

June 13, 1944.

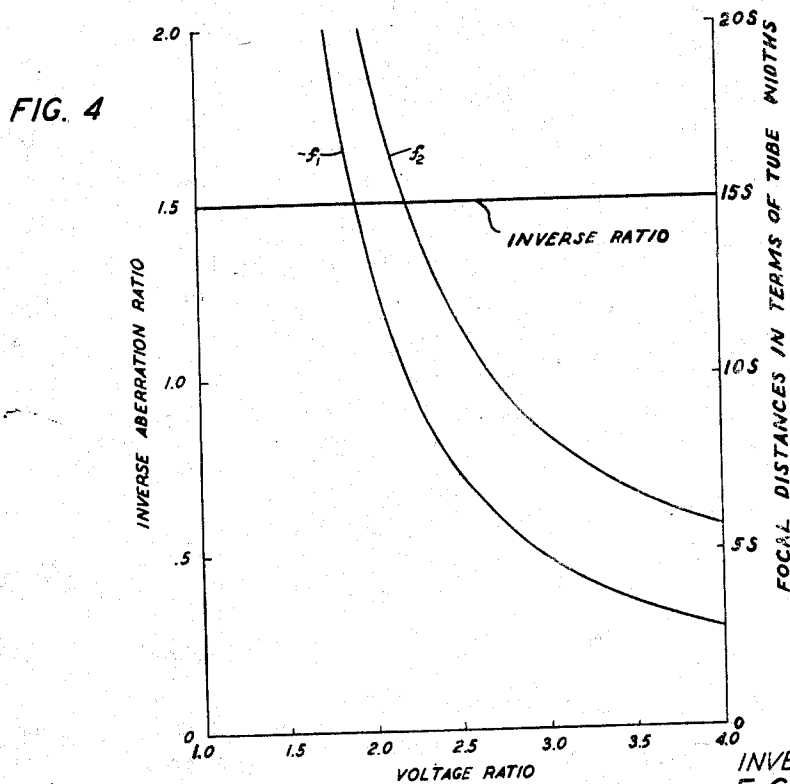
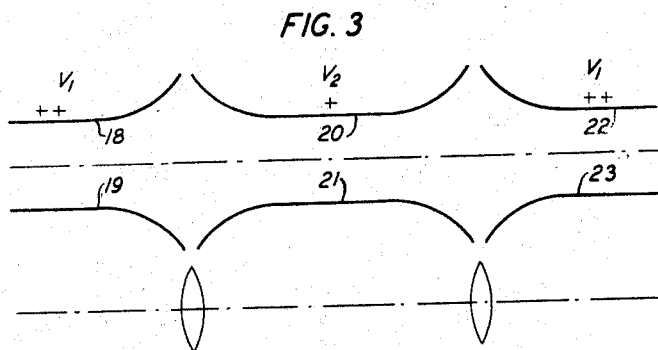
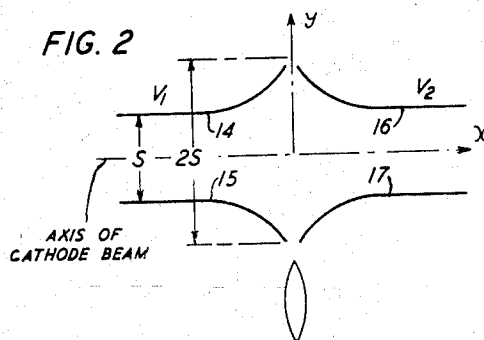
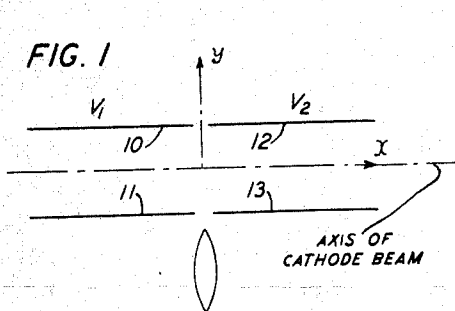
F. GRAY

