Sept. 3, 1946.

F. R. BIES

2,406,796

WAVE FILTER

Filed March 23, 1944

2 Sheets-Sheet 1

FIG. 1

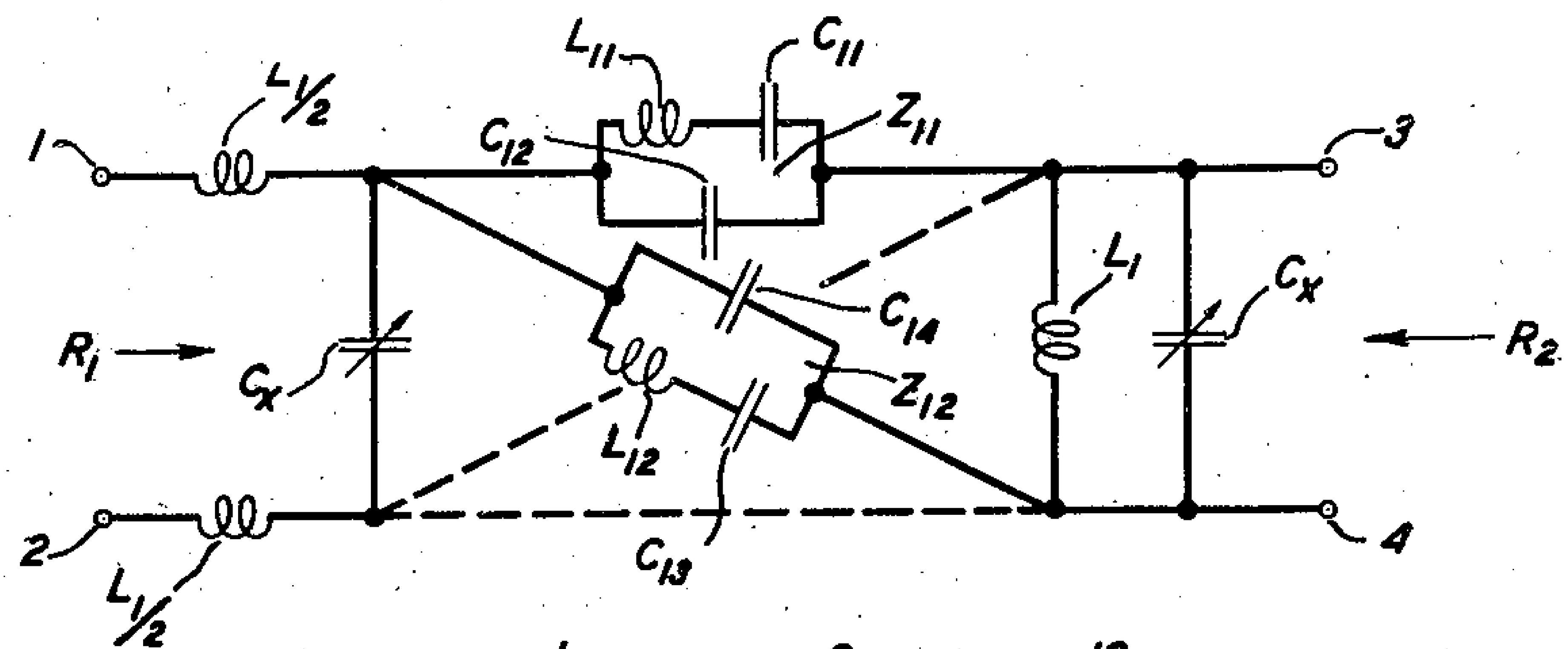


FIG. 2

