Dec. 28, 1943.

H. W. BODE

2,337,965

COUPLING NETWORK

Filed March 18, 1942

2 Sheets-Sheet 1

FIG. 1

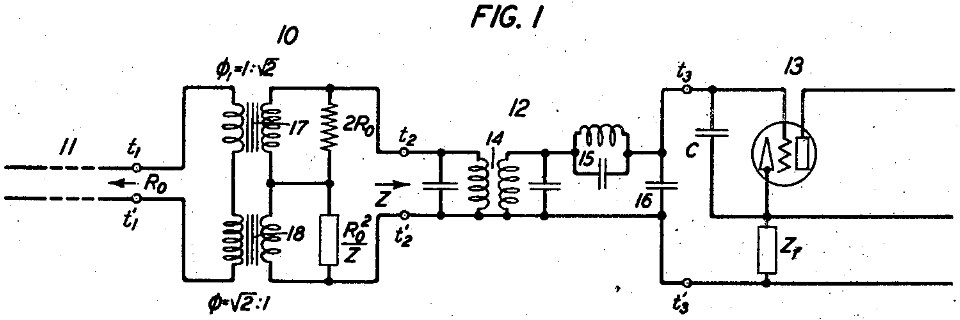
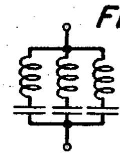


FIG. 2



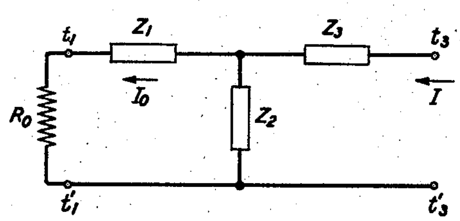


FIG. 3

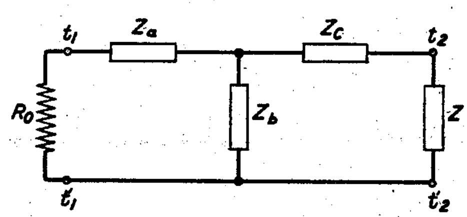


FIG. 4

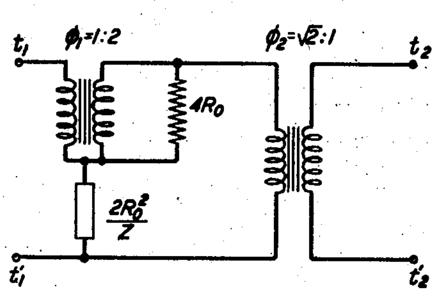


FIG. 5

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COUPLING NETWORK

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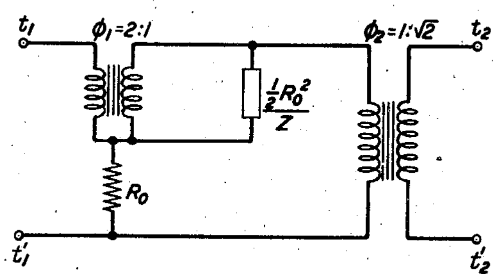

FIG. 6

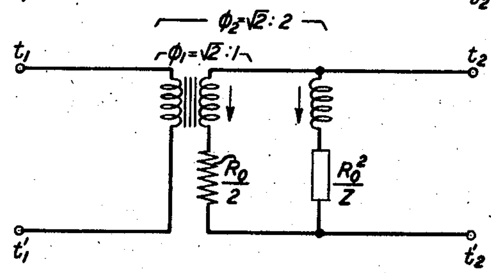

FIG. 7

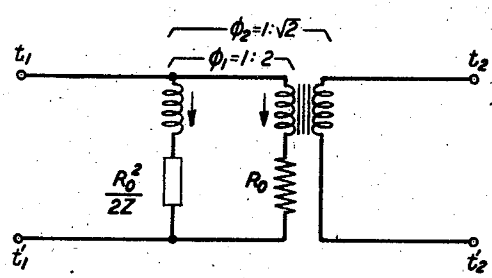

FIG. 8

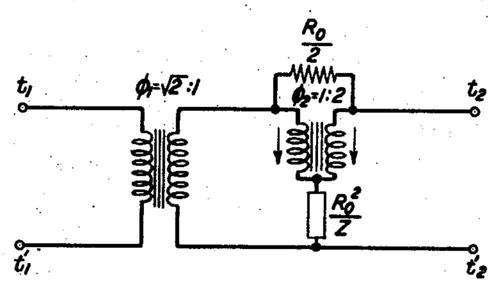

FIG. 9

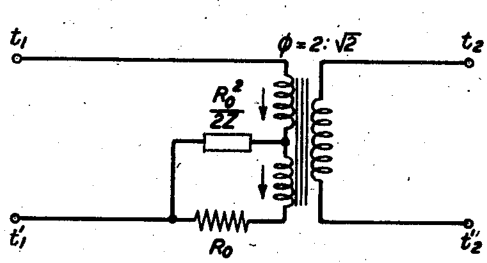

FIG. 10

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UNITED STATES PATENT OFFICE

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COUPLING NETWORK

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Application March 18, 1942, Serial No. 435,171