2,351,501

ELECTRON LENS STRUCTURE

Filed May 31, 1941

2 Sheets-Sheet 1



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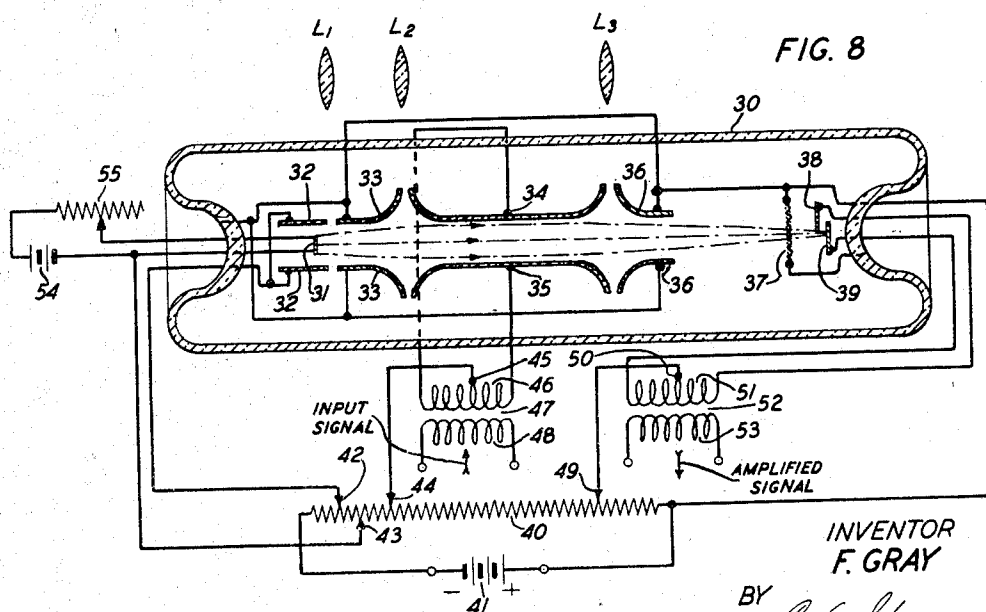
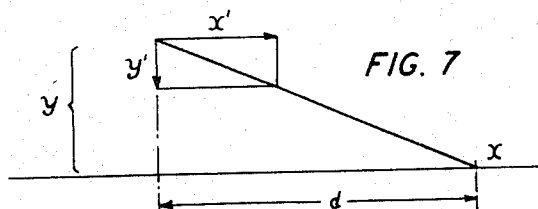
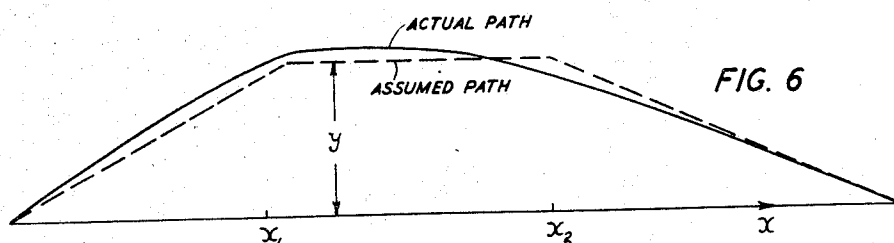
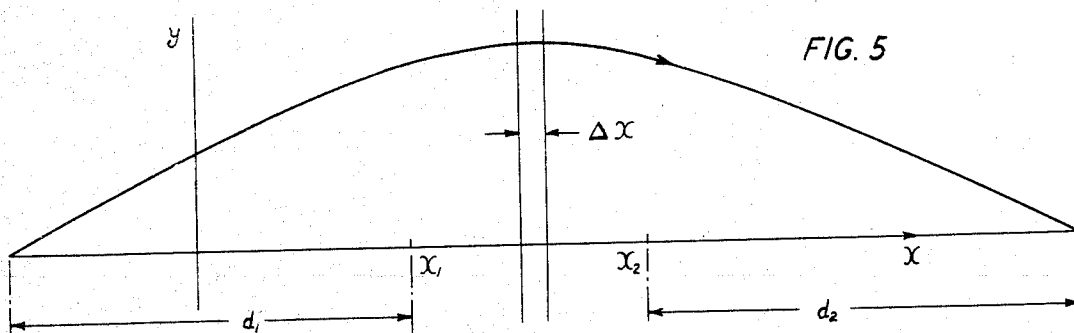
F. GRAY

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ELECTRON LENS STRUCTURE

Filed May 31, 1941

2 Sheets-Sheet 2



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## UNITED STATES PATENT OFFICE

2,351,501

## ELECTRON LENS STRUCTURE

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Application May 31, 1941, Serial No. 395,978

21 Claims. (Cl. 315-17)

This invention relates to electron discharge devices and more particularly to the reduction of aberration in electron lens systems.

It is an object of this invention to provide novel electron lens structures.

The two-dimensional analog of the three-dimensional electron lens comprising two concentric tubes, which analog may be either a pair of parallel plates or a rectangular tube providing that the beam passes through the tube at distances from the side walls of the tube greater than the width thereof, is an important form of electron lens, because it permits a compact combination of the electrodes in deflection amplifiers and similar tubes. Its aberration is, however, quite objectionable when the full aperture of the lens is used, or, in other words, when the beam substantially fills the space between the plates. The aberration of a lens is usually stated in terms of the aberration ratio which is

$$\frac{f}{f_0}$$

where  $f$  is the focal distance for rays entering the lens parallel to the axis and at some distance from the axis, and  $f_0$  is the focal distance for an electron path very near the axis. For a perfect lens the aberration ratio is equal to unity. The inverse aberration ratio is the reciprocal of the aberration ratio. The present invention, in one of its more important aspects, relates to a two-dimensional electron lens structure in which the inverse aberration ratio at the edge of the lens is reduced from infinity (the inverse aberration ratio at the edge of the lens in the usual two-dimensional analog) to a value near that of a perfect lens.

