

May 4, 1943.

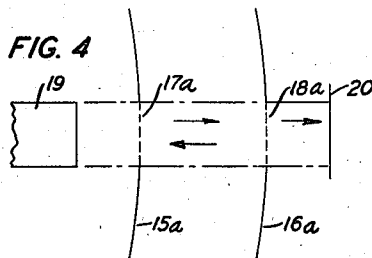
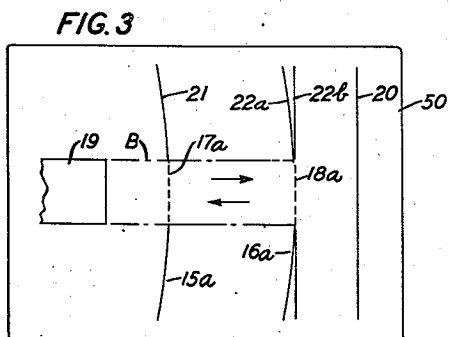
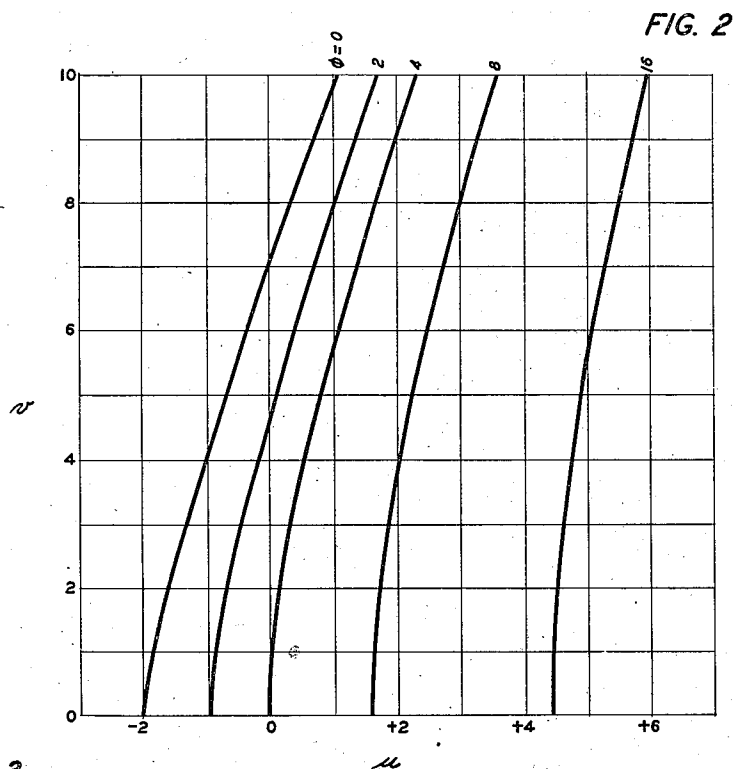
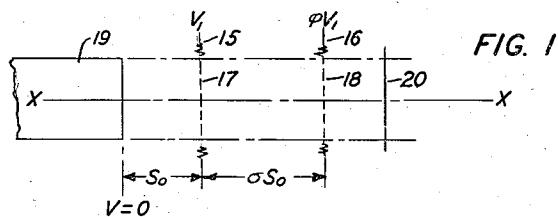
J. R. PEIRCE