7 Claims. (Cl. 178—44)

This invention relates to wave transmission networks and more particularly to networks for coupling a transmission line to a terminal device or load having a different impedance characteristic. Its objects are to diminish wave reflections in a transmission line terminated by a load having an impedance characteristic different from that of the line; to simplify the equalization of the line attenuation; and to improve the operation of transmission systems in which reactance networks are employed for the equalization of the line attenuation.

The networks of the invention have the property of minimum transmission loss consistent with the requirement of physical realizability. This minimum loss is equal to three decibels and is uniform at all frequencies. It is approximately half the loss encountered in impedance correcting networks of types heretofore used.

It has been found advantageous in certain types of repeatered transmission systems to equalize the line attenuation by means of pure reactance networks, such as resonant circuits, connected directly to the input and output terminals of the repeater amplifiers. The advantage in the use of such equalizers arises from the fact that, since they contain no resistance elements, they do not introduce any noise into the system and, therefore, do not limit the amount of amplification that can be used in the repeaters. Their use, however, entails a substantial amount of wave reflection at the line terminals, since the reactive impedances can absorb no wave energy. The reflected waves in their passage along the line have been found, under certain conditions, to give rise to undesirable disturbances, such as excessive cross-talk in adjacent circuits.

The networks of the invention are designed to be interposed between the line and the reactive equalizer and, when so connected, to present to the line a resistive impedance equal to the resistive impedance of the line. At the junction point to the equalizer the impedance presented to the equalizer is equal to the line impedance. Accordingly, the conditions under which the equalizer functions are not altered by the insertion of the network, but the wave reflections back into the line are substantially suppressed.

In the accompanying drawings,

Fig. 1 shows the application of the invention in the input circuits of a telephone repeater;

Fig. 2 refers to a detail of Fig. 1;

Figs. 3 and 4 are diagrams illustrating the principles of the invention, and

Figs. 5 to 10 inclusive, show additional forms of the networks of the invention.

In Fig. 1, the coupling network of the invention is shown in one of its forms at 10 coupling a transmission line 11 with a reactive equalizing network 12 in the input circuit of a feedback amplifier 13. The showing is representative of the input side of a repeater in a multiplex carrier telephone system using high frequency carriers. The line 11 may be an open wire line, a cable, or a coaxial conductor line, its impedance at the operating frequencies being a substantially pure resistance of constant value. The equalizing network, which in the form illustrated comprises a tuned transformer 14, a series antiresonant circuit 15, and a shunt capacity 16, is so designed as to produce at its output terminals t_3, t_3' a voltage which increases with frequency in such manner as to compensate the attenuation in the line. The design of equalizers of this type is described in my earlier United States Patent No. 2,242,878, issued May 20, 1941, wherein it is pointed out that the logarithm of the effective resistance measured at terminals t_3, t_3' when the equalizer is connected to the line should have the same frequency variation as the attenuation in the line. The first stage only of the repeater amplifier 13 is shown together with the feedback impedance Z_f, the feedback being of the series type. The input impedance of the first amplifier tube is indicated by the capacity C which represents the electrode capacity and such other parasitic capacities as are effective between the control grid and the cathode. This capacity is ordinarily small and its effective impedance is greatly multiplied by the feedback action. Consequently, the impedance into which the equalizer works may be regarded as infinite or open circuit in so far as it affects the voltage developed at the output terminals t_3, t_3'.

The coupling network 10 comprises two transformers 17 and 18, the two primary windings and the two secondary windings of which are connected in series. These transformers should have closely coupled high impedance windings and should be designed so that throughout the operating frequency range they behave substantially like ideal transformers. For this purpose the use of magnetic cores of high permeability nickel-iron alloys is advantageous. Transformer 17 has a voltage transformation ratio equal to $1:\sqrt{2}$ from the primary, or line side, to the secondary and transformer 18 has a voltage transformation of the reciprocal value $\sqrt{2}$:1. Shunted across the secondary of transformer 17 is a resistance of value $2R_0$, where R_0 denotes the value of the resistive line impedance. Across the secondary of transformer 18 is connected a reactive impedance of value R_0^2/Z which is inversely related to the input impedance, Z, of equalizer 12 measured at terminals t_2, t_2'. The form of this inverse impedance and the values of its elements can be derived from the known input impedance of the equalizer by well-known principles. A configuration suitable for the particular system illustrated in Fig. 1 is shown in Fig. 2.

