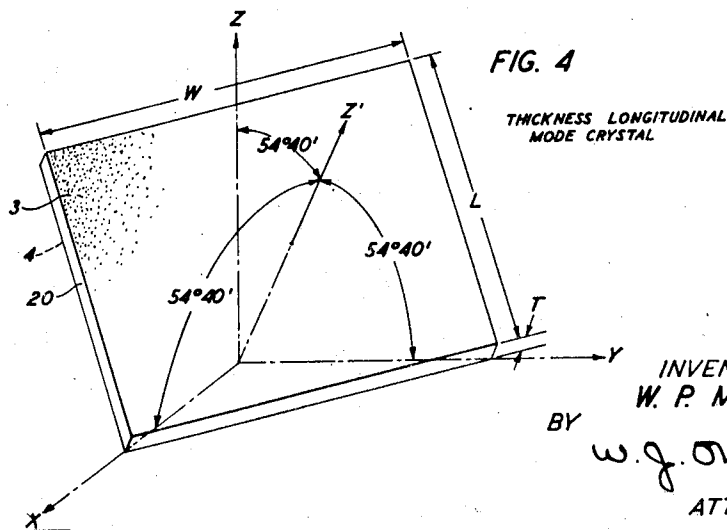


FIG. 4

THICKNESS LONGITUDINAL
MODE CRYSTAL

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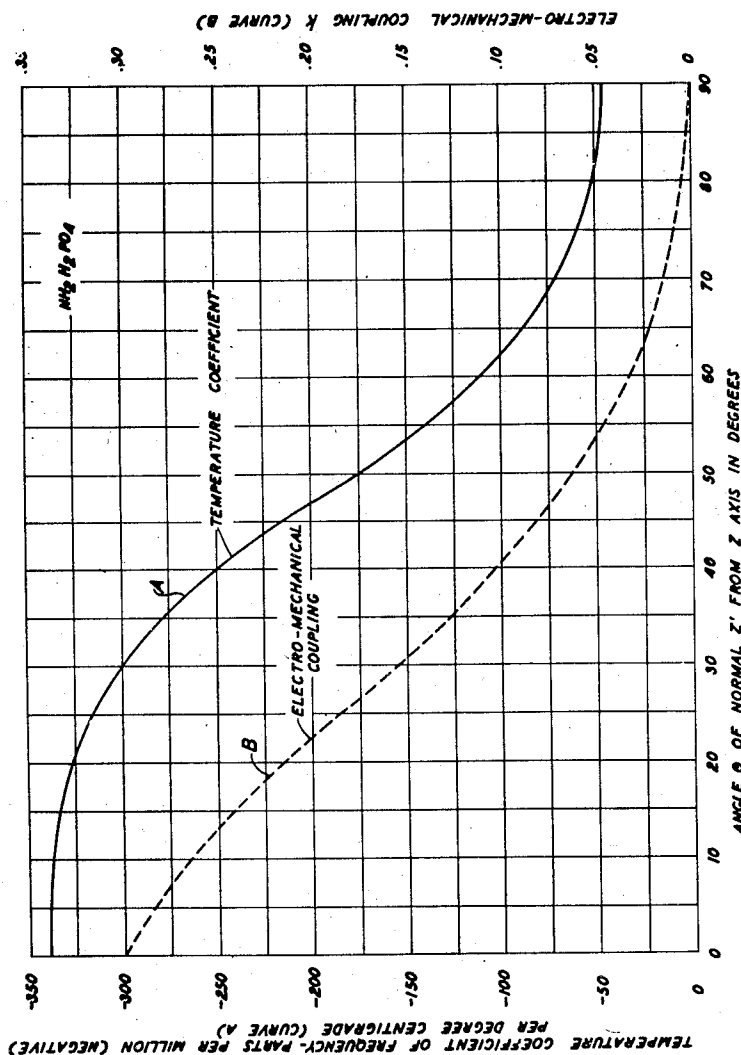
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3 Sheets-Sheet 2

FIG. 3A



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PIEZOELECTRIC CRYSTAL APPARATUS

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3 Sheets-Sheet 3

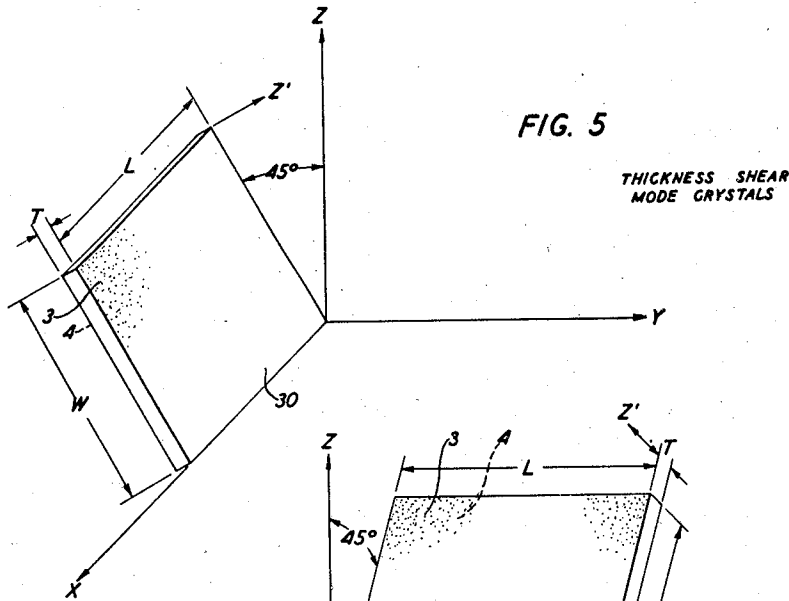


FIG. 6

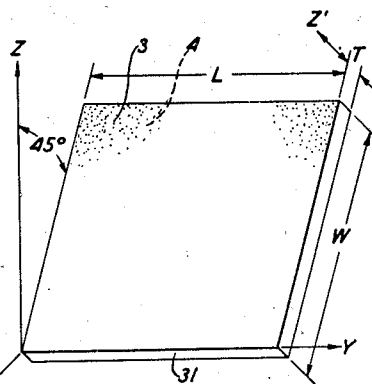
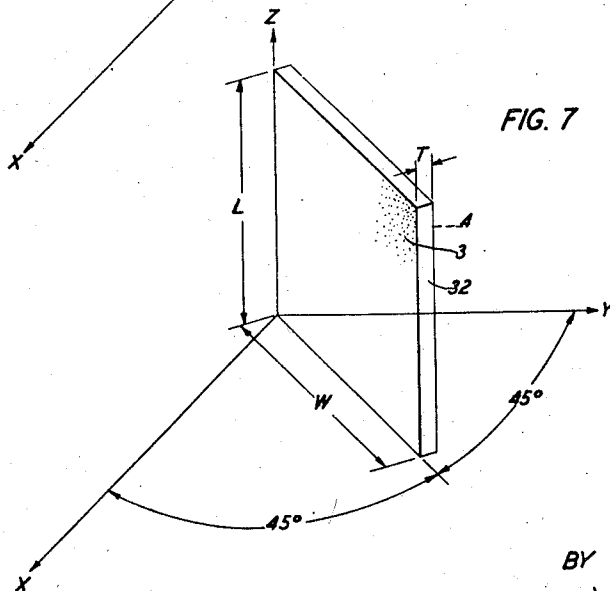


FIG. 7



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2,450,010

PIEZOELECTRIC CRYSTAL APPARATUS

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Application August 9, 1943, Serial No. 497,883

9 Claims. (Cl. 171-327)

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This invention relates to piezoelectric crystal apparatus and particularly to piezoelectric crystal elements comprising crystalline ammonium dihydrogen phosphate ($\text{NH}_4\text{H}_2\text{PO}_4$), potassium dihydrogen phosphate (KH_2PO_4), ammonium dihydrogen arsenate ($\text{NH}_4\text{H}_2\text{AsO}_4$), potassium dihydrogen arsenate (KH_2AsO_4) and isomorphous combinations. Such crystal elements are useful as electromechanical transducers utilized, for example, in sonic or supersonic projectors, microphones, pick-up devices and detectors. Also, they may be utilized as frequency control elements in electric wave filter systems, oscillation generator systems and amplifier systems. Other applications for such crystal elements may include harmonic producers, and, in general, any application where either a resonant or non-resonant piezoelectric crystal element may be utilized. The non-linear hysteresis loop characteristics of the non-resonant crystal may be made use of to produce overtones or harmonics therefrom.

One of the objects of this invention is to provide useful orientations and modes of motion in crystal elements made from crystalline ammonium dihydrogen phosphate, potassium dihydrogen phosphate, ammonium dihydrogen arsenate, potassium dihydrogen arsenate and isomorphous combinations.

Other objects of this invention are to provide crystal elements comprising dihydrogen phosphate and arsenate substances that may possess useful characteristics, such as large piezoelectric constants, large vibrational motion, minimum coupling of the desired mode of motion with undesired modes of motion therein, and temperature coefficients of frequency that may have the relatively lower values.

Another object of this invention is to take advantage of the high piezoelectric activity, the low cost and other advantages of crystalline ammonium dihydrogen phosphate and similar dihydrogen phosphate and arsenate crystals.