2,318,418

ELECTRON DISCHARGE DEVICE

Filed Oct. 28, 1941

3 Sheets-Sheet 1



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ELECTRON DISCHARGE DEVICE

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FIG. 5

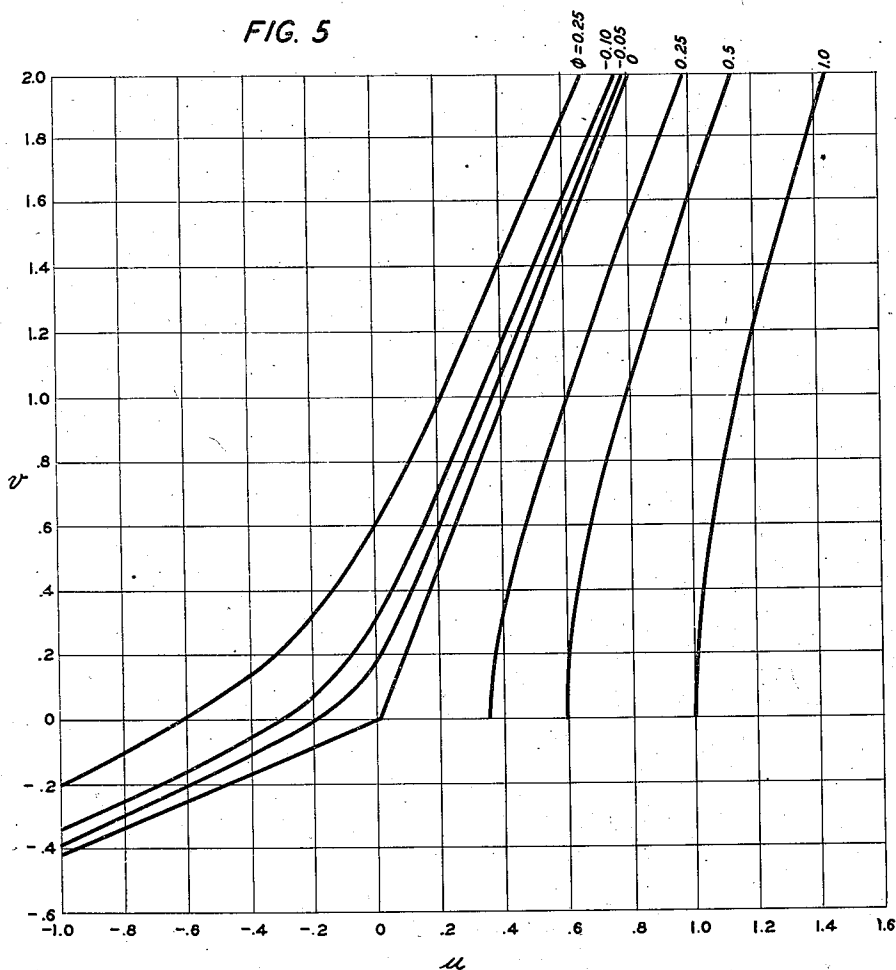


FIG. 6

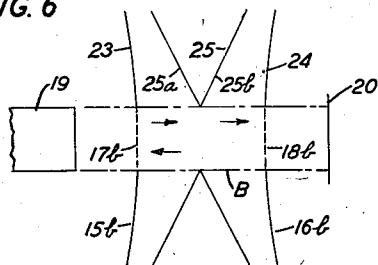
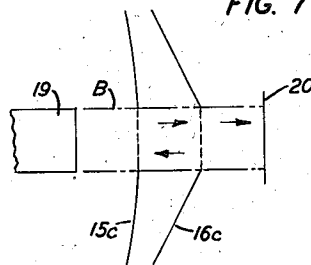


FIG. 7



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FIG. 8

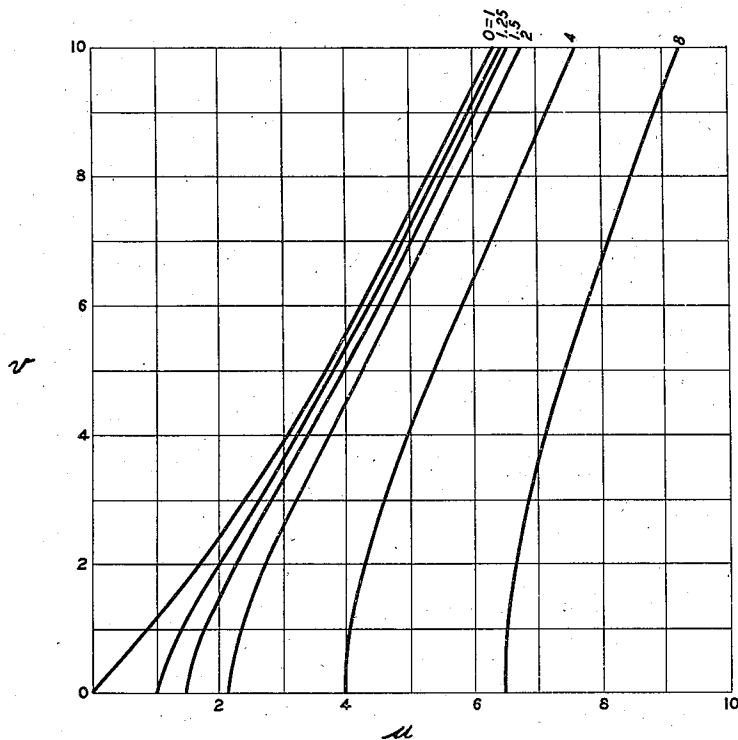


FIG. 9

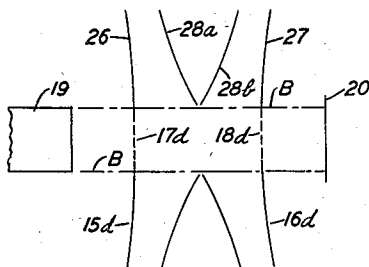
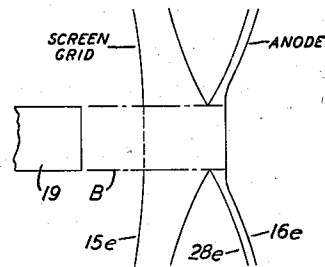


FIG. 10



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UNITED STATES PATENT OFFICE

2,318,418

ELECTRON DISCHARGE DEVICE

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Application October 28, 1941, Serial No. 416,778

12 Claims. (Cl. 250—155)

This invention relates to electron discharge devices and more particularly to electron discharge devices of the electron beam type.

One object of this invention is to obtain rectilinear flow of electrons in a desired region between a pair of electrodes in an electron discharge device whereby predictable performance, high efficiency, stable operation, and other desiderata are realized.

In accordance with one feature of this invention, an electron discharge device comprises a pair of electrodes, at least one of which is centrally apertured, and an electrode system or electron gun for projecting an electron stream into the region between the two electrodes, through the aperture in one of them and parallel to the axis of alignment of these electrodes, and these electrodes are so constructed and arranged that in the region noted parallel electron flow in the form of a beam surrounded by charge free space is obtained. More specifically, in accordance with one feature of this invention, the electrodes are so constructed and arranged that the opposed surfaces thereof conform to equipotential boundaries of a field which satisfies Laplace's equation subject to the conditions that at the boundary of the beam the gradient normal thereto is zero and that the potential along this boundary varies as a function of distance normal to the plane of injection of the electron stream into the region between the electrodes, in a prescribed manner according to a solution of the space charge equation for the region traversed by the beam.

