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ELECTRICAL PHASE INDICATING APPARATUS

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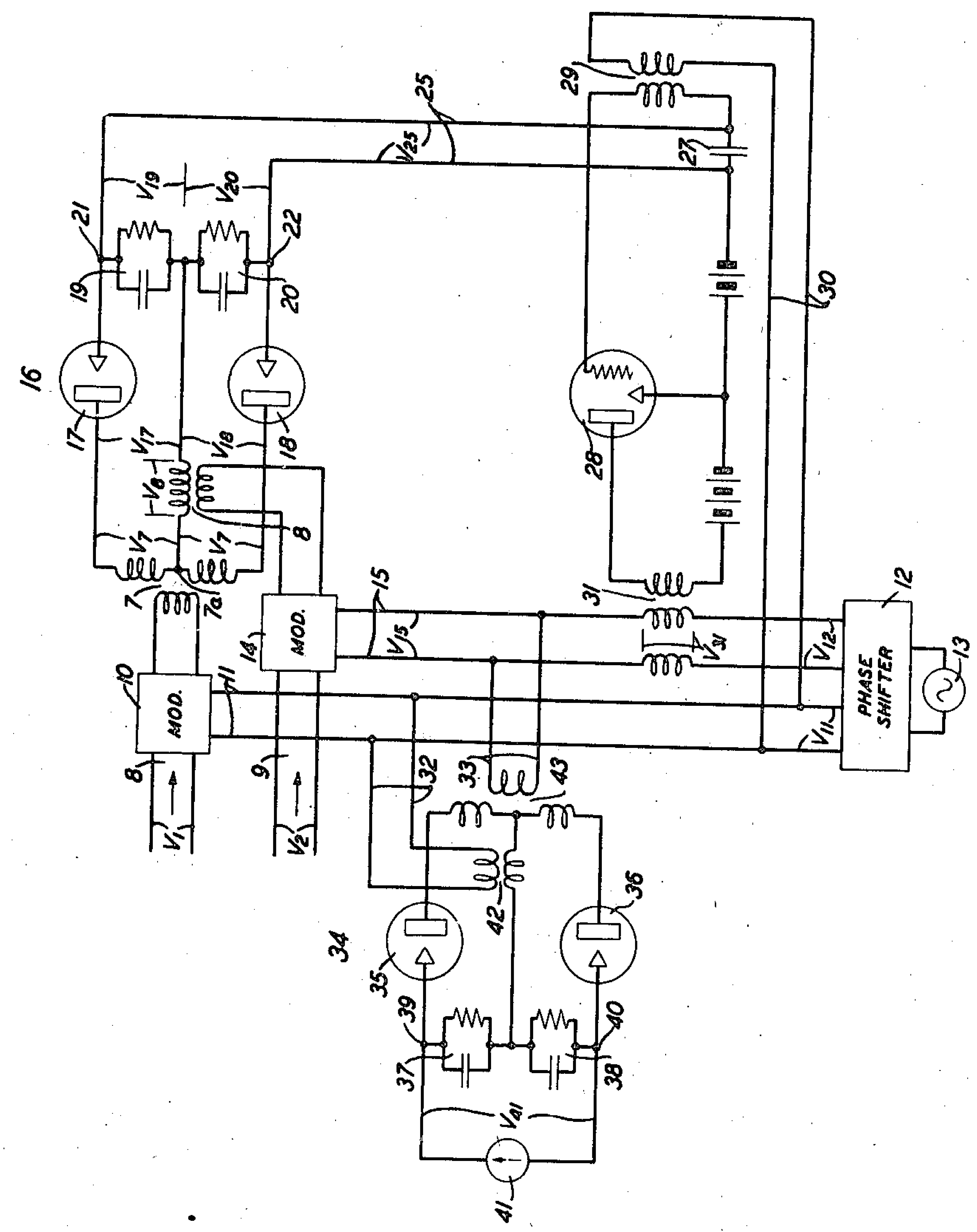

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ELECTRICAL PHASE INDICATING APPARATUS

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This invention relates to apparatus for indicating the phase relationship between two alternating quantities.

The general object of the invention is to indicate continuously the phase angle between two electrical quantities in a manner which substantially obviates the effect of the amplitudes of such two quantities.

In a specific embodiment, the present invention comprises two independent sources of electrical quantities, means for translating the phase difference between such two quantities into a direct current voltage whose magnitude is proportional to sin a, where a is the phase difference between the two quantities, and independent of the amplitudes thereof, and means responsive to such direct current voltage for continuously indicating the phase angle between the two quantities.

The invention will be readily understood from the following description taken together with the drawing, the single figure of which illustrates a specific embodiment of the invention.

