

Nov. 10, 1942.

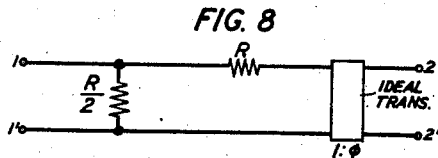
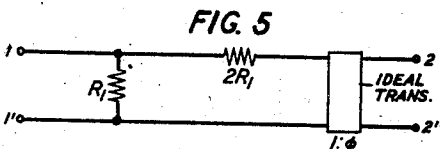
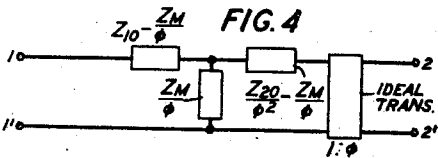
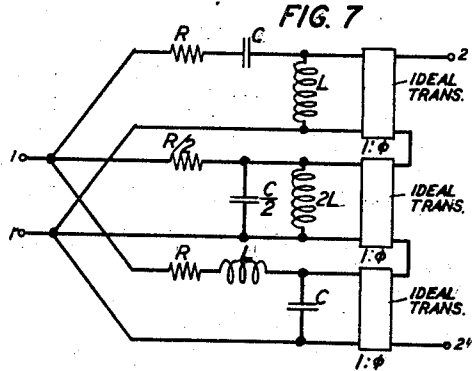
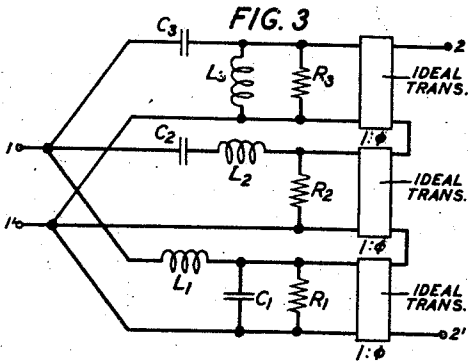
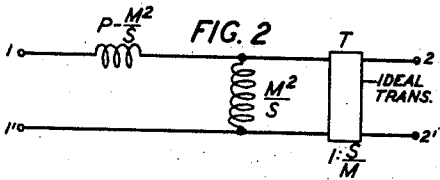
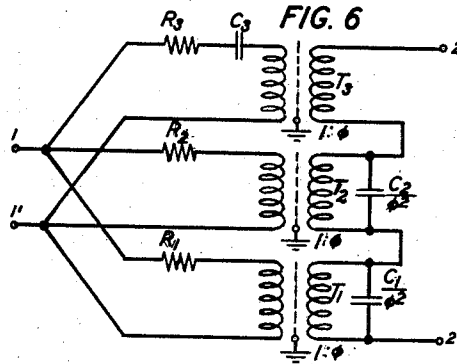
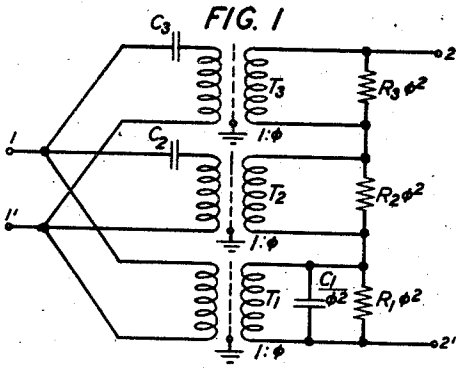
H. W. BODE