In accordance with the invention, in one embodiment thereof, chosen by way of example to illustrate the principles of the invention there is provided an electron lens structure comprising two pairs of plates, one pair being closer to the beam generating means than the other pair and the respective plates of each pair being located on opposite sides of the beam axis and the plates of each pair being flared outwardly in the region where they face each other. An abrupt flare is required particularly in tubes where the walls are used as sweep plates and they should therefore be plane-parallel electrodes over most of their length.

As a modification, two lenses are combined in a structure comprising three pairs of flared plates spaced along the axis of the tube, the potential of the plates of the middle pair being negative

with respect to the potential of the plates of both of the outer pairs. The potential of each of the negative plates is chosen so that the beam is focussed at a desired point.

The invention will be more fully understood by referring to the following description taken in accordance with the accompanying drawings forming a part thereof in which:

Fig. 1 shows a two-dimensional analog of a spherical electron lens system comprising two concentric tubes;

Fig. 2 shows a two-dimensional electron lens system corrected for aberration;

Fig. 3 shows a combination of two corrected two-dimensional electron lenses;

Figs. 4 to 7, inclusive, are graphical representations to aid in explaining the principles of the invention; and

Fig. 8 shows a tube in which a lens system made in accordance with the principles of this invention is used.

Referring more specifically to the drawings, Fig. 1 shows two pairs of plates 10, 11 and 12, 13 constituting, by means of the potentials  $V_1$  and  $V_2$  applied thereto, a two-dimensional electron lens structure analogous to a cylindrical positive optical lens shown below the electron lens structure. The plates 10, 11 of the first pair are located on opposite sides of the axis of the tube and are closer to the beam generating means than are the plates 12, 13 which are also placed on opposite sides of the axis of the cathode beam. The ratio of the potentials  $V_1$  and  $V_2$  is chosen so that the cathode rays passing successively between the pairs of plates 10, 11 and 12, 13 are focussed to a line in a desired plane to the right of the pair of plates 12, 13. The manner in which this focussing takes place is well understood in the art and will not be described in detail in this specification.

In a two-dimensional electron lens such as is shown in Fig. 1, let  $x$  be a coordinate along the axis of the lens and  $y$  be a coordinate normal to the axis, as shown in Fig. 5. Consider an elementary slice of the electric field constituting the lens. Let its thickness along the axis be  $\Delta x$ . When an electron passes through this slice, its component of velocity  $y$  is changed by an amount  $\Delta y$ , given by the equation

$$\Delta y = -E_y \Delta t \quad (1)$$

where  $E_y$  is the electric force in the  $y$  direction and  $\Delta t$  is the time required for the electron to travel through the element. In Equation 1 elec-

trical units are defined in a manner that eliminates the ratio

$$\frac{e}{m}$$

of an electron.

The equation of the kinetic energy of the electron is

$$\frac{\dot{x}^2 + \dot{y}^2}{2} = \Phi \quad (2)$$

where  $\Phi$  is the electric potential at points in the electric field. For beams of reasonable angular width,  $y$  is small compared with  $x$  and may be neglected in Equation 2. This gives

$$\dot{x} = \sqrt{2\Phi} \quad (3)$$

The element of time  $\Delta t$  in Equation 1 is

$$\Delta t = \frac{\Delta x}{\dot{x}} \quad (4)$$

or from Equation 3

$$\Delta t = \frac{\Delta x}{\sqrt{2\Phi}} \quad (5)$$

The component of electric force  $E_y$  is given by:

$$E_y = -\frac{\partial \Phi}{\partial y} \quad (6)$$

Substitution of Equations 5 and 6 in Equation 1 gives

$$\Delta \dot{y} = \frac{1}{\sqrt{2\Phi}} \cdot \frac{\partial \Phi}{\partial y} \cdot \Delta x \quad (7)$$

Let  $x_1$  and  $x_2$  designate the substantial boundaries of the electric field constituting the lens. Let  $y_1$  be the  $y$  component of velocity at  $x_1$ , and  $y_2$  be the corresponding component at  $x_2$ . Then

$$y_2 = y_1 + \int \Delta \dot{y} \text{ for all the elemental slices} \quad (8)$$

that is,

$$y_2 = y_1 + \int_{x_1}^{x_2} \frac{1}{\sqrt{2\Phi}} \cdot \frac{\partial \Phi}{\partial y} \cdot \Delta x \quad (8A)$$

Where the integral is to be taken along the particular electron path that is under consideration, the values of

$$\sqrt{2\Phi}$$

and

$$\frac{\partial \Phi}{\partial y}$$

are the ones that occur along that particular path. As an approximation, variations in  $y$  along an electron path through the lens can be neglected and  $y$  considered as constant along the path, that is, the electron is treated as if it goes through the lens along a path parallel to the axis at a distance  $y$ , that substantially corresponds to the actual path (see Fig. 6).

The focal distance  $d$  is given by (see Fig. 7)

$$d = -\frac{y\dot{x}}{\dot{y}} \quad (9)$$

The substitution of this equation and Equation 3 in Equation 8 gives

$$\frac{\sqrt{2\Phi_2}}{d_2} = \frac{\sqrt{2\Phi_1}}{d_1} - \frac{1}{y} \int_{x_1}^{x_2} \frac{1}{\sqrt{2\Phi}} \cdot \frac{\partial \Phi}{\partial y} \cdot \Delta x \quad (10)$$

where the subscripts again refer to values at  $x_1$  and  $x_2$ .