The invention is of general application to a pair of electrodes, irrespective of the potentials of the electrodes and the potential distribution extant therebetween. The distinct types of combinations of electrode potentials and potential distributions between the pair of electrodes are four in number and, for convenience of discussion herein, are designated as types A, B, C and D. Also, for convenience of discussion, the electrode through which the electron stream is injected into the region throughout which the parallel electron flow is to be realized is referred to as the first electrode and the other electrode of the pair is referred to as the second electrode. The four distinct types noted above are as follows:

Type A.—The second electrode is at a negative potential and, with this potential constant, all of the injected current is reflected.

Type B.—Both the first and second electrodes are at positive potentials and a potential min-

imum, zero, exists in the region between the electrodes. With the potentials constant, a fraction of the injected current is reflected and the remaining fraction is transmitted.

Type C.—Both the first and second electrodes are at positive potentials and a potential minimum at a positive potential exists in the region between the electrodes, and all the injected current is transmitted.

Type D.—Both the electrodes are at positive potentials and no potential minimum exists in the region therebetween, so that all of the injected current is transmitted.

The space charge equations for the region traversed by the beam, for parallel electron flow, in the four cases noted above are known and are given, for example, in the article by C. E. Fay et al., On the theory of space charge between parallel plane electrodes, in the Bell System Technical Journal, vol. 17, 1938, pages 49 et seq.

The invention and the above-noted and other features thereof will be understood more clearly and fully from the following detailed description with reference to the accompanying drawings in which:

Fig. 1 is a diagram illustrating generally the combination of a pair of electrodes mounted between an electron projecting system, such as an electron gun, and another electrode;

Fig. 2 is a graph illustrating appropriate electrode forms in accordance with this invention for devices of type A noted hereinabove;

Figs. 3 and 4 are diagrammatic views of electron discharge devices including electrodes of the configurations illustrated in Fig. 2;

Fig. 5 is a graph illustrating the form of electrodes constructed in accordance with this invention for devices of type B noted above;

Figs. 6 and 7 are diagrammatic views of electron discharge devices comprising electrodes having the configurations illustrated in Fig. 5;

Fig. 8 is a graph showing electrode forms in accordance with this invention for devices of type C noted hereinabove; and

Figs. 9 and 10 are diagrammatic views of electron discharge devices including electrodes of the configurations illustrated in Fig. 8.

In Figs. 4, 6, 7, 9 and 10 of the drawings, the enclosing vessel of the device has been omitted for purposes of simplicity of illustration, it being understood that the electrodes shown in these figures are mounted within a suitable evacuated vessel.

Referring now to the drawings, the general construction illustrated in Fig. 1 comprises a pair

of electrodes 15 and 16, the outer portions of which are broken away, having parallel, aligned, central apertured, reticulated or grid portions 17 and 18, an electrode system 19, which may be an electron gun of the constructions disclosed in the applications of John R. Pierce, Serial No. 307,233, filed December 2, 1939, and Serial No. 319,393, filed February 17, 1940, for projecting a parallel electron stream through the grid portion 17 and parallel to the axis of alignment of the grid portions 17 and 18, and another electrode 20.

In general, in order that the electron flow in the region between the grid portions 17 and 18 will be parallel and in the form of a beam surrounded by charge free space, it is necessary that the portions of the electrodes 15 and 16 beyond the beam boundary, i. e., the portions outside of the grid portions 17 and 18, conform to equipotentials of a field which satisfies Laplace's equation, for which the gradient normal to the beam boundary is zero and for which the potential along the beam boundary varies according to a solution of the equation:

$$\frac{\partial^2 \Phi}{\partial x^2} = -\frac{j}{p_0 \sqrt{2 \frac{e}{m}}} \quad (1)$$

where Φ is the potential, x is distance along the beam boundary, j is the current density in the beam, e and m are the electron charge and mass, respectively, and p_0 is the permittivity of a vacuum. In one system of units, x is in centimeters, j is in amperes/square centimeter, Φ is in volts, $e=1.591 \times 10^{-9}$, $m=9.0 \times 10^{-36}$ and $p_0=8.85 \times 10^{-14}$.

Equation 1 is derived in the following manner. As is known, Poisson's equation in rectangular coordinates is

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{-p}{p_0} \quad (1a)$$

where x , y and z are the coordinates in centimeters, Φ is potential in volts, p is charge density in coulombs/cm.³ and p_0 is as defined hereinabove.

