RELAY DESIGN CALCULATOR

Filed Aug. 23, 1944 3 Sheets-Sheet 1

FIG. 1

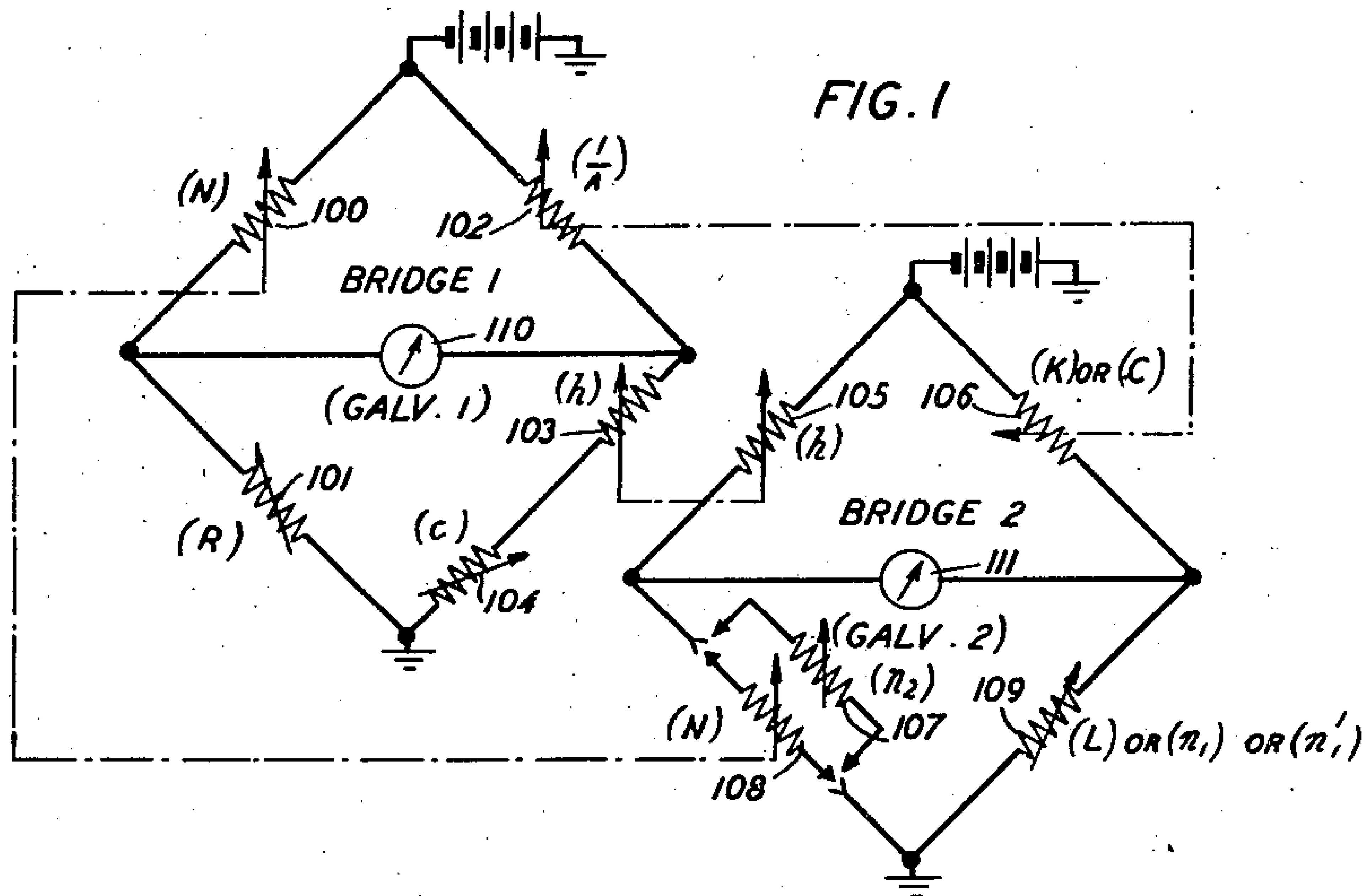

FIG. 2

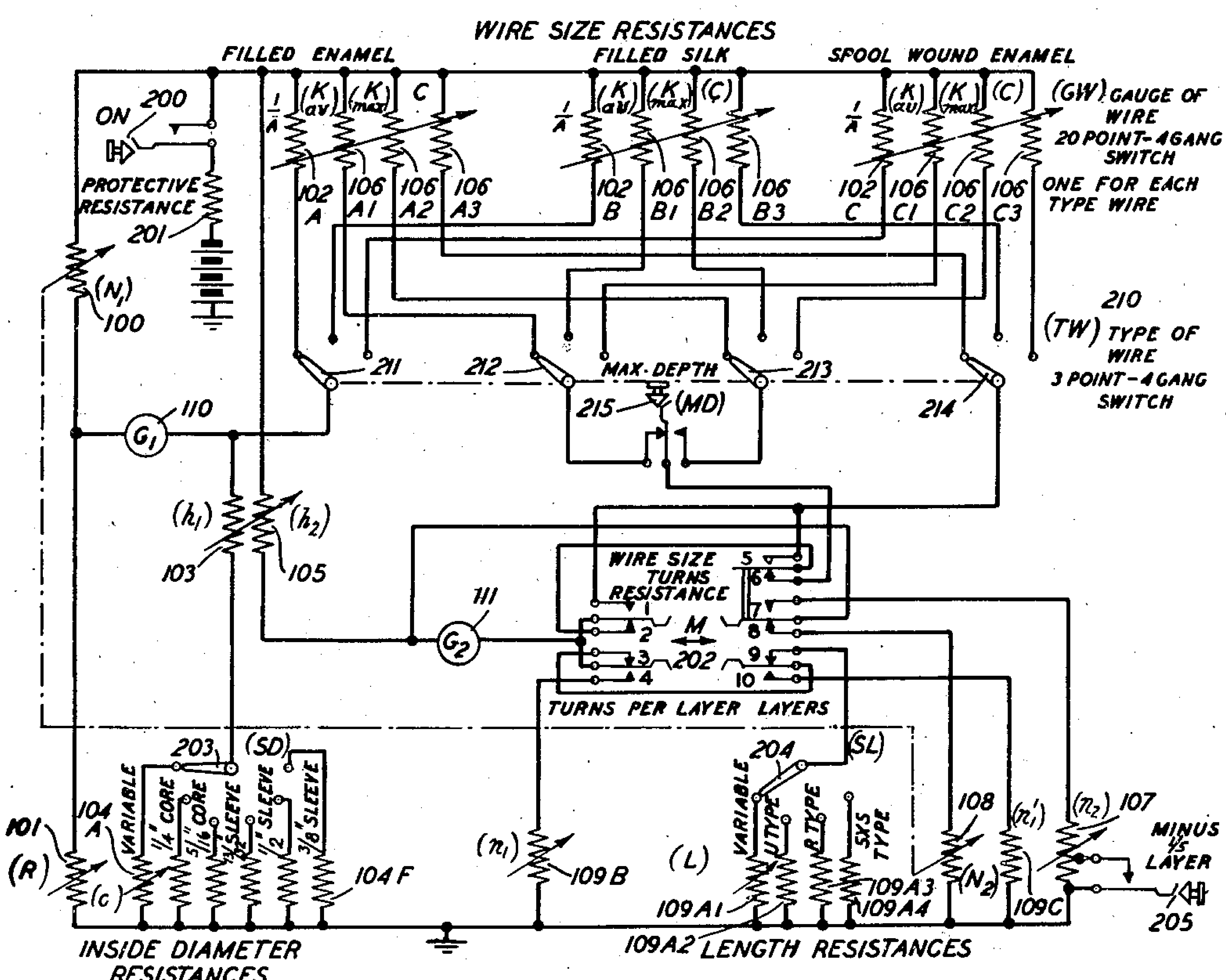