2,301,245

TRANSFORMER SYSTEM

Filed Aug. 13, 1940

3 Sheets-Sheet 1



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2,301,245

TRANSFORMER SYSTEM

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3 Sheets-Sheet 2

FIG. 9

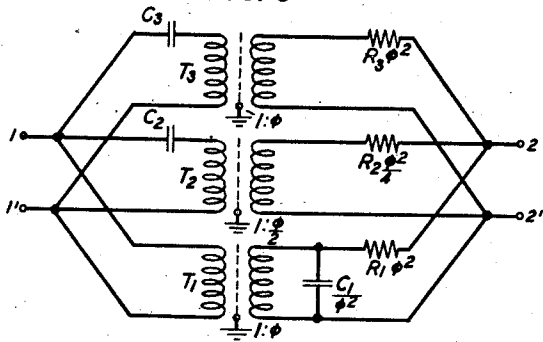


FIG. 12

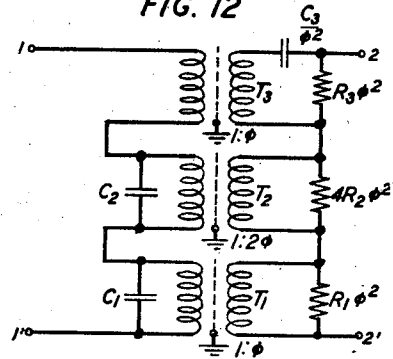


FIG. 10

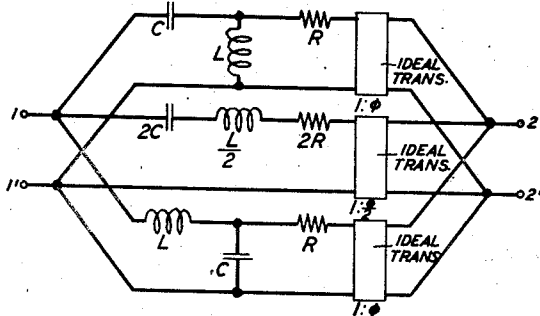


FIG. 13

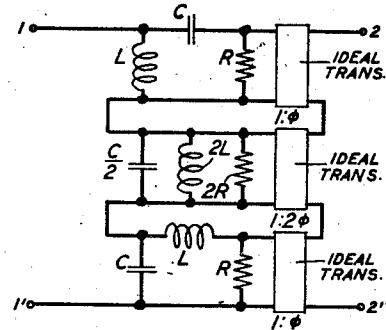


FIG. 11

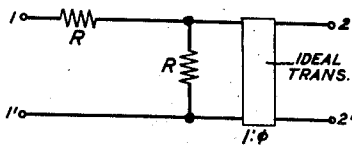
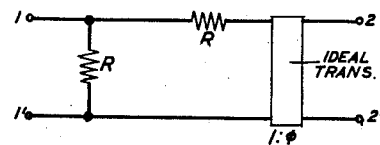


FIG. 14



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2,301,245

TRANSFORMER SYSTEM

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3 Sheets-Sheet 3

FIG. 16

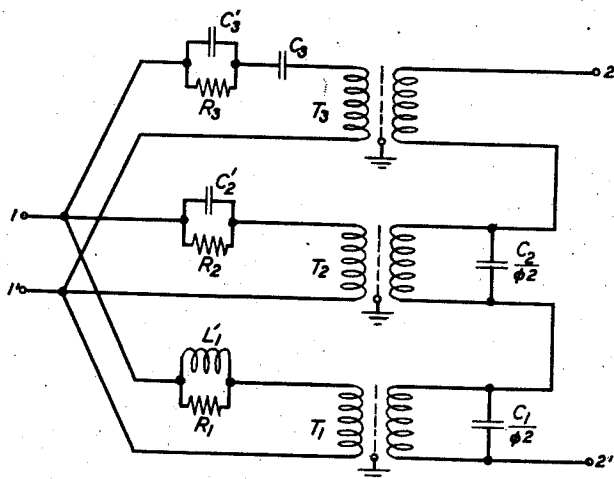
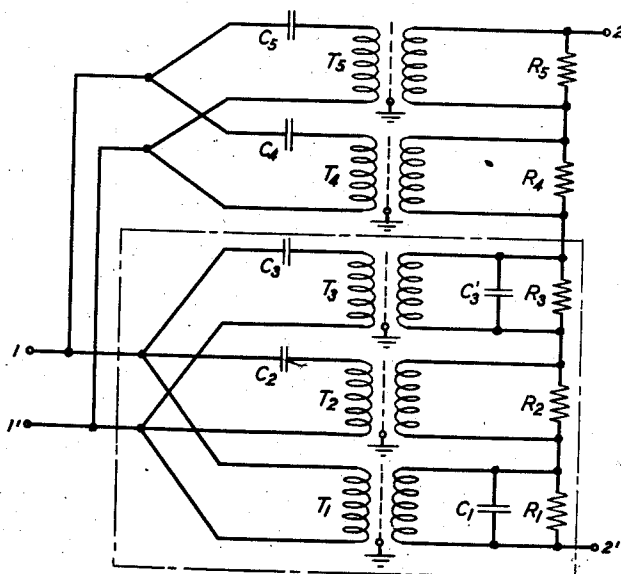


