

Feb. 29, 1944.

H. W. BODE

2,342,638

WAVE TRANSMISSION NETWORK

Filed Oct. 9, 1942

3 Sheets-Sheet 1

FIG. 1

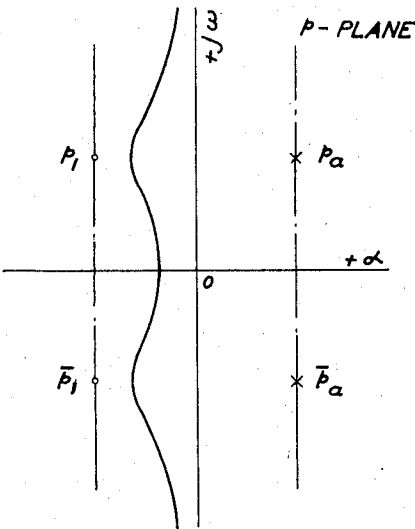


FIG. 2

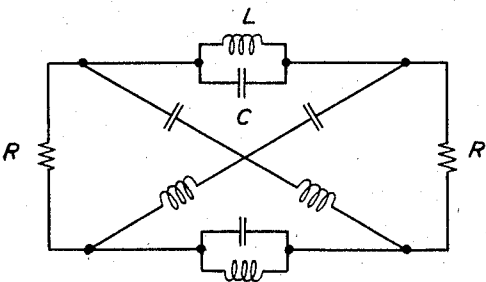
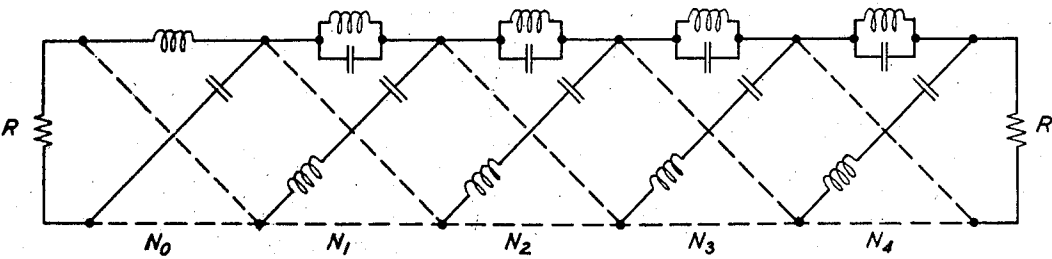


FIG. 3



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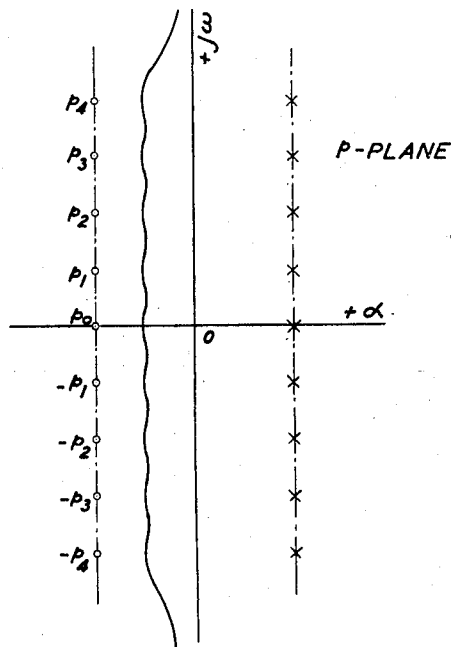