When the elements have the values given above, the coupling network 10 has these properties: When terminated by the equalizer 12 it presents to the line a matching impedance equal to R_0; when terminated by the line it presents to the equalizer a constant resistive impedance equal to R_0; and its inclusion between the line and the equalizer reduces the voltage at the equalizer output terminals by a constant amount equal to three decibels at all frequencies. Likewise, if the equalizer and the coupling network are located in the output circuit of a high impedance amplifier, the current delivered to the line is reduced uniformly by three decibels because of the inclusion of the network. This loss is the minimum uniform loss that can be achieved with a physically realizable network.

The networks of the invention may take various forms other than that shown in Fig. 1, certain of which will be described later. All have the properties described above and all have certain characteristic features in common. These characteristic features are developed in the following analysis of the principles underlying the operation of the invention.

It is convenient to begin the analysis by establishing the three-decibel loss as the theoretical minimum for an impedance matching network coupling a resistive source, represented by the line, to a reactive load, represented by the equalizer. This may be done most simply by considering the converse problem of the delivery of power to the line from a current source having infinite impedance connected to the terminals of the equalizer. This condition is illustrated in Fig. 3 in which the complete system is shown reduced to its simplest schematic form. The line is represented by its resistive impedance R_0 and the remainder of the circuit is replaced by its equivalent T-network comprising impedance Z_1, Z_2 and Z_3. The symbols R_1, R_2, etc. and X_1, X_2, etc. will be used to represent the real and imaginary components of the Z's with the corresponding subscripts. A current I is assumed to be delivered to the circuit at terminals t_3, t_3' and to appear in the line with a value I_0.

The power delivered to the circuit and the power which reaches the line are equal to $|I|^2R_e$ and $|I_0|^2R_0$, respectively, R_e denoting the resistance component of the total impedance opposed to the current I. If the reactive equalizer were used alone then the two powers would be equal, since no energy can be absorbed by the reactance elements. When the coupling network is included, the resistance elements, the presence of which is necessary for the impedance matching, absorb some power and the ratio of the two powers defined above then becomes a measure of the loss penalty incurred in effecting an impedance match to the line. From the geometry of the circuit this ratio may be shown to be given by

$$\frac{|I|^2R_e}{|I_0|^2R_0}=\frac{1}{R_0(R_2^2+X_2^2)}[R_3((R_0+R_1+R_2)^2+(X_1+X_2)^2)+(R_0+R_1+R_2)(R_0R_2+R_1R_2-X_1X_2)+(X_1+X_2)(R_0X_2+R_1X_2+R_2X_1)] \quad (1)$$

Since Z_1, Z_2 and Z_3 are merely the branches of the equivalent T of the actual circuit, it is not necessary that all of their resistance components R_1, R_2 and R_3 be positive for the circuit as a whole to be physically realizable. If, for example, R_3 could be made sufficiently negative, it would be possible to make the power ratio as favorable as desired and even to convert it to a gain. It is known from general network theory, however, that a four-terminal network is physically realizable as a passive structure only if the determinant of the matrix of the real parts of the open circuit driving point and transfer impedances is positive at every frequency. As applied to the T-network of Fig. 3, this condition is equivalent to

$$R_1R_2+R_2R_3+R_3R_1\geq 0 \quad (2)$$

The limiting case in which the condition is just met so that the equality sign in Equation 2 applies is obviously the most favorable one. Assuming this limiting case and using Equation 2 to eliminate R_3, Equation 1 becomes

$$\text{Power ratio}=1+\frac{R_0^2R_2^2+(X_1R_2-X_2R_1)^2}{R_0(R_1+R_2)(R_2^2+X_2^2)} \quad (3)$$

The further requirement that the line impedance be matched by the network may now be introduced. Since the circuit is assumed to open at terminals t_3, t_3', this requirement is simply that

$$R_1+R_2=R_0$$

and

$$X_1+X_2=0$$

When these relations are introduced the right-hand side of Equation 3 becomes numerically equal to two. Thus when the limiting condition of physical realizability is just met and the network matches the line impedance, exactly half of the power applied to the circuit reaches the line. This corresponds to a loss of three decibels and represents the minimum obtainable. Since the reactive equalizer, which was included in the T-network of Fig. 3, can absorb no power, all of the power loss must occur in the coupling network. The limiting condition of physical realizability set forth in Equation 2 therefore applies to the coupling network per se in its function of coupling the resistive line to the finite reactive terminal impedance provided by the equalizer.