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FIG. 4

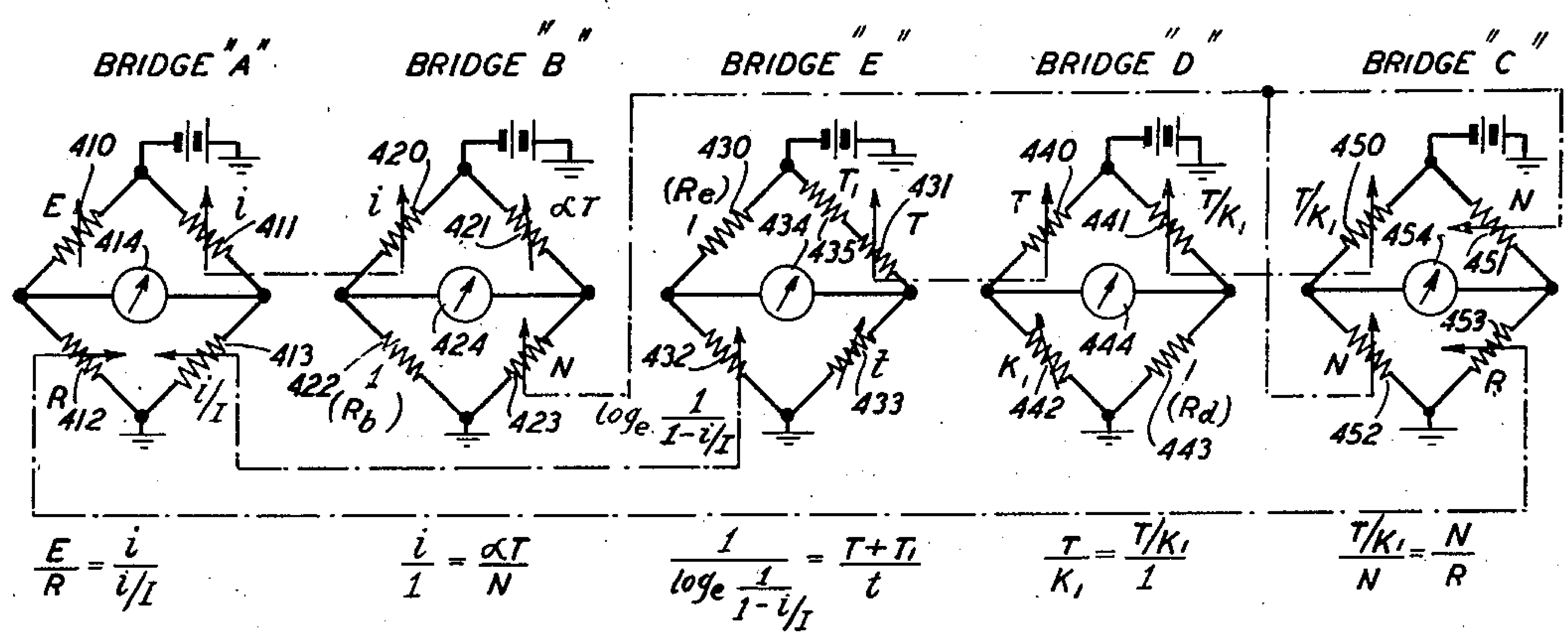
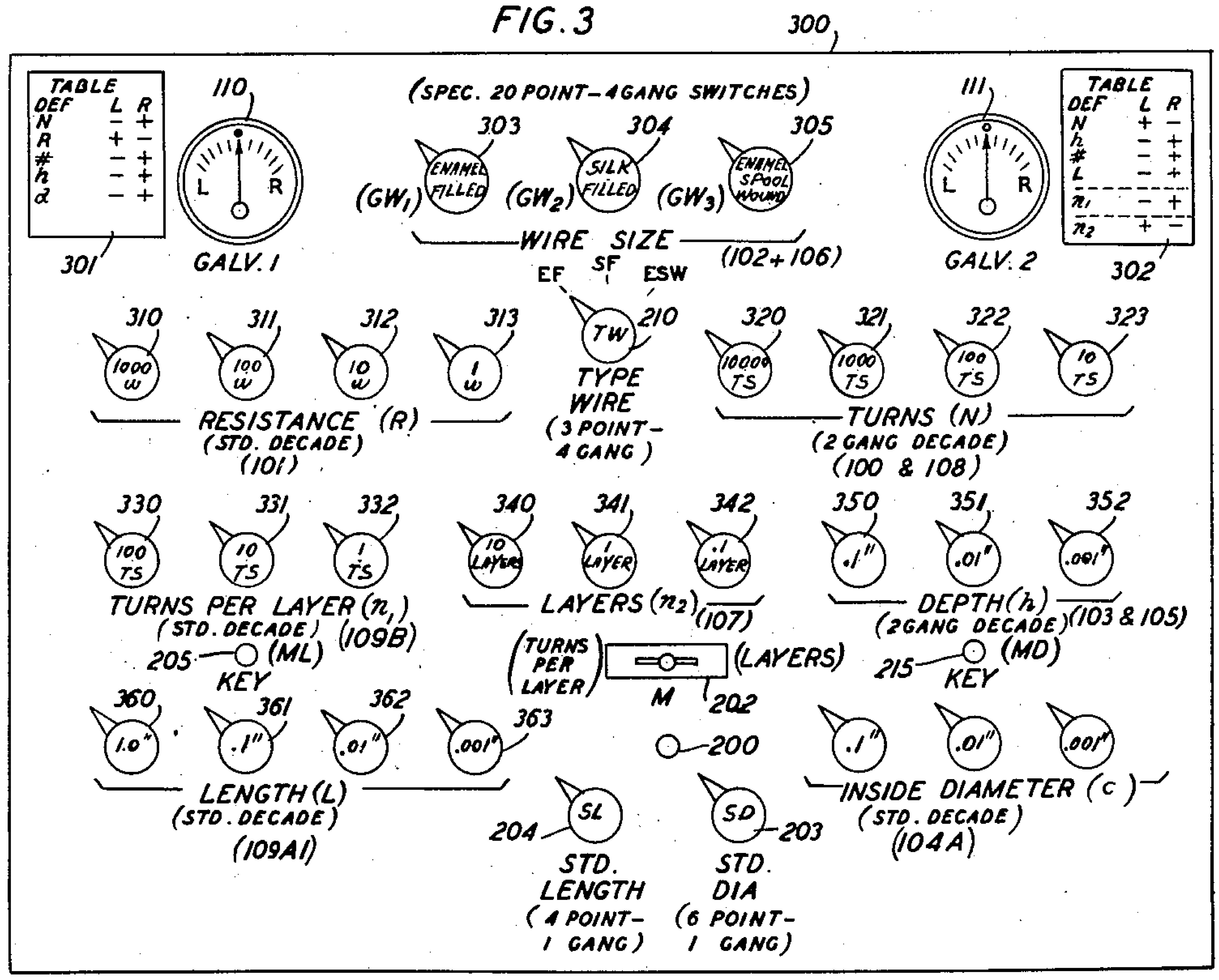