FIG. 15



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UNITED STATES PATENT OFFICE

2,301,245

TRANSFORMER SYSTEM

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York, N. Y., a corporation of New York

Application August 13, 1940, Serial No. 352,422

6 Claims. (Cl. 178—44)

This invention relates to transformers and more particularly to multiple transformer systems for the transmission of currents of all frequencies in a continuous wide range. Its principal objects are to improve the phase shift characteristics of multiple transformer systems and to increase the frequency range in which uniform transmission may be accomplished.

The image or video currents in a television system, for example, may range in frequency from a few cycles per second to several million. Faithful reproduction of the visual image requires that these currents be transmitted at all frequencies without variation of their amplitude relationships and without change of their relative phases. This imposes severe requirements on the attenuation and phase shift characteristics of any transformer that may be used to increase or decrease the voltages of the currents and, in particular, requires a high degree of linearity in the phase shift characteristic, since image distortions resulting from any departure from linearity are very readily perceived by the eye.

Systems of the type contemplated by the invention comprise a plurality of tuned transformers connected to provide a corresponding number of parallel transmission paths between common input and common output terminals and so proportioned that each operates efficiently in a separate part of the whole frequency range. I have found that, in general, the existence of parallel transmission paths may give rise to excessive and irregular phase shifts and that, when the parallel paths are constituted by transformers, the winding inductances are particularly troublesome sources of phase shift irregularities.

In accordance with the invention, improved transmission characteristics are obtained by organizing and proportioning the elements of the several paths in such manner that the phase shift produced by the complete transformer system is substantially the minimum consistent with the attenuation characteristic and so that the attenuation is substantially constant at all frequencies in the operating range. The term minimum phase shift as applied to networks is defined in my earlier Patent 2,123,178, issued June 12, 1938, wherein it is shown that for any prescribed attenuation characteristic the minimum possible phase shift is uniquely determined by the variation of the attenuation with frequency. Networks realizing this minimum phase shift characteristic have been called minimum phase shift networks. The conditions of minimum phase shift and uniform attenuation are achieved substantially in the systems of the invention by tuning certain effective inductances of the several transformers in a manner described hereinafter and by the inclusion of suitably proportioned resistances in each of the transmission paths.

A feature of the invention is that the uniformity of the attenuation and the minimum phase shift characteristic are independent of the values of the terminal impedances to which the transformer systems may be connected. Another feature is the substantially pure resistance character of the several short circuit and open circuit driving point and transfer impedances of the system.

These and other features of the invention will be more fully understood from the detailed description which follows and by reference to the accompanying drawings, of which:

Fig. 1 represents one form of the invention;

Figs. 2, 3, 4 and 5 are diagrams used in the explanation of the principles of the invention;

Fig. 6 represents a second form of the invention;

Figs. 7 and 8 are diagrams explanatory of Fig. 6;

Figs. 9, 10 and 11 illustrate a third form of the invention;

Figs. 12, 13 and 14 illustrate a fourth form of the invention;

Fig. 15 represents a modification of the invention for extending the frequency range of operation; and

Fig. 16 represents another modification for the same purpose.

The transformer system shown in Fig. 1 is of a type suitable for stepping up the voltage from a low impedance source connected to terminals 1, 1' and delivering the increased voltage to a high impedance load connected to terminals 2, 2'. It comprises three separate shielded transformers T_1 , T_2 and T_3 the secondary windings of which are connected in series between the output terminals and the primary circuits of which are connected in parallel at the input terminals. All of the transformers have the same transformation ratio, designated by ϕ , and all have their windings poled in the same way. Grounded shields, which are indicated by the dotted lines between the transformer windings, serve to eliminate any electrostatic coupling between the primary and secondary circuits.