The condition for parallel rectilinear electron motion is that forces act on the electrons only in the direction of motion, which, in the systems under consideration is the x direction. Hence, the electric gradient in the y and z directions must be zero everywhere in the beam; that is

$$\frac{\partial \Phi}{\partial y} \text{ and } \frac{\partial \Phi}{\partial z}$$

must be zero everywhere in the beam. Hence

$$\frac{\partial^2 \Phi}{\partial y^2} \text{ and } \frac{\partial^2 \Phi}{\partial z^2}$$

must be zero everywhere on the beam, so that

$$\frac{\partial^2 \Phi}{\partial x^2} = \frac{-p}{p_0} \quad (1b)$$

Φ may be measured with respect to the cathode. An electron leaving the cathode with zero velocity must have, at a point of potential Φ , a velocity such that the kinetic energy is equal to the potential energy, i. e.

$$\frac{1}{2}mv^2 = e\Phi \quad (1c)$$

where v is the velocity and is given by

$$v = \sqrt{2 \frac{e}{m} \Phi} \quad (1d)$$

The current density j has the dimensions coulombs/cm.²/second; the velocity v has the dimensions cm./second. The current density di-

vided by the velocity has the dimensions coulombs/cm.³ and is the charge density p . Hence, from Equation 1b

$$\frac{\partial^2 \Phi}{\partial x^2} = \frac{-p}{p_0} = \frac{-j}{vp_0} = \frac{-j}{p_0 \sqrt{2 \frac{e}{m} \Phi}}$$

In a device of type A and wherein the grid portions 17 and 18 are parallel planes and the injected electron stream is in the form of a plane sheet, and wherein the first electrode 15 is at a potential V_1 and the second electrode 16 is at a potential ϕV_1 , where $\phi < 0$, the relationship between potential and distance within the beam and in the region between the electrodes is given by the equation

$$\sigma = \sqrt{2}[(1-2\beta^4)(1+\beta^4)^{1/2} - (\phi^4 - 2\beta^4)(\phi^4 + \beta^4)^{1/2}] \quad (2)$$

where σ is distance from the plane of the grid 17 expressed in units of the distance between this plane and the cathode, zero potential plane and β is an integration constant, values of which are given, for example, in the Fay et al. article noted hereinabove.

As shown hereinafter, for the type A devices under consideration, wherein parallel electron flow in the form of a beam exists in the region between the grids 17 and 18, the potential outside of the beam boundary is given by the relation

$$u + iv = [(\Phi + i\psi)^4 - 2][(\Phi + i\psi)^4 + 1]^{1/2} \quad (3)$$

where u and v are the coordinates of equipotential lines of the field between the two electrodes,

$$\Phi = \frac{\phi}{\beta}$$

$i = \sqrt{-1}$, and ψ is a complementary function u being the coordinate in the direction of the axis of alignment of the electrodes and v being the coordinate normal to this axis and equal to zero at the beam boundary. u and v may be expressed as

$$u = -\frac{\sqrt{2x}}{S_0 \beta^4} + \frac{(1-2\beta^4)(1+\beta^4)^{1/2}}{\beta^4} \quad (4)$$

where x is distance from the plane of the grid 17;

$$v = \frac{\sqrt{2}}{S_0 \beta^4} y \quad (5)$$

The solution for Equation 1, for the case under consideration, then, is

$$u = (\Phi^{1/2} - 2)(\Phi^{1/2} + 1)^{1/2} \quad (6)$$

The derivation of Equation 3 will be apparent from the following. Consider the equation, which is similar to Equation 6:

$$\mu = \left[\left(\frac{\phi}{\beta} \right)^{1/2} - 2 \right] \left[\left(\frac{\phi}{\beta} \right)^{1/2} + 1 \right]^{1/2} = A x \quad (3a)$$

Here A is a proportionality factor between x , distance in centimeters, and μ , distance in units as defined by the equation. By differentiation of Equation 3a

$$A = \frac{3}{4} \frac{\frac{\partial \phi}{\partial x}}{\beta \left[\left(\frac{\phi}{\beta} \right)^{1/2} + 1 \right]^{1/2}} \quad (3b)$$

$$\frac{\partial \phi}{\partial x} = \frac{4A\beta \left[\left(\frac{\phi}{\beta} \right)^{1/2} + 1 \right]^{1/2}}{3} \quad (3c)$$

Differentiating Equation 3c gives

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{A\beta \left(\frac{\partial \phi}{\partial x} \right)}{3(\phi^4 \beta^4)^{1/2} \left[\left(\frac{\phi}{\beta} \right)^{1/2} + 1 \right]^{1/2}} \quad (3d)$$

From Equations 3c and 3d it will be seen that

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{4}{9} \frac{A^2 \beta^{4/3}}{\phi^{1/3}} \quad (3e)$$

From a comparison of Equations 1 and 3e it is seen that they are identical if

$$\frac{4}{9} \frac{A^2 \beta^{4/3}}{\phi^{1/3}} = - \frac{j}{p_0 \sqrt{2 \frac{e}{m} \phi}} \quad (3f)$$

Hence, if A and β have such magnitudes that Equation 3f is satisfied 3a is a solution of Equation 1.

In an electron beam wherein parallel electron flow exists and the potential is as given in Equation 3a, the boundary conditions at the edge of the beam ($y=0$) are determined in the following manner. As is known from electrostatics, the field outside of the beam must obey the Laplacian equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (3g)$$

It will be noted that Equation 3g is Equation 1a with zero charge density and no variation with z .

Assume that ϕ is in accordance with the relation:

$$\phi + ix = f(x + iy) \quad (3h)$$

where $f(x + iy)$ is any differentiable function of $(x + iy)$. ϕ is the real part of the function. The imaginary part may be designated as $i\psi$. Both ϕ and ψ are real.

