

Aug. 11, 1942.

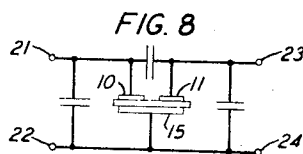
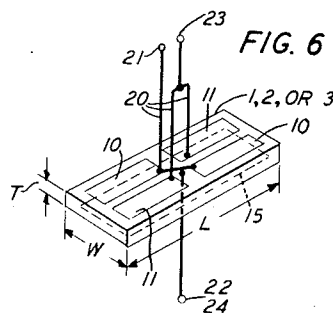
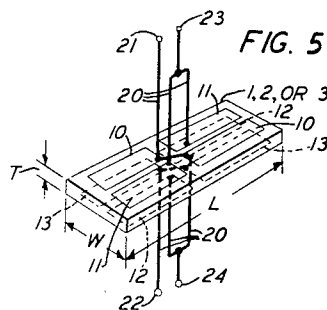
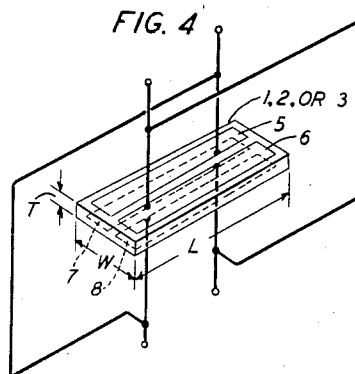
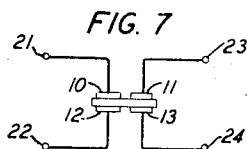
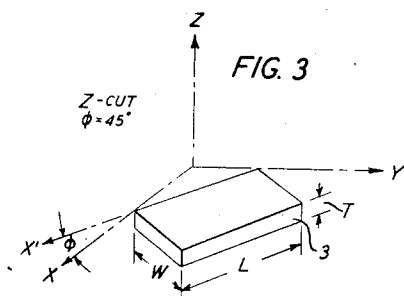
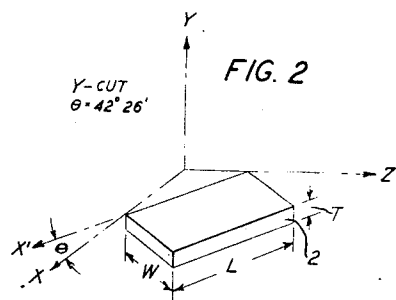
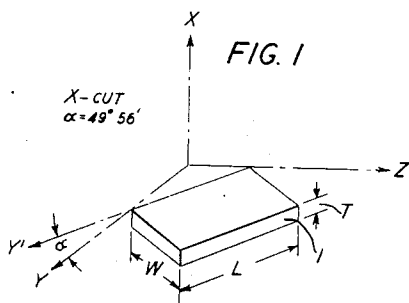
W. P. MASON

2,292,886

ROCHELLE SALT PIEZOELECTRIC CRYSTAL APPARATUS

Filed May 22, 1941

2 Sheets-Sheet 1



INVENTOR
W. P. MASON
BY
W. J. O'Neill
ATTORNEY

Aug. 11, 1942.