FIG. 4

FIG. 5

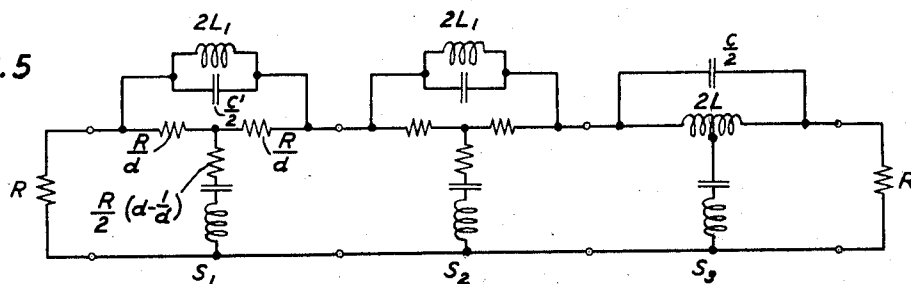
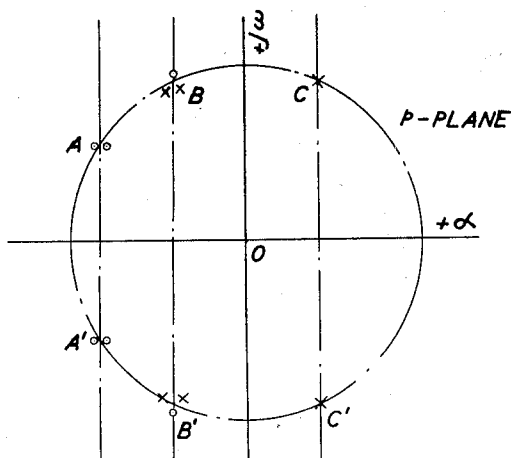


FIG. 6



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FIG. 7

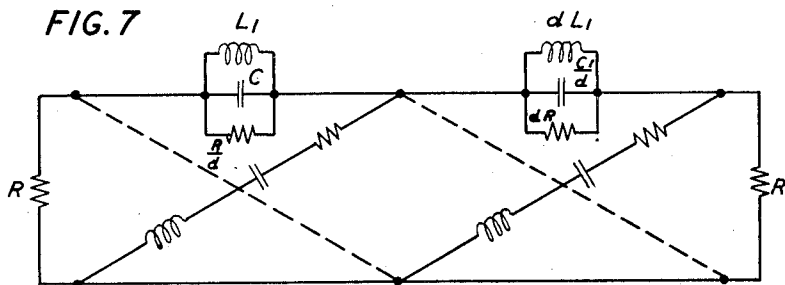


FIG. 8

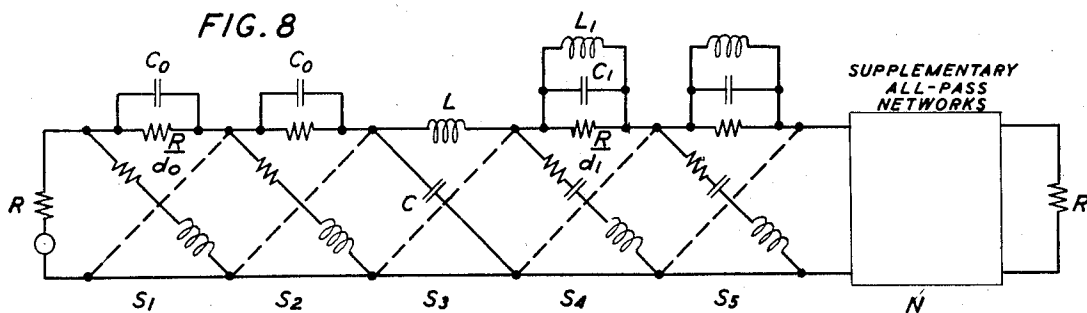


FIG. 9

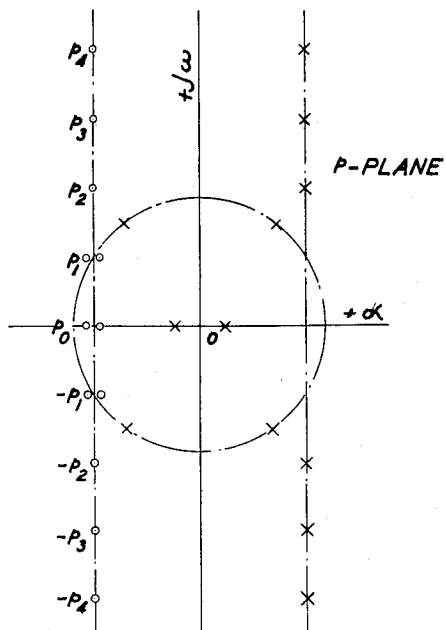
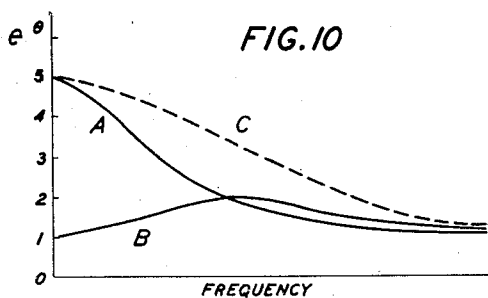


FIG. 10



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UNITED STATES PATENT OFFICE

2,342,638

WAVE TRANSMISSION NETWORK

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Application October 9, 1942, Serial No. 461,476

6 Claims. (Cl. 178—44)

This invention relates to wave transmission networks and more particularly to networks intended for use in broad band transmission systems. It has for its principal objects improving the phase shift characteristics of attenuating networks and delay networks. Another object is to compensate the delay distortion which is inherently present in networks having frequency dependent attenuation.