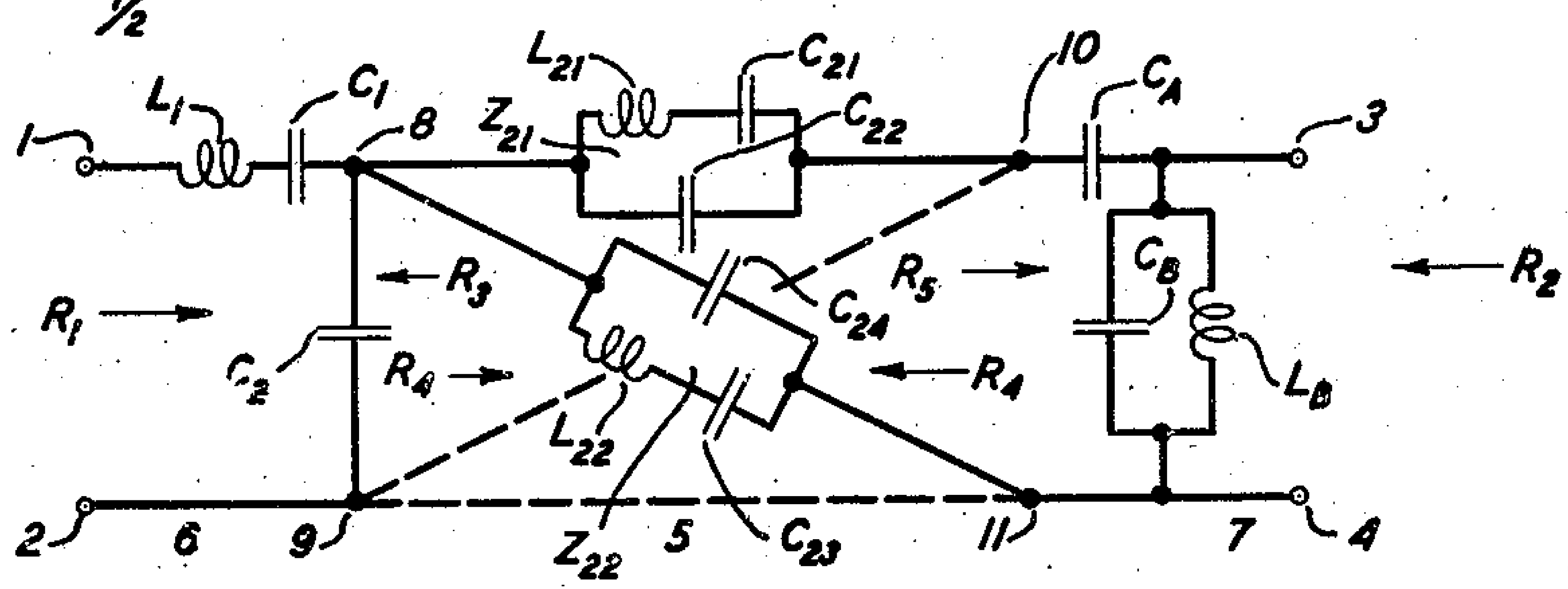


FIG. 3

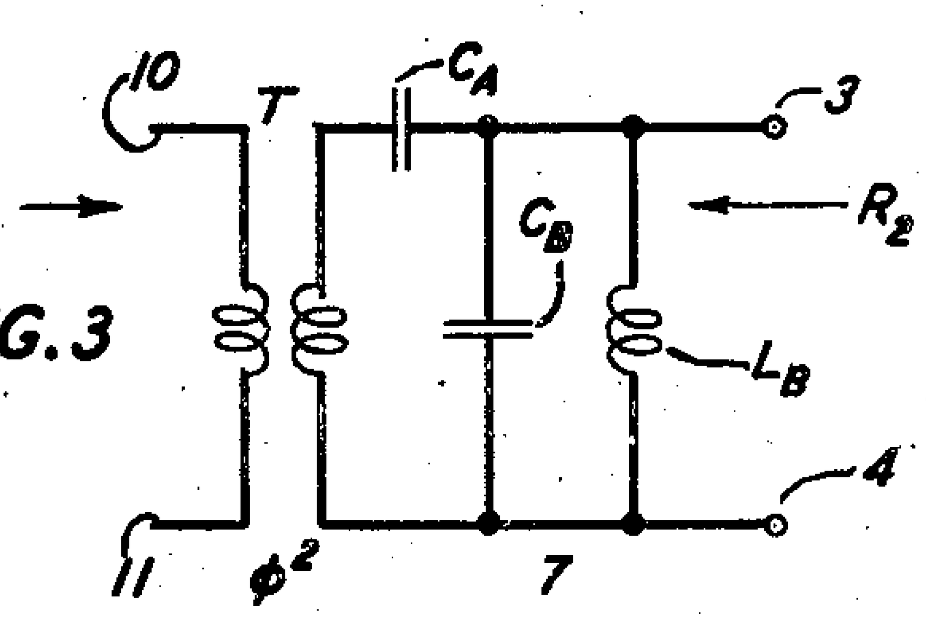


FIG. 4

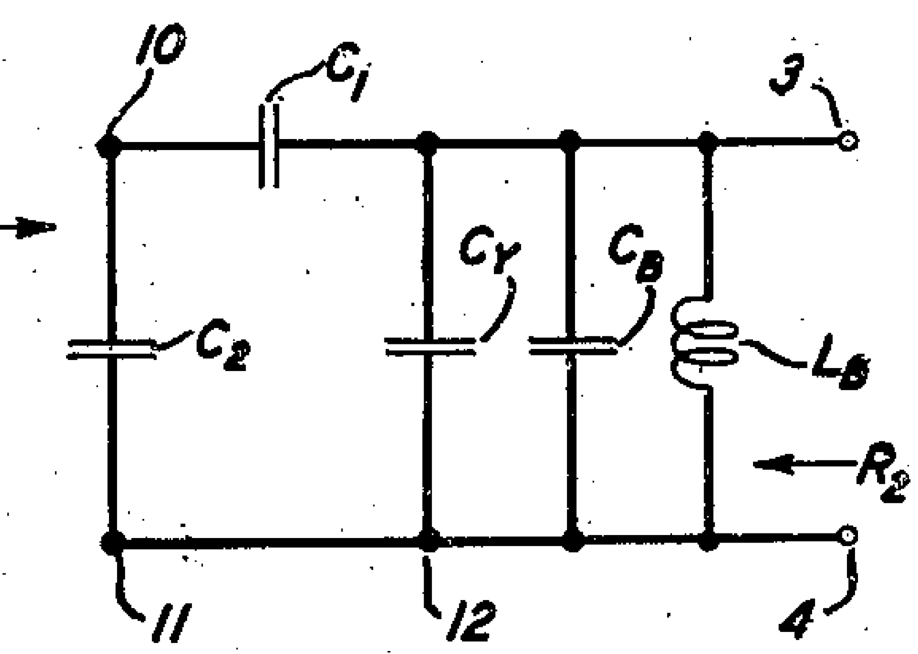
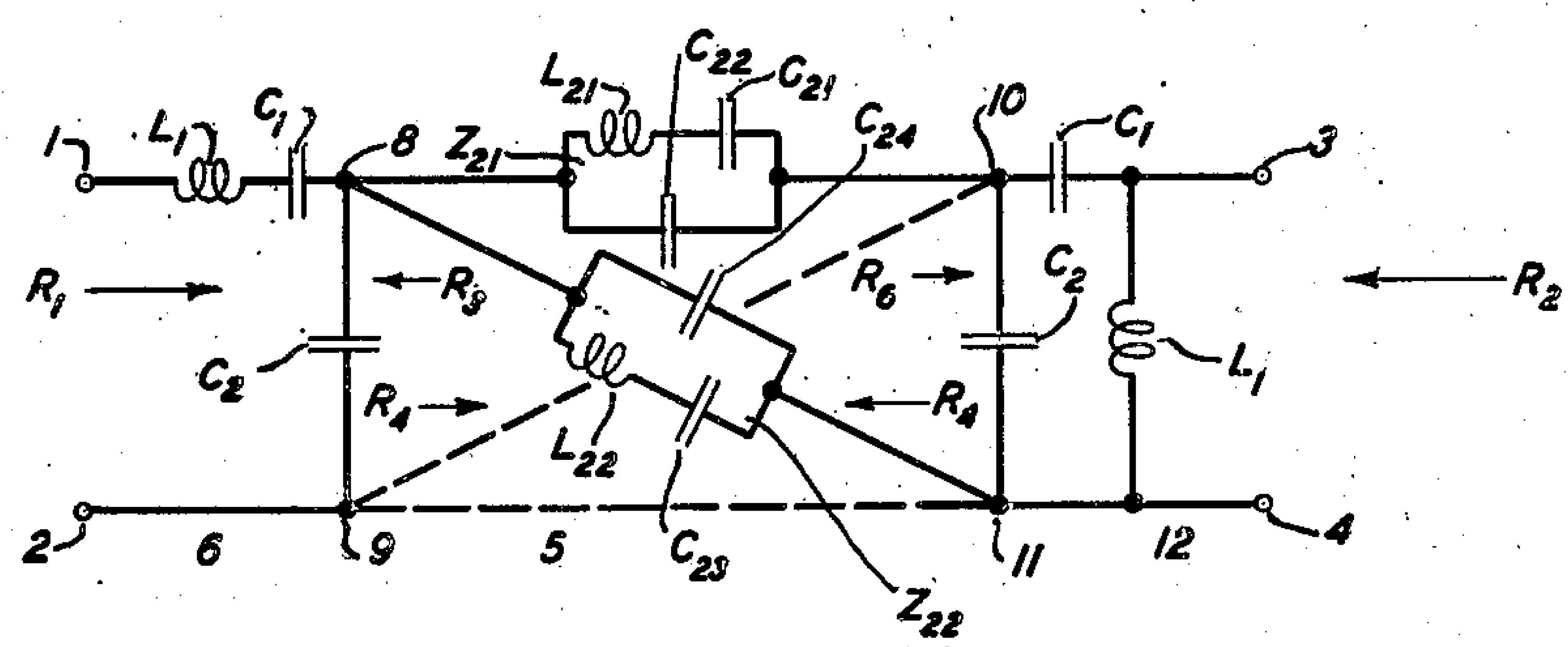