W. P. MASON

2,292,886

ROCHELLE SALT PIEZOELECTRIC CRYSTAL APPARATUS

Filed May 22, 1941

2 Sheets-Sheet 2

FIG. 9

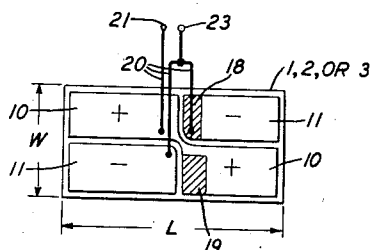


FIG. 10

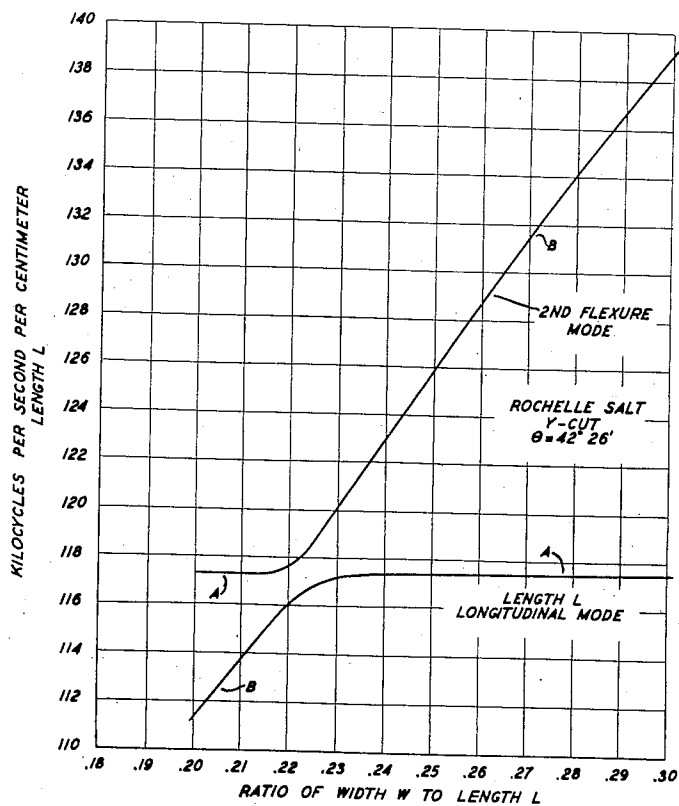
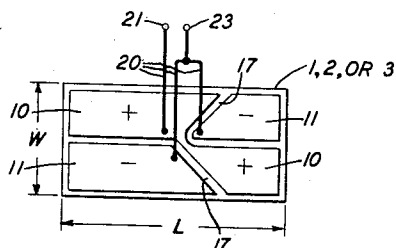


FIG. 11

INVENTOR
W. P. MASON
BY
W. J. O'Neill
ATTORNEY

UNITED STATES PATENT OFFICE

2,292,886

ROCHELLE SALT PIEZOELECTRIC
CRYSTAL APPARATUS

Warren P. Mason, West Orange, N. J., assignor to
Bell Telephone Laboratories, Incorporated, New
York, N. Y., a corporation of New York

Application May 22, 1941, Serial No. 394,571

21 Claims. (Cl. 171-327)

This invention relates to piezoelectric crystal apparatus and particularly to piezoelectric Rochelle salt or sodium potassium tartrate crystal elements suitable for use as circuit elements in electric wave filter systems and oscillator systems, for example.

This application is a continuation in part of my copending application for Rochelle salt piezoelectric crystal apparatus, Serial No. 392,688, filed May 9, 1941 (Case 65). In the present application, the Rochelle salt type crystal elements described may have the same or similar orientation or orientations as those described in the earlier filed application mentioned, but herein they are adapted for different vibrational modes of motion including the fundamental or first flexural mode of motion, and overtones of the fundamental flexural mode of motion such as the second flexural mode of motion.

One of the objects of this invention is to provide Rochelle salt type piezoelectric crystal elements having one or more useful low frequency flexural modes of motion that may be utilized alone, without interference or coupling with other modes of motion therein.

Another object of this invention is to provide Rochelle salt crystal elements having a plurality of simultaneously useful and independently controlled frequencies that may be substantially uncoupled with each other and free from spurious or undesired frequencies.

Another object of this invention is to provide Rochelle salt crystal elements of such an orientation that the longitudinal length and width modes of motion thereof may be uncoupled to the face flexural modes of motion thereof.

Another object of this invention is to reduce the number and the cost of crystals used in electric wave filter systems and other wave transmission networks, and to take advantage of the high piezoelectric activity and low cost of Rochelle salt.