The minimum loss condition has been discussed above in connection with a circuit in which power is supplied to a line from a source having infinite impedance. This is representative of the output circuit of a high impedance amplifier, for example, one using high impedance pentode tubes. The analysis may be extended to an input circuit such as shown in Fig. 1 by the principle of reciprocity. In that case the complete termination comprising the coupling network and the equalizer together when subject to the realizability condition may be regarded as one providing the maximum output voltage consistent with impedance matching.

The general characteristics of the coupling network may now be considered. In Fig. 4 the coupling network is represented by its own equivalent T comprising impedances Z_a, Z_b and Z_c and the input impedance of the equalizer at terminals t_2, t_2' is represented by Z. For the determination of Z_a, Z_b and Z_c the coupling network may be specified by three requirements. The first is the requirement that the impedance presented to the line should equal the line resistance R_0 when the structure is terminated at its other end by the equalizer impedance Z. The second is the requirements of minimum loss, which when applied to the coupling network itself becomes

$$R_aR_b+R_bR_c+R_cR_a=0 \qquad (4)$$

where R_a, R_b and R_c denote the resistance parts of Z_a, Z_b and Z_c. The third is that the impedance presented to the equalizer shall also equal R_0, which is desirable in order that the gain characteristic produced by the equalizer may be independent of the presence or absence of the coupling network.

By this choice of the requirements, particularly the third, I have been able to devise various network structures in which the performance is substantially independent of frequency. That is, the impedance matching and the achievement of minimum loss obtain through a very wide range limited only by the imperfections of the transformers.

The values of the branch impedances can be obtained from the specified requirements by routine algebra. They are given by

$$\begin{aligned} Za&=-(\sqrt{2}-1)R_0+(2-\sqrt{2})\frac{R_0^2}{Z}\\ &=\left(R_0+2\frac{R_0^2}{Z}\right)-\sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right)\\ Zb&=\sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right) \end{aligned} \qquad (5)$$

and

$$\begin{aligned} Z_c&=(2-\sqrt{2})R_0+(1-\sqrt{2})\frac{R_0^2}{Z}\\ &=\left(2R_0+\frac{R_0^2}{Z}\right)-\sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right) \end{aligned}$$

In terms of its open circuit driving point and transfer impedances the structure can also be specified by the matrix:

$$\left\|\begin{matrix} R_0+2\frac{R_0^2}{Z};\ \sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right)\\ \sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right);\ 2R_0+\frac{R_0^2}{Z} \end{matrix}\right\|$$

in which the upper left-hand term and the lower right-hand term denote the open circuit driving point impedances at the line and equalizer terminals respectively and the other two terms denote the open circuit transfer impedance. These values may be readily verified from the values of the equivalent T impedances given in Equation 5.

The equivalent T is not a physical network, but by the employment of highly efficient transformers and by selection of proper transformation ratios I have been able to devise structures having the required system of impedances. One of these is illustrated in Fig. 1. Others are shown in Figs. 5 to 10, inclusive.

In Figs. 5 to 10, the values of the resistances and the reactive impedances are indicated in the drawings in terms of the line resistance and the reactance of the equalizer. The voltage ratios, φ, of the various transformers are also given. The poling of the transformer windings is not material in the networks of Figs. 1, 5 and 6.

In Figs. 7 to 10, the arrows indicate windings that should be wound in the same direction on the core. The network of Fig. 10 may be derived from the network shown in Fig. 8 by substituting a single winding tapped at its mid-point for the two primary windings.

The open circuit driving point and transfer impedances can be computed for each of the networks by well-known methods of network analysis. It is to be observed that in each network two impedance transformations of different magnitudes are involved. The circuit configurations and the magnitudes of the transformation ratios are so chosen that the value of the resistance is subject to a step-up of two to one from the line terminals to the output terminals, while the value of the reactive impedance is decreased in the same ratio.

To the extent that the various transformers act like ideal transformers, all of the coupling networks of the invention have open circuit driving point impedances substantially equal to

$$R_0+2\frac{R_0^2}{Z}$$

at their line terminals and to

$$2R_0+\frac{R_0^2}{Z}$$

at their output terminals and have open circuit transfer impedances equal to

$$\sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right)$$

Departures from these values may be caused by low impedance of the transformer windings or by imperfect coupling of the windings. However, with the high efficiency transformers that are now available, the errors arising from these causes are usually negligible over a very wide frequency range.