It is shown in my earlier United States Patent 2,123,178, issued July 27, 1938, that a frequency dependent attenuation produced by a network of lumped impedances is necessarily accompanied by a varying phase shift the value of which for any frequency must be at least as great as a certain minimum determined by the course of the attenuation characteristic. This varying phase shift, unless the variation be linear with frequency, gives rise to delay distortion and to other effects which may seriously interfere with the reproduction of a transmitted signal. Similar effects may also be produced by delay networks where these are used for control purposes, unless the delay be quite uniform at all frequencies in the operating range. Television systems are particularly sensitive to troubles arising from delay distortion because of the extreme complexity of the signal wave and the necessity for maintaining the correct phase relationships of all of the signal wave components in order that a clear visual image may be reproduced. The presence of delay distortion in such systems disturbs the phase relationships of the signal components in such a way as to produce marked peaking of the wave form and consequent overloading of the amplifiers of the system. This results in the generation of modulation voltages which mar the reproduced image and which cannot be removed by any subsequent filtering process.

In many types of systems it has been practicable to compensate the effects of delay distortion by the use of delay equalizers located at terminal points and designed to equalize the over-all distortion. This method, however leaves intermediate portions of the system uncompensated and is therefore not well suited for television and other systems where the phase shift requirements are particularly severe. Accordingly, it has become desirable to maintain such systems free from delay distortion at all points and, to this end, it is necessary that all networks inserted for control or equalizing purposes should themselves be free from delay distortion.

In accordance with the invention, improved delay and attenuating networks are provided the

delay characteristics of which are uniform to a high degree over any assigned frequency range and which therefore may be inserted in a transmission system without introducing delay distortion in that range. The networks of the invention comprise a plurality of constant resistance sections connected in tandem and having their elements proportioned in a novel manner hereinafter described to produce certain unique relationships among the zeros and poles of the insertion loss. By virtue of the particular coordination of the network parameters, the delay of the whole network may be made uniform to any desired degree in an assigned frequency range with the use of a minimum number of component sections. In the case of the attenuating networks of the invention, uniformity of the delay is made possible by the use of attenuating sections of special character and configuration and the combination therewith or the inclusion therein of an all-pass section or its equivalent whereby certain components of the delay that are uniquely characteristic of the attenuation are exactly neutralized.

These and other aspects of the invention will be more fully understood from the detailed description which follows and by reference to the accompanying drawings, of which

Figs. 1 and 2 are illustrative of the principles underlying the invention;

Fig. 3 is a schematic representation of a delay network in accordance with the invention;

Fig. 4 shows a characteristic of the network of Fig. 3;

Fig. 5 shows the circuit configuration of a neutralized attenuating network in accordance with the invention;

Fig. 6 shows a characteristic of the network of Fig. 5;

Fig. 7 shows another form of neutralized network according to the invention; and

Figs. 8, 9 and 10 show the configuration and certain characteristics of a uniform delay attenuating network of the invention.

It will be convenient, before discussing particular embodiments of the invention, to develop the general principles on which the method of securing uniformity of delay is based. The application of these principles to the design of representative networks will then be described, but it will be understood that the invention is not limited to the particular examples illustrated.

The mathematical expression for the insertion loss of a lumped impedance network takes the

form of the ratio of two polynomials in p , the differential operator

$$\frac{d}{dt}$$

in which the coefficients are functions of the element parameters, inductance, capacity, and resistance. In the most typical case the expression may be written in the form

$$e^{\theta} = \frac{(p-p_1)(p-p_2) \dots (p-p_n)}{(p-p_a)(p-p_b) \dots (p-p_k)} \quad (1)$$

wherein θ denotes the insertion loss and the p 's with subscripts are the several roots of the two polynomials. The roots are subject to restrictions consistent with the fact that the network is linear and passive, namely, that complex roots must occur in conjugate pairs and that the real parts of the roots of the numerator must be negative and not zero. The roots of the numerator are generally referred to as the zeros of the insertion loss and the roots of the denominator as the poles. In accordance with well-known principles their real parts represent damping constants or logarithmic decrements and their imaginary parts correspond to frequencies. Each root therefore represents the frequency or vector angular velocity and the damping of an input oscillation that will make the output of the network either infinite or exactly zero. The zeros are also identified with the several free or transient oscillatory modes of the system, the real parts giving the dampings and the imaginary parts the angular velocities or frequencies.