Crystal elements of suitable orientation cut from crystalline ammonium dihydrogen phosphate, potassium dihydrogen phosphate, potassium dihydrogen arsenate, ammonium dihydrogen arsenate and isomorphous combinations thereof may be excited in different modes of motion, such as longitudinal length, longitudinal width or longitudinal thickness modes of motion, shear face modes of motion controlled mainly by the width and length major face dimensions, or thickness shear modes of motion controlled mainly by the thickness dimension. Also, low frequency flexural modes of motion of either the

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width bending flexure type or the thickness bending flexure type may be utilized. The contour or face modes of motion may be either the face shear mode of motion, or the width or length face longitudinal modes of motion. The thickness modes of motion may be either the thickness longitudinal mode of motion or the thickness shear mode of motion. These modes of motion are similar in the general form of their motion to those of corresponding names that are already known in connection with quartz, Rochelle salt, and other known piezoelectric crystals.

Crystal elements composed of crystalline ammonium dihydrogen phosphate, potassium dihydrogen phosphate, potassium dihydrogen arsenate, ammonium dihydrogen arsenate and isomorphous combinations may have piezoelectric and elastic constants or moduli of considerable interest for use in electromechanical transducers, filter systems and oscillator systems, for example. In accordance with this invention, a number of crystal orientations or cuts are provided that may be utilized for these purposes and others. The types of crystal cuts may be divided into several categories, such as (a) crystal cuts that have relatively large piezoelectric constants and hence may be driven strongly piezoelectrically, (b) crystal cuts that have advantageous elastic properties, such that the longitudinal face modes of motion therein are free from coupling to the face shear modes of motion therein, and face shear mode crystal elements that are free from coupling with other modes of motion therein, and (c) crystal cuts that may have the relatively lower values of temperature coefficients of frequency.

Crystal elements comprising ammonium dihydrogen phosphate, potassium dihydrogen phosphate, ammonium dihydrogen arsenate, potassium dihydrogen arsenate and isomorphous combinations also possess ferroelectric properties such as large dielectric constants, hysteresis loops and non-linearity of charge field relationships below their critical or Curie temperatures of about 115°K ., 91°K ., 155°K and 220°K ., respectively. These crystal substances also possess high piezoelectric constants at room temperatures, which, in general, are larger than those of most other piezoelectric crystals except Rochelle salt. Ammonium dihydrogen phosphate crystals have relatively the largest piezoelectric constants of any of the four isomorphous dihydrogen crystal substances mentioned, and are relatively easy to grow in shapes and sizes that are suitable for cutting useful plates or elements therefrom.

The ammonium dihydrogen phosphate crystals, for example, may have properties somewhat similar to 45-degree Y-cut type Rochelle salt crystals but will stand a much higher operating temperature which may be of the order of about 180° C. or much higher, and also have no water of crystallization and hence will not dehydrate when operated in air or in vacuum. The temperature coefficients of frequency for certain of the principal cuts are roughly of the order of 100 to 300 parts per million per degree centigrade. The dielectric constants decrease slightly with an increase in temperature while the piezoelectric constants relating charge and stress are nearly independent of temperature. Since the ammonium dihydrogen phosphate crystals have the relatively higher values of electromechanical coupling, and are free from water of crystallization which eliminates dehydration in the crystal, and will stand relatively high operating temperatures of the order of 180° C., or more, they are useful as driving elements for all transducer applications, such as projectors and microphones in under-water sound work, for example. Also, this type of crystal may be used as a substitute for quartz frequency control elements in filter and oscillator applications, especially when used with temperature control. For the lower frequency filter applications, the crystal cuts having the relatively lower temperature coefficients of frequency may be used at ordinary temperatures without temperature control.

Although all four of the crystalline dihydrogen substances particularly mentioned herein have relatively large piezoelectric constants and other useful characteristics, the ammonium dihydrogen phosphate crystal elements may be constructed to have the largest values of piezoelectric constants of the four crystalline dihydrogen phosphate and arsenate salts mentioned, and also generally, are relatively more easy to grow in the sizes and shapes that are useful for cutting crystal elements therefrom.

The crystal elements disclosed in this specification may have conductive electrode coatings on their major surfaces of any suitable composition, shape, and arrangement, such as those already known in connection with quartz, Rochelle salt and other piezoelectric crystals; and they may be mounted and electrically connected by any suitable means, such as for example, by pressure type clamping pins or by conductive supporting wires cemented by conductive cement to the crystal coatings at or near the nodal regions, as already known in connection with quartz, Rochelle salt and other crystals having similar or corresponding modes of motion.

Spurious modes of motion may be avoided in these crystal elements by a suitable dimensioning of the crystal element, such as by adjusting the thickness dimension thereof relative to the length and width dimensions thereof, in the case of face shear mode or face longitudinal mode crystals, and by adjusting the length and width dimensions relative to the thickness dimension in the case of thickness mode crystals, such as thickness shear mode crystals or thickness longitudinal mode crystals. Also the effect of spurious modes in these face mode and thickness mode dihydrogen crystals may be reduced by the use of centrally disposed electrodes partially covering the major faces of the crystals, in the manner of such partial electrodes as are now used in connection with quartz crystals, for example.

For a clearer understanding of the nature of

this invention and the additional advantages, features and objects thereof, reference is made to the following description taken in connection with the accompanying drawings, in which like reference characters represent like or similar parts and in which:

Fig. 1 is a perspective view illustrating the prismatic tetragonal-scalenohedral form in which ammonium dihydrogen phosphate, potassium dihydrogen phosphate, potassium dihydrogen arsenate, ammonium dihydrogen arsenate and isomorphous combinations thereof crystallizes, and also illustrating the relation of the prism faces and cap faces of such crystalline substances with respect to the mutually perpendicular electric axis X, mechanical axis Y, and optic axis Z thereof;

Fig. 2 is a perspective view illustrating the orientation, in terms of the angles ϕ , θ and ψ , of a crystal element cut from any of the dihydrogen crystalline substances illustrated in Fig. 1, and may be taken to illustrate the orientation of any dihydrogen salt crystal element disclosed in this specification;

Fig. 3 is a perspective view illustrating X-cut, Y-cut and Z-cut type contour or face mode crystal elements of the face shear mode type and of the face longitudinal mode type, cut from any of the crystalline substances ammonium dihydrogen phosphate, potassium dihydrogen phosphate, potassium dihydrogen arsenate, ammonium dihydrogen arsenate and isomorphous combinations, as illustrated in Fig. 1;

Fig. 3A is a graph illustrating the characteristics of the 45-degree Z-cut type ammonium dihydrogen phosphate crystal element 10 of Fig. 3 when rotated in effect about its width dimension W;

Fig. 4 is a perspective view illustrating the orientation of a thickness longitudinal mode crystal element cut from any of the crystalline substances ammonium dihydrogen phosphate, potassium dihydrogen phosphate and corresponding arsenate crystal substances, as illustrated in Fig. 1; and

Figs. 5, 6 and 7 are perspective views illustrating the orientations of several types of thickness shear mode crystal elements cut from any of the substances ammonium dihydrogen phosphate, potassium dihydrogen phosphate and corresponding arsenate crystal substances, as illustrated in Fig. 1.

This specification follows the conventional terminology as applied to piezoelectric crystalline substances, which employs three mutually perpendicular X, Y and Z axes, as shown in the drawings, to designate an electric axis, a mechanical axis and an optic axis, respectively, of piezoelectric crystalline substances, and which employs three orthogonal axes X', Y' and Z' to designate the directions of the axes of a crystalline piezoelectric body or element that is angularly oriented with respect to such X, Y and Z axes thereof.

This specification also follows the conventional terminology used to designate the elastic constants s and c , the piezoelectric constants d and other constants of piezoelectric crystalline substances. As an illustrative example, the d_{31} piezoelectric constant means that a Z axis field represented by the numeral 3 may produce XY shear motion represented by the numeral 6. If the d_{31} piezoelectric constant of the substance has a large value, as it does in the case of the several dihydrogen salt crystals here considered, then a Z

axis field applied thereto may produce a strong shear motion in the XY plane of the crystal body.

Referring to the drawing, Fig. 1 is a perspective view illustrating the form in which ammonium dihydrogen phosphate, potassium dihydrogen phosphate, ammonium dihydrogen arsenate, and potassium dihydrogen arsenate crystallizes. As illustrated in Fig. 1, such isomorphous dihydrogen crystal substances crystallize in the prismatic tetragonal scalenohedral form and are formed with four major prism faces and with four cap faces at each end. The optic axis Z extends between each apex of the cap faces, and the mutually perpendicular X and Y axes, extend perpendicular to the four major prism faces. The several cuts or orientations of dihydrogen phosphate and arsenate crystal elements hereinafter disclosed may be cut from the mother crystal 1 of the substances and form illustrated in Fig. 1.