The transformers provide three parallel transmission paths between the input and output terminals, each of which in accordance with the invention, is effective in a different portion of the whole frequency range. The individual frequency ranges of the several paths are controlled by tuning condensers included in circuit with each transformer the capacities of which are suitably proportioned with respect to the leakage and mutual inductances of the windings. In the figure the designations of the condensers represent their capacities. Transformer T_1 is tuned by a capacity $C_1 \rightarrow \phi^2$ in shunt to the secondary winding which resonates with the effective leak-

age inductance at a preassigned frequency. The mutual inductance of the windings is made sufficiently great to provide efficient transmission at frequencies well below the desired minimum frequency of the system. Its effect at frequencies within the operating range is then negligible and the combination of the transformer and tuning condenser may be treated as a portion of a low-pass filter having a cut-off at the preassigned resonance frequency. The leakage inductance of transformer T_2 is likewise tuned to the same preassigned frequency by a series capacity C_2 in the primary circuit. The mutual inductance of this transformer is also made sufficiently great to have negligible effect on the transmission throughout the operating range, in which case the transformer and condenser combination may be treated as a simple resonant circuit. The path including transformer T_3 is the same as the middle path in configuration but in this case the windings are of low inductance and are so proportioned that the capacity C_3 resonates with the effective mutual inductance at the preassigned frequency. The leakage inductance is made sufficiently small so that its effect is negligible at any frequency in the operating range, thus reducing the network effectively to a high-pass filter.

The control of the phase shift and attenuation characteristics is effected by means of suitably proportioned resistances, designated as $R_1\phi^2$, $R_2\phi^2$ and $R_3\phi^2$, connected in shunt to the several paths across the secondary windings of the transformers. The presence of these resistances in the circuit permits the construction of the system as a minimum phase shift network. By suitably proportioning their values in relation to each other and to the inductances and capacities of the system, substantially uniform attenuation and zero phase shift can be attained.

The relationship of the element values which produce the desired characteristics will now be developed. For this purpose the circuit diagram may be simplified by substituting for each of the transformers an equivalent network of the type shown in Fig. 2 which is the equivalent of a transformer having a primary inductance P , a secondary inductance S , and a mutual inductance M . The series and shunt inductances represent respectively the total effective leakage inductance and the effective mutual inductance, both referred to the primary circuit. These inductances may be described as parasitic inductances, since they represent undesired but unavoidable reactances and susceptances which absorb part of the energy of the oscillations impressed on the transformer and cause its performance to depart from that of its ideal prototype. The reactance of the leakage inductance absorbs part of the impressed voltage and the susceptance of the mutual inductance absorbs part of the current supplied to the transformer.

The application of the equivalence theorem to the network of Fig. 1 produces the simplified system of Fig. 3 when the mutual inductances of transformers T_1 and T_2 and the leakage inductance of T_3 are disregarded. The resistances and the capacity C_1 have all been transferred to the primary sides of the ideal transformers with appropriate changes in their values indicated by the removal of the impedance transformation factor ϕ^2 .

The relationships of the element values may be determined by computing certain of the open circuit or short-circuit characteristics of the net-

work and subjecting these to particular conditions consistent with the desired characteristics of the system. For the system of Figs. 1 and 3 it is convenient to compute first the open circuit voltage at terminals 2, 2' in response to a voltage applied to terminals 1, 1'. This is evidently equal to the sum of the separate voltages across resistances R_1 , R_2 and R_3 multiplied by the common transformation ratio ϕ . If the voltage at terminals 1, 1' be denoted by V_1 and the voltages across R_1 , R_2 and R_3 by e_1 , e_2 and e_3 , respectively, it is readily shown that:

$$\frac{e_1}{V_1} = \frac{1}{1 + p \frac{L_1}{R_1} + p^2 L_1 C_1} \quad (1)$$

$$\frac{e_2}{V_1} = \frac{p C_2 R_2}{1 + p C_2 R_2 + p^2 L_2 C_2} \quad (2)$$

and

$$\frac{e_3}{V_1} = \frac{p^2 L_3 C_3}{1 + p \frac{L_3}{R_3} + p^2 L_3 C_3} \quad (3)$$

where p denotes the quantity $j\omega$, ω being the pulsance or 2π times frequency. If the element values are subject to the relationships

$$L_1 C_1 = L_2 C_2 = L_3 C_3 = 1/\omega_0^2 \quad (4)$$

and

$$\frac{L_1}{R_1} = C_2 R_2 = \frac{L_3}{R_3} a/\omega_0 \quad (5)$$

where ω_0 corresponds to a common resonance frequency f_0 which may be assigned arbitrarily, the denominator of all three of the expressions for the voltage ratios become the same and the sum of the ratios becomes equal to