FIG. 3



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Dec. 2, 1947. W. KEISTER 2,431,696

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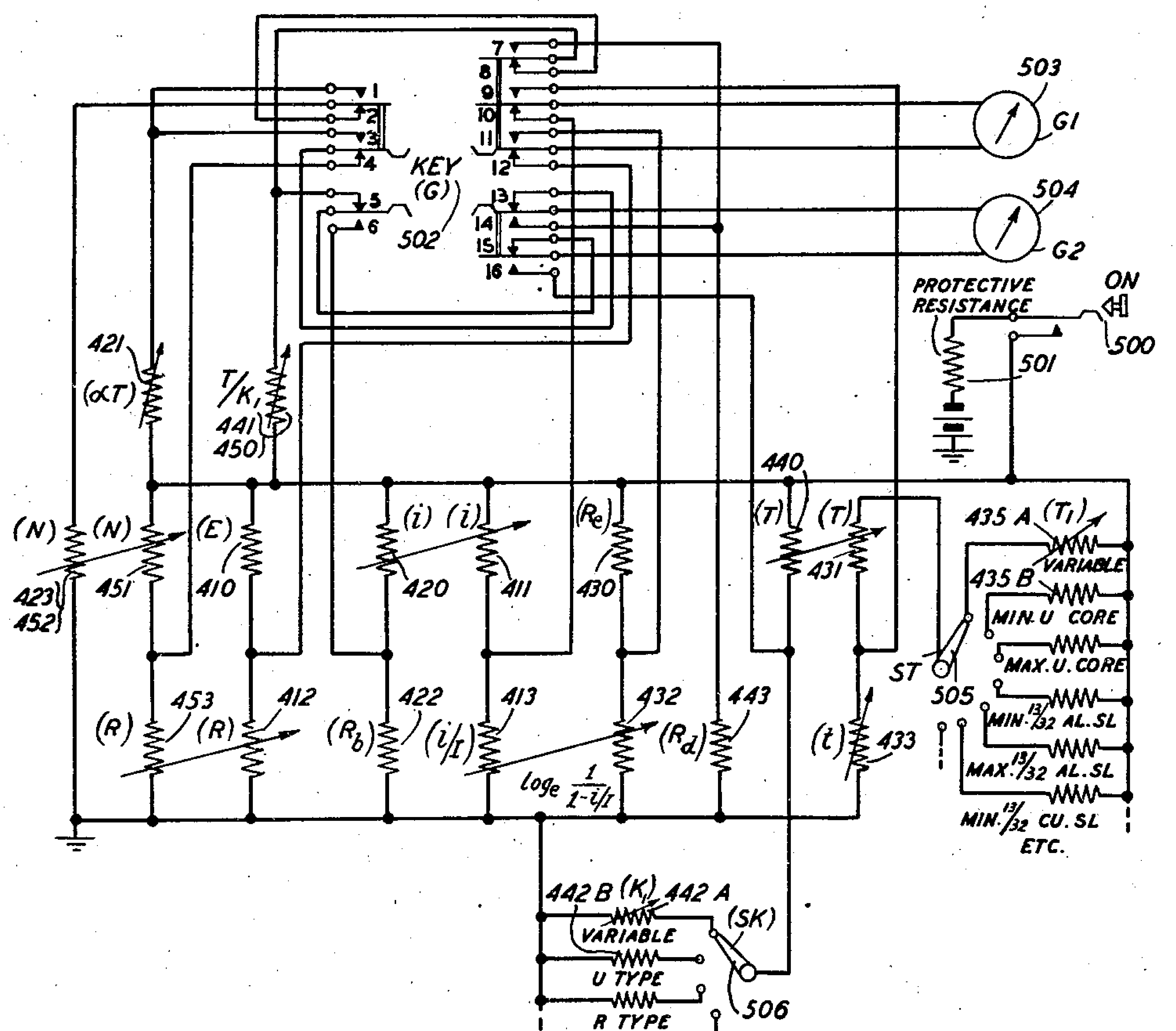
FIG. 5



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UNITED STATES PATENT OFFICE

2,431,696

RELAY DESIGN CALCULATOR

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11 Claims. (Cl. 235—61)

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This invention relates to the design of relays and has for its object to facilitate the calculations necessary for such design.

A large number of variables must be considered both in designing the winding of a relay and in determining the operating time thereof after a tentative design has been selected.

For use in the design of relays, tables and curves have been worked out and formulae set up which relate the data contained therein in such a way that the designer may determine the dimensions necessary for an operative relay under given circuit conditions. However, this determination necessitates considerable calculation because the data and formulae cannot be of a positive nature and the proper winding must be obtained by balancing the various factors involved.

Similarly the operating time of the relay can be calculated directly when the design of the relay and the circuit conditions are known, but when a relay is to be designed to operate in a given time, large numbers of calculations may be required before a design is arrived at which will satisfy all of the requirements.

In accordance with the present invention, means is provided for making the mathematical calculations necessary for relay winding design by electrical means. More specifically, a multiple Wheatstone bridge is provided whereby the bridge when balanced in connection with given quantities will indicate the corresponding values of the remaining variables required for the design of that relay winding. In the multiple Wheatstone bridge forming the subject-matter of this invention at least two of the legs are varied simultaneously in a plurality of the individual bridges. According to one form of the invention, the circuits are so arranged that the same galvanometer may serve a plurality of individual bridges.