FIG. 5



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WAVE FILTER

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FIG. 6

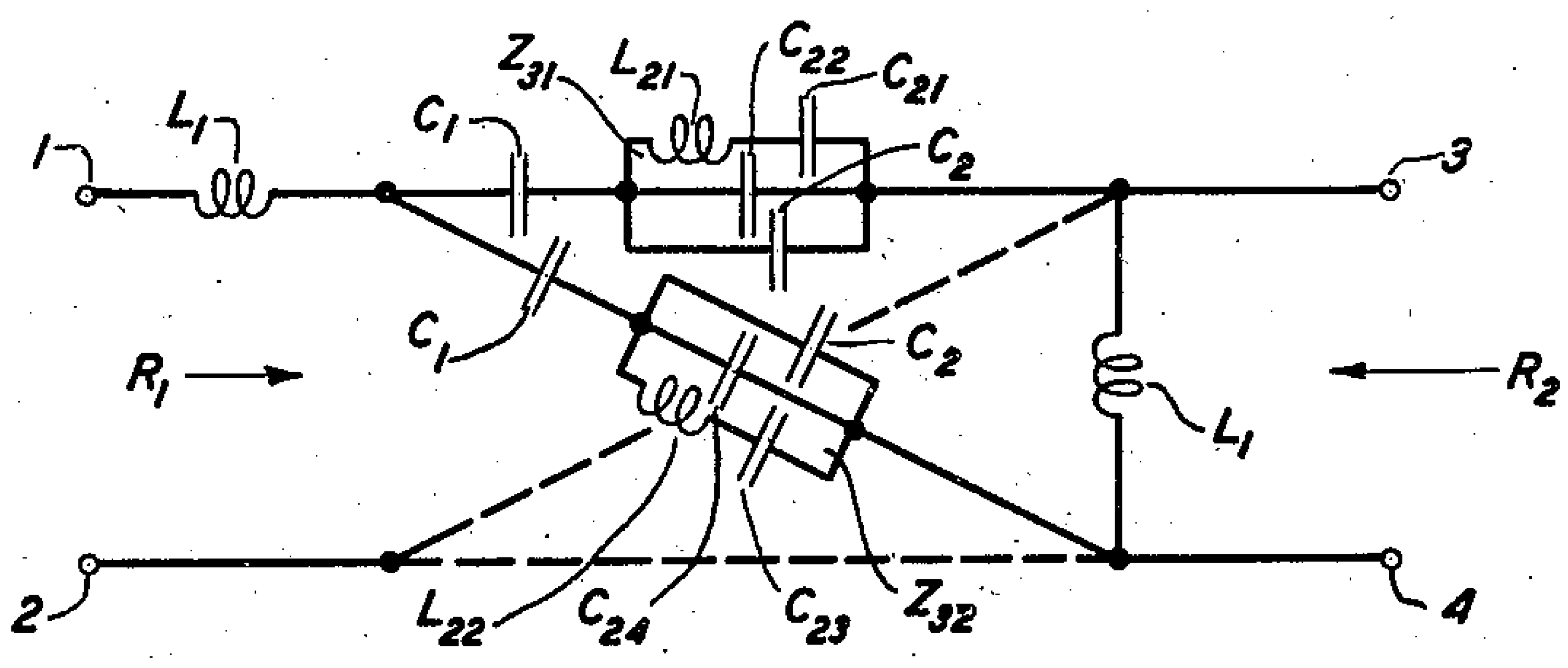