The mother crystal 1 illustrated in Fig. 1 may be grown from any suitable substances, and in any suitable manner such as, for example, by either the circulation method or the rocking method. As an illustrative example, the potassium salts, used in growing the mother crystal 1 illustrated in Fig. 1, may be obtained from potassium hydroxide and phosphoric or arsenic acid, and the ammonium salts may be obtained from ammonium carbonate and the corresponding acids. Saturated solutions may be prepared from these salts and the crystal 1 grown from watery solutions at a gradually decreasing temperature in any suitable manner. The crystal shape illustrated in Fig. 1 may be varied somewhat to obtain either needle-shaped crystals, or the more compact or short prism form as illustrated in Fig. 1. Ammonium dihydrogen phosphate produces short and thick prismatic crystals at room temperature. If liquor is added in excess, all of these salts may be crystallized in short prisms at room temperature. The short thick form of crystal 1, as illustrated in Fig. 1, is generally the more convenient form for cutting the various orientations of crystal plates therefrom.

Fig. 2 is a diagram illustrating the system, recently defined by the Institute of Radio Engineers, for specifying the orientation for a piezoelectric crystal element or body 2 in relation to its mutually perpendicular X, Y and Z axes. As shown in Fig. 2, the X' axis is taken along the length dimension L of the crystal element 2, the Y' axis is taken along the width dimension W of the crystal element 2, and the Z' axis is taken along the thickness or thin dimension T of the crystal element 2. The angle θ is, as shown in Fig. 2, the angle between the optic axis Z and the plate normal or Z' axis, and the angle ϕ is the angle between the + X axis (+ by tension) and the intersection of the plane containing the Z and Z' axes with the XY plane, while ψ is the angle between the length L axis X' and the tangent of the great circle containing the Z and Z' axes as measured in a plane perpendicular to the Z' axis. All angles are positive when measured in a counter-clockwise direction. Fig. 2 is applicable to a right-hand crystal, such as quartz, following the crystallographer's definition and the earlier Biot convention. The positive (+) X axis is the X axis for which a positive charge develops on a tensional stress being applied thereto.

The crystal element 2 of Fig. 2 may be cut from any of the crystalline phosphate and arsenate substances illustrated in Fig. 1, and, by specifying the values for the three angles θ , ϕ and ψ of Fig. 2 may generally designate the orientation of

any of the several crystal elements disclosed in this specification and illustrated in Figs. 3 to 7 of the drawing.

Suitable conductive electrodes such as the crystal electrodes 3 and 4 of Fig. 2 may be placed on or adjacent to or formed integral with the opposite major faces of any of the crystal plates disclosed herein in order to apply electric field excitation thereto. The crystal electrodes 3 and 4 when formed integral with the surfaces of any of the crystal elements 2 may consist of gold, platinum, aluminum, silver or other suitable conductive material deposited upon the crystal surfaces by evaporation in vacuum, painting, spraying, or by other suitable process. If desired, the crystal element 2 may be electroplated to the desired frequency by nickel plating or otherwise.

Fig. 3 is a perspective view of six differently oriented, relatively low frequency face mode crystal elements 5, 6, 7, 8, 9 and 10 which may be cut from any of the crystalline substances ammonium dihydrogen phosphate, potassium dihydrogen phosphate, potassium dihydrogen arsenate, ammonium dihydrogen arsenate, or isomorphous combinations thereof, each crystal element 5 to 10 being made into a plate of substantially rectangular parallelepiped shape with its major faces having a length dimension L and a width dimension W, the thickness or thin dimension T being measured between the opposite major faces. Either the length dimension L or the width dimension W or both may be made of values to suit a desired frequency. The thickness or thin dimension T may be made of a value to suit the impedance of the system in which the crystal element may be utilized, or it may be made of a suitable value to avoid a nearby spurious mode of motion which, by proper dimensioning of the thickness T relative to the width W and length L dimensions, may be placed in a location that is relatively remote from the desired face mode of motion.

The contour or face mode X-cut, Y-cut and Z-cut type crystal elements 5, 6, 7, 8, 9 and 10 of Fig. 3 may be cut from crystalline ammonium dihydrogen phosphate, potassium dihydrogen phosphate, ammonium dihydrogen arsenate or potassium dihydrogen arsenate of the form and symmetry illustrated in Fig. 1, and may be driven by the electrodes 3 and 4 which may partially or wholly cover the opposite major faces of each of the crystal elements and which thereby produce an electric field in the direction of the thickness or thinnest dimension T thereof.

As illustrated in Fig. 3, the crystal elements 5, 6 and 7 are three differently oriented face shear mode crystal elements of X-cut, Y-cut and Z-cut orientations, respectively, the frequency of which may be controlled mainly by the major face length L and width W dimensions; and the crystal elements 8, 9 and 10 are three differently oriented face longitudinal length or width mode crystal elements of X-cut, Y-cut and Z-cut type orientations, respectively, the frequency of which may be controlled mainly by either the length dimension L or by the width dimension W or by both simultaneously or separately. The arrows, illustrated in Fig. 3, may be taken to indicate that the crystal elements 5 to 10 may be rotated in effect about their length or width dimensions or both to positions where their major faces are no longer perpendicular to the X, Y or Z axes thereof. In addition, the longitudinal face mode crystal elements 8, 9, and 10 of Fig. 3 may be rotated in effect about their respective thickness

dimensions T to either side of the 45-degree angular position particularly illustrated in Fig. 3.

It will be noted that when the length dimension L of the face shear mode crystal elements 5, 6 and 7 is substantially parallel to the X, Y or Z axes as illustrated in Fig. 3, all of the motion in the crystal element 5, 6 or 7 will be a face shear motion and none longitudinal; while if the length dimension L is mid-way or at the bisecting angle of 45 degrees between the X and Y, and Y and Z or the X and Z axes as illustrated by the crystal elements 8, 9, and 10 of Fig. 3, the shear motion will be zero or of small value and the longitudinal motion a maximum.

The X-cut, Y-cut, and Z-cut type crystal elements 5, 6 and 7 of Fig. 3 illustrate face shear mode types of crystal elements which are controlled by the d_{14} , the d_{25} and the d_{36} piezoelectric constants, respectively, following the conventional terminology used for expressing the relation between the applied field direction and the resulting stress or type of motion. In each of the four dihydrogen phosphate and arsenate crystal substances particularly disclosed in this specification, the d_{36} piezoelectric constant thereof is of larger value than the d_{14} and the d_{25} piezoelectric constants thereof and, consequently, the Z-cut face shear mode crystal element 7 may be driven more strongly piezoelectrically than the X-cut or Y-cut face shear mode crystal elements 5 and 6, and since ammonium dihydrogen phosphate has relatively the largest values of piezoelectric constants d_{14} , d_{25} and d_{36} , it consequently may be driven more strongly than the other three dihydrogen crystalline substances mentioned.

As illustrated in Fig. 3, the X-cut type face shear mode crystal element 5 has its square or rectangular shaped major faces disposed perpendicular or nearly perpendicular to the X axis, the length dimension L, the width dimension W, and the peripheral edges of the major faces being made parallel or nearly parallel to the Z and Y axes thereof in order to obtain freedom from coupling of its desired face shear mode of motion with longitudinal face modes of motion therein. Similarly, the Y-cut type face shear mode crystal element 6 of Fig. 3 has its square or rectangular shaped major faces disposed perpendicular or nearly perpendicular to the Y axis, the length dimension L, the width dimension W, and the edges of the major faces being made parallel or nearly parallel to the X or the Z axes thereof in order to obtain freedom from coupling of its desired face shear mode of motion with longitudinal face modes of motion therein. Similarly, the Z-cut type face shear mode crystal element 7 of Fig. 3 has its rectangular shaped major faces disposed perpendicular or nearly perpendicular to the Z axis, the length dimension L, the width dimension W, and the edges thereof being made parallel or nearly parallel to the X or the Y axes thereof in order to obtain substantially no coupling of its desired face shear mode of motion with longitudinal modes of motion therein.

The frequency constants for the fundamental face shear mode of motion of the X-cut, Y-cut and Z-cut type crystal elements 5, 6 and 7 of Fig. 3 are of the order of from 127 to 133 kilocycles per second per centimeter of width W or length L for crystal elements constructed from crystalline ammonium dihydrogen phosphate, and are roughly of the same order for potassium dihydrogen phosphate and the corresponding arsenates.

While the face shear mode crystal elements 5, 6 and 7 of Fig. 3 are shown as having nearly square shaped major faces, they may be cut in elongated rectangular form as illustrated in W. P. Mason United States Patent 2,309,467, dated January 26, 1943, may have a selected width W to length L dimensional ratio, and may be adapted to vibrate either simultaneously or independently in the first and second shear face modes of motion controlled by the width W and length L dimensions of the crystal element, as disclosed in the last-mentioned Mason patent.

The X-cut, the Y-cut, and the Z-cut type longitudinal length L or width W mode crystal elements 8, 9 and 10 of Fig. 3 may be cut from the X-cut, Y-cut and Z-cut type face shear mode crystal elements 5, 6 and 7, respectively, as illustrated in Fig. 3, or they may be cut separately. The X-cut type crystal element 8 of Fig. 3 may have its opposite major faces disposed perpendicular or nearly perpendicular to the X axis, the length L and width W dimensions thereof being inclined at an angle of substantially 45 degrees or other angles with respect to the other two axes, namely, the Y and Z axes. Similarly, the Y-cut and the Z-cut type longitudinal mode crystal elements 9 and 10 of Fig. 3 may have their major faces disposed perpendicular or nearly perpendicular to the Y axis and the Z axis, respectively, the length L dimensions thereof being inclined at an angle of substantially 45 degrees or other angle with respect to the other two of the three crystallographic axes. The electric field may be applied in the thickness direction T by the electrodes 3 and 4 which may be placed on or adjacent the opposite major faces in order to drive the crystal elements 8, 9 or 10 in the face longitudinal length L or width W modes of motion.