By differentiation:

$$\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + i \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) =$$

$$f''(x + iy) - f''(x + iy) = 0 \quad (3j)$$

Hence, it will be seen that the real part of any such function of $(x + iy)$ is a solution of Laplace's equation and a possible two-dimensional potential.

Defining $v = Ay$,

$$\Phi = \frac{\phi}{\beta} \text{ and } \Psi = \frac{\psi}{\beta}$$

we may write, to obtain from Equation 3a a potential which is a solution of Laplace's equation,

$$(u + iv) = A(x + iy) = [(\Phi + i\Psi)^4 - 2[(\Phi + i\Psi)^4 + 1]^{1/2}]^{1/2} \quad (3k)$$

In this relation ψ is a variable representing the imaginary part to make Equation 3k valid.

In order that two fields may exist side by side, the boundary conditions are that each field have the same potential and the potential gradient normal to the boundary be the same for each field. In the case under consideration, the latter condition is that

$$\frac{\partial \phi}{\partial y}$$

be the same for the two fields. At the boundary, in the case under consideration, $y=0$, $\psi=0$ and Equations 3k and 3a are the same. Hence, these two equations give the same potential at the boundary.

Everywhere in the electron beam in the case

under consideration, ϕ is independent of y and, hence

$$\frac{\partial \phi}{\partial y} = 0$$

In the potential relation given by Equation 3k, it will be noted that at $\mu=v=0$,

$$\frac{\partial \phi}{\partial y} = \frac{\partial \Phi}{\partial y} = 0$$

Hence, $\phi = \beta \Phi$ as given by Equation 3k is a potential satisfying Laplace's equation and consistent with the potential inside the beam, along the beam boundary. Therefore, the potential relation as given by Equation 3k is that requisite to insure the desired parallel rectilinear electron motion in the beam.

To obtain the equipotential lines, x may be varied and values of u and v computed. Equipotentials for various values of Φ are shown in Fig. 2.

Equation 3 supra is valid only for cases where ϕ is equal to or greater than 0. For cases where ϕ is less than 0, the potential varies linearly with distance and the equipotentials are parallel planes.

An illustrative device of type A is shown in Fig. 3 wherein the first electrode 15a has a plane central grid portion 17a and a dished outer portion 21, the electrode 16 has a plane central grid portion 18a parallel to the grid portion 17a and two outer portions 22a and 22b, one, 22a, of which is dished and the other, 22b, of which is plane and parallel to the grid portion 17a and the electrode 20 is plane and parallel to the outer portion 22b of the electrode 16a. The electrodes are mounted within an evacuated enclosing vessel 50. In order to simplify the drawing, the leading-in conductors for the several electrodes have been omitted. In this construction, the electrode 15a is operated at a positive potential, the electrode 16a is operated at zero potential and the electrode 20 is operated at a negative potential.

The appropriate forms for the electrode portions 21 and 22a requisite to obtain the desired parallel flow between the electrodes 15a and 16a in the form of a beam, the boundaries of which are indicated by the broken lines B in Fig. 3, are determined by obtaining the proper value of β for the desired potentials and spacings from tables such as given in the Fay et al. article noted hereinabove, calculating u from Equation 4, and deriving the appropriate equipotential either from Fig. 2 or by calculation from Equation 3.

In the electrode system illustrated in Fig. 3, all of the injected electron current is reflected so that none flows to either of the electrodes 16a or 20, the arrows in the drawing illustrating this action. Such a system may be utilized, for example in devices, particularly high frequency devices, wherein it is desired to act on the electron stream by means of an electrode to which no electron current flows.

Another illustrative device of type A is illustrated in Fig. 4 wherein the first electrode 15a and the electrode 20 are maintained at positive potentials and the electrode 16a is biased at about zero potential. A portion of the injected current may be transmitted to the electrode 20 and the remaining portion returned. The grid portion 18a allows a discontinuity of the field of the plane thereof. Hence, this system may be utilized in stopping potential devices to reduce or prevent loss in transconductance due to space

charge divergence in the stopping potential region. The requisite shapes for the outer portions of the electrodes 15a and 16a may be obtained in the manner described heretofore utilizing, however, the following modification of Equations 4 and 5.

$$u = \frac{(2-Z)^{1/2}x}{S_0\beta^{1/2}} + \frac{(1-2\beta^{1/2})(1+\beta^{1/2})}{\beta^{1/2}} \quad (4a)$$

and

$$v = \frac{(2-Z)^{1/2}y}{S_0\beta^{1/2}} \quad (5a)$$

where Z is the fraction of the injected current transmitted and y is distance from the beam boundary.

In systems of type B, as noted hereinabove, both the electrodes of the pair between which the rectilinear electron flow is desired are at positive potentials and a potential zero exists between the electrodes. In such systems, a fraction (Z) of the injected current is transmitted and the remaining fraction (1-Z) of the injected current is reflected. The relationships between potential and distance within the beam and on the two sides of the zero potential point, i. e., between the first electrode and this point, and between this point and the second electrode, are given, respectively, by the equations

$$\sigma_- = \frac{1}{(2-Z)^{1/2}} - \frac{\phi^{1/2}}{(2-Z)^{1/2}} \quad (7)$$

and

$$\sigma_+ = \frac{1}{(2-Z)^{1/2}} + \frac{\phi^{1/2}}{Z^{1/2}} \quad (8)$$

Both of these equations, it can be shown, are reducible to

$$u = \phi^{1/2} \quad (9)$$

which is the solution for Equation 1 for the case under consideration, where, in the case of Equation 7,

$$u_- = \frac{-(2-Z)^{1/2}x}{S_0} + 1$$

$$v_- = \frac{(2-Z)^{1/2}y}{S_0}$$

and in the case of Equation 8,

$$u_+ = \frac{Z^{1/2}x}{S_0} - \frac{Z^{1/2}}{(2-Z)^{1/2}}$$

$$v_+ = \frac{Z^{1/2}y}{S_0}$$

where y is distance from the beam boundary.