$$\frac{e_1 + e_2 + e_3}{V_1} = \frac{1 + a \frac{p}{\omega_0} + \frac{p^2}{\omega_0^2}}{1 + a \frac{p}{\omega_0} + \frac{p^2}{\omega_0^2}} \quad (6)$$

which is equal to unity and independent of frequency. The quantity a appearing in Equations 5 and 6 is a simple numeric, the value of which is so far undetermined. Under the conditions of Equations 4 and 5, the open circuit voltage V_2 at terminals 2, 2' is given by

$$V_2 = \phi V_1 \quad (7)$$

Since there are nine impedance elements in the system, aside from the ideal transformers, the relationships of Equations 4 and 5 are insufficient for the complete determination of the network. They do, however, suffice to establish the minimum phase shift character of the system as may be seen from the following considerations.

A general transmission equivalent of the complete system of Fig. 3 comprising a T-network and a single ideal transformer is shown in Fig. 4. The symbols Z_{10} , Z_{20} and Z_M represent respectively the primary open circuit impedance, the secondary open circuit impedance and the open circuit transfer impedance. The transformation factor ϕ appearing in the designations refers the impedance values to the primary side of the system. Equations 6 and 7 show that, as a result of the relationships of the element values, the ratio of the open circuit output voltage to the impressed voltage is constant and equal to ϕ . Examination of Fig. 4 shows that the voltage ratio is also equal to Z_M/Z_{10} . It follows then, that

$$Z_M = \phi Z_{10} \quad (8)$$

Since Z_{10} , the primary open circuit impedance, is

necessarily physically realizable, that is, can be constructed out of real elements with positive impedance coefficients, it follows that the open circuit transfer impedance Z_M is likewise physically realizable. The left-hand series impedance, $Z_{10} - Z_M/\Phi$, of the T-network therefore becomes zero, thereby reducing the network to an L structure. The right-hand series impedance, $Z_{20}/\Phi^2 - Z_M/\Phi$ becomes equal to the secondary short-circuit impedance, referred of course to the primary side, and is therefore also physically realizable. The fact that the system of Fig. 3 is equivalent to a ladder type network of real elements is sufficient to establish its minimum phase shift character as pointed out in my earlier Patent 2,123,178 of June 12, 1938. This result has been made possible by the inclusion of the resistance elements in the several transmission paths.

Equations 4 and 5 provide five independent relationships of the nine impedance coefficients of the system, the quantity a being undetermined, leaving three relationships to be imposed for the complete determination of the network proportions. In accordance with the invention, the additional relationships are such as to make the constancy of the voltage ratio independent of the magnitudes of the impedances between which the network is connected. This is accomplished by proportioning the relative values of the corresponding elements in the different paths so that the primary open circuit impedance, and therefore the open circuit transfer impedance, and the secondary short-circuit impedance become purely resistive. The network of Fig. 3 is then equivalent to a simple resistance pad together with an ideal transformer.

The primary open circuit impedance is given by

$$Z_{10} = R_1 \frac{1 + a \frac{p}{\omega_o} + \frac{p^2}{\omega_o^2}}{1 + p R_1 (C_1 + C_2 + C_3) + \frac{R_1 p^2}{R_3 \omega_o^2}} \quad (9)$$

and the secondary short-circuit impedance, Z_{20} , by

$$\frac{Z_{20}}{\Phi^2} = R_2 \frac{1 + p \frac{L_3}{R_2} + \frac{L_1}{R_2} + \frac{p^2}{\omega_o^2}}{1 + a \frac{p}{\omega_o} + \frac{p^2}{\omega_o^2}} \quad (10)$$

These become respectively equal to R_1 and R_2 when

$$\begin{aligned} R_1 &= R_3 \\ R_1 (C_1 + C_2 + C_3) &= \frac{a}{\omega_o} \end{aligned} \quad (11)$$

and

$$\frac{L_1 + L_3}{R_2} = \frac{a}{\omega_o}$$

With the help of Equations 4 and 5, the relationship expressed in the above equations may be reduced to

$$\begin{aligned} R_3 &= R_1 \\ R_2 &= 2R_1 \end{aligned} \quad (12)$$

and

$$a = 2$$

which together with Equations 4 and 5 serve to determine completely the proportions of the network elements. The values of the inductances

and capacities are given below in terms of R_1 and ω_o , which may be assigned arbitrarily.