If a lens is to be free of aberration, the second focal distance  $d_2$  must be independent of the distance  $y$  at which an electron passes through the lens, otherwise electrons from a common focal point at distance  $d_1$  would come to a focus at

a different distance  $d_2$ , and the focus would be diffused along the axis. Now  $\Phi_1$  and  $\Phi_2$  are the potentials just outside of the lens, that is, the potential at the entrance side of the lens and the potential at the exit side, and they are independent of  $y$ . Thus if the last term of Equation 10 is independent of  $y$ , the lens will be substantially free from aberration. This demands that

$$\frac{1}{y} \int_{x_1}^{x_2} \frac{1}{\sqrt{2\Phi}} \cdot \frac{\partial \Phi}{\partial y} \cdot \Delta x = \text{constant} \quad (11)$$

or, since  $y$  is considered constant along an electron path, that

$$\int_{x_1}^{x_2} \frac{1}{y\sqrt{2\Phi}} \cdot \frac{\partial \Phi}{\partial y} \cdot \Delta x = \text{constant} \quad (12)$$

This would be true if

$$\frac{1}{\sqrt{2\Phi}} \cdot \frac{\partial \Phi}{\partial y}$$

were proportional to  $y$ , and it is more likely to be true if that factor is roughly proportional to  $y$ . In any practical case the factor cannot be made exactly proportional to  $y$  but it can be made of such a nature that it is more than proportional in some regions and less in others and so that the departures from proportionality cancel each other in the integral. This seems more likely to occur when the departures from proportionality are not large.

The electric potential obeys the equation

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \quad (13)$$

To obtain a lens with reduced aberration, a solution of Equation 13 which also obeys Equation 12 is necessary. A further requirement is that the electron lens structure correspond to two sets of parallel plates over most of their extent.

The aberration of a lens is usually stated in terms of the aberration ratio. It is based on the focal distance  $f$  of rays entering the lens parallel to the axis. From Equation 10

$$\frac{\sqrt{2\Phi_2}}{f} = \frac{\sqrt{2\Phi_1}}{\infty} - \frac{1}{y} \int_{x_1}^{x_2} \frac{1}{\sqrt{2\Phi}} \cdot \frac{\partial \Phi}{\partial y} \cdot \Delta x \quad (14)$$

or

$$\frac{1}{f} = -\frac{1}{\sqrt{2\Phi_2}} \int_{x_1}^{x_2} \frac{1}{y\sqrt{2\Phi}} \cdot \frac{\partial \Phi}{\partial y} \cdot \Delta x \quad (15)$$

The focal length for an electron path near the axis is designated as  $f_0$ . The focal length for an electron path near the edge of the lens is calculated and designated as  $f$ . The aberration ratio at the edge of the lens is

$$\frac{f}{f_0}$$

For a perfect lens this ratio would be equal to unity, and this condition is at least approximately attained in the structure shown in Fig. 2.

In Fig. 2 each of the plates 14, 15 and 16, 17 has its edge facing the corresponding plate in the other pair flared. An abrupt flare, such as is shown in the drawings, is required because the plates in some tubes may be used as sweep plates and they should therefore be plane parallel electrodes over most of their length. Moreover, for design reasons, it is desirable to calculate the focal lengths of the lens without undue labor and this requires the electric potential to be an analytic function of the coordinates  $x$  and  $y$ . The electric potential  $\Phi$  in such a lens structure may be combined with its conjugate potential  $\Psi$  to give a complex potential function  $\Phi + i\Psi$  of

$x+iy$ . One such complex potential has been found and it may be written in the form

$$\Phi + i\Psi = \frac{V_2 + V_1}{2} + \frac{V_2 - V_1}{2} \tanh \omega (x + iy) \quad (16)$$

where  $V_1$  is the potential of the first pair of plates 14, 15 and  $V_2$  is the potential of the second pair of plates 16, 17 and

$$\omega = \frac{\pi}{2S}$$

where  $S$  is the distance between the pairs of plates 14, 15 and 16, 17 in the region where the plates are substantially parallel. The electric potential is

$$\Phi = \frac{V_2 + V_1}{2} + \frac{V_2 - V_1}{2} \frac{\sinh 2\omega x}{\cosh 2\omega x + \cos 2\omega y} \quad (17)$$

The equation of the equipotential surface at  $V_1$  is

$$y = \pm S/\pi \cos^{-1}(-e^{\pm \pi x/S}), x < 0 \quad (18)$$

and the equation of the equipotential surface at  $V_2$  is

$$y = \pm S/\pi \cos^{-1}(-e^{-\pi x/S}), x > 0 \quad (19)$$

These equipotential surfaces are shown in Fig. 2 and it is seen that they correspond to two pairs of plates with flared ends. The shapes of the physical electrodes for the lens shown in Fig. 2 are consequently given by Equations 18 and 19.