In its logarithmic form Equation 1 becomes

$$\theta = \log_e (p-p_1) + \log_e (p-p_2) + \dots - [\log_e (p-p_a) + \log_e (p-p_b) + \dots] \quad (2)$$

The real parts of the logarithms give the components of the attenuation for each pole and each zero and the imaginary parts give the component phase shifts. The values for real frequencies are obtained by taking for the variable p the pure imaginary $j\omega$, where ω denotes 2π times frequency.

The poles and zeros of the insertion loss constitute a set of parameters by which the operating characteristics of the network are determined and their array, when plotted in the complex plane of p , may itself be regarded as an important characteristic of the network. Experience has shown that salient features of the performance of the network may be ascertained by interpretation of this characteristic and that changes in the performance due to changes in the locations of the poles and zeros can be readily translated into corresponding changes in the physical structure of the network.

The interpretation of the array of the poles and zeros in the p -plane to show the delay characteristic of a network is facilitated by the use of an analogy that exists between the logarithm of the insertion loss and the electric potential in the field of a particular system of electrostatic charges. It will be observed that each of the terms in Equation 2 has the same form as the expression for the potential at a point in the field of an infinitely long straight filament carrying a uniformly distributed electric charge, the quantity $(p-p_x)$ corresponding to the radial distance of the point from the filament. The insertion loss for a given array of zeros and poles is therefore analogous to the potential field of a corresponding system of parallel filaments carrying unit charges of appropriate sign and disposed

with respect to each other in the same relative positions as the poles and zeros in the p -plane. The variation of the potential from point to point in the plane perpendicular to the filaments is the same as the variation of the real part of the insertion loss for varying values of p including complex values. The potential variation along the line corresponding to the imaginary axis of the p -plane represents the attenuation characteristic at real frequencies.

Since the insertion loss θ is a function of the complex variable p , its rate of change in any given direction in the p -plane will be complex also and will be subject to the condition that the imaginary part is equal in magnitude to the real part of the rate of variation in the perpendicular direction. It follows from this relationship that the delay of the network, which is the imaginary part of the rate of variation of the insertion loss with real frequency, is represented by the electric force in the analogous electrostatic system perpendicular to the line corresponding to the real frequency axis. The delay characteristic of a network can thus be studied by examining the electric force in an analogous electrostatic system of simple and familiar configuration.

Fig. 1 shows the array of the zeros and poles in the p -plane for the all-pass network shown in Fig. 2. The network is a symmetrical lattice connected between equal terminal resistances of value R and is characterized by an inverse relationship with respect to R^2 of the series and the diagonal impedances. For the example illustrated, in which the series impedances comprise the parallel combination of an inductance L and a capacity C , the insertion loss has the value

$$e^{\theta} = \frac{p^2 + \frac{1}{CR}p + \frac{1}{LC}}{p^2 - \frac{1}{CR}p + \frac{1}{LC}} = \frac{(p-p_1)(p-\bar{p}_1)}{(p-p_a)(p-\bar{p}_a)} \quad (3)$$

where

$$\begin{aligned} p_1 &= -\frac{1}{2CR} + j\sqrt{\frac{1}{LC} - \left(\frac{1}{2CR}\right)^2} \\ &= -\alpha + j\omega_0 \\ \bar{p}_1 &= -\alpha - j\omega_0 \\ p_a &= \alpha + j\omega_0 = -\bar{p}_1 \\ \bar{p}_a &= \alpha - j\omega_0 = -p_1 \end{aligned} \quad (4)$$

In Fig. 1 the positions of the zeros p_1 and $\bar{p}_1$ in the complex plane of p are indicated by circles and the positions of the poles by crosses. The real parts of the p 's, denoted by α , are plotted horizontally and the imaginary parts, denoted by $j\omega$, vertically, the vertical axis through O being the real frequency axis. The symmetry of the array is characteristic of all-pass networks.