$$\begin{aligned} L_1 &= L_3 = 2L_2 = \frac{2R_1}{\omega_o} \\ C_1 &= C_3 = 2C_2 = \frac{1}{2\omega_o R_1} \end{aligned} \quad (13)$$

When given the proportions indicated by the above equations, the network of Fig. 3 becomes exactly equivalent to that of Fig. 5 which comprises a single shunt resistance of value R_1 , a single series resistance equal to $2R_1$ and an ideal transformer. Its phase shift is zero at all frequencies and the attenuation is constant when connected between resistive terminal impedances of any values. The three paths have respectively low-pass, band-pass, and high-pass transmission characteristics, the band of the middle path being, in general, relatively narrow and serving principally to sustain the transmission at the region of the common cut-off frequency of the low-pass and high-pass paths.

Since the circuit of Fig. 3 has been derived from that of Fig. 1 on the assumption that one of the parasitic inductances of each transformer may be disregarded, the system of Fig. 1 will not behave exactly as indicated by the equations, but will realize the results to a close degree of approximation. The susceptance of the mutual inductance of transformer T_1 sets a low frequency limit to the range of uniform transmission and the leakage inductance of transformer T_3 operates to establish a high frequency limit. By using transformers having magnetic cores of high permeability material, such as the alloys of iron and nickel disclosed in U. S. Patents 1,586,884, June 1, 1926, and 1,768,443, June 24, 1930, both to G. W. Elmen, the effects of these parasitic inductances may be made negligibly small in a frequency range extending from a lower limit of less than twenty cycles per second to an upper limit greater than three million cycles per second.

The modified form of the invention shown in Fig. 6 differs from that of Fig. 1 in the disposition of the resistance elements in the circuit and in the tuning of the band-pass transformer T_2 . The resistance elements are connected in the primary circuit as series impedances and the mutual inductance of transformer T_2 is tuned by a condenser connected in shunt in the secondary circuit. This transformer has a relatively low mutual inductance and should have negligibly small leakage. As in the system of Fig. 1, all of the transformers have the same transformation ratio and are tuned by their associated condensers to the same resonance frequency.

The equivalent network is shown in Fig. 7, the mutual inductance of transformer T_1 and the leakage inductances of T_2 and T_3 being disregarded. The impedance coefficients of the significant elements in the low-pass path are denoted by R , L and C , the subscripts being dropped as unnecessary. The values of the corresponding elements in the other paths necessary for the realization of the desired characteristics are indicated in the figure in terms of the low-pass element values. The corresponding elements in the high-pass and low-pass paths have the same values while in the band-pass path the values are modified by the factor two.

These proportions may be established by a procedure similar to that followed in connection with Figs. 1 and 3. The voltage transfer ratio with the secondary terminals open is readily com-

puted as the sum of the transfer ratios for the individual paths. Setting this equal to the transformation ratio of the transformers fixes the condition that the open circuit transfer impedance must be equal to the primary open circuit impedance or, in other words, that the equivalent T-network corresponding to Fig. 4 has its left-hand series impedance equal to zero and therefore reduces to a simple L-network. As in the case of Fig. 3, the right-hand series impedance of the equivalent T-network becomes equal to the secondary short-circuit impedance referred to the primary side of the ideal transformer. The further step of making the primary open circuit impedance and the secondary short-circuit impedance both purely resistive leads to the proportions indicated and to values of L and C in terms of the resistance R and the common resonance frequency as follows:

$$L = \frac{2R}{\omega_0} \quad (14)$$

$$C = \frac{1}{2\omega_0 R}$$

A third form of the invention shown in Fig. 9 is characterized by the parallel connection of the three paths at both primary and secondary terminals. In the preceding forms, where the circuits are connected in parallel on one side and in series on the other, the three-element networks constituted by the significant impedances of each path have the configurations of L-networks with the series branches on the same side as the parallel connections and the shunt branches on the same side as the series connections. This is in accordance with a common practice in the connection of multiple filter systems. In the system of Fig. 9, since the circuits are connected in parallel at both terminals, the individual networks in each path have series terminations at both ends. The significant elements of the high-pass and low-pass paths form T-networks and those of the middle path form a simple series circuit. For the realization of the desired characteristics the transformer T₂ in the middle or band-pass path is required to have a transformation ratio equal to one half that of the other two transformers. Its significant inductance is the leakage inductance, the mutual inductance being made sufficiently large to offer a negligibly small shunt susceptance. As in the case of the preceding systems, the mutual inductance of T₁ and the leakage inductance of T₃ are assumed to have negligibly small effects on the characteristics in the operating range.