It follows from Equations 18 and 19 that the maximum separation of a pair of plates occurs at  $x=0$ . The substitution of this value of  $x$  in either equation gives

$$y = \pm \frac{S}{\pi} \cos^{-1}(-1) = \pm \frac{S}{\pi} \cdot \pi = \pm S$$

The maximum separation is thus  $2S$ , which is twice the minimum separation  $S$ .

It also follows from these equations that, at  $x=S$ , the separation  $2y$  is  $1.03S$  which is only 3 per cent greater than the minimum separation  $S$ . Thus the flare is substantially confined to a region along the axis equal to the minimum plate separation.

The

$$\frac{\partial \Phi}{\partial y}$$

is obtained from Equation 16. The appreciable values of this derivative are confined to a narrow region, where the pairs of plates 14, 15 and 16, 17 face each other, and covering a distance along the  $x$  axis about equal to  $S$ . In cases where the conjugate focal distances of the electron lens are large compared to  $S$ , it may be treated as a thin lens. The inverse focal term is then given approximately by the integral

$$\frac{1}{F} = - \int_{-\infty}^{+\infty} \frac{1}{y} \frac{\partial \Phi}{\partial y} \frac{dx}{\sqrt{2\Phi}} \quad (20)$$

where  $y$  is considered a constant in the integration.

For paraxial electron paths—paths near the axis— $\cos 2y$  is approximately equal to unity, and  $\sin 2y$  is approximately equal to  $2y$ . With these approximations the integration for the inverse focal term gives

$$\frac{1}{F} = \frac{\sqrt{2}}{S} \left[ \frac{4(V_2^{3/2} - V_1^{3/2})}{3(V_2 - V_1)} - \sqrt{V_2} - \sqrt{V_1} \right] \quad (21)$$

The first principal focal distance  $f_1$  is  $-F\sqrt{2V_1}$ ,

and from Equation 21 it may be written in the form

$$f_1 = -\frac{3S(\sqrt{\beta} + 1)}{\pi(\sqrt{\beta} - 1)^2} \quad (22)$$

where  $\beta$  is the voltage ratio

$$\frac{V_2}{V_1}$$

10 The second principal focal distance  $f_2$  is

$$f_2 = -f_1\sqrt{\beta} \quad (23)$$

The values of these principal focal distances  $f_1$  and  $f_2$  are given as a function of voltage ratio in the curves shown in Fig. 4. These plotted values may be used in the lens equation

$$\frac{f_2}{d_2} + \frac{f_1}{d_1} = 1 \quad (24)$$

for calculating a conjugate focal distance  $d_1$  or  $d_2$ .

Equation 20 for the inverse focal term cannot in general be integrated for electron paths at a distance from the axis, but it can be integrated in series for the particular case of an electron path through the edge of the lens, that is, a path at  $y$  equal to  $S/2$ . For this path the integral (20) may be written in the form

$$\frac{1}{F} = -A \int_{-\pi/2}^{+\pi/2} \frac{\sin \theta d\theta}{\sqrt{1 + K \sin \theta}} \quad (25)$$

where

$$A = -\frac{V_2 - V_1}{S} (V_2 + V_1)^{-1/2}$$

$$K = \frac{V_2 - V_1}{V_2 + V_1} \quad (26)$$

$$\theta = \sin^{-1} \tanh 2\omega x$$

40 The integral is transformed to

$$\frac{1}{F} = A \int_0^{\pi/2} \left[ \frac{1}{\sqrt{1 - K \sin \theta}} - \frac{1}{\sqrt{1 + K \sin \theta}} \right] \sin \theta d\theta \quad (27)$$

45 and its two terms are expanded as power series in  $K \sin \theta$  and subtracted from each other. Each term of the resulting series constitutes a known definite integral, and the integration of the series gives

$$\frac{1}{F} = \frac{\pi A}{2} \left[ \frac{1}{2} K + \frac{3.5}{4.6} \frac{3}{2.4} K^3 + \frac{3.5 \cdot 7.9}{4.6 \cdot 8.10} \frac{3.5}{2.4 \cdot 6} K^5 + \dots \right] \quad (28)$$

55 The first principal focal distance for an electron path through the edge of the lens is  $-F\sqrt{2V_1}$  and it may be written in the form

$$\frac{1}{f_1} = -\frac{\pi}{4S\sqrt{2}} \frac{(\beta - 1)^2}{(\beta + 1)^{3/2}} \left[ 1 + \frac{15}{32} \frac{(\beta - 1)^2}{(\beta + 1)} + \frac{315}{1024} \frac{(\beta - 1)^4}{(\beta + 1)^2} + \dots \right] \quad (29)$$

The second principal focal distance is again equal to

$$-f_1\sqrt{\beta}$$

From the preceding equations the aberration ratio  $f/f_0$  is obtained at the edge of the lens; it is the ratio of a focal distance  $f$  at the edge of the lens to the corresponding focal distance  $f_0$  at the center of the lens. The inverse ratio  $f_0/f$  is, however, more descriptive in the present case, and this ratio is plotted as a function of voltage ratio in a curve shown in Fig. 4. This

curve shows that the correction improves the lens in a decided manner. It reduces the inverse aberration ratio from infinity—for an uncorrected lens—to values near the unity ratio of a perfect lens.