These and other features of the invention will be more clearly understood from a consideration of the following description read in connection with the drawings in which:

Fig. 1 shows the basic relationship between the physical quantities employed in designing relay windings;

Fig. 2 shows the practical circuits employed in such a bridge; while,

Fig. 3 shows a typical panel board for such a bridge;

Fig. 4 shows a theoretical multiple bridge for determining the time of operation of relays; and

Fig. 5 shows a practical circuit employed in a bridge of the type of Fig. 4.

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Structurally, the design of a relay winding involves the size of the space in which the winding is to be located and the size of the wire. These functions in turn involve the diameter of the core, the length of the core on which the wire is to be wound and the depth of the winding as well as the gauge and insulation of the wire itself, which determines the number of turns and number of layers which may be placed in the space available. Electrically, the voltage and current being used and the resistivity of the wire must be considered.

The principal formulae used in the calculations necessary for relay winding design are as follows:

Relationship between turns and fullness of winding

The number of turns of wire which may be wound in one layer may be expressed by the equation

$$n_1=\frac{L}{B} \qquad (1)$$

where

L=length of coil in inches
B=effective breadth of one wire.

The number of layers which may be wound in a given space may be expressed by the equation

$$n_2=\frac{h}{C} \qquad (2)$$

where

h=depth of coil space
C=effective depth of one wire including insulating filler.

The number of turns which may be used may be expressed by the equation

$$N=n_1n_2=\frac{L}{B}\times\frac{h}{C} \qquad (3)$$

Calculations relating to resistance

To obtain resistance when the number of turns, gauge of wire and dimensions of winding space are given:

From the formulas set forth at page 504 of the Standard Handbook for Electrical Engineers, seventh edition, the total resistance of a relay winding may be stated

$$R=\rho Nf \qquad (3a)$$

where

ρ=ohms per unit length of the wire, and
f=length of a mean turn in inches.

From geometrical considerations, it is evident that

$$f=\pi\left(\frac{[d+2h]+d}{2}\right)=\pi(h+d) \qquad (3b)$$

where d= diameter of the core.

From Equation 3 the value of h may be derived as

$$h=\frac{NBC}{L}=\frac{NK}{L} \qquad (3c)$$

where $K=B\times C$=effective cross-section of one turn of wire.

Hence Equation 3a may be written

$$R=\pi\rho N\left(\frac{NK}{L}+d\right)$$

A "bulge factor" d_c is sometimes used to allow for the fact that the first layer of the coil may not fit snugly against the core and has the effect of increasing the apparent core diameter. Therefore,

Total resistance R is in accordance with the equation

$$R=AN\left(\frac{NK}{L}+d+d_c\right) \qquad (4a)$$

where

A=resistivity factor for unit length=$\pi\rho$
K=effective cross-section of one turn of wire= $B\times C$
d=diameter of core
d_c=bulge correction to be added to diameter of core.

When the winding includes only whole layers, this calculation may take the form, employing the value of h derived from Equation 2

$$R=AN(n_2C+d+d_c) \qquad (4b)$$

The proper size of wire may be determined from the resistivity factor A which may be determined from one of the following equations:

When the resistance, number of turns and coil dimensions are given

$$A=\frac{R}{N(h+d+d_c)} \qquad (5)$$

When the desired voltage, ampere turns and coil dimensions are given

$$A=\frac{E}{(NI)(h+d+d_c)} \qquad (6)$$

where

E=voltage
I=current in amperes.

When the resistance and coil dimensions are given

$$\frac{A}{K}=\frac{R}{Lh(h+d+d_c)} \qquad (7)$$

The general theory on which is based the use of the Wheatstone bridge for solving mathematical equations may be set forth as follows:

The balance condition for an ordinary Wheatstone bridge is represented by the equations

$$\frac{R_1}{R_2}=\frac{R_3}{R_4} \qquad (8a)$$

or

$$R_1R_4=R_2R_3 \qquad (8b)$$

where R_1, R_2, R_3 and R_4 are the values of the resistances in the four arms of the bridge. If a mathematical equation can be put in the form of either Equations 8a or 8b it can be solved by means of a bridge. It is only necessary to choose "scale factors" and calibrate the various resistances to represent corresponding variables. Then any setting of the resistances which balances the bridge gives concurrent values of the variables.

The "scale factor" is a constant which relates ohms resistance in the bridge to units of the corresponding variable in the mathematical equation. For example, if the resistance R_1 represents a variable X and it is determined that 10 ohms shall represent one unit of X we have

$$R_1=10X \qquad (9a)$$

where R_1 is in ohms. This may be expressed

$$R_1=\mu X \qquad (9b)$$

Here μ is the scale factor of X and in this example μ=10.

Scale factors are of two types. In one case the scale factor is an arbitrary constant chosen to give reasonable values of resistance or to obtain satisfactory balance over a given range of variables. In the other case it is necessary to hold one or more members of the bridge at a fixed value in order to make the bridge Equation 8 conform to a given mathematical formula. This scale factor depends on the value chosen for the fixed resistance.

For scale factors of the first type consider a mathematical equation of the form

$$\frac{W}{X}=\frac{Y}{Z} \qquad (10)$$

It is obvious that the validity of this equation is not affected by the introduction of arbitrary constants μ_1, μ_2, μ_3 and μ_4 in any of the following ways,

$$\frac{\mu_1 W}{\mu_1 X}=\frac{Y}{Z} \qquad (10a)$$

$$\frac{\mu_3 W}{X}=\frac{\mu_3 Y}{Z} \qquad (10b)$$

or, in general

$$\frac{\mu_1\mu_3 W}{\mu_1\mu_4 X}=\frac{\mu_2\mu_3 Y}{\mu_2\mu_4 Z} \qquad (10c)$$

Comparing (10c) with (8a) we see that,

$$R_1=\mu_1\mu_3 W$$
$$R_2=\mu_1\mu_4 X$$
$$R_3=\mu_2\mu_3 Y$$
$$R_4=\mu_2\mu_4 Z$$

Here, $\mu_1\mu_3$ is the scale factor of W, $\mu_1\mu_4$ the scale factor of X, etc., and R_1 . . . R_4 are in units of resistance (ohms).