The equation for the equipotential lines outside of the parallel beam is, then,

$$u + iv = (\phi + i\psi)^{1/2} \quad (10)$$

from which the configurations of the electrode surfaces, requisite to produce the desired parallel electron flow, are determinable. The requisite configurations, for various values of ϕ , are illustrated in Fig. 5.

Two constructions illustrative of type B are illustrated in Figs. 6 and 7. In Fig. 6, the first electrode 15b includes a central plane grid portion 17b and an outer dished portion 23, the elements of which are of the appropriate form, depending upon the potential at which it is to be operated, as determined from Equation 9 or from Fig. 5, and the second electrode 16b includes a central grid portion 18b, parallel and aligned with the grid portion 17b, and an outer portion 24, the elements of which likewise are of the ap-

propriate form, depending upon the potential at which it is operated. Intermediate these two electrodes is the zero potential electrode 25, the surfaces 25a and 25b of which have straight elements inclined to the axis of the system in accordance with the $\phi=0$ line in Fig. 5.

In the device illustrated in Fig. 6, the electron flow between the first and second electrodes is in the form of a sheet beam, the boundaries of which are indicated by the broken lines B, and in the electrode system the electrons traverse rectilinear parallel paths. As noted previously some of the injected electrons are reflected and others are transmitted. A virtual cathode is thus produced at the plane passing through the junction of the surfaces 25a and 25b. The injected current may be varied in known ways to obtain a contrary variation in the current transmitted. If the injected current is fixed, the electrode 25 may be utilized as a control electrode to vary the transmitted current.

In the construction illustrated in Fig. 7, the electrodes 15c and 16c, similarly to the electrodes 15b and 16b, have outer portions conforming to appropriate equipotentials as determined from Equation 9 or Fig. 5. The electrode 16c may be biased at zero potential and utilized as a control grid to vary the current transmitted.

In type C systems, it is known that the potential distance relations within the beam and on opposite sides of the region of potential minimum are given by the equations

$$\sigma_- = -(\phi^{1/2} + 2\alpha^{1/2})(\phi^{1/2} - \alpha^{1/2})^{1/2} + (1 + 2\alpha^{1/2})(1 - \alpha^{1/2})^{1/2} \quad (11)$$

for the region between the first electrode and the potential minimum and

$$\sigma_+ = (\phi^{1/2} + 2\alpha^{1/2})(\phi^{1/2} - \alpha^{1/2})^{1/2} + (1 + 2\alpha^{1/2})(1 - \alpha^{1/2})^{1/2} \quad (12)$$

for the region between the potential minimum and the second electrode. In Equations 10 and 11, α is an integration constant, values of which may be obtained from tables such as given in the Fay et al. article, and the remaining characters are as defined heretofore. These equations can be rewritten as

$$u = (\phi^{1/2} + 2)(\phi^{1/2} - 1)^{1/2} \quad (13)$$

the solution for Equation 1 for the case under consideration, by treating the equations using the following relationships:

$$\frac{V}{V_1} = \phi - \alpha\phi$$

in both,

$$u_- = -\frac{x}{S_0\alpha^{1/2}} + \frac{(1+2\alpha^{1/2})(1-\alpha^{1/2})^{1/2}}{\alpha^{1/2}}$$

$$v_- = \frac{y}{S_0\alpha^{1/2}}$$

in Equation 10, and

$$u_+ = \frac{x}{S_0\alpha^{1/2}} - \frac{(1+2\alpha^{1/2})(1-\alpha^{1/2})^{1/2}}{\alpha^{1/2}}$$

and

$$v_+ = \frac{y}{S_0\alpha^{1/2}}$$

in Equation 11. Hence, the potential outside of the beam requisite to obtain parallel electron flow between the two electrodes in the type C case is

$$u + iv = [(\phi + i\psi)^{1/2} + 2][(\phi + i\psi)^{1/2} - 1]^{1/2} \quad (14)$$

and the equipotential lines or boundaries to which

the outer portions of the electrodes shall conform are determinable from this equation. Typical equipotential lines for a series of values of Φ are shown in Fig. 8.

One illustrative construction including electrodes having surfaces conforming to equipotential boundaries as determined by Equation 13 is shown in Fig. 9 wherein the first and second electrodes 15d and 16d have central, aligned parallel grid portions 17d and 18d, respectively, and outer portions 26 and 27, respectively, conforming to appropriate equipotentials as given by Equation 13 or in Fig. 8. Intermediate these two electrodes are a pair of electrodes 28a and 28b, at the minimum potential, the surfaces of which likewise conform to appropriate equipotentials and the edges of which adjacent the beam boundaries, indicated by the broken lines B in Fig. 9, are slightly spaced. Parallel electron flow is maintained between the electrodes 15d and 16d. The minimum potential electrodes 28a and 28b may be utilized as a velocity variation element. A similar construction may be employed as the energy extracting or output element in velocity variation devices. In the system disclosed and described, it will be appreciated that parallel electron flow may be maintained without a net increase in potential.