FIG. 7

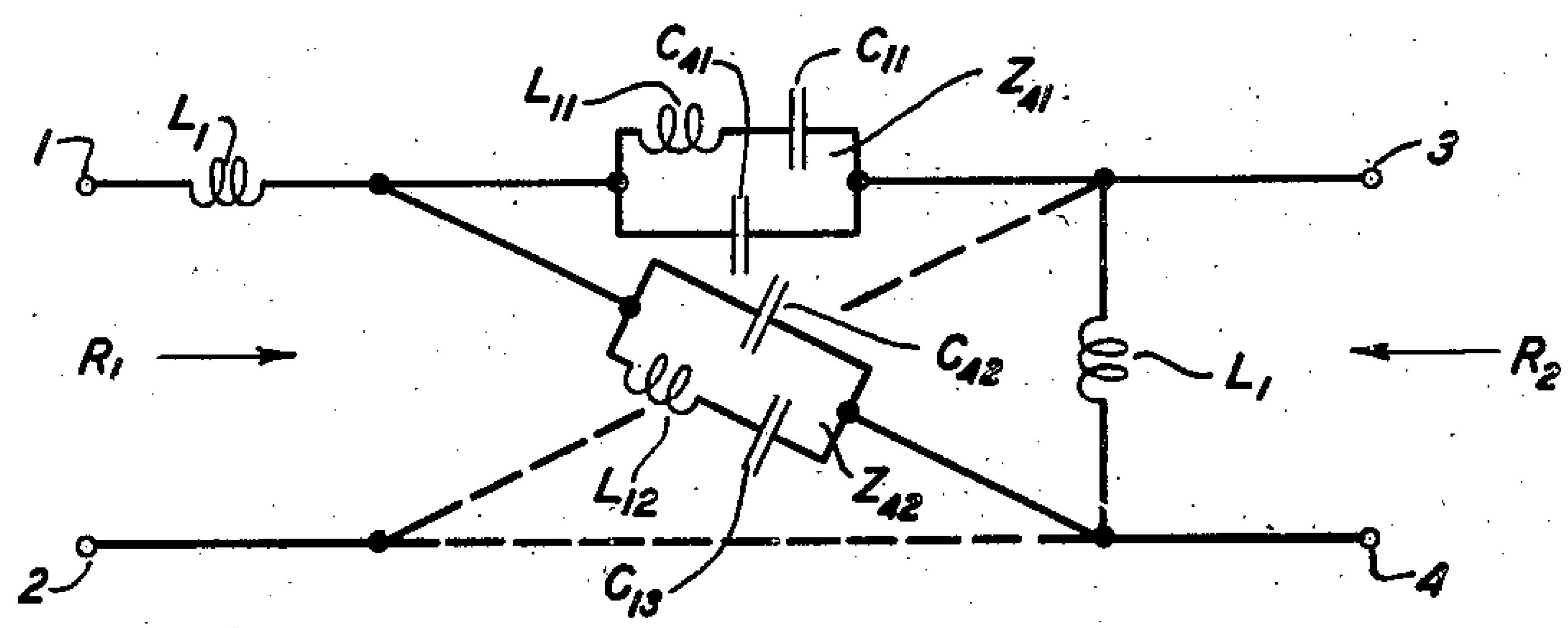


FIG. 8

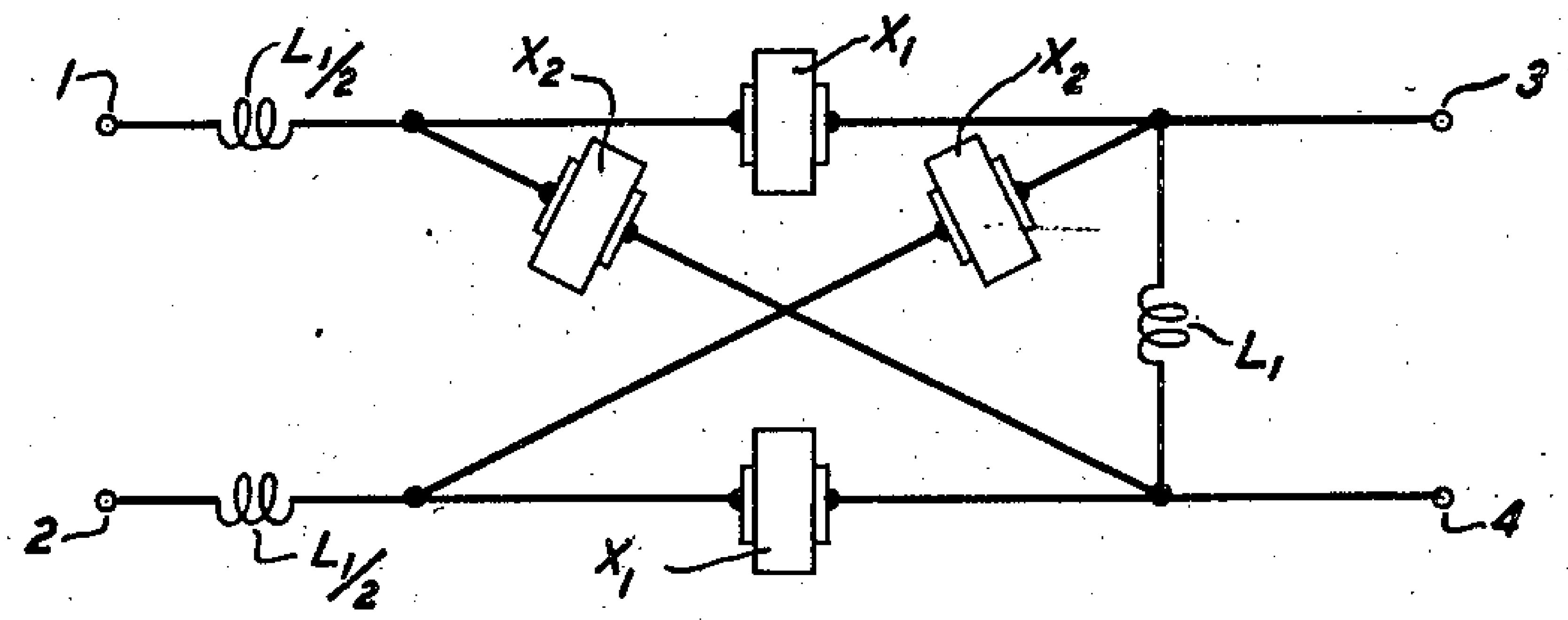
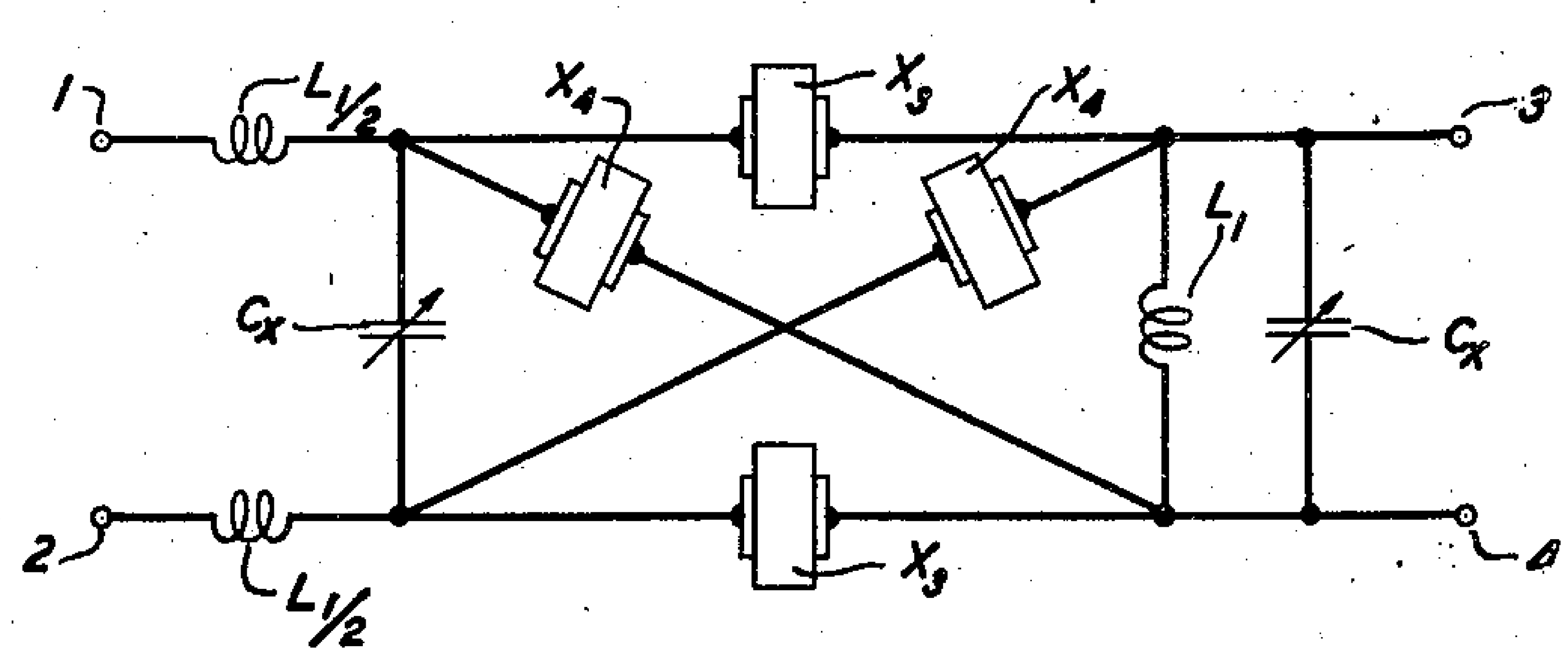