The dimensional ratio of the width W with respect to the length L of the face longitudinal mode crystal elements 8, 9 and 10 of Fig. 3 may be made of any suitable value. The smaller values decreasingly less than a width W to length L ratio of about .5 may be utilized to obtain a length L longitudinal mode of motion that is less affected by the face shear mode of motion therein, particularly when the length L dimension of the X-cut, Y-cut and Z-cut type crystal elements 8, 9 and 10 of Fig. 3 is disposed at an angle other than the angle which gives zero coupling to a face shear mode which in the case of the Z cut is the bisecting 45-degree angle particularly illustrated at 10 in Fig. 3, and which in the case of the X and Y cuts 8 and 9 is a ψ angle of about 24° 30'.

A feature of special interest is that at the bisecting 45-degree angle as illustrated at 10 in Fig. 3, the 45-degree Z-cut crystal element 10 of Fig. 3 has not only maximum motion for the desired longitudinal length L mode of motion therein but also that mode of motion has substantially no coupling with the face shear mode of motion therein, corresponding to a similar freedom from coupling effect that obtains in the 42° 26' Y-cut Rochelle salt crystal element of W. P. Mason Patent 2,292,885 dated August 11, 1942, Fig. 2, and to the -18.5-degree X-cut quartz crystal element of W. P. Mason et al., Patent 2,173,589 dated September 19, 1939.

If desired, the longitudinal mode crystal elements, such as the crystal elements 8, 9 and 10 of Fig. 3, may be operated alone or simultaneously in the length L longitudinal mode of motion and in the width W flexure mode of motion, as disclosed in W. P. Mason Patent 2,292,886

dated August 11, 1942; also they may be operated simultaneously in the length L and width W longitudinal modes of motion as disclosed in W. P. Mason Patent 2,292,885 dated August 11, 1942.

It will be understood that the longitudinal mode crystal elements 8, 9 and 10 of Fig. 3 may comprise crystalline ammonium dihydrogen phosphate, potassium dihydrogen phosphate, potassium dihydrogen arsenate, ammonium dihydrogen arsenate and isomorphous combinations, and may be rotated in effect about their thickness dimensions T to any position at either side of and other than at the 45-degree bisecting angular position that is particularly illustrated in Fig. 3. Illustrative examples of the characteristics of a few of such longitudinal mode crystal elements 8, 9, 10 of Fig. 3 rotated in effect about the thickness dimension T are given as follows, in connection with the substance ammonium dihydrogen phosphate particularly.

As an illustrative example for X-cut crystals, the 45-degree X-cut length L longitudinal mode ammonium dihydrogen phosphate crystal element 8 of Fig. 3 having a length dimension L, a width dimension W and a thickness dimension T of about 1.89, .725 and .222 centimeters, respectively, giving a dimensional width W to length L ratio of about .373, has a frequency constant for the length L longitudinal mode of motion of about 182.5 kilocycles per second per centimeter of length dimension L, an elastic constant of s_{11}^E value of about 4.16×10^{-12} ; a resonant frequency f_R of about 96.456 kilocycles per second, an anti-resonant frequency f_A of about 96.466 kilocycles per second, a static capacitance of C_0 of about 31.1 micromicrofarads, a ratio of capacities

$$r = \frac{C_0}{C_1} = 4650$$

a dielectric constant K_1^{LC} of about 57.1, a resistance at resonance of about 42,500 ohms, a Q of about 5,800, a coupling coefficient k of about .0163, a piezoelectric constant d_{14} of about 14.8×10^{-8} , a piezo electric coefficient D_{14} of about 0.39×10^4 , a temperature coefficient of frequency for its length L longitudinal motion of about -145 parts per million per degree centigrade, and a temperature coefficient of about +340 for its elastic constant s_{11}^E .

As another illustrative example for X-cut crystals, a 22.5-degree X-cut length L longitudinal mode ammonium dihydrogen phosphate crystal plate 8 similar to the 45-degree X-cut crystal plate 8 of Fig. 3 but having its length dimension L inclined at an angle of 22.5 degrees instead of 45 degrees from the Z axis, and having a length L, a width W and a thickness T of about 1.844, .608 and .23 centimeters, respectively, giving a width W to length L dimensional ratio of about .329, has a frequency constant for its length L longitudinal mode of motion of about 175.9 kilocycles per second per centimeter of length dimension L, an elastic constant s_{11}^E of value about 4.49×10^{-12} , a temperature coefficient of frequency for its length L longitudinal mode of motion of about -71.4 parts per million per degree centigrade, and a temperature coefficient of about +214 for its elastic constant s_{11}^E .

As another illustrative example for X-cut crystals, a 67.5-degree X-cut length L longitudinal mode ammonium dihydrogen phosphate crystal element 8 similar to the 45-degree X-cut crystal element 8 of Fig. 3 but having its length dimension L inclined at an angle of 67.5

degrees instead of 45 degrees from the Z axis, and having a length L, a width W and a thickness T of about 1.851, .637 and .25 centimeters, respectively, giving a width W to length L dimensional ratio of about .354, has a frequency constant for its length L longitudinal mode of motion of about 211.5 kilocycles per second per centimeter of length dimension L, an elastic constant s_{11}^E value of about 3.10×10^{-12} , a temperature coefficient of frequency for its length L longitudinal mode of motion of about -299 parts per million per degree centigrade, and a temperature coefficient of about 626.8 for its elastic constant s_{11}^E .

As an illustrative example for Z-cut crystals, a 22.5-degree Z-cut length L longitudinal mode ammonium dihydrogen phosphate crystal element 10 similar to the 45-degree Z-cut crystal element 10 of Fig. 3 but having its length dimension L inclined at an angle of 22.5 degrees instead of 45 degrees from the X axis, and having a length L, a width W and a thickness T of about 1.575, .464 and .220 centimeters, respectively, giving a width W to length L dimensional ratio of about .294, has a frequency constant of about 182.2 kilocycles per second per centimeter of length dimension L, an elastic constant s_{11}^E of value about 4.17×10^{-12} , a temperature coefficient of frequency for its length L longitudinal mode of motion of about -344 parts per million per degree centigrade, and a temperature coefficient for its elastic constant s_{11}^E of about +708.

As another example, the 45-degree Z-cut length L longitudinal mode ammonium dihydrogen phosphate ($\text{NH}_4\text{H}_2\text{PO}_4$) crystal element 10 of Fig. 3 having a length L, a width W and a thickness T of about 1.730, 0.486 and 0.21 centimeter, respectively, giving a width W to length L dimensional ratio of about .278, has a frequency constant of about 162.1 kilocycles per second per centimeter of length dimension L, an elastic constant s_{11}^E of about 5.28×10^{-12} , a resonant frequency of about 93.369 kilocycles per second, an anti-resonant frequency of about 96.695 kilocycles per second, a static capacitance C_0 of about 4.94 micromicrofarad, a ratio of capacities r of about 11.65, a dielectric constant K_1^{LC} of about 14.2, a resistance at resonance of about 740 ohms, a Q of about 5,600, a coefficient of coupling k of about .306 as shown by the curve B of Fig. 3A at $\theta=0$ degrees, a piezoelectric constant d_{36} of about 155×10^{-8} , a piezoelectric coefficient D_{36} of about 12.20×10^4 , a temperature coefficient of frequency of about -338 parts per million per degree centigrade as shown by the curve A of Fig. 3A at $\theta=0$ degrees, and a temperature coefficient for its elastic constant s_{11} of about +692.

As an illustrative example for crystalline potassium dihydrogen phosphate (KH_2PO_4), a 45-degree Z-cut length L longitudinal mode potassium dihydrogen phosphate (KH_2PO_4) crystal element 10 of Fig. 3 having a length L, a width W and a thickness T of about 1.04, .492 and .112 centimeters, respectively, has a resonant frequency of 141.9 kilocycles per second, an anti-resonant frequency of about 142.7 kilocycles per second, a resistance at resonance of about 2,500 ohms, a capacity of about 8.5 micromicrofarads, a Q of about 3,160, a frequency constant of about 147.6 kilocycles per second per centimeter of length dimension L, a velocity of propagation of about 2.952×10^5 ; a density of about 2.338, a dielectric constant K of about 21.0, a ratio of capacities r of about 58.5, a coefficient of electromechanical coupling k of about .1438, a value

of Young's modulus Y_0 along the length L of the crystal element of about 1.945×10^{11} dynes per square centimeter, a piezoelectric coefficient D relating force to charge of about 4.9×10^4 e. s. u., and a piezoelectric constant d_{31} relating potential to shear strains of about 84.2×10^{-8} .

A potassium dihydrogen arsenate (KH_2AsO_4) 45-degree Z-cut length L longitudinal mode crystal element 10 of Fig. 3 has a piezoelectric constant d_{31} of about 84×10^{-8} , a dielectric constant K of about 16.0, a Young's modulus along the length dimension L of about 1.98×10^{11} , and a density of about 2.85.