What is claimed is:

1. Means for coupling a line of resistive impedance R_0 to a load having a prescribed frequency dependent reactive impedance Z with minimum attenuation and with impedance matching at the line terminals throughout a wide frequency range, comprising a four-terminal network having open circuit driving point impedances at its line and load terminals substantially equal to

$$R_0+2\frac{R_0^2}{Z}$$

and

$$2R_0+\frac{R_0^2}{Z}$$

respectively, and an open circuit transfer impedance substantially equal to

$$\sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right)$$

said network including a resistor, a reactor having an impedance inversely related to Z, and transforming means whereby the effective impedances of said resistor and said reactor measured at the network terminals are subject to reciprocally related transformations.

2. A four-terminal network for coupling a line of resistive impedance R_0 to a load having a prescribed frequency dependent reactive impedance of value Z with minimum attenuation and with impedance matching at the line terminals throughout a wide frequency range, comprising

a resistor, a reactive impedance inversely related to the prescribed impedance Z, and a plurality of transformer means having different transformation ratios coupling said resistor and said inverse impedance to the input and output terminals of the network, the impedances of the said network elements and the transformation ratios of said transformer means being proportioned to make the open circuit driving point impedances of the network at the line and load terminals substantially equal to

$$R_0+2\frac{R_0^2}{Z}$$

and

$$2R_0+\frac{R_0^2}{Z}$$

respectively, and to make the open circuit transfer impedance substantially equal to

$$\sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right)$$

at all frequencies in a wide range.

3. A four-terminal network for coupling a line of resistive impedance R_0 to a terminal device having a prescribed frequency dependent reactive impedance of value Z with minimum attenuation and with impedance matching at the line terminals, comprising a resistor, a reactive impedance inversely related to the prescribed impedance Z, and a plurality of transformer means having different transformation ratios coupling said resistor and said inverse impedance to the terminals of the network, the connections of the said transformer means being so arranged that the impedances of said resistor and said inverse impedance are subject to reciprocally related impedance transformations between the line and load terminals of the network, and the network elements being proportioned to make the open circuit transfer and driving point impedances at the line and load terminals substantially equal to

$$\sqrt{2}\left(R_0+\frac{R_0^2}{Z}\right)$$

$$R_0+\frac{2R_0^2}{Z}$$

and

$$2R_0+\frac{R_0^2}{Z}$$

respectively, at all frequencies in a wide range.

4. A four-terminal coupling network for coupling a line of resistive impedance R_0 to a terminal device having a prescribed frequency dependent reactive impedance of value Z comprising a pair of transformers having their primary windings connected in series between the line terminals of the network and having their secondary windings connected in series between the other terminals of the network, one of said transformers having a voltage transformation ratio of $1:\sqrt{2}$ between its primary and secondary terminals and the other transformer having a corresponding transformation ratio of $\sqrt{2}:1$, a resistor connected across one winding of the said one transformer, and a reactive impedance inversely related to the prescribed impedance Z connected across a winding of said other transformer, said resistor and said inverse impedance being proportioned to make the open circuit impedance of the network at the line terminals equal to

$$R_0+\frac{2R_0^2}{Z}$$

5. A coupling network in accordance with claim 4 in which the resistor is connected across the second winding of the said one transformer and has a resistance $2R_0$ and in which the inverse impedance is connected across the secondary winding of the said other transformer and has a impedance equal to

$$\frac{R_0^2}{Z}$$

6. A network for coupling a line of resistive impedance R_0 to a terminal device having a reactive impedance of value Z comprising transformer means having a secondary winding arranged for connection to said terminal device, a first primary winding and a second primary winding, said windings having turns in the ratios $\sqrt{2}:1:2$, respectively, a reactive impedance of value

$$\frac{R_0^2}{2Z}$$

connected in series with said first primary winding, a resistance of value R_0 connected in series with said second primary winding, a pair of input terminals for connection to the line, and circuits connecting said primary windings and their respectively associated impedances in parallel between said terminals.

7. A network for coupling a line of resistive impedance R_0 to a terminal device having a reactive impedance of value Z comprising a transformer having a secondary winding arranged for connection to said terminal device and a centrally tapped primary winding, the two portions of said primary winding and said secondary winding having turns in the ratios $1:1:\sqrt{2}$, a pair of input terminals for connection to the line, a resistance of value R_0 connected between one of said terminals and one end of said primary winding, a reactive impedance of value

$$\frac{R_0^2}{2Z}$$

connected between said one terminal and the center tap of said primary winding and a connection from the other of said terminals to the other end of said primary winding.

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