The type of lens covered by this invention is used as a component in the two-lens combination shown in Fig. 3. Such a combination comprises pairs of plates 18, 19 and 20, 21 and 22, 23 flared as in the arrangement shown in Fig. 2, and spaced along the axis of the tube. To these pairs of plates are applied potentials  $V_1$ ,  $V_2$  and  $V_1$ , respectively,  $V_1$  being more positive than  $V_2$ , although it is possible for the potentials to be reversed and  $V_2$  to be more positive than  $V_1$ . The two lens systems (that is, one comprising pairs of plates 18, 19 and 20, 21, and the other comprising pairs of plates 20, 21 and 22, 23) tend to correct each other and any residual aberration in the corrected lenses will be still further reduced when they are used in this manner. The pair of corrected lenses shown in Fig. 3 gives a clean-cut line image.

Throughout the description the term "pairs of plates" has been used but it is to be understood that the arrangement operates just as well (except when the pair of plates is replaced by a single tube of rectangular cross section, parallel edges of which are flared as in the case of the parallel plates in all regions at a distance greater than  $S$  away from the side walls of the rectangular tube. The design is similar for these two types of structures.

In Fig. 4 focal distances in terms of tube width and inverse aberration ratio are plotted against voltage ratios. The principal focal distances  $f_1$  and  $f_2$  are shown for values of the voltage ratio

$$\frac{V_2}{V_1} > 1$$

The curves may also be used for voltage ratios less than 1 by interchanging  $V_2$  and  $V_1$ , and interchanging  $-f_1$  and  $f_2$  at the same time.

The lens systems described above may be used in any suitable electron discharge device wherein it is desired to have two-dimensional electron lens systems of high quality but they are particularly applicable in the case of amplifiers which operate by deflecting an electron beam between a pair of plates. Such a tube is shown in Fig. 8.

In Fig. 8 there is disclosed a deflection amplifier tube with two-dimensional electron lenses corrected for aberration in accordance with this invention. The structural details of the electrode mountings are omitted for simplicity in description. The tube comprises an evacuated container 30 enclosing a linear cathode 31 to furnish a stream of electrons, a pair of plates 32, 32 electrically connected together and placed at a negative potential with respect to the cathode, a lens element comprising a pair of plates 33, 33 flared in accordance with the structure shown in Fig. 3, a lens element comprising flanged parallel deflecting plates 34, 35 the average potential of which is placed at a value which is less than that of the plates 33, 33, a lens element comprising the plates 36, 36 flanged as shown in Fig. 3 and which are connected together and placed at the same potential as the plates 33, 33, and a split target 38, 39 over which the focussed ribbon stream of electrons is moved by the deflecting plates 34, 35. In front of the split target 38, 39

is a positive grid element 37 to draw secondary electrons from the split target and this is connected to a point of the same potential as the plates 36, 36 and the plates 33, 33. This potential is obtained from the positive terminal of the potentiometer 40 which is connected across a suitable source 41. The most negative tap 42 of the potentiometer resistance 40 is connected to the plates 32, 32, the next tap 43 is connected to the cathode 31, and the tap 44 is connected to the mid-point 45 of the secondary winding 46 of a transformer 47, the primary winding 48 of which is connected to a source of input signals for deflecting the sheet of electrons in the tube. The tap 49 is connected to the mid-point 50 of the primary winding 51 of the transformer 52 the secondary winding 53 of which is connected to an output circuit. The terminals of the primary winding 51 are connected to the plates 38, 39 forming the split target. The intensity of the flow of secondary electrons from the target 38, 39 to the mesh grid 37 determines the output current of the amplifier. The cathode 31 is heated by current from a source 54 through resistance 55.

The operation of the tube 30 is as follows:

The electron stream from the cathode 31 is partially focussed by the field near the negative plates 32, 32 which acts as a lens  $L_1$ . This lens need not be corrected for aberration because the stream does not pass near the edges of the plates. The beam is then focussed into a parallel stream of electrons by the corrected lens  $L_2$  which is located in the flared regions of plates 33, 33 and deflecting plates 34, 35. The sheet of electrons is deflected by signals applied to the sweep plates 34, 35 which signals may be sine waves, sawtoothed waves, or other types of complex signal waves. The stream then passes through the electron lens  $L_3$  which is located in the flared regions of the sweep plates 34, 35 and 36, 36. This lens  $L_3$  focusses the beam to a narrow ribbon-like image on the split target 38, 39. The motion of this image across the split target gives an amplified signal output in a manner well known by varying the potential of the plates 38 and 39 with respect to each other and thus setting up a current in the primary winding 51 of the transformer 52 which is proportional to the input signal applied to the primary winding 48 of the transformer 47.

Since the two electron lenses  $L_2$  and  $L_3$  are substantially corrected for aberration, the beam can substantially fill the region between the sweep plates 34 and 35. The sweep plates can thus be positioned near each other and this gives a gain in sensitivity with a resultant gain in amplification.