Rochelle salt piezoelectric crystal elements generally may be excited in many different modes of motion such as, for example, extensional or longitudinal modes, flexural modes, and shear modes of motion. When crystal elements are to be applied to filter systems for example, it is generally desirable to have all of the undesired or extraneous modes of motion therein uncoupled with and considerably higher, or lower, in frequency than the desired main mode or modes of motion of the crystal element since otherwise the extraneous resonance frequencies therein may introduce undesirable frequencies or pass bands in the filter characteristic. Accordingly, it is often desirable in filter systems and elsewhere that the desired main mode or modes of motion of a crystal element be substantially uncoupled to other modes of motion and independently con-

trolled in order that such mode or modes of motion may be given any desired frequency values to obtain prescribed frequency characteristics.

In accordance with this invention, wave filters and other systems may comprise as a component element thereof, a piezoelectric crystal element of Rochelle salt which may be adapted to vibrate simultaneously in a plurality of substantially uncoupled modes of motion in order to provide either separately or simultaneously a plurality of useful effective resonances which may be independently controlled and placed at predetermined frequencies of the same or different values for use in an electric wave filter or elsewhere.

The crystal element may be a Rochelle salt crystal plate of suitable orientation with respect to the X, Y and Z axes thereof, and of suitable dimensional proportions, and provided with a suitable electrode arrangement and connections for separately driving either, or simultaneously driving both, of two uncoupled modes of motion therein and independently controlling the relative strengths of such resonances.

In particular embodiments, the orientation of the crystal element may be that of either an X-cut or a Y-cut or a Z-cut Rochelle salt crystal plate rotated in effect a selected angle in degrees about its X-axis or Y-axis or Z-axis thickness dimension, respectively. The width dimension of the major surfaces and the length dimension thereof may be made of selected values in order to obtain from the crystal element separately or simultaneously either or both of two useful independently controlled resonant frequencies resulting from two independently controlled face modes of motion, one particular set of which is described herein as the fundamental length or longest axis dimension longitudinal or extensional mode of motion and the other as the second harmonic length axis flexural mode of motion. Both the length longitudinal mode of motion and the flexural mode of motion are in the major plane of the crystal element and due to the crystal orientation have substantially no mechanical coupling with each other.

Such Rochelle salt crystal elements when provided with suitable electrodes may be connected into a filter circuit in such a way that one of the resonances of each crystal element is effective in the line branch and another of the resonances is, effective in the diagonal branch of the lattice portion of the equivalent network thereof, in order to obtain filter circuits using a single crystal which are electrically equivalent to circuits requiring two crystals thereby reducing the number and cost of crystals therein. Such Rochelle salt crystal elements may be utilized for example in either balanced or unbalanced filter structures such as those disclosed, for example, in W. P. Mason U. S. Patent 2,271,870, granted February

3, 1942, on my application Serial No. 303,757, filed November 10, 1939 (Case 58), and in the H. J. McSkimin and R. A. Sykes U. S. Patent 2,277,709, granted March 31, 1942, on application Serial No. 369,694, filed December 12, 1940 (Case 1-8).

By using two of such doubly resonant Rochelle salt crystal elements with or without temperature control, filter systems may be inexpensively constructed of frequencies as low as 16 kilocycles per second. With the relatively lower values of frequency, there is less absolute shift in the pass band due to temperature change and therefore temperature control often is not needed at the lower values of frequency, and electric wave filters may be made using Rochelle salt crystal elements as the vibrating elements thereof, with characteristics nearly as good as those obtained when using quartz crystal elements. However, due to the relatively high temperature frequency coefficient of Rochelle salt, it is often desirable to have temperature control to about 1 or 2° centigrade to hold the pass bands to their required frequency. By the use of both wet and dry Rochelle salt placed in the same enclosing crystal container, the Rochelle salt vibratory crystals may be preserved indefinitely, or for a long period of time without change in the characteristics thereof.