The 45-degree Z-cut length L longitudinal mode ammonium dihydrogen phosphate crystal element 10 of Fig. 3 is useful as an electromechanical transducer in sonic and supersonic projectors and microphones, for example, where it may be used either as a resonant or a non-resonant crystal element unit to obtain a maximum extensional motion along its length dimension L and a maximum piezoelectric excitation. Ammonium dihydrogen phosphate cut in the 45-degree Z-cut orientation 10 of Fig. 3 has the largest piezoelectric constant of the four isomorphous dihydrogen crystalline substances mentioned herein, although the other three may also be used similarly. When used as an electromechanical transducer in a supersonic projector, the 45-degree Z-cut crystal element 10 of Fig. 3 may conveniently be made of a resonant length corresponding to the frequency to be projected.

From the electromechanical equivalent circuit as described in applicant's book "Electromechanical Transducers and Wave Filters," Chapter VI, D. Van Nostrand Co. Inc., New York, 1942, the performance of such a crystal as an electromechanical transducer may be calculated.

Applied to transducers, the principal constants of interest are the dielectric constant K , the piezoelectric coefficient D relating the stress to the applied charge, the value of Young's modulus Y_0 , and the density ρ . In c. g. s. units, these constants for the 45-degree Z-cut ammonium dihydrogen phosphate crystal 10 of Fig. 3 have values at room temperature of about $K=14.2$; $D=12.2 \times 10^4$ dynes/e. s. u. charge;

$$Y_0 = 1.89 \times 10^{11}$$

dynes/centimeters²; $\rho = 1.8$ grams/centimeters cubed.

Applied to the conventional electromechanical equivalent circuit for a piezoelectric crystal, from which the response of the crystal unit as a sonic projector may be calculated, the constants of 45-degree Z-cut ammonium dihydrogen phosphate crystals glued to steel resonators, as measured from the electrical side, become shunt capacity

$$C_0 = 1.26 \times 10^{-12} \frac{nLW}{T}$$

farads; series capacity

$$C_1 = .0782 \times 10^{-12} \frac{nLW}{T}$$

farads; inductance

$$L_1 = 49 \frac{LT}{nW}$$

henries; resistance

$$R_E = \frac{47.6TR_M}{nW}$$

ohms or about

$$\frac{7.15 \times 10^4 T}{nW}$$

ohms; where L is the length, W the width, T the thickness of each individual crystal element 10 of Fig. 3 expressed in centimeters, n is the total number of crystal elements 10 of Fig. 3 connected in parallel, and R_M is the mechanical resistance load per square centimeter placed on the radiating end of the crystal unit.

The four parallel-connected 45-degree Z-cut crystal elements 10 of Fig. 3 referred to, or other number of such crystal elements, may each be provided with electrodes of evaporated gold or of metal foil cemented to their large faces by conductive Bakelite cement, for example, or by other suitable adhesive means and then cemented together with tabs of gold foil for intermediate leads to the four parallel-connected crystals which may be cemented onto a ceramic wafer and a steel resonator to form a supersonic radiator or projector. Such dihydrogen phosphate crystals will endure relatively high temperatures and relatively larger cavitation and power output in castor oil, as compared with similar ones constructed from Rochelle salt. Ammonium dihydrogen phosphate crystals do not have a tendency to burn up at high power levels. Power applied up to the cavitation value of castor oil, or about 5 watts per square centimeter, may be continuously radiated from such ammonium dihydrogen crystals that are well glued or cemented to the steel resonators used in projectors in under-water sound work, for example.

Measurements on the electrical resistivity of 45-degree Z-cut crystal elements 10 of Fig. 3 are useful in the evaluation of purely conduction power loss in the material, and may be made by applying direct current voltage to the crystal electrodes which may be provided with peripheral guard rings around the edges to separate out the effect of surface leakage. The electrodes may be of tin foil or gold foil cemented thereon, or gold plating or otherwise. The electrical leakage of ammonium dihydrogen phosphate crystals and potassium dihydrogen phosphate crystals is due mainly to volume conduction. The volume resistivity ρ varies with the absolute temperature T according to the following relation, which follows the law usual for semiconductors:

$$\log_{10} \rho = a + \frac{b}{T}$$

where a and b are constants which may have some small measure of variation between different crystals. The surface electrical resistivity is of the order at least fifteen times the volume resistivity ρ , and varies in approximately the same way with temperature change. The leakage conduction in these crystals is, therefore, quite different from that in crystals of Rochelle salt. In Rochelle salt, the conductivity is almost entirely along the surface and depends on the relative humidity of its environment. In dry liquid, Rochelle salt may furnish its own moisture from its own water of crystallization causing lowered surface resistance and eventually electrical breakdown under conditions of continued piezoelectric drive. The dihydrogen crystals, however, have no water of crystallization that may escape to the surface and accordingly the surface resistance remains high in air or in oil even at relatively high temperatures.

As used in a supersonic projector, the length L

longitudinal mode of motion and the width W longitudinal mode of motion of the face longitudinal mode crystal element 10 of Fig. 3 may be used either alone or together simultaneously to radiate sound. For the latter purpose, the value of the width W with respect to the length L of the several similar crystals comprising the crystal mosaic generally used in such sound projectors may be proportioned so that the length L mode frequency response may cover one frequency range and the width W mode thereof may cover an adjoining higher frequency range with substantially an equal or flat response in decibels for both ranges over a relatively wider transmission band than can be obtained from the use of length L longitudinal mode of motion therein alone.

The constants of the 45-degree Z-cut longitudinal mode crystal element 10 of Fig. 3 comprising ammonium dihydrogen phosphate, potassium dihydrogen phosphate, potassium dihydrogen arsenate, ammonium dihydrogen arsenate, or isomorphous combinations are such that the crystal element 10 of Fig. 3 has a maximum value of piezoelectric constant d_{31}' , a maximum value of longitudinal motion along its length dimension L , and substantially zero coupling of the length L longitudinal mode of motion with the face shear motion in the major faces thereof, and, consequently, is a useful crystal element for many applications.

In Z-cut crystal elements 2 of Fig. 2 and 10 of Fig. 3, $\theta=0$, $\psi=0$ and the piezoelectric constant

$$d_{31}' = \frac{d_{36}}{2} (\cos 2\varphi)$$

Accordingly, the value of d_{31}' is a maximum and equal to

$$\frac{d_{36}}{2}$$

when the Z-cut longitudinal mode crystal element 10 of Fig. 3 has its length dimension L or X' axis inclined at the bisecting angle of 45 degrees with respect to the X and Y axes, as illustrated by the crystal element 10 in Fig. 3.

Similarly, for the X-cut and the Y-cut longitudinal mode crystal elements 8 and 9 of Fig. 3 where the length dimension L is disposed at the bisecting angle of 45 degrees between the Y and Z axes, and the X and Z axes, respectively, the corresponding face longitudinal mode piezoelectric constants d_{11} and d_{21} thereof, respectively, have their maximum values and are equal to

$$\frac{d_{14}}{2}$$

or

$$\frac{d_{25}}{2}$$

For the dihydrogen crystals herein described, the piezoelectric constants d_{14} and d_{25} are of equal value, and are of relatively smaller value than the value of the d_{36} piezoelectric constant referred to in connection with Z-cut type crystal element 10 of Fig. 3.

To obtain an additional series of length L longitudinal mode dihydrogen salt crystal elements, the 45-degree X-cut, the 45-degree Y-cut and the 45-degree Z-cut longitudinal length mode crystal elements, 8, 9 and 10 of Fig. 3, instead of having their major faces perpendicular or nearly perpendicular to the X , Y and Z axes, respectively, as illustrated in Fig. 3 may be inclined in either direction by rotation in effect about the respective

width dimensions W thereof to obtain a whole series of new positions such that the major faces are not perpendicular to any of the X , Y and Z axes. As an example, the 45-degree Z-cut crystal element 10 of Fig. 3 rotated in effect about its width dimension W so that its length dimension L is inclined at an angle between 0 and 90 degrees with respect to the Z axis, as indicated in Fig. 3 by the arrow, is of special interest because of its relatively large piezoelectric constant d_{31}' with resulting large magnitude of motion along its length dimension L , and because of the lower values of temperature coefficients of frequency that may be obtained for its longitudinal length L mode of motion as shown by the curve A of Fig. 3A for ammonium dihydrogen phosphate. Expressed in terms of the angles illustrated in Fig. 2, the 45-degree Z-cut crystal element 10 of Fig. 3 rotated in effect about its width dimension W has an orientation of $\varphi=0$ or 90 degrees, $\psi=45$ degrees and θ is a variable angle between 0 and 90 degrees, which may be an angle θ of about 60 degrees, for example, to obtain a relatively low value of temperature coefficient of frequency for the longitudinal length L mode vibration of the order of -110 parts per million per degree centigrade as shown by the curve A of Fig. 3A.