Various changes may be made in the embodiments disclosed above without changing the scope of the invention, as indicated by the appended claims.

What is claimed is:

1. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent, a second pair of metallic plate members which are generally parallel throughout most of their extent and are also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members being adjacent each other and having a common longitudinal axis which passes between the metallic members of each pair and each of said members being flared outwardly

in a portion thereof near the other pair of members, and means for maintaining at all times the potential of one member of one of said pairs equal to that of the other member of the same pair.

2. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent, and a second pair of metallic plate members which are generally parallel throughout most of their extent and are also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members being adjacent each other and having a common longitudinal axis which passes between the metallic members of each pair and each of said plates being flared outwardly in a portion thereof near the other pair of members, the plates of at least one of said pairs of members constituting throughout most of their extent two of the four walls of an open box-like member having a rectangular cross-section, the entire box-like member being at one potential.

3. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent, a second pair of metallic plate members which are generally parallel throughout most of their extent and are also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members being adjacent each other and having a common longitudinal axis which passes between the metallic members of each pair and each of said plates being flared outwardly in a portion thereof near the other pair of members, the maximum distance between the metallic members in a pair being substantially twice that of the minimum distance between said members, and means for maintaining at all times the potential of one member of one of said pairs equal to that of the other member of the same pair.

4. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent, and a second pair of metallic plate members which are generally parallel throughout most of their extent and are also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members being adjacent each other and having a common longitudinal axis which passes between the metallic members of each pair and each of said plates being flared outwardly in a portion thereof near the other pair of members, and means for maintaining one plate of each pair at the same potential as the other plate of the same pair, said potentials being different.

5. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent, a second pair of metallic plate members which are generally parallel throughout most of their extent, and a third pair of metallic plate members which are generally parallel throughout most of their extent, the plates of the second and third pairs being also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members having a common longitudinal axis which passes between the metallic members of each pair and said pairs of members being spaced in the direction of said

axis, each of the members of the first pair being flared outwardly in portions thereof nearer the second pair of members, each of the members of the second pair being flared outwardly in portions thereof nearer said first pair of members and nearer said third pair of members and each of said third pair of members being flared outwardly in portions thereof nearer the second pair of members.

6. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent, a second pair of metallic plate members which are generally parallel throughout most of their extent, and a third pair of metallic plate members which are generally parallel throughout most of their extent, the plates of the second and third pairs being also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members having a common longitudinal axis which passes between the metallic members of each pair and said pairs of members being spaced in the direction of said axis, each of the members of the first pair being flared outwardly in portions thereof nearer the second pair of members, each of the members of the second pair being flared outwardly in portions thereof nearer said first pair of members and nearer said third pair of members, and each of said third pair of members being flared outwardly in portions thereof nearer the second pair of members, the maximum distance between the metallic members in a pair being substantially twice that of the minimum distance between said members.

7. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent, a second pair of metallic plate members which are generally parallel throughout most of their extent, and a third pair of metallic plate members which are generally parallel throughout most of their extent, the plates of the second and third pairs being also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members having a common longitudinal axis which passes between the metallic members of each pair and said pairs of members being spaced in the direction of said axis, each of the members of the first pair being flared outwardly in portions thereof nearer the second pair of members, each of the members of the second pair being flared outwardly in portions thereof nearer said first pair of members and nearer said third pair of members, and each of said third pair of members being flared outwardly in portions thereof nearer the second pair of members, means for maintaining the first pair of members and the third pair of members at substantially the same average potential, and means for maintaining the second pair of members at an average potential which is different from that of the first and third pairs of members.

8. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent, a second pair of metallic plate members which are generally parallel throughout most of their extent and are also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members being adjacent each other and having a common longitudinal axis which passes between the metallic members of each



pair and each of said members being flared outwardly in portions thereof near the other pair of members, the flare being substantially confined to a region along the axis equal to the minimum separation between the members of a pair, and means for maintaining at all times the potential of one member of one of said pairs equal to that of the other member of the same pair.

9. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent and a second pair of metallic plate members which are generally parallel throughout most of their extent and are also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members being adjacent each other and having a common longitudinal axis which passes between the metallic members of each pair and each of said members being flared outwardly in portions thereof near the other pair of members, the flare being substantially confined to a region along the axis equal to the minimum separation between the members of a pair, the flare of one of said members of the first pair being in accordance with the equation

$$y = \pm \frac{S}{\pi} \cos^{-1} \left( -e^{-\frac{\pi x}{S}} \right)$$

where  $x < 0$  and the flare of one of the members of the second pair being in accordance with the equation

$$y = \pm \frac{S}{\pi} \cos^{-1} \left( -e^{-\frac{\pi x}{S}} \right)$$

where  $x > 0$ , in which equations  $y$  is the perpendicular distance from the axis  $x$  is the distance along the axis from a point thereon equidistant from the two pairs, and  $S$  is the distance between the members in the parallel portions thereof.