FIG. 9



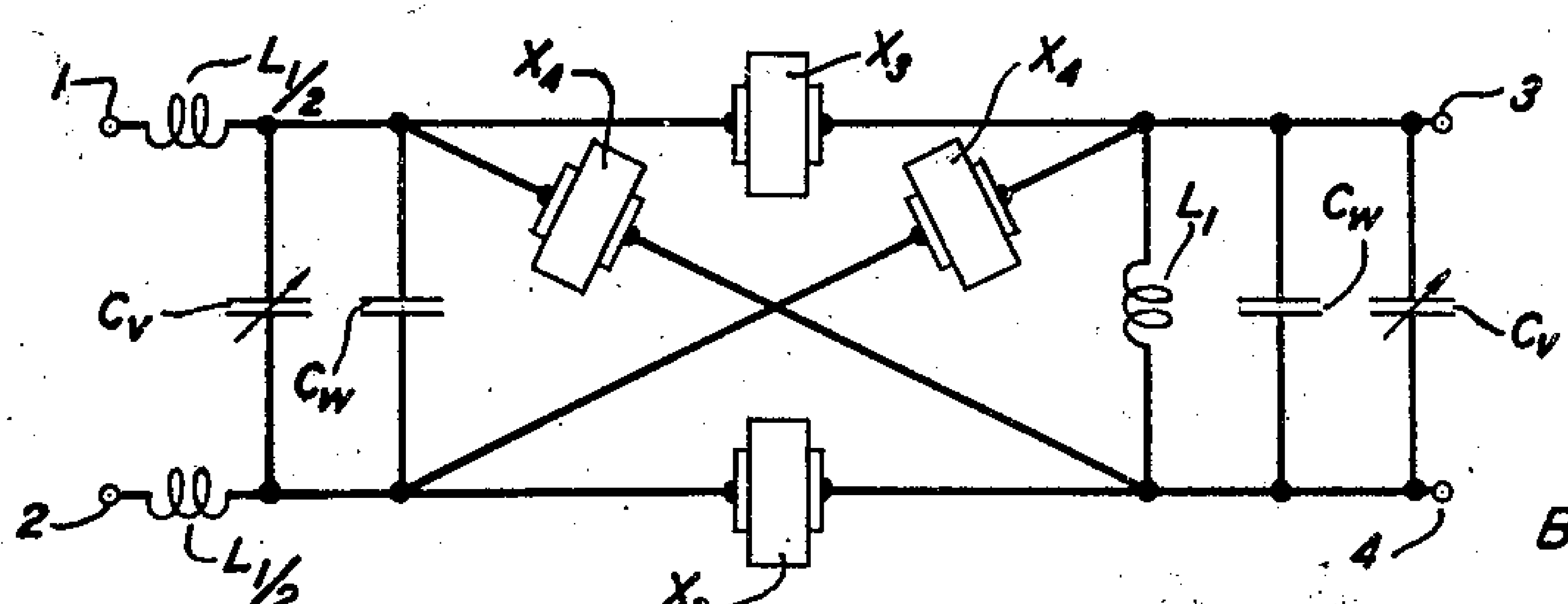


FIG. 10

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UNITED STATES PATENT OFFICE

2,406,796

WAVE FILTER

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Application March 23, 1944, Serial No. 527,764

17 Claims. (Cl. 178—44)

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This invention relates to wave filters and more particularly to those of the impedance-transforming variety.

The principal object of the invention is to provide impedance transformation in a wave filter of the lattice type. A further object is to employ piezoelectric crystal elements in the impedance branches of an impedance-transforming filter of the lattice type.