The relative values of the elements may be established by a procedure similar to that followed in the preceding examples, but differing therefrom in detail. To begin with the transformation ratio of transformer T₂ may be left undetermined. The primary short-circuit transfer admittance and the primary short-circuit input admittance may be computed readily and by setting these respectively equal to $1/\phi R$, and $1/R$, respectively, it turns out that the effective mutual impedance must be equal to the secondary open circuit impedance, both referred to the primary circuit, and hence must be physically realizable. This suffices to establish the minimum phase shift character of the system. A further consequence is that the equivalent T-network of the whole system reduces to an L-network of the configuration shown in Fig. 11. The voltage transfer ratio from the secondary terminals on open circuit also becomes constant and equal to the transformation

ratio ϕ . Certain relationships of the circuit elements are likewise fixed, but not sufficient for the complete determination of their proportions. Computing the secondary short-circuit input admittance and setting its value equal to $1/R\phi^2$ establishes the ratio of the transformer T₂ as one half that of the others and reduces the transmission equivalent of the system to the purely resistive system of Fig. 11.

The condition of constancy of the open circuit voltage transfer ratio might have been applied as in the preceding examples, but the computation of this ratio is rather complex, consequently the indirect method indicated above is preferable.

The modification shown in Fig. 12 is characterized by the series connection of the several paths at both the primary and secondary terminals. Transformer T₂ has a transformation ratio twice as great as that of the other transformers and is tuned by a shunt condenser C₂ which resonates with the effective mutual inductance. Transformers T₁ and T₃ are similar to the corresponding transformers in the other figures. The equivalent network is shown in Fig. 13 with the proportions of the element coefficients indicated by their designations. The establishment of the proportions to provide the desired characteristics is most conveniently effected by computations of the primary open circuit transfer impedance, and the primary and secondary open circuit input impedances. The transmission equivalent of Fig. 13 is shown in Fig. 14.

The principal factors which limit the operating frequency range of transformer systems are the leakage inductance of the high frequency transformers and the mutual inductance of the low frequency transformers. The former gives rise to a high frequency limit and the latter to a low frequency limit to the range of substantially uniform transmission. The self-capacities of the transformer windings also tend to limit the transmission range, their principal effect being to sharpen the cut-off at the upper frequency limit of the operating range. The capacity which is mainly effective in this is that of the high impedance winding of the high frequency transformer. The winding capacities of the low frequency transformer are absorbed in the capacity of the tuning condenser and those of the intermediate transformer have substantially no effect since this transformer contributes to the transmission of the system only in a narrow frequency range.

For most purposes the three-transformer systems provide adequately wide transmission bands, but a substantial addition to the frequency range may be obtained by an extension of the principle already discussed to systems including larger numbers of transformers. Such a system comprising five transformers is illustrated in Fig. 15. The lower part of the figure, enclosed by broken lines, represents a three-transformer group like that of Fig. 1, but including an additional capacity C_{3'} connected across the secondary terminals of transformer T₃. This capacity may be the self-capacity of the secondary winding or it may include that of an added condenser. Additional transformers T₄ and T₅, tuned by condensers C₄ and C₅ and having associated resistances R₄ and R₅, are provided to extend the operating range to higher frequencies, the two together with the lower group constituting in effect a three-transformer group also like that of Fig. 1.