Fig. 3A is a graph giving experimentally obtained data on the values of the temperature coefficient of frequency and of the electromechanical coupling k for 45-degree Z-cut type length longitudinal mode ammonium dihydrogen phosphate crystal elements 10 of Fig. 3 which are rotated in effect about the width dimension W thereof from the position shown at 10 in Fig. 3 where the angle θ between the plate normal Z' and the Z axis is 0 degrees to a position 90 degrees therefrom where the angle θ of Fig. 2 is 90 degrees. As shown in Fig. 3A, the values of the temperature coefficient of frequency are given by the curve A as a function of the angle θ of Fig. 2 and the values of the electromechanical coupling k are given by the curve B of Fig. 3A as a function of the angle θ of Fig. 2. When the angle θ is 0 degrees, the ammonium dihydrogen phosphate crystal element 10 of Fig. 3 has a temperature coefficient of frequency for its length L longitudinal mode of motion of about -340 parts per million per degree centigrade as shown by the curve A of Fig. 3A, and has a strong value of electromechanical coupling k of about .30 as shown by the curve B of Fig. 3A. When the 45-degree Z-cut crystal element 10 of Fig. 3 is rotated in effect about its width dimension W , the values of its temperature coefficient of frequency and its electromechanical coupling decrease with increasing values of the angle θ . At an angle θ of 60 degrees or greater, the relatively lower values of temperature coefficient of frequency of the order of about 100 parts per million per degree centigrade or less may be obtained as shown by the curve A of Fig. 3A. The value of the electromechanical coupling k is rather low for θ angles much greater than 60 degrees as shown by the curve B of Fig. 3A.

In such 45-degree Z-cut type crystal elements 10 of Fig. 3, $\varphi=0$ or 90 degrees, $\psi=45$ degrees, and θ is a variable angle, and the value of the piezoelectric constant d_{31}' thereof is:

$$d_{31}' = \frac{d_{36} \cos^2 \theta - d_{14} \sin^2 \theta}{2} \quad (\text{for } \varphi=0^\circ)$$

$$d_{31}' = \frac{-d_{36} \cos^2 \theta + d_{14} \sin^2 \theta}{2} \quad (\text{for } \varphi=90^\circ)$$

In this equation, d_{14} is of opposite sign to d_{36} , and d_{14} being of considerably smaller value than

d_{33} , the 45-degree Z-cut dihydrogen salt crystal element 10 of Fig. 3 may have a good coupling piezoelectrically.

Figs. 4 to 7 illustrate piezoelectric crystal elements 20, 30, 31 and 32 having thickness modes of motion which may be either the longitudinal thickness mode of motion controlled by the d_{33}' piezoelectric constant as illustrated by the crystal element 20 in Fig. 4 and as disclosed and claimed in my copending divisional application, Serial No. 637,126, filed December 24, 1945, now Patent Number 2,450,011, or the shear thickness modes of motion controlled by the piezoelectric constants d_{34}' or d_{35}' as illustrated by the crystal elements 30, 31 and 32 in Figs. 5 to 7 and as disclosed and claimed in my copending divisional application, Serial No. 637,127 filed December 24, 1945. The thickness mode crystal elements 20, 30, 31 and 32 of Figs. 4 to 7 may be utilized at the relatively high thickness mode frequencies, fundamental or harmonic, to generate high frequency waves in liquids for submarine detection and also may be used as frequency control elements in electric wave filter systems, oscillation generator systems and for other purposes where a relatively high frequency or thickness mode crystal element may be desired.

Fig. 4 is a perspective view of a longitudinal thickness mode piezoelectric crystal element 20 cut from crystallized ammonium dihydrogen phosphate, potassium dihydrogen phosphate or from the corresponding arsenates. The longitudinal thickness mode of motion of the crystal element 20 of Fig. 4 is controlled mainly by the value of the thinnest or thickness dimension T thereof, and may be used, for example, to generate high frequency longitudinal waves in liquids as in high frequency supersonic projectors for submarine detection, and for other purposes where a relatively high frequency crystal element may be desired.

The longitudinal mode of motion which is utilized in the thickness mode crystal element 20 of Fig. 4 is controlled by the piezoelectric constant d_{33}' . The value of the piezoelectric constant d_{33}' is given by the relation:

$$d_{33}' = (2d_{14} + d_{36}) l_3 m_3 n_3 = (2d_{14} + d_{36}) \sin \varphi \cos \varphi \sin^2 \theta \cos \theta$$

The value of the piezoelectric constant d_{33}' of the last equation is a maximum when all of the direction cosines thereof are equal which occurs when $\theta = 45$ degrees and $\varphi = 54^\circ 40'$ giving a longitudinal thickness mode crystal element 20 of Fig. 4, the normal Z' of which makes equal angles with all three of the crystallographic axes X, Y and Z, as illustrated in Fig. 4. The major faces of the longitudinal thickness mode crystal element 20 of Fig. 4 may be of square, rectangular, circular

$$d_{34}' = (d_{14} + d_{36}) \cos \theta \left[\cos 2\varphi \cos \psi \sin \theta - \frac{\sin 2\varphi \sin 2\theta \sin \psi}{2} \right] + d_{14} \sin 2\varphi \sin^2 \theta \sin \psi$$

$$d_{35}' = (d_{14} + d_{36}) \cos \theta \left[\cos 2\varphi \sin \theta \sin \psi + \frac{\sin 2\varphi \sin 2\theta \cos \psi}{2} \right] - d_{14} \sin 2\varphi \sin^2 \theta \sin \psi$$

or other desired shape, and the length L and width W dimensions thereof may be proportioned with respect to the frequency determining thickness dimension T thereof to reduce the effect of spurious face modes of motion on the desired thickness motion thereof.

In the equation last given, the piezoelectric constant d_{14} is opposite in sign to the piezoelectric constant d_{36} , and the piezoelectric drive of

the crystal element 20 of Fig. 4, represented by the resultant difference of these values, may be strongly driven since the value of the piezoelectric constant d_{36}' is some ten times that of the piezoelectric constant d_{14}' for ammonium dihydrogen phosphate, and is more than two times that of the piezoelectric constant d_{14} for potassium dihydrogen phosphate, and also has useful values in the corresponding arsenates.

The fundamental thickness longitudinal mode frequency of the crystal element 20 of Fig. 4 when composed of ammonium dihydrogen phosphate has a frequency constant of about 2670 kilocycles per second per millimeter of thickness dimension T and a temperature coefficient of frequency of about -400 parts per million per degree centigrade for its thickness T longitudinal mode of motion.

While in Fig. 4, a single orientation is illustrated, it will be understood that other longitudinal thickness mode crystal elements may be cut in the general region of the orientation illustrated by the crystal element 20 of Fig. 4.

Figs. 5, 6 and 7 are perspective views of thickness shear mode crystal elements 30, 31 and 32 which may be cut from crystalline ammonium dihydrogen phosphate, potassium dihydrogen phosphate, potassium dihydrogen arsenate, ammonium dihydrogen arsenate and isomorphous combinations, and made into a plate of substantially rectangular parallelepiped shape with its major faces having a length dimension L and width dimension W which may be of equal dimensions or with one dimension either longer or shorter with respect to the other. The frequency determining thickness or thin dimension T between the major faces of the crystal elements 30, 31 and 32 is perpendicular to the other two dimensions L and W, which may be dimensionally related to the thickness dimension T to remove spurious face mode frequencies from the region of the desired thickness mode frequency.

The thickness shear modes of motion in the piezoelectric crystal elements 30, 31 and 32 of Figs. 5, 6 and 7 are similar to the same type of shear motion that obtains in quartz crystals and may be similarly utilized in filter systems and oscillator systems, for example.

The thickness shear modes in the four isomorphous dihydrogen crystal substances mentioned hereinbefore are generated by the piezoelectric constants d_{34}' and d_{35}' . In terms of the direction cosines, these are given by the formulae:

$$d_{34}' = (d_{14} + d_{36}) n_3 (l_2 m_3 + m_2 l_3) + 2d_{14} l_3 m_3 n_3$$

$$d_{35}' = (d_{14} + d_{36}) n_3 (l_3 m_1 + m_3 l_1) + 2d_{14} l_3 m_3 n_1$$

Inserting the values of the direction cosines in these equations, the piezoelectric driving constants d_{34}' and d_{35}' for thickness shear mode crystal elements become:

From these equations, the piezoelectric constants that have the larger values are obtained in the three orientations for the thickness shear mode crystal elements 30, 31 and 32 illustrated in Figs. 5, 6 and 7. In the crystal elements 30, 31, 32 of Figs. 5, 6 and 7, the frequency is controlled mainly by the relatively thin thickness dimension T, and the major faces thereof may be of square or rectangular shape as illustrated in

Figs. 5, 6 and 7, or of circular or other shape if desired.