The second type of scale factor has a value depending on some fixed resistance in the bridge. For example, consider simple multiplication,

$$XY=Z \qquad (11)$$

Introducing a constant, K_1, this may be given the form of the bridge equations (8).

$$XY=K_1\frac{Z}{K_1} \qquad (11a)$$

or

$$\frac{X}{\frac{Z}{K_1}}=\frac{K_1}{Y} \qquad (11b)$$

Type one scale factors may now be introduced in any of several ways as discussed above. For example:

$$\frac{\mu_3 X}{\mu_4 \frac{Z}{K_1}} = \frac{\mu_3 K_1}{\mu_4 Y} \qquad (11c)$$

Referring to (8a), $\mu_3 K_1$ corresponds to R_3 and by choosing an appropriate fixed value R_K for this resistance the value of K_1 can be determined as follows:

$$\mu_3 K_1 = R_K$$

$$K_1 = \frac{R_K}{\mu_3}$$

where R_K is expressed in units of resistance. Now, substituting in (11c)

$$\frac{\mu_3 X}{\mu_3 \mu_4 \left(\frac{Z}{R_K}\right)} = \frac{R_K}{\mu_4 Y} \qquad (11d)$$

$$(\mu_3 X)(\mu_4 Y) = \left(\frac{\mu_3 \mu_4}{R_K} Z\right)(R_K) \qquad (11e)$$

Comparing this with (8b)

$$R_1 = \mu_3 X$$

$$R_4 = \mu_4 Y$$

$$R_2 = \frac{\mu_3 \mu_4}{R_K} Z$$

$$R_3 = R_K$$

In a particular example suppose that X has values between 0.1 and 0.5 and Y has values between 20 and 100 and it is desired to represent X and Y with resistances varying from 1000 to 5000 ohms.

If 1000 ohms is to represent a value of 0.1 for X, then

$$\mu_3 = \frac{1000}{0.1} = 10{,}000$$

Likewise

$$\mu_4 = \frac{1000}{20} = 50$$

Choosing a resistance of 2500 ohms as appropriate for R_K, then (11e) gives,

$$(10{,}000X)(50Y) = \left(\frac{10{,}000 \times 50}{2500} Z\right)(2500)$$

$$(10{,}000X)(50Y) = (200Z)(2500)$$

Comparing this with (8b) and taking the unit of resistance as one ohm.

$R_1 = 10{,}000X$ ohms
$R_4 = 50Y$ ohms
$R_2 = 200Z$ ohms
$R_3 = 2500$ ohms

The values for the bridge for solving (11), $XY = Z$ then may be tabulated as follows:

Resistance	Variable	Scale Factor	Calibration 1 ohm=	Range of Variable	Range of Resistance
					Ohms
R_1	X	10,000	.001	0.1–0.5	1,000–5,000
R_4	Y	50	.02	20–100	1,000–5,000
R_2	Z	200	.005	2–50	400–10,000
R_3	2500 ohms Fixed Resistance				

An equation which includes addition in conjunction with multiplication can be solved by connecting two variable resistances in series in one or more arms of the bridge. For example, a formula such as

$$X(Y+Z) = W \qquad (12)$$

may be handled by introducing a constant, K_1; as in (11),

$$X(Y+Z) = K_1 \frac{W}{K_1}$$

$$\frac{X}{\frac{W}{K_1}} = \frac{K_1}{Y+Z}$$

Further scale factors may be introduced and both Y and Z will have the same scale factors.

Quadratic equations such as

$$x^2 + Bx - C = 0 \qquad (13)$$

may be solved by using two resistances calibrated in terms of x and coupled together to be controlled by the same set of knobs. Equation 13 must first be put in the form of (8), thus

$$x^2 + Bx - C = 0$$

$$x(x+B) = K_1 \frac{C}{K_1}$$

$$\frac{x}{\frac{C}{K_1}} = \frac{K_1}{(x+B)} \qquad (13a)$$

Scale factors may now be introduced

$$\frac{\mu_1 \mu_3 x}{\mu_1 \mu_4 \frac{C}{K_1}} = \frac{\mu_2 \mu_3 K_1}{\mu_2 \mu_4 (x+B)}$$

Now, choose an appropriate fixed resistance R_K and set $\mu_2 \mu_3 K_1 = R_K$, then,

$$K_1 = \frac{R_k}{\mu_2 \mu_3}$$

Hence,

$$\frac{\mu_1 \mu_3 x}{\frac{\mu_1 \mu_4 \mu_2 \mu_3}{R_K} C} = \frac{R_K}{\mu_2 \mu_4 (x+B)} \qquad (13b)$$

Comparing (13b) with (8a) gives the values and calibrations of the four arms of the bridge. Note that the scale factors for the two appearances of x are not the same for each appearance and the coefficient B has the same scale factor as one appearance of x.

Using the methods outlined above, a dual Wheatstone bridge has been developed for solving the winding formula (4a) which may be repeated in the form

$$R = AN\left(\frac{NK}{L} + c\right) \qquad (14)$$

where $c = d + d_c$ = inside diameter of coil in inches including bulge allowance.

From Equation 3 it is apparent that the depth, h, of the winding in inches is given by,

$$h = \frac{NK}{L} \qquad (15)$$

So that (14) becomes,

$$R = AN(h+c) \qquad (16)$$

or

$$\frac{N}{R} = \frac{\frac{1}{A}}{h+c} \qquad (16a)$$

This is of the form (8a) and may be solved by means of a bridge.

The value of h can be determined by a second independent bridge because formula (15) can be expressed as

$$\frac{h}{N}=\frac{K}{L} \qquad (15a)$$

If the two h and two N resistances of (15a) and (16a) are mechanically coupled, a dual bridge results as shown schematically in Fig. 1 and, since the resistance representing

$$\frac{1}{A} \text{ and } K$$

are dependent on wire size they may also be mechanically coupled.

A form wound coil is wound to even layers and in specifications of such coils it is usual to give the turns per layer n_1, and the number of layers, n_2. These calculations may be made by modifications of bridge 2 as follows:

The equation of bridge 2 is,

$$\frac{h}{N}=\frac{K}{L} \qquad (15a)$$

Values of n_1 and n_2 may be obtained from the following equations:

$$\frac{h}{N}=\frac{C}{n_1} \qquad (17)$$

with resistance C replacing K and n_1 replacing L;

$$\frac{h}{n_2}=\frac{C}{1} \qquad (18)$$

with C replacing K, n_2 replacing N and L replaced by a fixed resistance which may be designated n_1'.

A circuit for the dual bridge of Fig. 1 is shown in Fig. 2 and the arrangement of the control dials is shown in Fig. 3, the same reference characters being used in Figs. 1 and 2. The resistances which represent the values of N, R, n_1, n_2, h, L and c are in the form of decade resistances for convenience in reading the setting. The dials which control the resistances, representing N and h, vary two sets of resistances simultaneously, while the resistances which represent

$$\frac{1}{A},\ K_{av},\ K_{max} \text{ and } C$$

are varied simultaneously by a single dial. Since the effective size of the wire has a different range for different types of insulation, a switch 210 is provided which may be set in any one of three positions to render a different set of resistances available in accordance with the insulation to be used. A switch 203 is used to include one of the resistances 104A to 104F in a circuit depending on the inside diameter of the coil to be designed. While it would be possible to use the variable resistance 104A for all calculations, certain core sizes recur so frequently as to make it convenient to use fixed resistances for their values. Similarly, a number of fixed resistances 109A2 to 109A4 are used to correspond to the length of the most common types of relays with a variable resistance 109A1 for unusual types, the switch 204 serving to include the desired resistance.