The wave filter in accordance with the invention comprises a lattice network and two inductances, one of which is connected in series at one end of the lattice and the other of which is connected in shunt at the other end. If certain design limitations are observed the impedance branches of the lattice may be constituted by piezoelectric crystals. Variable shunt capacitances may be provided at the ends of the lattice to facilitate the adjustment of the transmission characteristic of the filter.

The nature of the invention will be more fully understood from the following detailed description and by reference to the accompanying drawings, in which like reference characters refer to similar or corresponding parts and in which:

Fig. 1 is a schematic circuit of a wave filter in accordance with the invention;

Fig. 2 shows the initial network comprising a central lattice and two three-element ladder-type end sections, from which the circuit of Fig. 1 is derived;

Fig. 3 shows the combination of the right-hand ladder section and an ideal transformer;

Fig. 4 is an impedance-transforming section equivalent to the combination of Fig. 3;

Fig. 5 shows the network of Fig. 2 with the right-hand ladder section replaced by the impedance-transforming section of Fig. 4 to provide equal series and equal shunt capacitances external to the lattice;

Fig. 6 is the network of Fig. 5 with the external capacitances taken into the lattice;

Fig. 7 is a network equivalent to the one of Fig. 6 with the redundant capacitances eliminated;

Fig. 8 is the network of Fig. 7 with the branches of the lattice constituted by piezoelectric crystal elements;

Fig. 9 shows the filter of Fig. 8 with variable capacitances in shunt at the ends of the lattice for adjustment of the transmission characteristic; and

Fig. 10 shows the filter of Fig. 9 with each shunt capacitance constituted by a fixed portion and a variable portion.

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Taking up the figures in more detail, Fig. 1 is a schematic circuit of an impedance-transforming wave filter in accordance with the invention. The filter has two pairs of terminals 1, 2 and 3, 4, either pair of which may be considered the input and the other pair the output. The network comprises a lattice having two equal series impedance branches Z_{11} and two equal diagonal impedance branches Z_{12}. For the sake of simplicity, in Fig. 1 and also in Figs. 2, 5, 6 and 7 only one series impedance branch and one diagonal impedance branch of the lattice are shown in detail, the other branches being indicated by dashed lines between the appropriate terminals. At the right-hand end of the lattice is a shunt inductance of value L_1. At the left-hand end there is a series inductance, also of value L_1, which may be equally divided, as shown, and $L_{1/2}$ connected in each side of the line to provide a balanced structure. At each end of the lattice there is a variable shunt capacitance of value C_x, provided for adjusting the transmission characteristic of the filter.

Each series branch Z_{11} of the filter is made up of a capacitance of value C_{12} shunted by the series-connected combination of an inductance of value L_{11} and a capacitance of value C_{11}. Each diagonal branch Z_{12} comprises a capacitance of value C_{14} shunted by the series-connected combination of an inductance of value L_{12} and a capacitance of value C_{13}. It will be understood, of course, that in some cases the end shunt capacitances C_x may be incorporated in the lattice by increasing the value of each of the capacitances C_{12} and C_{14} by an amount equal to C_x. Formulas for evaluating the various elements of the filter are given hereinafter. The filter transmits a band extending between selected frequencies, has peaks of attenuation only at zero frequency and at infinite frequency and provides an impedance step-up from left to right which at the mid-band frequency is equal to the ratio R_2 to R_1.

The derivation of the filter of Fig. 1 will now be taken up. Fig. 2 shows schematically the initial network, comprising a central lattice section 5 with ladder-type half sections 6 and 7 connected one at each end. The lattice 5 has the same configuration as the lattice in Fig. 1 but will have different element values, as indicated by the use of the different subscripts. It has a transmission band extending between a lower cut-off frequency f_2 and an upper cut-off frequency f_3 and an image impedance R_4 at the mid-band frequency f_m, which is the geometric mean of f_2 and f_3. For the case where peaks of attenuation occur only at zero and infinite frequency, which is the one most frequently used, the values of the component reactance elements may be found from the following formulas:

$$C_{21}=C_{23}=\frac{f_3-f_2}{2\pi R_4 f_2(f_2+f_3)} \quad (1)$$

$$C_{22}=C_{24}=\frac{1}{2\pi R_4(f_2+f_3)} \quad (2)$$

$$L_{21}=\frac{R_4(f_2+f_3)}{2\pi f_2(f_3-f_2)} \quad (3)$$

$$L_{22}=\frac{R_4(f_2+f_3)}{2\pi f_3(f_3-f_2)} \quad (4)$$

The capacitances are given in farads and the inductances in henries. The proper value to use for R_4 is derived hereinafter and is given by Equation 27.

Each of the end ladder half sections **6** and **7** transmits a band extending between a lower cut-off frequency f_1 and an upper cut-off frequency f_4, with the same mid-band frequency f_m as the lattice **5**. The band may be of the same width as that of the lattice or it may be wider, as explained below. The section **6** to the left is of the type designated III_1 in Fig. 168B on page 316 of Shea's "Transmission Networks and Wave Filters," published by D. Van Nostrand Company. The series impedance branch comprises an inductance L_1 and a capacitance C_1 connected in series and the shunt impedance branch consists of a capacitance C_2. The section **7** to the right is of the type designated III_4 by Shea in the above reference. The series impedance branch is a capacitance C_A and the shunt impedance branch comprises a capacitance C_B and an inductance L_B connected in parallel. These elements may be evaluated from the following formulas:

$$C_1=\frac{f_4-f_1}{2\pi f_1^2 R_1} \quad (5)$$

$$C_2=\frac{1}{2\pi(f_1+f_4)R_1} \quad (6)$$

$$L_1=\frac{R_1}{2\pi(f_4-f_1)} \quad (7)$$

$$C_A=\frac{f_1+f_4}{2\pi f_1 f_4 R_2} \quad (8)$$

$$C_B=\frac{f_1}{2\pi f_4(f_4-f_1)R_2} \quad (9)$$

$$L_B=\frac{(f_4-f_1)R_2}{2\pi f_1 f_4} \quad (10)$$

where R_1 is the mid-series image impedance at mid-band for the section **6** and R_2 is the mid-shunt image impedance at mid-band for the section **7**.