Referring to the drawing, a signal transmission line 8 transmits a signal voltage V_1 and a signal transmission line 9 transmits a signal voltage V_2. The input circuit of modulator 10 is connected to the line 8 and the corresponding ends of leads 11, 11 whose opposite ends are connected to the output circuit of phase shifter 12. The input circuit of this phase shifter is connected to a source 13 of carrier voltage of preselected frequency. The input circuit of modulator 14 is connected to the line 9 and corresponding ends of leads 15, 15 whose opposite ends are also joined to the output circuit of the phase shifter 12. This phase shifter serves to establish initially a 90-degree phase difference between the carrier voltage portions transmitted on the leads 11, 11 and 15, 15.

The output circuits of modulators 10 and 14 are connected via transformers 7 and 8, respectively, to the input circuit of a balanced rectifier 16 comprising individual diode tubes 17 and 18. In the output circuits of the tubes 17 and 18 are disposed R.-C. networks 19 and 20, respectively. Leads 25, 25 connect output terminals 21 and 22 of the rectifier 16 across a capacitor 27 in the input circuit of a variable-μ electronic tube 28, in series with the secondary winding of a transformer 29. This transformer has its primary winding connected by leads 30, 30 to leads 11, 11. The output circuit of amplifier 28 embodies the primary winding of transformer 31 which includes two coupled secondary windings each of which is interposed in series in one of the leads 15, 15. Leads 32, 32 connect leads 11, 11 to the

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primary winding of a transformer 42 whose secondary winding is positioned in the input circuit of balanced rectifier 34. Leads 33, 33 joined to leads 15, 15 at points intermediate the input of modulator 14 and the secondary windings of transformer 31, connect the leads 15, 15 to the primary winding of transformer 43 whose secondary winding is embodied in the input circuit of rectifier 34. This rectifier comprises diode tubes 35 and 36. In the output circuits of the individual tubes 35 and 36 are R.-C. networks 37 and 38, respectively An indicator 41 is connected across the output terminals 39, 40 of rectifier 34.

The circuit shown in the drawing operates to produce across the terminals 39 and 40 a rectified voltage which has a magnitude proportional to sin a, where a is the phase angle between the signal voltages V_1 and V_2 and which is independent of the amplitudes of both these voltages, in a manner that will now be explained.

Let it be assumed that the signal voltage $V_1=V_1 \sin (pt)$; the signal voltage $V_2=V_2 \sin (pt+a)$, where a is the phase angle between the two signal voltages V_1 and V_2; the carrier voltage V_{11} (input to modulator 10) $=V_{11} \sin (Qt)$; and the carrier voltage V_{15} (input to the modulator 14) $=V_{11} \sin (Qt+\theta)$, where θ is the phase difference established between the carrier voltages V_{11} and V_{12} in a manner to be explained later, the voltage V_{12} being identical in phase and magitude with the voltage V_{15} except as hereinafter pointed out. Then, $V_7=KV_1 \sin [(p-Q)t]$ and $V_8=KV_2 \sin [(p-Q)t+a-\theta]$. Hence,

$$V_{17}=V_8+V_7=K\left(V_2 \sin [(p-Q)t+a-\theta]+V_1 \sin [(p-Q)t]\right)$$

$$V_{18}=V_8-V_7=K\left(V_2 \sin [(p-Q)t+a-\theta]-V_1 \sin [(p-Q)t]\right)$$

Thus, the tube 17 rectifies the input voltage V_{17} which comprises the voltage across the secondary winding of transformer 8 plus the voltage across the upper secondary winding of transformer 7. The tube 18 rectifies the input voltage which comprises the voltage across the secondary winding of transformer 8 minus the voltage across the lower secondary winding of transformer 7. The transformer 7 is so designed that voltages produced across its individual secondary windings are equal in magnitude.

The voltage across R.-C. network 19 is proportional to the peak of the effective input voltage V_{17} rectified by tube 17, which rectified voltage

$$V_{19}=K_1\sqrt{V_2^2+V_1^2+2V_1V_2 \cos (a-\theta)}$$

The voltage across the R.-C. network 20 is proportional to the peak of the effective input voltage V_{18} rectified by the tube 18 which rectified voltage

$$V_{20}=K_1\sqrt{V_2^2+V_1^2-2V_1V_2\cos(a-\theta)}$$

As the R.-C. networks 19 and 20 are connected in opposition in the output circuits of the respective tubes 17 and 18, the rectified voltage V_{25} effective across terminals 21 and 22 and supplied over leads 25, 25 to capacitor 27 is the difference between the rectified voltage V_{19} and V_{20}. Thus,

$$V_{25}=\frac{2K_1V_1V_2}{\sqrt{V_1^2+V_2^2}}\cos(a-\theta) \qquad (1)$$

Also, let it be assumed that the parameters of the tube 28 and the circuit associated therewith are so chosen that the gain of tube 28 is n_0, that is, the voltage V_{31} (across secondary windings of transformer 31) $=n_0V_{11}$, where the rectified voltage $V_{25}=0$, and the gain of tube 28 changes proportionally to variations of the rectified voltage V_{25} with reference to zero. Accordingly, the voltage $V_{31}=(n_0+n_1V_{25})V_{11}$. Hence

$$\begin{aligned}V_{15}&=V_{12}+V_{31}\\&=V_{12}+(n_0+n_1V_{25})V_{11}\end{aligned} \qquad (2)$$