Key 200 when operated supplies battery to the circuits through resistance 201. Key 215 is used to include either the average or the maximum value of the resistance corresponding to K in the circuit. When key 215 is normal, the average value of K is employed while with key 215 operated the maximum value of K is employed. Key 202 is a three-way key. When in its normal position it serves to place bridge 2 in condition to solve the fundamental equation of the bridge,

that is, Equation 14. With key 202 moved to the left it arranges the circuit to solve the equation involving turns per layer (n_1) that is Equation 17 and with key 202 moved to the right the bridge is arranged to solve Equation 18 which involves the number of layers (n_2). It is customary with certain types of relays to allow one-fifth layer for possible variation and key 205 is used to make this allowance when required.

The solution of a winding problem by the use of the dual bridge of Fig. 2 requires that both bridges be balanced. In most cases this is not difficult since several of the variables are known and these values can be set and only the remaining dial need be manipulated to obtain a balance. For convenience in adjusting the bridges, tables 301 and 302 located on the face of the panel show the adjustments which will tend to bring the bridge into balance when the deflections of the galvanometers are to the right or the left of zero.

A typical problem will indicate the manner in which the bridge may be used. Assume first that it is desired to know the resistance of the coil which will result from the use of a given number of turns of wire of a given size in a coil of known dimensions. None of the keys 200, 215 or 205 is operated. Switches 203 and 204 are set on the proper dimensions of coils, switch 210 is set on the type of wire, for example, in the position shown, which corresponds to filled enamel wire, and the switch 303 is positioned to the proper gauge thereby simultaneously adjusting resistances 102A of the bridge 1 and resistances 106A1, 106A2 and 106A3 which belong to the bridge 2. Dials 320 to 323 are then operated to positions indicating the desired number of turns resulting in the simultaneous adjustment of resistance 100 of bridge 1 and resistance 108 of bridge 2. Key 200 is then operated to supply battery to the bridges and the dials 350, 351 and 352, corresponding to depth of coil, are adjusted until bridge 2 is balanced, at which time Equation 15a will be satisfied. Thereafter dials 310 to 313, corresponding to resistance, are adjusted until bridge 1 is balanced. The values of the resistance may then be read from the corresponding dials. If, after the solution has been obtained as above, it is desired to find the number of layers necessary to produce the desired winding, the dial and switches are left in the positions obtained and key 202 is thrown to its right-hand position. In this position it will be apparent that resistance 107 is substituted for resistance 108 and resistance 106A3 for resistance 106A1, while the fixed resistance 109C (n_1') is substituted for the variable resistance 109A1. Adjustment of the dials 340, 341, 342, corresponding to (n_2), until bridge 2 is balanced will indicate the number of layers by the positions assumed by the dials. On the other hand if it is desired to know the number of turns per layer, key 202 is thrown to the left substituting resistance 109B (n_1) for one of the resistances 109A (L) and resistance 106A3 (C) for resistance 106A1 (K_{av}). Adjustment of dials 330, 331, and 332, corresponding to (n_1), until bridge 2 is balanced will give an indication of the number of turns per layer under the conditions set up.

A form of the multiple Wheatstone bridge which may be used in determining the time of operation of relays is shown in Figs. 4 and 5. A large number of variables must be considered in determining the operating time of a relay. The determination of the operating time is fairly simple when the relay design and circuit conditions are known but the design of a relay to operate in a given time is difficult because of this large number of variables. For example, if only the spring load and the circuit voltage are known, a relay may be designed to meet a given operating time requirement by assuming a resistance and determining the corresponding number of turns but this may result in a relay which is impossible to obtain in a standard relay structure. Other resistance values must then be tried until a practical relay is found or it is determined that it is impossible to meet a given time requirement. With the multiple bridge of Figs. 4 and 5 many of the individual calculations are made simultaneously and automatically so that trials may be made quickly and the complete range of possible coils explored to obtain the most satisfactory ones in a reasonable length of time.

The fundamental equations employed in the determination of relay operating time have been derived in the Journal of the Institute of Electrical Engineers, volume 66, No. 376, April 1928, are as follows:

$$t=(T+T_1)\log_e\frac{1}{1-\frac{i}{I}} \qquad (19)$$

where

t=operating time of the relay
T=time constant of the relay coil
T_1=time constant of core structure including sleeve, if any
i=current on which the relay just operates after adjustment
I=circuit current

$$T=\frac{K_1N^2}{R} \qquad (20)$$

where

K_1=inductance constant of the relay coil

$$\frac{i}{I}=\frac{i}{\frac{E}{R}}=\frac{Ri}{E} \qquad (21)$$

Equations 19, 20 and 21 may be combined into

$$t=\left(\frac{K_1N^2}{R}+T_1\right)\log_e\frac{1}{1-\frac{R_i}{E}} \qquad (22)$$

These equations may be restated as the following bridge type equations

$$\frac{1}{\log_e\frac{1}{1-\frac{i}{I}}}=\frac{T+T_1}{t} \qquad (23)$$

$$\frac{\frac{T}{K_1}}{N}=\frac{N}{R} \qquad (24)$$

$$\frac{E}{R}=\frac{i}{\frac{i}{I}} \qquad (25)$$

Two other equations are of interest

$$\frac{i}{1}=\frac{\alpha T}{N} \qquad (26)$$

where

αT=ampere turns corresponding to current i,

$$\frac{T}{K_1}=\frac{\frac{T}{K^1}}{1} \qquad (27)$$

i may be derived from T and N by means of Equation 26; the value of the ratio of i/I may be determined from the quantities E, R and i by means of Equation 25; the ratio T/K_1 may be derived from N and R by means of Equation 24 and the value of T derived from that ratio by means of Equation 27. Equation 23 combines the results secured from the other four equations to obtain the operating time t. These five equations are applied to the five bridges A, B, C, D and E forming the multiple bridge of Fig. 4.

The arrangement shown in Fig. 4 requires five galvanometers 414, 424, 434, 444 and 454. It would, of course, be possible to use a single galvanometer with a switch to connect it into any one of the bridges, in which case the bridges would be balanced one at at ime. A more satisfactory arrangement is that shown in Fig. 5 which employs two galvanometers 503 and 504 permitting two bridges to be balanced simultaneously.