Each of these half sections **6** and **7** introduces an impedance transformation which, in terms of the cut-off frequencies f_1 and f_4, may be expressed as

$$\frac{R_3}{R_1}=\frac{R_2}{R_5}=\frac{f_1+f_4}{f_4} \quad (11)$$

where R_3 is the mid-shunt image impedance at mid-band for the section **6** and R_5 is the mid-series image impedance at mid-band for the section **7**. The mid-shunt image impedance of the section **6**, and the mid-series image impedance of the section **7**, which face each other, will match perfectly throughout the band, and also will match the image impedance of the lattice **5**, if the impedance levels are properly chosen.

In accordance with the invention it is desired that the lattice **5** have at its right end a shunt capacitance equal to the capacitance C_2 shunting the left terminals **8**, **9** and a series capacitance equal to the capacitance C_1 at the left, in order that these external capacitances may be taken into the lattice **5** either in whole or in part. In order to accomplish this an ideal transformer T is inserted between the capacitance C_A and the terminals **10**, **11** of the lattice **5**, as shown in Fig. 3, giving a new mid-band image impedance equal to R_6. The network of Fig. 3 is now transformed into the equivalent network **12** shown in Fig. 4 by replacing the transformer T and the capacitance C_A by the equivalent π comprising the shunt capacitance C_2, the series capacitance C_1 and the shunt capacitance C_Y which have the values

$$C_1=\frac{C_A}{\Phi} \quad (12)$$

$$C_2=\frac{(1-\Phi)C_A}{\Phi^2} \quad (13)$$

$$C_Y=\frac{(\Phi-1)C_A}{\Phi} \quad (14)$$

where Φ is the square root of the impedance transformation ratio from right to left in the transformer T. In this connection reference is made to the United States patents to E. L. Norton 1,681,554, issued August 21, 1928, and 1,708,950, issued April 16, 1929. From Equations 12 and 13 Φ is found, in terms of C_1C_2, to be

$$\Phi=\frac{C_1}{C_1+C_2} \quad (15)$$

Substituting in Equation 15 the values of C_1 and C_2 from Equations 5 and 6 gives Φ in terms of the cut-off frequencies f_1 and f_4 as

$$\Phi=1-\left(\frac{f_1}{f_4}\right)^2 \quad (16)$$

The next step is to determine R_2 in terms of R_1, f_1 and f_4. This may be done by substituting in Equation 12 the values of C_1, C_A and Φ as given respectively by Equations 5, 8 and 16. The result is

$$R_2=\frac{R_1 f_1 f_4}{(f_4-f_1)^2} \quad (17)$$

Now, by substituting this value of R_2 in Equation 9, C_B is found to have the value

$$C_B=\frac{f_4-f_1}{2\pi f_4^2 R_1} \quad (18)$$

But from Equation 14, by the aid of Equations 5, 12 and 16, it is found that

$$C_Y=-\frac{f_4-f_1}{2\pi f_4^2 R_1} \quad (19)$$

It is seen from Equations 18 and 19 that the sum of C_B and C_Y is zero. These capacitances may, therefore, be omitted from the network **12**.

From Equations 10 and 17 it is found that

$$L_B=\frac{R_1}{2\pi(f_4-f_1)} \quad (20)$$

which is the same as the value given in Equation 7 for L_1. Therefore, when the network **12** is substituted for the section **7** in Fig. 2 the circuit shown in Fig. 5 is obtained.

The next step is to determine the image impedance R_4 to be used in evaluating the component elements used in the lattice **5**. The impedances between which the lattice **5** works are

R_3 at the left end and R_6 at the right end. These will be found in terms of R_1 and the cut-off frequencies f_1 and f_4. We know that

$$R_6=\Phi^2 R_5 \qquad (21)$$

But from Equation 11

$$R_5=\frac{R_2 f_4}{f_1+f_4} \qquad (22)$$

Now from Equation 21, using Equations 16, 17 and 22

$$R_6=\frac{R_1 f_1(f_1+f_4)}{f_4^2} \qquad (23)$$

From Equation 11

$$R_3=\frac{R_1(f_1+f_4)}{f_4} \qquad (24)$$

Dividing Equation 23 by Equation 24 gives

$$\frac{R_6}{R_3}=\frac{f_1}{f_4} \qquad (25)$$

If the end sections **6** and **12** have a comparatively narrow band (f_4-f_1) the impedances R_3 and R_6 will not differ much from each other. If the image impedance R_4 of the lattice **5** is taken as the geometric mean of the impedances R_3 and R_6 the lattice **5** will be terminated substantially in its image impedance and the reflection losses at the junctions **8**, **9** and **10**, **11** will be negligible. The value of R_4 is, therefore, taken as

$$R_4=\sqrt{R_3 R_6} \qquad (26)$$

and by the aid of Equations 23 and 24 is found to be

$$R_4=\frac{R_1(f_1+f_4)}{f_4}\sqrt{\frac{f_1}{f_4}} \qquad (27)$$

This value of R_4 is used in Equations 1, 2, 3 and 4 to evaluate the component elements of the lattice **5**.

From Equation 17 the over-all impedance step-up of the filter at the mid-band frequency is

$$\frac{R_2}{R_1}=\frac{f_1 f_4}{(f_4-f_1)^2}=\left(\frac{f_m}{f_4-f_1}\right)^2 \qquad (28)$$

Since the mid-band frequency f_m does not change, it is apparent that the impedance ratio is inversely proportional to the square of the band width (f_4-f_1) of the end sections. The width of the band may, therefore, be chosen to correspond to the desired impedance ratio. For example, a five per cent band will give a ratio of 400 and a two per cent band a ratio of 2500. In some cases it may be desirable to make the band of the end sections wider than that of the lattice in order to get a more suitable image impedance outside of the band. This is of importance where two or more filters are to be operated in parallel. However, as the band width of the end sections is increased the attenuation near the cut-off is reduced.