The electrostatic analogue in this case would consist of four charged filaments normal to the plane and passing through the pole and zero points. All filaments would carry unit charges, those corresponding to the zeros having charges of opposite sign from those corresponding to the poles. It is evident from the symmetry of the system that the potential at all points on the vertical axis through the origin would be zero, which corresponds to the fact that the all-pass network has no attenuation at real frequencies. The electric force at points along the axis and in the direction perpendicular thereto would vary in some such manner as indicated

by the undulating curve, the values of the force being represented by the horizontal distances of the curve from the vertical axis. By the analogy, this curve represents the variation of the delay of the network at real frequencies.

Delay networks having delays of very great uniformity in a wide band of frequencies are provided, in accordance with the invention, by coupling in tandem a plurality of all-pass networks so proportioned that the real parts of the zeros and poles of their individual insertion losses are all equal in magnitude and the imaginary parts increase in equal steps. The zeros and poles of the insertion loss of the coupled system are the same as the total of those for the individual sections, since the sections all have the same constant-resistance image impedances and the insertion loss is therefore the sum of the individual insertion losses.

Fig. 3 shows the circuit configuration of a typical uniform delay network according to the invention. The network contains five sections, designated N_0 to N_4 , but it will be understood that greater numbers of sections may be used as required. Each section is a symmetrical all-pass lattice, four of the sections being of the type shown in Fig. 1 and the other being a simple lattice with single element branches. For the sake of clarity only two branches of each lattice are shown in detail, the symmetrical branches being indicated by dotted lines. The elements of the successive sections are graduated in such manner as to provide the array of zeros and poles of the insertion loss shown in Fig. 4. Here the zeros are indicated by the small circles in the left half of the plane and the poles by symmetrically located crosses in the right half. The zeros are designated by p 's with subscripts indicating the sections of the network to which they correspond. Each section except the first gives rise to a pair of conjugate zeros and a symmetrical pair of conjugate poles disposed as in Fig. 1. The first section is aperiodic and gives rise to only a single zero and a single pole, both located on the real axis. All of the zeros and all of the poles have real parts of equal magnitudes and have imaginary parts which increase by equal steps from one section to the next. The array in the p -plane is thus in the form of two parallel columns equi-distant from the imaginary axis.

The electrostatic analogue of the network of Fig. 3 would consist of two parallel grids of uniformly spaced filaments disposed to correspond to the array of zeros and poles in Fig. 4, one set of the filaments carrying unit positive charges and the other set carrying unit negative charges. The slightly undulating vertical line to the left of the imaginary axis in Fig. 4 represents the variation of the electric force normal to the central plane in the electrostatic system and therefore represents the frequency variation of the delay of the analogous network. The undulations are uniform, because of the special characteristics of the array, and are readily made very small. It can be shown that so long as the uniform intervals between the zeros is less than the common value of their real parts, the total extent of the undulation will be less than one per cent of the average delay.

Reducing the frequency intervals between the poles and the zeros by the addition of extra sections progressively reduces the undulations of the delay characteristic. In the extreme case, where the spacing is reduced to zero, substantially per-

fect uniformity would be obtained. Under this condition the delay characteristic would follow the same variation as the electric force normal to the central plane between two extended parallel condenser plates. As indicated above, the result is very closely approximated with relatively great separations between the successive zeros.

The operating range of the network of Fig. 3 extends from zero to an upper frequency limit corresponding approximately to the highest zero p_4 . At the limits of the frequency range, delay variations analogous to the end effects in the corresponding electrostatic systems will, of course, be present, but these may be taken care of, if necessary, by the addition of one or two sections having insertion loss zeros and poles at points beyond the desired operating frequency range. This is analogous to the use of a guard ring around a circular condenser plate for the purpose of maintaining a uniform electric field at all points under the plate. The location of the zeros and poles of such guard sections may also correspond to other known electrostatic devices for maintaining uniformity of the electric field in a given region.

It will be evident that the range in which the delay is uniform may be confined to any assigned band between finite frequency limits by appropriate location of the zeros and poles. For example, the sections giving the zeros at p_0 and p_1 may be omitted and additional sections giving zeros at frequencies above p_4 added, thereby shifting the operating range away from zero frequency. In other words, the imaginary parts of the zeros and poles may be confined to the desired frequency range and spaced uniformly therethrough.