For a clearer understanding of the nature of this invention and the additional advantages, features and objects thereof, reference is made to the following description taken in connection with the accompanying drawings, in which like reference characters represent like or similar parts and in which:

Figs. 1, 2 and 3 are respectively perspective views of X-cut, Y-cut and Z-cut type piezoelectric Rochelle salt type crystal elements in accordance with this invention, and illustrate particularly the orientation thereof with respect to the X, Y and Z axes of the Rochelle salt crystal material from which the crystal elements may be cut;

Figs. 4 to 10 are views illustrating types of electrodes and connections therewith which may be utilized with any of the Rochelle salt crystal elements of Fig. 1, 2 or 3 to drive the crystal element separately in either, or simultaneously in both, of two independently controlled modes of motion fundamental and harmonic, in order to obtain the desired resonance frequency or frequencies.

Fig. 4 is a perspective view of an electrode arrangement that may be used to drive the piezoelectric crystal element of Fig. 1, 2 or 3 in the fundamental flexural mode of motion bending the length or longest dimension thereof in the direction of the width dimension W;

Figs. 5 and 6 are perspective views of electrode arrangements that may be used to drive the crystal element of Fig. 1, 2 or 3 in the second harmonic flexural mode of motion and in the fundamental longitudinal length mode of motion, separately or simultaneously;

Fig. 7 is a schematic diagram illustrating an example of balanced filter connections that may be used with the crystal element electrodes of Fig. 5;

Fig. 8 is a schematic diagram illustrating an example of unbalanced filter connections that may be used with the electrodes of the crystal element of Fig. 6;

Fig. 9 is a top major face view of the electrode crystal element of Figs. 5 and 6, and illus-

trates particularly a divided electrode adjustment arrangement for reducing the piezoelectric drive on the longitudinal length L mode of motion, without appreciably changing the piezoelectric drive on the second flexural mode of motion of a doubly resonant crystal element;

Fig. 10 is a top major face view similar to that shown in Fig. 9 but illustrating particularly an oblique arrangement of the divided electrodes for reducing the piezoelectric drive on the second flexural mode of motion, without appreciably changing the drive on the longitudinal length L mode of motion of a doubly resonant crystal element;

Fig. 11 is a graph illustrating the frequency-dimension constants of the fundamental length L longitudinal mode of motion and the second flexural mode of motion in the Y-cut type Rochelle salt crystal element as illustrated in Fig. 2 having a dimensional ratio of width to length in the region of 0.2.

This specification follows the conventional terminology as applied to crystalline Rochelle salt, which employs three orthogonal or mutually perpendicular *a*, *b* and *c* axes or X, Y and Z axes, respectively, as shown in the drawings, to designate an electric axis, a mechanical axis and an optic axis, respectively, of piezoelectric Rochelle salt or sodium potassium tartrate crystal material, and which employs three orthogonal axes X', Y' and Z' to designate the directions of axes of a piezoelectric body angularly oriented with respect to such X, Y and Z axes thereof. Where the orientation is obtained in effect by a single rotation of the Rochelle salt crystal element, the rotation being in effect substantially about the thickness dimension axis X, Y or Z of the piezoelectric body as illustrated in Figs. 1, 2 and 3, respectively, the orientation angles, respectively, α =substantially 49° 56', θ =42° 26', and ϕ =0° or 90° between the X and Y axes designate in degrees the effective angular position of the length axis dimension L of the crystal plate as measured from one of the other two X, Y and Z axes. The relation of the X, Y and Z axes to the outer faces of a grown Rochelle salt crystal body are illustrated in W. P. Mason U. S. Patent 2,178,146, dated October 31, 1939. Rochelle salt belongs to the rhombic hemihedral class of crystals and has three orthogonal or mutually perpendicular axes generally designated as the *a*, *b*, and *c* axes or the X, Y and Z axes, respectively.

Referring to the drawings, Figs. 1, 2 and 3 represent perspective views of thin bare piezoelectric Rochelle salt crystal elements 1, 2 and 3 cut from crystalline Rochelle salt free from defects and made into a plate of substantially rectangular parallelepiped shape with its major surfaces having a length or longest dimension L and a width dimension W which is perpendicular to the length dimension L, the thickness or thin dimension T between the major surfaces being perpendicular to the other two dimensions L and W. In accordance with the particular mode or modes of motion selected, the final length dimension L and width dimension W of the Rochelle salt crystal element 1, 2 or 3 of Figs. 1, 2 and 3 may be made of suitable values according to the value of the desired resonant frequency. The width dimension W also may be related to the length dimension L in accordance with the value of the desired flexural mode resonant frequency. The thickness dimension T may be of the order of 1 millimeter or any other

suitable value, for example, to suit the impedance of the circuit in which the crystal element 1, 2 or 3 of Figs. 1, 2 and 3 may be utilized.