The equal shunt capacitances C_2 and the equal series capacitances C_1 of Fig. 5 may be taken into the lattice as shown in the equivalent network of Fig. 6. The impedance branches Z_{31} and Z_{32} of the lattice of Fig. 6 may then be transformed into the equivalent forms Z_{41} and Z_{42} as shown in Fig. 7. The branch Z_{41} comprises L_{11}, C_{11} and C_{41} and the branch Z_{42} is made up of L_{12}, C_{13} and C_{42}. These elements may be evaluated from the following formulas:

$$C_{11}=C_{13}=\frac{C_{21}C_1^2}{(C_1+C_2+C_{22})(C_1+C_2+C_{21}+C_{22})} \qquad (29)$$

$$C_{41}=C_{42}=\frac{C_1(C_2+C_{22})}{C_1+C_2+C_{22}} \qquad (30)$$

$$L_{11}=\frac{L_{21}(C_1+C_2+C_{22})^2}{C_1^2} \qquad (31)$$

$$L_{12}=\frac{L_{22}(C_1+C_2+C_{24})^2}{C_1^2} \qquad (32)$$

In Fig. 7 the elements C_{11}, C_{13}, L_1, L_{11} and L_{12} are the same as the ones shown in Fig. 1. The capacitances C_{41} and C_{42} will be the same as C_{12} and C_{14} if the end capacitances C_X are zero. However, it is sometimes desirable to have variable shunt capacitances such as C_X available to adjust the transmission characteristic of the filter. They may be provided by reducing the value of C_{41} and C_{42} each by an amount equal to C_X and connecting C_X at the ends, as shown in Fig. 1.

It is seen that each of the lattice branches Z_{41} and Z_{42} of Fig. 7 has the configuration of the equivalent circuit of a piezoelectric crystal element. If the ratio of the capacitances in these branches does not exceed a certain value K, that is, if

$$\frac{C_{41}}{C_{11}}=\frac{C_{42}}{C_{13}}\leq K \qquad (33)$$

the branches may be constituted by crystals. Fig. 8 shows the branches Z_{41} and Z_{42} replaced, respectively, by the crystal elements X_1 and X_2. The series inductance L_1 is also divided, as in Fig. 1. For quartz crystals K has a value of approximately 140. The limitation expressed in Equation 33 also places a limit on the maximum band width of the end sections **6** and **7** which, in terms of the mid-band frequency f_m and the band width (f_3-f_2) of the lattice may be expressed as

$$\frac{f_4-f_1}{f_1+f_4}=\frac{f_m}{K(f_3-f_2)} \qquad (34)$$

If K is taken as 140, it is found from Equation 34 that for a two per cent band for the lattice the maximum band for the end sections will be approximately seventy-six per cent and for a six per cent band for the lattice this maximum band for the end sections will be approximately twenty-five per cent. For approximately a twelve per cent band for the lattice the maximum band for the end sections will just equal the band of the lattice. Therefore, if quartz crystals are used the maximum band that can be transmitted by the filter as a whole is twelve per cent. If the lattice section is designed to have a wider band than this the band of the end sections will be narrower and thus limit the over-all band. Of course, if a type of crystal having a smaller value of K is used the maximum band of the end sections and the maximum band for the filter as a whole are correspondingly increased.

It is assumed that the filter of Fig. 8 has the maximum possible band width and therefore no shunt capacitances, such as C_X in Fig. 1, are required. However, if the filter has a band width less than maximum additional shunt capacitances are required. These may be added in shunt at the ends of the network as the capacitances C_X in Fig. 9 and may be made variable, as indicated, for adjustment. The crystals X_1 and X_2 of Fig. 8 will, of course, have to be replaced by slightly different crystals X_3 and X_4.

The filter shown in Fig. 10 is the same as the one of Fig. 9 except that each of the end capacitances C_X is replaced by two capacitances C_V and C_W of such value that

$$C_V+C_W=C_X \quad (35)$$

One of these capacitances, for example C_V, is made variable for adjusting the filter. This circuit may be used where it is inconvenient to make the entire capacitance C_X variable, as is done in Fig. 9.

What is claimed is:

1. An impedance-transforming wave filter comprising a lattice network and two inductances of equal value, one of said inductances being connected in series at one end of said lattice and the other of said inductances being connected in shunt at the other end of said lattice.

2. A wave filter in accordance with claim 1 in which said lattice is symmetrical.

3. A wave filter in accordance with claim 1 having a band width which is small compared to the mid-band frequency.

4. A wave filter in accordance with claim 1 in which the impedance branches of said lattice comprise piezoelectric crystal elements.

5. A wave filter in accordance with claim 1 which has peaks of attenuation only at zero and infinite frequency.

6. A wave filter in accordance with claim 1 which has peaks of attenuation only at zero and infinite frequency and a band width which is small compared to the mid-band frequency.

7. A wave filter in accordance with claim 1 in which said lattice is symmetrical and the band width is small compared to the mid-band frequency.

8. An impedance-transforming wave filter comprising two pairs of piezoelectric crystal elements and two inductances of equal value, said crystal elements being arranged to form a lattice network, one of said inductances being connected in series at one end of said lattice network and the other of said inductances being connected in shunt at the other end of said lattice network.

9. A wave filter in accordance with claim 8 having a band width which is small compared to the mid-band frequency.

10. A wave filter in accordance with claim 8 which has peaks of attenuation only at zero and infinite frequency.

11. A wave filter in accordance with claim 8 which has peaks of attenuation only at zero and infinite frequency and a band width which is small compared to the mid-band frequency.

12. An impedance-transforming wave filter comprising a lattice network, two inductances, and two variable capacitances, one of said inductances being connected in series at one end of said lattice, the other of said inductances being connected in shunt at the other end of said lattice, and said capacitances being connected in shunt at the respective ends of said lattice.

13. A wave filter in accordance with claim 12 in which said lattice is symmetrical.

14. A wave filter in accordance with claim 12 having a band width which is small compared to the mid-band frequency.

15. A wave filter in accordance with claim 12 in which said inductances are equal.

16. A wave filter in accordance with claim 12 in which the impedance branches of said lattice comprise piezoelectric crystal elements.

17. A wave filter in accordance with claim 12 which has peaks of attenuation only at zero and infinite frequency.

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