The values of the elements of the network of Fig. 3 are readily found with the help of Equation 4. Having determined the values of the zeros and poles for a desired delay characteristic, the real parts of these give at once the values of the capacities in the series branches of the several sections. The values of the inductances are then obtained in an obvious manner from the imaginary parts. In this connection it is to be noted that the poles and zeros for any particular section lie in the p -plane on a circle centered at the origin and of a radius proportional to the resonant frequency of the branch impedances. Since all of the zeros have the same real parts, it is evident that the capacities in the series branches must all be equal. The relative values of the series branch inductances follow from the fact that the imaginary parts of the p 's increase by equal steps. In the case of the simple section N_0 , the single zero is given by

$$p_0 = -\frac{R}{L} = -\frac{1}{CR} \quad (5)$$

where L and C denote, respectively, the series inductance and the diagonal capacity. From this it follows that the diagonal capacity has a value twice that of the series branch condensers of the other sections.

As already pointed out in connection with Equation 2, the imaginary part of each logarithmic term of the insertion loss expression represents the component phase shift at real frequencies associated with the corresponding zero or pole. Since the terms are all of the same form, the phase shift components will all have the same general characteristics, but the components corresponding to the poles will be of

opposite sign from those corresponding to zeros of respectively equal values. The phase shift corresponding to a pair of conjugate zeros with negative real parts, that is, to zeros in the left half of the p -plane, is taken conventionally as positive. It follows then that conjugate poles with negative real parts give rise to a negative phase shift and, conversely, that conjugate poles with positive real parts give rise to a positive phase shift. In the case of all-pass networks such as shown in Figs. 2 and 3, the poles and the zeros of the insertion loss contribute equally, both in magnitude and sign to the total phase shift.

The negative phase shifts associated with poles having negative real parts are characteristic of attenuating networks and are the significant factors entering into the delay distortion of these networks. I have found that by the use of particular circuit configurations for attenuating networks, it is possible to effect an exact neutralization of the negative phase shifts leaving a residual phase shift that is the same in its magnitude and direction as that of an all-pass network. These neutralized networks may then be combined with other neutralized networks or with appropriately coordinated all-pass networks to provide uniform delay systems in accordance with the invention.

A typical neutralized attenuating network according to the invention is shown in Fig. 5. It comprises two similar attenuating sections S_1 and S_2 and a phase neutralizing all-pass section S_3 . The attenuating sections are of the well-known bridged-T constant resistance type, a feature of this type being the possession of minimum phase shift characteristics as defined in my earlier Patent 2,123,178. The section S_3 is a bridged-T all-pass network which is the equivalent of the type of lattice shown in Fig. 2. The general formula for the insertion loss of the bridged-T attenuating network is given in an article by O. J. Zobel on Distortion correction in electrical circuits, Bell System Technical Journal, July 1928, page 438. For the case illustrated, the insertion loss of section 5, is

$$e' = \frac{p^2 + \frac{d+1}{C_1 R} p + \frac{1}{L_1 C_1}}{p^2 + \frac{d-1}{C_1 R} p + \frac{1}{L_1 C_1}} \quad (6)$$

$$= \frac{(p-p_1)(p-\bar{p}_1)}{(p-p_a)(p-\bar{p}_a)} \quad (7)$$

where d is a numerical factor which must be greater than unity, and where

$$p_1 = -\frac{d+1}{2C_1 R} + j\sqrt{\frac{1}{L_1 C_1} - \left(\frac{d+1}{2C_1 R}\right)^2} = -\alpha_1 + j\omega_1 \quad (8)$$

$$p_a = -\frac{d-1}{2C_1 R} + j\sqrt{\frac{1}{L_1 C_1} - \left(\frac{d-1}{2C_1 R}\right)^2} = -\alpha_a + j\omega_a$$

and $\bar{p}_1$ and $\bar{p}_a$ are the respective conjugates of p_1 and p_a . The poles and zeros for section S_2 are the same as those for section S_1 , the two sections together giving an insertion loss characterized by double poles and double zeros.