At the upper limit of its operating range the

lower group behaves like a low-pass filter having a cut-off frequency determined by the leakage inductance of transformer T_3 and the shunt capacity C_3' . All of the other elements of this group have substantially no effect at the upper frequency limit and may therefore be ignored. Transformer T_4 , the leakage inductance of which is tuned by capacity C_4 to the new cut-off frequency, and transformer T_5 , the mutual inductance of which is likewise tuned by capacity C_5 , provide respectively intermediate and high-pass paths coordinated in the manner already described with the low-pass path provided by transformers T_1 , T_2 and T_3 . Resistances R_4 and R_5 are equal and have the same value as those of the lower group. The winding inductances of T_4 and T_5 and their respective tuning capacities have values related to the resistances and the new cut-off frequency in accordance with Equation 13. Similar extensions of the systems of Figs. 6, 9 and 12 may also be made.

Another way of extending the frequency range is illustrated in Fig. 16 as applied to a system of the type shown in Fig. 6. Here the effect of the leakage inductance of the high frequency transformer is diminished by means of a small compensating capacity C_3' connected in shunt to the series resistance. As is well known, the effective reactance of a resistance-capacity combination of this type is the same as that of a negative inductance for a considerable range of frequencies above zero. By the proper choice of the capacity C_3' it is therefore possible to annul all or part of the effect of the transformer leakage inductance in the whole operating range of the system. The value of the shunted resistance should be increased somewhat so that the effective resistance of the combination may have the correct value in the high frequency range. The same method of compensation may be applied to the intermediate frequency path as shown, although in this case it is less necessary. In the low frequency path the shunting effect of the transformer mutual inductance may be compensated to some extent by shunting the series resistance by a relatively large inductance. At low frequencies the inductance becomes the dominant part of the impedance and the voltage impressed on the system tends to divide between this series branch and the transformer in proportion to their inductances. The added inductance L_4' may be proportioned to maintain approximately the same output voltage as obtains at higher frequencies.

What is claimed is:

1. A transformer system comprising a plurality of transformers connected to provide parallel transmission paths between two pairs of terminals, each of said transformers having closely coupled windings, a condenser connected in shunt to a winding of a first of said transformers and proportioned to resonate with the effective leakage inductance of the transformer at a preassigned frequency whereby the transmission of the transformer is limited substantially to frequencies below said assigned frequency, a condenser connected in series with a winding of a second of said transformers and proportioned to resonate with the effective mutual inductance of

said second transformer at said preassigned frequency, whereby the transmission range of said second transformer is limited substantially to frequencies above said preassigned frequency, a condenser connected in circuit with said third transformer, and resistors included in circuit with each of said transformers respectively, said third transformer being tuned by its associated condenser to provide transmission in a relatively narrow frequency range centered at said preassigned frequency and said resistors having resistance values proportioned with respect to the transformer inductances and the capacities of the said condensers to provide uniform attenuation and minimum phase shift for transmission in a wide range of frequencies above and below said preassigned frequency.

2. A system in accordance with claim 1 characterized by the connection of the primary circuits of the transformers in parallel and the connection of the secondary circuits in series.

3. A system in accordance with claim 1 in which the effective mutual inductance of the said second transformer is equal to the effective leakage inductance of the said first transformer.

4. A system in accordance with claim 1 in which the effective mutual inductance of the said second transformer is equal to the effective leakage inductance of the said first transformer and in which the said third transformer is tuned by a condenser in shunt to one of its windings and has an effective mutual inductance equal to twice the effective leakage inductance of the said first transformer.

5. A system in accordance with claim 1 in which the said third transformer is tuned by a series condenser having a capacity which resonates with the effective leakage inductance of the transformer at the said preassigned frequency and in which the effective leakage inductance of the said third transformer is equal to one half the effective leakage inductance of the said first transformer.

6. A transformer system for currents of a wide range of frequencies comprising a group of three transformers connected to provide three parallel transmission paths between a pair of input terminals and a pair of output terminals, a tuning condenser and a resistor included in circuit with each of said transformers, said condensers and resistors having capacity and resistance values proportioned with respect to the inductances of said transformer such that the open circuit output voltages of the several paths in response to an oscillation input of constant amplitude are substantially in the proportions

$$\frac{1}{F(p)}, \frac{a p}{\omega_0 F(p)}, \text{ and } \left(\frac{p}{\omega_0}\right)^2$$

where p denotes $j\omega$, being 2π times frequency, a is a numerical constant, ω_0 corresponds to an assigned fixed frequency, and $F(p)$ is the frequency function

$$1 + a \frac{p}{\omega_0} + \left(\frac{p}{\omega_0}\right)^2$$

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