The use of galvanometers 503 and 504 is controlled by a three-position switch 502. When this switch is normal the galvanometers are associated with bridges A and C, galvanometer 503 being connected over contact 12 of key 502 to the junction between resistances 410 and 412 representing the quantities E and R and over contact 10 of key 502 to the junction between resistances 411 and 413 representing the quantities

$$i \text{ and } \frac{i}{I}$$

respectively, galvanometer 504 being connected over contacts 15 and 5 of key 502 to the junction between resistances 450 and 452 representing quantities

$$\frac{T}{K_1} \text{ and } N$$

and over contacts 13 and 4 of key 502 to the junction between resistances 451 and 453 representing the quantities N and R. With the left-hand springs of key 502 operated, galvanometer 503 is still connected in bridge A but galvanometer 504 is transferred to bridge B being connected over contacts 13 and 3 of key 502 to the junction between resistances 421 and 423 representing the quantities αT and N, respectively, and over contacts 15 and 6 of key 502 to the junction between resistances 420 and 422 representing the quantities i and R_b, respectively. When the right-hand springs of key 502 are operated, galvanometers 503 and 504 are connected in bridges D and E, galvanometer 503 being connected over contact 9 of key 502 to the junction between resistances 431 and 433 representing the quantities $T+T_1$ and t and over contact 11 of key 502 to the junction between resistances 430 and 432 representing the quantities

$$R_e \text{ and } \log_e\frac{1}{1-\frac{i}{I}}$$

galvanometer 504 being connected over contact 14 of key 502 to the junction between resistances 441 and 443 representing the quantities

$$\frac{T}{K_1} \text{ and } R_d$$

and over contact 16 of key 502 to the junction between resistances 440 and 442 representing the quantities T and K_1, respectively.

As in the multiple bridge of Fig. 2, a switch 505 is provided for selecting the value of T_1 corresponding to the size of core and a switch 506 for selecting the value of K_1 corresponding to the various types of relays. Where the quantity such as R, i, T, etc., appears in more than one bridge, the two values are adjusted simultaneously by a common control dial. However, it is to be noted that the functions of the resistances shown as resistances **423** and **452** in Fig. 4 are performed by a single resistance in the circuit of Fig. 5 and likewise the functions of the resistances shown as resistances **441** and **450** in Fig. 4, are performed by a single resistance of Fig. 5. Resistances R_b, R_d and R_e are fixed resistances representing the value of the numeral **1** in bridges B, D and E in Fig. 4.

The general operating procedure would be as follows: In the usual problem the values of N, R, E and i are known and the value of t is desired. In addition the values of K_1 and T_1 are determined by the type of relay under consideration. The resistances representing the known quantities are set, after which key **500** is operated to supply battery to the bridges. With key **502** normal the resistance **413** representing the quantity

$$\frac{i}{I}$$

is adjusted to balance bridge A by means of galvanometer **503** at the same time setting the value of

$$\log_e \frac{1}{1-\frac{i}{I}}$$

in bridge E, and resistance **450** representing the quantity

$$\frac{T}{K_1}$$

is adjusted to balance the bridge C by means of galvanometer **504**, thereby fixing the value of this resistance for use in bridge D. Key **502** is now thrown to the right and resistance **440** representing the quantity T is adjusted to balance bridge D by means of galvanometer **504** at the same time setting resistance **431** in bridge E. Bridge E is then balanced, using galvanometer **503**, by adjusting resistance **433** to arrive at the value of t which gives the operating time.

Where the operate adjustment of the relay under consideration is known in terms of αT rather than i, the value of i is determined by means of bridge B. The left-hand contacts of key **502** are operated and the known values of αT and N set by the adjustment of resistances **421** and **423**. Bridge B is then balanced, using galvanometer **504**, by adjusting resistance **420** to arrive at the value of i, and set its value in bridge A, after which the calculations proceed as above described.

When insufficient variables are known, such as designing a relay to meet given time requirements, arbitrary values are assumed for the required variables and the remaining values determined, trials being made until a satisfactory relation is attained or until it is found that the requirement cannot be met with standard relay structures.

All of the variables with the exception of the quantities

$$\frac{i}{I} \text{ and } \log_e \frac{1}{1-\frac{i}{I}}$$

are represented by decade resistances such as indicated for the multiple bridge of Fig. 3 but these two arms will be special slide wire resistances. The

$$\frac{i}{I}$$

arm is a linear resistance having the slide directly coupled to the dial on the panel which is calibrated to correspond. The resistance used for the quantity

$$\log_e \frac{1}{1-\frac{i}{I}}$$

is wound on a flat strip, the contour of which is mathematically determined to conform to that quantity and the slide of which is mechanically coupled with the slide on the resistance representing the quantity

$$\frac{i}{I}$$

From the foregoing it is apparent that the bridges disclosed provide novel and efficient means for performing the calculations required by the design of relays.

What is claimed is:

1. A compound Wheatstone bridge for making calculations involving more than four variables, comprising a plurality of bridges, a lesser plurality of galvanometers, each of said bridges comprising resistances calibrated to represent certain of said variables, one of the variables represented in each bridge being also represented in another one of said bridges, each bridge indicating a known relationship between the included variables when balanced, means for simultaneously adjusting all resistances representing the same variable, means for adjusting the remaining resistances to balance said bridges, and means for individualizing said galvanometers to said bridges to indicate when balances are reached.

2. A compound Wheatstone bridge for making calculations involving more than four variables, comprising five bridges, two galvanometers, each of said bridges comprising resistances calibrated to represent certain of said variables, one of the variables represented in each bridge being also represented in another one of said bridges, each bridge indicating a known relationship between the included variables when balanced, means for simultaneously adjusting all resistances representing the same variable, means for adjusting the remaining resistances to balance said bridges and means for individualizing said galvancmeters to said bridges in pairs to indicate when balances are reached.

3. A compound Wheatstone bridge for making calculations involving more than four variables comprising five bridges, each composed of resistances calibrated to represent certain of said variables, at least two of the variables represented by arms of each of said bridges appearing as arms of another one of said bridges, each bridge indicating a known relationship between the included variables when balanced, means for simultaneously adjusting all resistances representing the same variables, means for adjusting the remaining resistances to balance said bridges, two galvanometers, and means for individualizing said galvanometers to said bridges to permit the successive balancing of said bridges.

4. A compound Wheatstone bridge for designing relay windings comprising one bridge composed of variable resistances calibrated to represent the number of turns, total resistance, the reciprocal of the resistivity factor and the diameter of the core plus the depth of the coil and a second bridge composed of variable resistances calibrated to represent the depth of the coil, the effective cross-section of a turn of wire, the num-

ber of turns and the length of the coil, means for simultaneously adjusting the value of the two resistances representing the number of turns, of the two resistances representing the depth of the coil and of the two resistances representing the reciprocal of the resistivity and the effective cross-section, means for individually adjusting the remaining resistances and a galvanometer for each bridge to indicate when a balance has been reached.