Electrode systems corresponding to a type C potential distribution may be utilized also in the screen grid-anode region of beam power tubes, one suitable construction being illustrated in Fig. 10 wherein the electrode 15e serves as the screen grid, the electrode 16e is the anode and the minimum potential electrode 28e cooperates with the screen grid and anode to produce the desired parallel electron flow between the screen grid and anode. The surfaces of the electrodes in this construction, of course, conform to equipotential boundaries as given by Equation 13 or shown in Fig. 8.

Potential distributions of type D, as indicated in the Fay et al. article noted hereinabove, have four possible solutions, namely,

- D₁, wherein the integration constant α is negative and the potential increases continuously as σ increases from 0;
- D₂, wherein the integration constant α is negative; and the potential decreases continuously as σ increases from zero;
- D₃, wherein the integration constant β is positive and ϕ is greater than unity; and
- D₄, wherein the integration constant β is positive and ϕ is less than unity.

It can be shown that for the D₁ and D₂ solutions, the equipotential boundaries are as given by Equation 13 hereinabove, and that for the D₃ and D₄ solutions the equipotential boundaries are as given by Equation 3 hereinabove. As will be apparent, electrodes conforming to equipotential boundaries for the type D solutions may be utilized to obtain parallel electron flow in the control grid-anode region in triodes or multigrid devices and in the control grid-screen or screen grid-anode regions of screen grid type electron discharge devices.

It will be appreciated from the foregoing that this invention enables attainment of parallel electron flow between any pair of electrodes. The attainment of such flow, in turn, enables the construction of electron discharge devices with quantitatively predeterminable and stable operating characteristics.

In the constructions described hereinabove, it will be noted that the electron flow is in the form

of a flat beam or sheet and that the electrodes and the beam are symmetrical with respect to a plane passing through the axis of the beam. The invention may be embodied also, as will be apparent, in devices wherein the electron flow is in the form of a cylindrical beam, in which case the several electrodes are symmetrical with respect to the longitudinal axis of the beam. The appropriate forms of the equipotentials and hence of the electrode surfaces requisite for the attainment of the parallel electron flow may be obtained conveniently by use of an electrolytic tank in the manner described in the application Serial No. 319,393 noted hereinabove.

It will be apparent also that various modifications may be made in the several illustrative embodiments shown and described without departing from the scope and spirit of this invention as defined in the appended claims.

What is claimed is:

1. An electron discharge device comprising a first electrode having a central apertured portion, means to one side of said electrode for projecting a parallel electron stream through said apertured portion, and means including said first electrode and a second electrode for directing the electrons in said projected stream along parallel paths within a boundary and throughout a region on the other side of said first electrode, said first and second electrodes having opposed surfaces extending outwardly from said boundary and conforming to equipotential boundaries of a field which satisfies Laplace's equation, which has a zero gradient normal to said boundary and for which, along said boundary, the potential varies according to a solution of the equation

$$\frac{\partial^2 \Phi}{\partial x^2} = -\frac{j}{p_0 \sqrt{\frac{2e}{m} \Phi}}$$

where Φ is potential, x is distance from the end of said region into which said stream is projected, j is the current density in said stream, p_0 is the permittivity of a vacuum, and e and m are the electron charge and mass, respectively.

2. An electron discharge device comprising a pair of aligned centrally apertured electrodes, and means for projecting a parallel electron beam through the aperture in one of said electrodes toward the other of said electrodes and parallel to the axis of alignment of said electrodes, said electrodes having outer portions the opposed surfaces of which conform to equipotential boundaries of a field which satisfies Laplace's equation subject to the conditions that at the boundary of said beam the potential gradient normal to the boundary is zero and that the potential along said boundary varies according to a solution of the equation

$$\frac{\partial^2 \Phi}{\partial x^2} = -\frac{j}{p_0 \sqrt{\frac{2e}{m} \Phi}}$$

where Φ is potential, x is distance from the end of said region into which said stream is projected, j is the current density in said stream, p_0 is the permittivity of a vacuum, and e and m are the electron charge and mass, respectively.

3. An electron discharge device comprising a pair of aligned electrodes having parallel central portions one of which is apertured, means for projecting a parallel electron stream into the region between said central portions through the apertured portion and normal to said parallel

portions, and means for directing the electrons in said stream along rectilinear paths within a boundary normal to said parallel portions, said directing means comprising outer portions of said electrodes having opposed surfaces corresponding to equipotential boundaries of a field satisfying Laplace's equation, for which the gradient normal to said boundary is zero and for which the potential along said boundary is equal to the solution of the space charge equation corresponding to parallel electron flow within said boundary.

4. An electron discharge device comprising an electrode system including a pair of aligned electrodes one of which has an aperture therein, means for projecting a parallel electron beam through said aperture toward the other of said electrodes and in the direction of the axis of alignment of said electrodes, said electrodes having opposed surfaces conforming to equipotential boundaries of the same field and each of said opposed surfaces being of the form given by the relation

$$u+iv=[(\Phi+i\psi)^N-2][(\Phi+i\psi)^N+1]^N$$

where u and v are the coordinates of the surface,

$$i=\sqrt{-1}$$

ψ is a complementary function, and Φ is a constant proportional to the potential at which the electrode is to be operated.