As illustrated in Fig. 5, the crystal element 30 has one pair of its edges along or nearly along the X axis, the rectangular major faces thereof and the normal Z' to the major faces being inclined at an angle of 45 degrees or nearly 45 degrees with respect to the Y and Z axes, which corresponds to the orientation angles, expressed in terms of the convention illustrated in Fig. 2 of $\varphi=0$ degrees, $\theta=45$ degrees and $\psi=0$ degrees. Introducing these angular values in the preceding equation, the value of the piezoelectric driving constant d_{34}' for the thickness shear mode crystal element 30 of Fig. 5 becomes:

$$d_{34}' = \left(\frac{d_{14} + d_{36}}{2} \right) \sin 2\theta = \frac{d_{14} + d_{36}}{2}$$

The crystal element 31 of Fig. 6 has one edge along or nearly along the Y axis, the rectangular major faces and the normal Z' to the major faces being inclined at an angle of 45 degrees or nearly 45 degrees with respect to the X and Z axes, which corresponds to the orientation angles of $\varphi=90$ degrees, $\theta=45$ degrees and $\psi=0$ degrees as expressed in terms of the convention illustrated in Fig. 2. Introducing these angular values in the foregoing general equation for d_{34}' , the value of the piezoelectric constant d_{34}' controlling the thickness shear mode of motion in the crystal element 31 of Fig. 6 becomes:

$$d_{34}' = - \left(\frac{d_{14} + d_{36}}{2} \right) \sin 2\theta = - \left(\frac{d_{14} + d_{36}}{2} \right)$$

The crystal element 32 of Fig. 7 has one edge along or nearly along the Z axis, the rectangular major faces and the normal Z' to the major faces being inclined at an angle of 45 degrees or nearly 45 degrees with respect to the X and Y axes, which corresponds to the orientation angles of $\varphi=45$ degrees, $\theta=90$ degrees and $\psi=90$ degrees as expressed in terms of the angles illustrated in Fig. 2. Introducing these angular values in the foregoing general equation for d_{34}' , the value of the piezoelectric constant d_{34}' controlling the thickness shear mode of motion in the crystal element 32 of Fig. 7 becomes:

$$d_{34}' = d_{14}$$

Values of the piezoelectric constants for all other angular orientations of thickness shear mode crystal elements may be similarly calculated from the foregoing general equation for d_{34}' .

The three thickness shear mode crystal elements 30, 31 and 32 of Figs. 5, 6 and 7, respectively, when constructed from crystalline ammonium dihydrogen phosphate, have frequency constants of about 1040, 1040, and 1015, respectively, expressed in kilocycles per second per millimeter of thickness dimension T, and have temperature coefficients of frequency of about -308, -308, and -184, respectively, expressed in parts per million per degree centigrade.

As an illustrative example of the characteristics of thickness shear mode crystals, an ammonium dihydrogen phosphate crystal element 30 of Fig. 5 having its width W or length L along the X axis and having its thickness axis T inclined 45 degrees from the Z axis and having a length L, a width W and a thickness T of about 1.25, 1.25 and 0.103 centimeters, respectively, has a fundamental thickness shear mode resonant frequency of about 1010 kilocycles per second and a frequency constant f/τ of about 1040 kilocycles per second per millimeter of thickness

dimension T for its fundamental thickness shear mode frequency, a shear elastic constant

$$c_{44}^{E'} = \frac{c_{44}^E + c_{66}^E}{2} = \text{about } 7.79 \times 10^{10}$$

a temperature coefficient of thickness shear mode frequency of about -308 parts per million per degree centigrade, and a temperature coefficient of about -666 for its shear elastic constant $c_{44}^{E'}$.

As another illustrative example of the characteristics of a thickness shear mode crystal element, an ammonium dihydrogen phosphate crystal element 32 of Fig. 7 having its width W or length L along the Z axis and having its thickness axis T inclined 45 degrees from the X axis and having a length L, a width W and a thickness T of about 1.848, 1.840 and 0.256 centimeters, respectively, has a fundamental resonant frequency of 396.81 kilocycles per second and a frequency constant f/τ of about 1015 kilocycles per second per millimeter of thickness T for its fundamental thickness shear mode frequency, a shear elastic constant c_{44}^E of about 7.42×10^{10} , a temperature coefficient of thickness shear mode frequency of about -184 parts per million per degree centigrade, and a temperature coefficient of about -388 for its shear elastic constant c_{44}^E .

The thickness shear mode crystal element 30 of Fig. 5 having its width W or length L along the X axis and having its thickness axis T inclined at an angle of about 45 degrees from the Y and Z axes, corresponding to angles in Fig. 2 of $\psi=0$ degrees, $\varphi=90$ degrees and $\theta=45$ degrees, is controlled by the elastic constant:

$$c_{44}^{E'} = c_{44}^E [\sin^2 \theta] + c_{66}^E [\cos^2 \theta] = \frac{c_{44}^E + c_{66}^E}{2}$$

The thickness shear mode crystal element 32 of Fig. 7 having its width W or length L along the Z axis and having its thickness axis T inclined at an angle of 45 degrees from the X and Y axes, corresponding to angles in Fig. 2 of $\theta=90$ degrees, $\psi=0$ degrees and $\varphi=45$ degrees, is controlled by the elastic constant c_{55}^E , and for all angles of rotation φ about the Z axis, the thickness shear modulus is given by:

$$c_{55}^{E'} = c_{44}^E [\cos^2 \varphi + \sin^2 \varphi] = c_{44}^E$$

The thickness shear mode frequency f of the crystal element 30 of Fig. 5, 31 of Fig. 6 and 32 of Fig. 7 is given by the following equations, respectively:

$$f = \frac{1}{2T} \sqrt{\frac{c_{44}^{E'}}{\rho}} = \frac{1}{2T} \sqrt{\frac{c_{44}^E + c_{66}^E}{2\rho}}$$

$$f = \frac{1}{2T} \sqrt{\frac{c_{44}^{E'}}{\rho}} = \frac{1}{2T} \sqrt{\frac{c_{44}^E + c_{66}^E}{2\rho}}$$

$$f = \frac{1}{2T} \sqrt{\frac{c_{55}^{E'}}{\rho}} = \frac{1}{2T} \sqrt{\frac{c_{44}^E}{\rho}}$$

where:

T is the thickness T in millimeters

ρ is the density which in the case of ammonium dihydrogen phosphate is about 1.8

$c_{44}^{E'}$, c_{44}^E , and $c_{55}^{E'}$ is the corresponding shear-
ing elastic constant

The thickness shear mode crystal elements 30, 31 and 32 of Figs. 5, 6 and 7 may be adapted to vibrate alone or simultaneously in two thickness shear modes of motion, one being the fundamental thickness shear mode and the other the second thickness shear mode, in the manner as disclosed in W. P. Mason Patent 2,303,375 dated December 1, 1942. Both the first and second

shear mode frequencies are controlled mainly by the thickness dimension T of the crystal element and vary inversely as the value of the thickness dimension T of the crystal element.

It will be understood that the crystal elements illustrated in Figs. 3 to 7 represent some orientations or cuts for which the piezoelectric constants are advantageous. In addition, some of these cuts possess advantageous elastic constants with zero coupling constants whereby the desired mode of motion may be free from coupling with undesired or extra modes of motion, as pointed out hereinbefore in connection with the face mode crystal elements 5 to 10 illustrated in Fig. 3.

In the case of the contour or face modes of motion of the crystal elements 5 to 10 of Fig. 3, the coupling between (a) the face shear mode of motion, which is an XY' shear mode of motion since the thickness T is always taken along the Z' axis as illustrated in Fig. 2, and (b) the length L longitudinal mode of motion, which is an $X'x$ mode of motion since the length L is always taken along the X' axis as illustrated in Fig. 2, is governed by the S_{16}' elastic constant. In terms of the direction cosines, the value of the elastic constant S_{16}' is given by the equation:

$$S_{16}' = 2S_{11}(l_1^2 l_2 + m_1^2 m_2) + (2S_{12} + S_{66})l_1 m_1 \\ (l_1 m_2 + m_1 l_2) + (2S_{13} + S_{56})n_1 \\ [n_1(l_1 l_2 + m_1 m_2) + n_2(l_1^2 + m_1^2)]$$

For the case of a face mode Z -cut crystal element having its plate normal Z' along the Z axis, $\theta=0$ degrees, $\psi=0$ degrees, and the direction cosines are given by:

$$l_1 = \cos \varphi, l_2 = -\sin \varphi, l_3 = 0, m_1 = \sin \varphi, \\ m_2 = \cos \varphi, m_3 = 0, n_1 = 0, n_2 = 0, n_3 = 1$$

Introducing these values into the foregoing equation for S_{16}' , the value of the coupling constant S_{16}' becomes:

$$S_{16}' = \left(-S_{11} + S_{12} + \frac{S_{66}}{2} \right) \sin 2\varphi \cos 2\varphi$$

which goes to zero when $\varphi=0$ degrees, $\varphi=45$ degrees and $\varphi=90$ degrees.

The first and last angles mentioned of $\varphi=0$ degrees and $\varphi=90$ degrees represent the Z -cut face shear mode crystal element 7 of Fig. 3 having the edges thereof parallel to the X and Y axes, to obtain a zero coupling coefficient S_{16}' and hence no coupling of its desired face shear mode of motion with the longitudinal modes of motion.

The φ angle of 45 degrees mentioned represents the face longitudinal mode Z -cut crystal element 10 of Fig. 3 which is excited longitudinally by the d_{31}' piezoelectric constant and which at the φ angle of 45 degrees has a zero coupling coefficient S_{16}' and hence its face longitudinal mode of motion is not coupled to the face shear mode of motion.