10. An electron optical system comprising a pair of metallic plate members which are generally parallel throughout most of their extent and a second pair of metallic plate members which are generally parallel throughout most of their extent and are also parallel throughout most of their extent to a plane parallel to the plates of the first pair throughout most of their extent, said pairs of members being adjacent each other and having a common longitudinal axis which passes between the metallic members of each pair and each of said members being flared outwardly in portions thereof near the other pair of members, the flare being substantially confined to a region along the axis equal to the minimum separation between the members of a pair, and the maximum separation being approximately twice the minimum separation.

11. An electron discharge device comprising an enclosure containing a ribbon cathode, at least two pairs of metallic plate members therein positioned so that electrons emitted from said cathode pass between the members of each of said pairs in succession, the members of each pair being generally parallel throughout most of their extent and flared outwardly in portions thereof near the other pair of members, and means for maintaining at all times the potential of one member of one of said pairs equal to that of the other member of the same pair.

12. An electron discharge device comprising a ribbon cathode, at least two pairs of metallic plate members therein positioned so that electrons from said cathode pass between the mem-

bers of each of said pairs in succession, the members of each pair being generally parallel throughout most of their extent and flared outwardly in portions thereof near the other pair of members, means for applying deflecting signals between the members of the pair farthest removed from said cathode, means for maintaining at all times the potential of one member of another one of said pairs equal to that of the other member of the same pair, and a pair of target elements.

13. An electron discharge device having a ribbon cathode for emitting a stream of electrons, a pair of metallic members on opposite sides of said cathode, means for applying a negative potential to said pair of members with respect to the potential of said cathode, a second pair of metallic members which are generally parallel throughout most of their extent and which are spaced from said first pair of members in the direction of the electron flow, a third pair of metallic members which are generally parallel throughout most of their extent and which are spaced from said second pair of members in the direction of electron flow, a fourth pair of metallic members which are generally parallel throughout most of their extent and which are spaced from said third pair in the direction of electron flow, means for applying deflecting signals between the members of the third pair, a pair of target members, and means for connecting said pair of targets to an output circuit, the second and third of said pairs of members being flared outwardly in portions thereof near the other and said third and fourth pairs of members being flared outwardly in the portions thereof near the other.

14. An electron lens comprising two pairs of metallic members, each member having a substantially plane portion and all said plane portions being parallel to a plane between the members of a pair, each of said members having an end portion flared outwardly in a direction away from said plane with its extremity spaced a short distance from that of the flared end portion of a member of the other pair, and means for maintaining at all times the potential of one member of one of said pairs equal to that of the other member of the same pair.

15. In combination, a pair of parallel deflecting plates, means for producing and causing to flow between said plates a sheet of electrons with substantially parallel sides which are at all times nearly parallel to said plates respectively and spaced therefrom by a distance much less than the thickness of said sheet of electrons, said means comprising a source of electrons and a pair of plates between said source and said deflecting plates and close to the latter and positioned in planes parallel to the sides of said sheet of electrons between said deflecting plates, said last-mentioned plates having their ends which are adjacent said deflecting plates curved outwardly and the ends of said deflecting plates adjacent thereto similarly curved outwardly, whereby the electrons from said source are made to converge to form said sheet by virtue of electron lens action between said pairs of plates.

16. The combination of elements as in claim 15 in further combination with another pair of plates placed on opposite sides of said sheet of electrons on the side of said pair of deflecting plates remote from said cathode, said last-mentioned plates having their ends which are adjacent said deflecting plates curved outwardly and the ends



of said deflecting plates which are adjacent said other pair of plates similarly curved outwardly, and means for applying a direct potential between said other plates and said deflecting plates whereby an electron lens is formed between said pair of deflecting plates and said other pair of plates.

17. The combination of elements as in claim 15 in further combination with a target for said sheet of electrons, and means for applying a potential between the plates of the pair of deflecting plates of such magnitude as to give said sheet a deflection at the target which is small compared with the thickness of said sheet between the deflecting plates.

18. An electron optical system comprising two pairs of metallic members, each member having a substantially plane portion and all said plane portions being parallel to a plane between the members of a pair, each of said members having an end portion flared outwardly in a direction away from said plane with its extremity spaced a short distance from that of the flared end portion of a member of the other pair, and means for applying a constant potential to one of said pairs of members and a varying deflecting potential between the members of the other of said pairs.

19. An electron lens comprising two pairs of

metallic members, each member having a substantially plane portion and all said plane portions being parallel to a plane between the members of a pair, each of said members having an outwardly flared portion adjacent the corresponding member of the other pair which flared portion presents a convex surface portion to said corresponding member and has its extremity spaced a short distance from that of the flared end portion of a member of the other pair, and means for maintaining at all times the potential of one member of one of said pairs equal to that of the other member of the same pair.

20. The combination of elements as in claim 18 in further combination with means for maintaining at all times the average of the potentials of the members of the pair to which the varying deflecting potential is applied different from the constant potential applied to the members of the other pair.

21. The combination of elements as in claim 19 in further combination with means for maintaining at all times the potential of one member of the other of said pairs equal to that of the other member of that pair which potential is different from that of the potential of the members of said one of said pairs.

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