The carrier voltages V_{11} and V_{15} may be initially selected by proper adjustment of the phase shifter 12 so that a phase quadrature relationship exists therebetween. Now, considering the condition $V_{25}=0$

$$V_{15}=V_{12}\sin(Qt+\Phi)+n_0V_{11}\sin(Qt)$$

or

$$=V_{11}\sin(Qt+\theta)$$

Since

$$\theta=\frac{\pi}{2}$$

then

$$V_{15}=V_{11}\sin\left(Qt+\frac{\pi}{2}\right)$$

that is,

$$V_{12}\cos\Phi+n_0V_{11}=V_{11}\cos\frac{\pi}{2}=0$$

$$V_{12}\sin\Phi=V_{11}\sin\frac{\pi}{2}=V_{11}$$

from which

$$\sin\Phi=\frac{V_{11}}{V_{12}}$$

$$\cos\Phi=-\sqrt{1-\sin^2\Phi}=-\sqrt{1-\frac{V_{11}^2}{V_{12}^2}}$$

$$V_{12}\sqrt{1-\frac{V_{11}^2}{V_{12}^2}}=-n_0V_{11}$$

$$\frac{V_{11}}{V_{12}}=\pm\frac{1}{\sqrt{1+n_0^2}}$$

in which obviously the (+) sign must be used. Substituting these values in Equation 2,

$$V_{15}=V_{12}\sin\left[(Qt+\sin^{-1})\left(\frac{1}{\sqrt{1+n_0^2}}\right)\right]+n_0V_{11}\sin(Qt)+n_1V_{25}V_{11}\sin(Qt)$$

$$=V_{11}\sin\left(Qt+\frac{\pi}{2}\right)+n_1V_{25}V_{11}\sin(Qt)$$

$$=V_{11}\sqrt{1+n_1^2V_{25}^2}\sin\left(Qt+\frac{\pi}{2}+\tan^{-1}n_1V_{25}\right)$$

so that

$$\theta=\tan^{-1}n_1V_{25}+\frac{\pi}{2}$$

Substituting this value of θ in Equation 1,

$$V_{25}=\frac{2K_1V_1V_2}{\sqrt{V_1^2+V_2^2}}\cos\left(a-\tan^{-1}\frac{1}{n_1V_{25}}\right)$$

$$a-\tan^{-1}\frac{1}{n_1V_{25}}=\cos^{-1}\frac{V_{25}\sqrt{V_1^2+V_2^2}}{2K_1V_1V_2}=\tan^{-1}\sqrt{\frac{4K_1^2V_1^2V_2^2}{V_{25}^2(V_1^2+V_2^2)}}-1$$

or

$$\tan a=\frac{\sqrt{\frac{4K_1^2V_1^2V_2^2}{V_{25}^2(V_1^2+V_2^2)}}-1+\frac{1}{n_1V_{25}}}{1-\frac{1}{n_1V_{25}}\sqrt{\frac{4K_1^2V_1^2V_2^2}{V_{25}^2(V_1^2+V_2^2)}}-1}$$

$$\tan a-\frac{1}{n_1V_{25}}=\sqrt{\frac{4K_1^2V_1^2V_2^2}{V_{25}^2(V_1^2+V_2^2)}}-1\left(1+\frac{1}{n_1V_{25}}\tan a\right)$$

$$\tan^2 a-2\tan a\frac{1}{n_1V_{25}}+\frac{1}{n_1^2V_{25}^2}=$$

$$\frac{4K_1^2V_1^2V_2^2}{V_{25}^2(V_1^2+V_2^2)}+8\frac{K_1^2V_1^2V_2^2\tan a}{n_1^2V_{25}^3(V_1^2+V_2^2)}+\frac{K_1^2V_1^2V_2^2\tan^2 a}{(V_1^2+V_2^2)n_1^2V_{25}^4}-1-2\frac{1}{n_1V_{25}\tan a}-\frac{1}{n_1^2V_{25}^2\tan^2 a} \qquad (3)$$