5. A compound Wheatstone bridge for designing relay windings comprising one bridge composed of variable resistances calibrated to represent the number of turns, the total resistance, the reciprocal of the resistivity factor and the diameter of the core plus the depth of the coil and a second bridge composed of variable resistances calibrated to represent the depth of the coil, the effective cross-section of a turn of wire, the number of turns and the length of the coil, means for simultaneously adjusting the values of the two resistances representing the number of turns, of the two resistances representing the depth of the coil and of the two resistances representing the reciprocal of the resistivity and the effective cross-section, means for individually adjusting the remaining resistances, a galvanometer for each bridge to indicate when a balance has been reached, and means for adapting said second bridge to express relay design in terms of layers of wire comprising a variable resistance calibrated to represent the number of layers, a variable resistance calibrated to represent the depth of one layer, a fixed resistance and switching means for simultaneously substituting said last three resistances for the resistances representing the number of turns, the cross-section of one wire and the length of the core respectively.

6. A compound Wheatstone bridge for designing relay windings comprising one bridge composed of variable resistances calibrated to represent the number of turns, the total resistance, the reciprocal of the resistivity factor and the diameter of the core plus the depth of the coil and a second bridge composed of variable resistances calibrated to represent the depth of the coil, the effective cross-section of a turn of wire, the number of turns and the length of the coil, means for simultaneuosly adjusting the values of the two resistances representing the number of turns, of the two resistances representing the depth of the coil and of the two resistances representing the reciprocal of the resistivity and the effective cross-section, means for individually adjusting the remaining resistances, a galvanometer for each bridge to indicate when a balance has been reached, and means for adapting said second bridge to express relay design in terms of layers of wire comprising a variable resistance calibrated to represent the number of layers, a variable resistance calibrated to represent the depth of one layer, a fixed resistance and switching means for simultaneously substituting said last three resistances for the resistances representing the number of turns, the cross-section of one wire and the length of the core respectively, said resistance representing the number of layers being adjustable independently of said first bridge.

7. A compound Wheatstone bridge for designing relay windings comprising one bridge composed of variable resistances calibrated to represent the number of turns, the total resistance, the reciprocal of the resistivity factor and the diameter of the core plus the depth of the coil and a second bridge composed of variable resistances calibrated to represent the depth of the coil, the effective cross-section of a turn of wire, the number of turns and the length of the coil, means for simultaneously adjusting the values of the two resistances representing the number of turns, of the two resistances representing the depth of the coil and of the two resistances representing the reciprocal of the resistivity and the effective cross-section, means for individually adjusting the remaining resistances, a galvanometer for each bridge to indicate when a balance has been reached, and means for adapting said second bridge to express relay design in terms of turns per layer, comprising a variable resistance calibrated to represent the number of turns in one layer, a variable resistance calibrated to represent the depth of one layer and switching means for simultaneously substituting said last two resistances for the resistance representing the number of turns and the resistance representing the cross-section of one wire respectively.

8. A compound Wheatstone bridge for designing relay windings comprising one bridge composed of variable resistances calibrated to represent the number of turns, the total resistance, the reciprocal of the resistivity factor and the diameter of the core plus the depth of the coil and a second bridge composed of variable resistances calibrated to represent the depth of the coil, the effective cross-section of a turn of wire, the number of turns and the length of the coil, means for simultaneously adjusting the values of the two resistances representing the number of turns, of the two resistances representing the depth of the coil and of the two resistances representing the reciprocal of the resistivity and the effective cross-section, means for individually adjusting the remaining resistances, a galvanometer for each bridge to indicate when a balance has been reached, and means for adapting said second bridge to express relay design in terms of turns per layer, comprising a variable resistance calibrated to represent the number of turns in one layer, a variable resistance calibrated to represent the depth of one layer and switching means for simultaneously substituting said last two resistances for the resistance representing the number of turns and the resistance representing the cross-section of one wire respectively, said resistance representing the number of turns in one layer being adjustable independently of said first bridge.

9. A compound Wheatstone bridge for designing relay windings comprising one bridge having as arms variable resistances calibrated to represent the number of turns, the total resistance, the reciprocal of the resistivity factor and the diameter of the core plus the depth of the coil and a second bridge having as arms variable resistances calibrated to represent the depth of the coil, the average effective cross-section of a turn of wire, the number of turns and the length of the coil, means for simultaneously adjusting the values of the two resistances representing the number of turns, of the two resistances representing the depth of the coil and of the two resistances representing the reciprocal of the resistivity and the average effective cross-section, means for individually adjusting the remaining resistances, a galvanometer for each bridge to indicate when a balance has been reached, a resistance representing the maximum effective cross-section of a wire and a resistance representing the depth of one layer of turns, the means for adjusting the

resistances representing the average effective cross-section of a wire being effective to adjust said last two resistances, switching means for substituting either one of said last two resistances for said resistance representing the average effective cross-section, means for correspondingly adjusting an adjacent arm of said bridge, said bridge when balanced indicating the relay design in terms of the substituted quantities.

10. A compound Wheatstone bridge for designing relay windings comprising one bridge having as arms variable resistances calibrated to represent the number of turns, the total resistance, the reciprocal of the resistivity factor and the diameter of the core plus the depth of the coil and a second bridge having as arms variable resistances calibrated to represent the depth of the coil, the effective cross-section of a turn of wire, the number of turns and the length of the coil, means for simultaneously adjusting the values of the two resistances representing the number of turns, of the two resistances representing the depth of the coil and of the two resistances representing the reciprocal of the resistivity and the effective cross-section, means for individually adjusting the remaining resistances, a galvanometer for each bridge to indicate when a balance has been reached, said resistance representing the effective cross-section of a turn of wire comprising a plurality of individual variable resistances each corresponding to a different type of insulation and means for rendering one of said individual resistances effective.

11. A compound Wheatstone bridge for designing relay windings comprising one bridge having as arms variable resistances calibrated to represent the number of turns, the total resistance, the reciprocal of the resistivity factor and the diameter of the core plus the depth of the coil and a second bridge having as arms variable resistances calibrated to represent the depth of the coil, the average effective cross-section of a turn of wire, the number of turns and the length of the coil, means for simultaneously adjusting the values of the two resistances representing the number of turns, of the two resistances representing the depth of the coil and of the two resistances representing the reciprocal of the resistivity and the effective cross-section, means for individually adjusting the remaining resistances and a galvanometer for each bridge to indicate when a balance has been reached, resistances representing the maximum effective cross-section of a wire and resistances representing the depth of one layer of turns, the means for adjusting the resistance representing the average effective cross-section of a wire being effective to also adjust said last two resistances, switching means for substituting either one of said last two resistances for said resistance representing the average effective cross-section and correspondingly adjusting an adjacent arm of the bridge, said resistances representing the average effective cross-section of wire, the maximum effective cross-section of wire, the depth of one layer and the reciprocal of the resistivity factor, each comprising a plurality of individual variable resistances arranged in sets corresponding to different types of insulation and means for rendering any one of said sets of individual resistances effective, said bridge when balanced indicating the relay design in terms of the substituted quantities.

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