For the face mode X -cut and face mode Y -cut crystal elements of Fig. 3 which are excited by the d_{14} piezoelectric constant, the elastic constant S_{16}' coupling the face shear mode of motion therein with the face longitudinal mode of motion thereof becomes:

$$S_{16}' = \sin 2\psi \left[S_{11} \sin^2 \psi + \left(S_{13} + \frac{S_{56}}{2} \right) (\cos^2 \psi - \sin^2 \psi) - S_{33} \cos^2 \psi \right]$$

The value of S_{16}' in the last equation goes to zero when

$$\tan^2 \psi = \frac{S_{33} - S_{13} - \frac{S_{44}}{2}}{S_{11} - S_{13} - \frac{S_{44}}{2}}$$

Introducing the values of the elastic constants for ammonium dihydrogen phosphate in to the last equation, the value of S_{16}' of the foregoing equation becomes zero when $\psi=24^\circ-30'$ for the X -cut and Y -cut crystal elements 8 and 9 of Fig. 3 when the length dimensions L thereof are inclined about $24^\circ 30'$ from the Z axis.

Other angles may be calculated from these equations that give for the more general rotations the value of the coupling constant S_{16}' .

The coupling between the face shear mode of motion and the thickness shear mode of motion is controlled by the elastic constant c_{46}' which is given by:

$$c_{46}' = c_{11}(l_1 l_2^2 l_3 + m_1 m_2^2 m_3) + c_{33} n_1 n_2^2 n_3 + \\ (c_{12} + c_{66}) l_2 m_2 (l_1 m_3 + m_1 l_3) + \\ (c_{13} + c_{44}) [n_2 m_3 (l_1 l_2 + m_1 m_2) + \\ n_1 n_2 (l_2 l_3 + m_2 m_3)] + c_{44} [m_1 m_3 n_2^2 + \\ n_1 n_3 m_2^2 + l_1 l_3 n_2^2 + n_1 n_3 l_2^2] + \\ c_{66} (m_1 m_3 l_2^2 + l_1 l_3 m_2^2).$$

The value of c_{46}' from this equation for the thickness shear mode crystal elements 30 and 31 of Figs. 5 and 6 rotated in effect about the X and Y axes, respectively, is given by:

$$c_{46}' = \left(\frac{c_{66} - c_{44}}{2} \right) \sin 2\theta$$

which goes to a zero value only at $\theta=0$ degrees and $\theta=90$ degrees where no piezoelectric thickness shear driving constant is present. On the other hand, for the orientation of Fig. 7,

$$-l_1 = l_3 = m_1 = m_3 = .707$$

$l_2 = m_2 = n_1 = n_3 = 0$ and $n_2 = 1$, so that c_{46}' vanishes, and this crystal element 32 of Fig. 7 will have no coupling to a face shear mode of motion.

Although this invention has been described and illustrated in relation to specific arrangements, it is to be understood that it is capable of application in other organizations and is, therefore, not to be limited to the particular embodiments disclosed, but only by the scope of the appended claims and the state of the prior art.

What is claimed is:

1. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said major faces being disposed substantially perpendicular to the Z axis of the three mutually perpendicular X , Y and Z axes, and said length axis dimension being inclined at an orientation angle of substantially 45 degrees with respect to said X and Y axes, said orientation angle being a value corresponding to the maximum value of piezoelectric constant for

said longitudinal mode of motion, to the maximum value of said motion along said length axis dimension, and substantially to zero value of coupling of said desired longitudinal motion with the undesired face shear mode of motion in said crystal body.

2. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said major faces being disposed substantially perpendicular to the Z axis of the three mutually perpendicular X, Y and Z axes, and said length axis dimension being inclined at an orientation angle of substantially 45 degrees with respect to said X and Y axes, said orientation angle being a value corresponding to the maximum value of piezoelectric constant for said longitudinal mode of motion, to the maximum value of said motion along said length axis dimension, and substantially to zero value of coupling of said desired longitudinal motion with the undesired face shear mode of motion in said crystal body, said length axis dimension expressed in centimeters being a value roughly of the order of 160 divided by the value of said frequency expressed in kilocycles per second.

3. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said major faces being disposed substantially perpendicular to the Z axis of the three mutually perpendicular X, Y and Z axes, and said length axis dimension being inclined at an orientation angle of substantially 45 degrees with respect to said X and Y axes, said orientation angle being a value corresponding to the maximum value of piezoelectric constant for said longitudinal mode of motion, to the maximum value of said motion along said length axis dimension, and substantially to zero value of coupling of said desired longitudinal motion with the undesired face shear mode of motion in said crystal body, and means comprising electrodes disposed adjacent said major faces and applying an electric field to said crystal body substantially perpendicular to said major faces for operating said crystal body in said longitudinal mode of motion along said length axis dimension with substantially no coupling to said face shear mode of motion.

4. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension

thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said major faces being disposed substantially perpendicular to the Z axis of the three mutually perpendicular X, Y and Z axes, and said length axis dimension being inclined at an orientation angle of substantially 45 degrees with respect to said X and Y axes, said orientation angle being a value corresponding to the maximum value of piezoelectric constant for said longitudinal mode of motion, to the maximum value of said motion along said length axis dimension, and substantially to zero value of coupling of said desired longitudinal motion with the undesired face shear mode of motion in said crystal body, said length axis dimension expressed in centimeters being a value roughly of the order of 160 divided by the value of said frequency expressed in kilocycles per second, and means comprising electrodes disposed adjacent said major faces and applying an electric field to said crystal body substantially perpendicular to said major faces for operating said crystal body in said longitudinal mode of motion along said length axis dimension with substantially no coupling to said face shear mode of motion.

5. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said width axis dimension being disposed substantially in the plane formed by the X and Y axes of the three mutually perpendicular X, Y and Z axes and inclined at an angle of substantially 45 degrees with respect to said X and Y axes, the normal axis perpendicular to said major faces being disposed at one of the angles in the range of angles substantially from 0 to 70 degrees with respect to said Z axis, said last-mentioned angle being a value corresponding to a controlled or reduced value of temperature coefficient of frequency for said longitudinal length mode of motion substantially as given by a point on the curve A of Fig. 3A for said value of said last-mentioned angle.

6. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said width axis dimension being disposed substantially in the plane formed by the X and Y axes of the three mutually perpendicular X, Y and Z axes and inclined at an angle of substantially 45 degrees with respect to said X and Y axes, the normal axis perpendicular to said major faces being disposed at one of the angles in the range

of angles substantially 30 to 70 degrees with respect to said Z axis, said last-mentioned angle being a value corresponding to a controlled or reduced value of temperature coefficient of frequency for said longitudinal length mode of motion substantially as given by a point on the curve A of Fig. 3A for said value of said last-mentioned angle.

7. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said width axis dimension being disposed substantially in the plane formed by the X and Y axes of the three mutually perpendicular X, Y and Z axes and inclined at an angle of substantially 45 degrees with respect to said X and Y axes, the normal axis perpendicular to said major faces being disposed at one of the angles in the range of angles substantially from 0 to 70 degrees with respect to said Z axis, said last-mentioned angle being a value corresponding to a controlled or reduced value of temperature coefficient of frequency for said longitudinal length mode of motion substantially as given by a point on the curve A of Fig. 3A for said value of said last-mentioned angle, and means comprising electrodes disposed adjacent said major faces and applying an electric field to said crystal body substantially perpendicular to said major faces for operating said crystal body in said longitudinal mode of motion along said length axis dimension at said frequency having said selected value of said temperature coefficient of frequency as given by said point on said curve.

8. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said width axis dimension being disposed substantially in the plane formed by the X and Y axes of the three mutually perpendicular X, Y and Z axes and inclined at an angle of substantially 45 degrees with respect to said X and Y axes, the normal axis perpen-

dicular to said major faces being disposed at one of the angles in the range of angles substantially 30 to 70 degrees with respect to said Z axis, said last-mentioned angle being a value corresponding to a controlled or reduced value of temperature coefficient of frequency for said longitudinal length mode of motion substantially as given by a point on the curve A of Fig. 3A for said value of said last-mentioned angle, and means comprising electrodes disposed adjacent said major faces and applying an electric field to said crystal body substantially perpendicular to said major faces for operating said crystal body in said longitudinal mode of motion along said length axis dimension at said frequency having said selected value of said temperature coefficient of frequency as given by said point on said curve.

9. Piezoelectric crystal apparatus comprising an ammonium dihydrogen phosphate piezoelectric crystal body adapted for longitudinal lengthwise motion at a frequency dependent mainly upon the value of the longest or length axis dimension thereof, said value of said elongated length axis dimension corresponding to the value of said frequency, said crystal body having substantially rectangular shaped major faces, the width axis dimension of said major faces being substantially perpendicular to said length axis dimension thereof, and the ratio of said width axis dimension with respect to said length axis dimension being a value less than 0.6, said width axis dimension being disposed substantially in the plane formed by the X and Y axes of the three mutually perpendicular X, Y and Z axes and inclined at an angle of substantially 45 degrees with respect to said X and Y axes, the normal axis perpendicular to said major faces being disposed at an angle of substantially 60 degrees with respect to said Z axis, said last-mentioned angle being a value corresponding to a controlled or reduced value of temperature coefficient of frequency for said longitudinal length mode of motion substantially as given by a point on the curve A of Fig. 3A for said value of said last-mentioned angle, and means comprising electrodes disposed adjacent said major faces and applying an electric field to said crystal body substantially perpendicular to said major faces for operating said crystal body in said longitudinal mode of motion along said length axis dimension at said frequency having said selected value of said temperature coefficient of frequency as given by said point on said curve.

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