The array of the poles and zeros in the p -plane is shown in Fig. 6, the double zeros being indicated at A and A' and the double poles at B and B'. As in the case of Fig. 1, the zeros and poles lie on a circle about the origin having a radius proportional to the resonance frequency of the bridging branch. The array is unsymmetrical with respect to the imaginary axis of

the plane, a consequence of this being that the attenuation varies with frequency. The conjugate double zeros at A and A' give rise to a phase shift of the same magnitude and sign as would be produced by an all-pass network having an insertion loss characterized by a pair of zeros at A and A' and a symmetrical pair of poles in the right-hand side of the plane. The double poles at B and B' give rise to a negative phase shift equal in magnitude to the positive phase shift of an all-pass network the insertion loss of which has single zeros at B and B' and symmetrical poles at C and C'. The negative phase shift may therefore be exactly neutralized by the addition of an all-pass network having this characteristic, the combination then having a phase shift characteristic of a single all-pass section. The neutralizing all-pass section is shown in Fig. 3 at S_3 .

The constants of the attenuating sections are readily calculated from the chosen values of the zeros and poles with the help of Equation 8. The difference of the real parts of the zeros and poles gives the capacity of the condenser in the bridging arm, the inductance being then obtainable from the imaginary parts of either the zeros or the poles. The values of the impedance elements of the neutralizing network may be obtained in terms of the inductances and capacities of the attenuating sections with the help of Equations 4 and 8. The inductance and the capacity of series branch impedances are found to be

$$C = \frac{C_1}{d-1} \quad (9)$$

and

$$L = L_1(d-1)$$

The insertion loss of the complete neutralized network is given by

$$e' = \left[\frac{(p-p_1)(p-\bar{p}_1)}{(p-p_a)(p-\bar{p}_a)} \right] \left[\frac{(p-p_1)(p-\bar{p}_1)}{(p-p_a)(p-\bar{p}_a)} \right] \quad (10)$$

where p_1 and p_a are defined by Equation 8 and where $\bar{p}_1$ and $\bar{p}_a$, the conjugate poles of the insertion loss of the neutralizing section are, respectively, the negatives of $\bar{p}_a$ and p_a . The zeros for the neutralizing section and one set of the poles for the attenuating networks do not appear in the equation, the factors including these quantities having cancelled each other.

It will be seen that Equation 10 has the same form as the insertion loss of a network comprising two constant resistance sections, one characterized by insertion loss zeros at A and A' in Fig. 6 and by poles at B and B' and the other characterized by zeros at the same points and poles at C and C'. The modified form of the invention shown in Fig. 7 represents the physical realization of a neutralized attenuating network having two such sections. The network cannot ordinarily be realized in the unbalanced or bridged-T form without the use of transformers, but can always be realized in symmetrical lattice sections as shown. The left-hand section is the lattice equivalent of either section S_1 or section S_2 in Fig. 3. The right-hand section is so proportioned that the zeros and poles of its insertion loss are the same as those for the left-hand section except that the real parts of the poles are positive. The relative values of the series branch elements of the two sections are indicated in the figure, the factor d being the same numerical factor that appears in Equations 6 and 8. The

values of the diagonal branch impedances follow from the inverse relationship imposed by the constant resistance impedances of the sections. The right-hand section of the network of Fig. 7 is the equivalent of the combination of the left-hand section with an appropriate neutralizing all-pass section. When the rules for the transformation of two tandem lattices into a single equivalent lattice are applied to this combination, the relationship of the elements established by the neutralization condition permit the reduction of the circuit to the simple form of the right-hand section. This section therefore includes within itself the means for neutralizing the negative component of the phase shift due to the attenuation characteristics of both sections.

The networks of Figs. 5 and 7 both have phase shift characteristics the same as that of a single all-pass network of the same type as shown in Fig. 1. They may therefore be used as component sections of a constant delay network of the general type described in connection with Figs. 3 and 4, but having a frequency dependent attenuation. A composite network of this type is shown in Fig. 8. Sections S_1 , S_2 and S_3 constitute one neutralized attenuating network of the type shown in Fig. 5 and sections S_4 and S_5 constitute another of the type shown in Fig. 7. The supplementary network N comprises a plurality of coordinated all-pass sections proportioned to make the delay of the whole system uniform throughout any desired band. The first attenuating network is of the aperiodic type having only single reactances in the branches of each section. The zeros and poles of its insertion loss have no imaginary parts and appear in the p -plane on the real axis. The values of the zeros and poles can be obtained from Equation 8 by making the value of the inductance infinite.