5. An electron discharge device comprising a first electrode having a central apertured portion and an outer portion, a plane electrode mounted to one side of and in alignment with said first electrode, means mounted to the other side of said first electrode for projecting a parallel electron beam through said aperture and in the direction of the axis of alignment of said first and plane electrodes, and an electrode intermediate and aligned with said first and plane electrodes having a central aperture therein and having an outer portion one surface of which is opposite and parallel to said plane electrode and the opposite surface of which faces said first electrode, said opposite surface and the surface of the outer portion of said first electrode facing theretoward each being of the form given by the relation

$$u+iv=[(\Phi+i\psi)^N-2][(\Phi+i\psi)^N+1]^N$$

where u and v are the coordinates of the surface,

$$i=\sqrt{-1}$$

ψ is a complementary function, and Φ is a constant proportional to the potential at which the electrode is to be operated.

6. An electron discharge device comprising an electrode system including a pair of aligned electrodes one of which has an aperture therein, means for projecting a parallel electron beam through said aperture toward the other of said electrodes and in the direction of the axis of alignment of said electrodes, said electrodes having opposed surfaces conforming to equipotential boundaries of the same field and each of said opposed surfaces being of the form given by the relation

$$u+iv=(\Phi+i\psi)^N$$

where u and v are the coordinates of the surface,

$$i=\sqrt{-1}$$

ψ is a complementary function, and Φ is a constant proportional to the potential at which the electrode is to be operated.

7. An electron discharge device comprising a first electrode having a central apertured por-

tion and an outer portion, a second electrode mounted to one side of and in alignment with said first electrode, means for projecting a parallel electron stream through said apertured portion, parallel to the axis of alignment of said electrodes and toward said second electrode, and electrode means intermediate and in alignment with said first and second electrodes and having a central aperture, said electrode means having a first surface facing a surface of said outer portion and having a second surface facing a surface of said second electrode, and the facing surfaces of said first electrode and said electrode means and of said second electrode and said electrode means conforming to equipotential boundaries of a field which satisfied Laplace's equation, for which the gradient normal to the boundary of said stream in the region between said facing surfaces is zero and for which the potential along said boundary varies according to a solution of the equation

$$\frac{\partial^2 \Phi}{\partial x^2} = -\frac{j}{\epsilon_0 \sqrt{2e/m} \Phi}$$

where Φ is potential, x is distance from the end of said region into which said stream is projected, j is the current density in said stream, ϵ_0 is the permittivity of a vacuum, and e and m are the electron charge and mass, respectively.

8. An electron discharge device comprising a first electrode having a central apertured portion and an outer portion, a second electrode mounted to one side of and in alignment with said first electrode, means for projecting a parallel electron stream through said apertured portion, parallel to the axis of alignment of said electrodes and toward said second electrode, and electrode means intermediate and in alignment with said first and second electrodes and having a central aperture, said electrode means having a first surface facing a surface of said outer portion and having a second surface facing a surface of said second electrode, and the facing surfaces of said first electrode and said electrode means and of said second electrode and said electrode means being of the form given by the relation

$$u+iv(\Phi+i\psi)^N$$

where u and v are the coordinates of the surface,

$$i=\sqrt{-1}$$

ψ is a complementary function and Φ is a constant proportional to the potential at which the electrode is to be operated.

9. An electron discharge device comprising an electrode system including a pair of aligned electrodes one of which has an aperture therein, means for projecting a parallel electron beam through said aperture toward the other of said electrodes and in the direction of the axis of alignment of said electrodes, said electrodes having opposed surfaces conforming to equipotential boundaries of the same field and each of said opposed surfaces being of the form given by the relation

$$u+iv=[(\Phi+i\psi)^N+2][(\Phi+i\psi)^N-1]^N$$

where u and v are the coordinates of the surface,

$$i=\sqrt{-1}$$

ψ is a complementary function and Φ is a constant proportional to the potential of the electrode.

10. An electron discharge device comprising a

first electrode having a central apertured portion and an outer portion, a second electrode mounted to one side of and in alignment with said first electrode, means for projecting a parallel electron stream through said apertured portion, parallel to the axis of alignment of said electrodes and toward said second electrode, and electrode means intermediate and in alignment with said first and second electrodes and having a central aperture, said electrode means having a first surface facing a surface of said outer portion and having a second surface facing a surface of said second electrode, and the facing surfaces of said first electrode and said electrode means and of said second electrode and said electrode means being of the form given by the relation

$$u+iv=[(\Phi+i\psi)^2+2]_m[(\Phi+i\psi)^2-1]^{\frac{1}{2}}$$

where u and v are the coordinates of the surface,

$$i=\sqrt{-1}$$

ψ is a complementary function and Φ is a constant proportional to the potential of the electrode.

11. An electron discharge device in accordance with claim 7 wherein said first and second surfaces of said intermediate electrode means are electrically integral.

12. An electron discharge device in accordance with claim 7 wherein said first and second surfaces of said intermediate electrode means are electrically separate and have juxtaposed edges adjacent said beam boundary.

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