Equation 3 may be solved for V_{25} and then the factor (n_1) chosen so that the voltage V_{25} is made very nearly equal to zero. In such case, $\cos(a-\theta)$ in Equation 1 is also made very nearly equal to zero since K_1, V_1 and V_2 individually have values other than zero; and therefore under the equilibrium condition

$$a-\theta=\frac{\pi}{2}$$

very nearly or

$$\theta=a-\frac{\pi}{2} \qquad (3a)$$

By a similar analysis, it can be demonstrated that

$$V_{41}\text{ (across terminals 39, 40)}=\frac{2K_1V_{11}V_{15}}{V_{11}^2+V_{15}^2}\cos\theta=\frac{2K_1V_{11}V_{15}}{\sqrt{V_{11}^2+V_{15}^2}}\sin a$$

As $V_{15}=V_{11}$, very nearly, then

$$V_{41}=K_1V_{11}\sin a \qquad (4)$$

Referring to Equation 1 and assuming

$$\theta=\frac{\pi}{2}$$

then

$$V_{25}=\frac{2K_1V_1V_2}{\sqrt{V_1^2+V_2^2}}\sin a \qquad (5)$$

The factor (n_1) tends to vary the amount of current in the output circuit of the tube 28 with reference to the amount of current prevailing therein under the equilibrium condition and thereby the phase of the carrier voltage V_{15}. As the phase of carrier voltage V_{15} differs initially from the phase of carrier voltage V_{11} by 90 degrees, the effect of the factor (n_1) is to vary the phase of carrier voltage V_{15}, with reference to the 90 degrees, in a proportional manner. Hence, the variation of the phase of carrier voltage V_{15} with reference to 90 degrees is proportional to the voltage (V_{25}). As above pointed out, the gain (n_1) of tube 28, or the voltage V_{25}, tends to reduce the factor $\cos(a=\theta)$ in Equation 1 to a very small quantity. This tends to establish a phase relationship between V_{11} and V_{15} such as to set up a quadrature relationship between the voltages V_7 and V_8, and hence makes the phase relationship between the carrier voltages V_{11} and V_{15} differ from

$$\frac{\pi}{2}$$

by an amount a as indicated in Equation 3a.

Referring to Equation 4, the factors K_1 and V_{11} are design constants of the circuit. Hence, the voltage V_{41} depends only upon these constants and sin a where a is the phase difference between the two signal voltages V_1 and V_2, and is independent of their amplitudes. The rectified voltage V_{41} produces appropriate phase readings on indicator 41. This indicator is calibrated to produce a zero reading when sin a is zero, that is, when phase quadrature substantially occurs between the carrier voltages V_{11} and V_{15}.

It will be understood that the variable $-\mu$ tube 28 is illustrative of one type of device capable of utilizing rectified voltage for accomplishing phase control, and other devices equally suitable for the same purpose will occur to those skilled in the art.

What is claimed is:

1. A phase detecting circuit comprising two modulating devices, means for producing two carrier voltages in phase quadrature, circuit means for applying each of two signal voltages to the input circuit of a respective one of said modulating devices, additional circuit means for applying each of said two carrier voltages to the input circuit of a respective one of said modulating devices, differential detecting means connected to the output circuits of both said modulating devices for producing a rectified voltage, phase changing means controlled by said first-mentioned rectified voltage to vary the relative phases of the carrier voltages applied to said modulating devices, further differential detecting means connected to said additional circuit means for producing a second rectified voltage, and an indicator connected to said further detecting means and responsive to the second rectified voltage.

2. In combination in a circuit for indicating the phase difference between two signal voltages, two discrete sources of signal voltage whose phase difference is to be indicated, means for producing two carrier voltages one of which differs in phase from the other by 90 degrees, means for translating said two signal voltages and one portion of each of said two carrier voltages into two further voltages, differential means for detecting said two further voltages to produce a rectified

voltage, means for utilizing such rectified voltage to control the phase of the translated portion of one of said two carrier voltages with reference to 90 degrees, and further differential means for detecting the phase difference between the two translated portions of the two carrier voltages, with reference to the initial 90-degree phase difference therebetween, to obtain an indication of the phase difference between said two signal voltages.

3. In a system for measuring the phase difference between two signal voltages, two independent sources of signal voltage, means for producing two carrier voltages differing in phase by 90 degrees, means for translating each signal voltage and a first portion of each carrier voltage into a a further voltage, differential means for detecting said further voltages to produce a rectified voltage, means for utilizing such rectified voltage and a second portion of one carrier voltage to control the phase of the translated first portion of the second carrier voltage, further differential means for utilizing a third portion of the one carrier voltage and a further portion of the translated first portion of the second carrier voltage to produce a rectified voltage proportional to sin a, where a is the phase difference between the two signal voltages, and independent of the amplitudes of the two signal voltages, and means responsive to the last rectified voltage for indicating the phase difference between the two signal voltages.

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