The array of the zeros and poles in the p -plane is shown in Fig. 9, the zeros being indicated by circles and the poles by crosses. The double zeros for the first attenuator appear at p_0 on the real axis. The residual poles after neutralization appear on the real axis at points equally distant to the right and to the left from the origin. The double zeros of the second attenuator appear at p_1 and $\bar{p}_1$ and the residual poles at symmetrical points on the circle about the origin passing through the zeros. The zeros for the supplementary all-pass sections constituting the network N are shown at p_2 , p_3 and p_4 and the poles at symmetrically located points in the right-hand half of the diagram. In accordance with the principles discussed in connection with Figs. 3 and 4, uniformity of delay is obtained by making the real parts of all of the zeros equal and making the imaginary parts increase by equal steps from one section to the next. As already explained, the double zeros p_0 and p_1 act like the symmetrical zeros and poles of all-pass networks in so far as the production of phase shift or delay is concerned. The symmetrical poles associated with p_0 and p_1 neutralize each other in this respect.

Fig. 10 shows the general form of the attenuation characteristic for the network of Fig. 9. Curve A represents the attenuation for the first attenuating network S_1 , S_2 , S_3 for a value of the factor d_0 equal to about 3. Curve B represents the attenuation of the second network for a value of d_1 equal to 1.5 and curve C represents the resultant attenuation of the two networks. A network of this type would be suitable for use

in television systems for preemphasizing the higher frequency components of the signal to enable them to over-ride interfering currents. Its delay characteristic would be the same as that of the all-pass network of Fig. 3 as shown in Fig. 4. A total of five component networks or sections is shown in Fig. 8, but this number may be increased as desired, either to extend the frequency range of uniform delay or to increase the uniformity in a given range. In the case of the network of Fig. 8, for example, increased uniformity of the delay might be obtained by adding all-pass sections to give additional insertion loss zeros midway between the zeros indicated in Fig. 9.

Since each neutralized attenuating network has the same phase and delay characteristics as a related all-pass network, it will be evident that uniform delay networks according to the invention may comprise attenuating sections and all-pass sections in many combinations and, in fact, might comprise only neutralized attenuating networks. Where several attenuating networks are used, the over-all attenuation characteristic may be shaped by using different values of the factor d in the several networks as was done in the case of Fig. 8.

What is claimed is:

1. A four-terminal wave transmission network comprising a plurality of tandem connected sections, all of said sections having constant resistive image impedances of the same magnitude, said sections including reactance elements determining the zeros and poles of the insertion loss of the network, the elements of said sections being proportioned so that the zeros of the insertion loss of the network when connected between impedance matching terminal resistances have equal real parts and have imaginary parts increasing progressively by equal amounts through an assigned range of frequencies, whereby the time delay for wave transmission through said network is maintained substantially uniform throughout the said assigned frequency range.
2. A network in accordance with claim 1, characterized in this that the intervals between the imaginary parts of the zeros of the insertion loss are less than the common magnitude of the real parts.
3. A four-terminal wave transducer comprising a plurality of tandem connected networks all having constant resistance image impedances of the same magnitude, said networks including reactance elements determining the values of the zeros and poles of the insertion loss of the transducer, the elements of said networks being so proportioned that the zeros of their respective insertion losses have equal real parts and have imaginary parts increasing progressively from section to section by equal amounts through a range of values corresponding to an assigned frequency range, whereby the time delay for wave transmission through the transducer is maintained substantially uniform throughout the same frequency range.
4. A transducer in accordance with claim 3, in which at least one of the component networks has a frequency dependent attenuation and includes impedance means substantially exactly neutralizing the negative phase shift component due to the frequency variation of the attenuation.
5. A four-terminal attenuating network comprising a pair of similar tandem connected attenuating sections, said sections having equal constant resistance image impedances and to-

gether having an insertion loss when terminated by matching impedances characterized by double zeros and by double poles having negative real parts, and an all-pass network having an insertion loss characterized by single zeros coincident with the double poles of the insertion loss of the attenuating sections.

6. A four-terminal attenuating network comprising two sections connected in tandem, said sections having equal constant resistance image impedances and both being proportioned to give equal frequency dependent attenuations when terminated by impedance matching terminal

circuits, reactance elements included in said sections determining the values of the zeros and poles of their respective insertion losses, one of said sections having an insertion loss characterized by zeros and poles both having negative real parts and the other of said sections having an insertion loss characterized by zeros coincident with the insertion loss zeros of the first of said sections and by a corresponding number of poles which have values differing from those of the first section only in that their real parts are positive.

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