As shown in Fig. 1, the length dimension L of the X-cut type crystal element 1 illustrated in Fig. 1 lies along a Y' axis in the plane of the mechanical axis Y and the optic axis Z of the Rochelle salt crystal material from which the element 1 is cut and is inclined at an angle of α degrees with respect to said Y axis, the angle α being preferably one of the values in the region of substantially 49 degrees 56 minutes (49° 56'). The major surfaces and the major plane of the Rochelle salt crystal element 1 of Fig. 1 are disposed parallel or nearly parallel with respect to the plane of the Y and Z axes, the length dimension L and the width dimension W lying along the Y' axis and the Z' axis, respectively, both of which lie in the plane of the Y and Z axes mentioned, the Y' axis and the Z' axis being inclined at the angle α with respect to the Y axis and the optic axis Z, respectively. The axis Y' is accordingly the result of a single rotation of the length dimension L about the X axis α degrees. It will be noted that the crystal element 1 of Fig. 1 is in effect an X-cut Rochelle salt crystal plate rotated α degrees about the X axis.

Fig. 2 is a perspective view of a Y-cut type Rochelle salt piezoelectric crystal element 2 having its longest or length dimension L along the X' axis and inclined at an angle of θ =substantially 42 degrees 26 minutes (42° 26') with respect to the X axis, the major surfaces of the crystal element 2 being parallel or nearly parallel to the plane of the Z axis and the X axis.

Fig. 3 represents a Z-cut type Rochelle salt crystal element 3 having its length or longest dimension L along the X' axis and inclined at an angle ϕ which may be any angle intermediate the X and Y axes, the major surfaces of the crystal element 3 being parallel or nearly parallel to the plane of the X and Y axes.

The orientations illustrated in Figs. 1, 2 and 3 accordingly represent X-cut, Y-cut and Z-cut type Rochelle salt piezoelectric crystal elements 1, 2 and 3, respectively, which may be adapted for independently controlled longitudinal length L mode vibrations and flexural face mode vibrations. Such low frequency or face mode vibrations may be utilized either alone or simultaneously, according to the arrangement of the electrodes and connections that are used therewith, and the dimension-frequency constants that are selected therefor.

Suitable conductive electrodes, such as the crystal electrodes of Figs. 4, 5 or 6, for example, may be placed on or adjacent to or formed integral with the opposite major surfaces of the crystal plate 1, 2 or 3 of Figs. 1, 2 and 3, in order to apply electric field excitation to the Rochelle salt plates 1, 2 or 3 which may be vibrated alone or simultaneously in the desired length L fundamental longitudinal mode of motion and/or the fundamental or harmonic flexural mode of motion at independently controlled resonant response frequencies which depend upon different sets of dimensions involving the width dimension W and the length dimension L of the crystal element 1, 2 or 3. The fundamental longitudinal mode frequency of the X-cut, Y-cut, and Z-cut type crystal elements 1, 2 and 3, respectively, of Figs. 1, 2 and 3 has a value roughly of about 160, 118, and 186, respectively, kilocycles per second per centimeter of the length dimension L. For the 49° 56' X-cut

type element 1 mentioned, the frequency is a function of temperature but averages around the value of 160 mentioned for its frequency constant. The value of the flexural mode frequency depends upon the ratio and values of the width dimension W and the length dimension L, and the numerical order thereof such as the fundamental flexural mode frequency or the second, third, etc., overtone flexural mode frequency.

The crystal electrodes and interconnections therebetween, if any, when formed integral with the major surfaces of any of the crystal elements of Figs. 1, 2 or 3 may consist of thin coatings of colloidal carbon, silver, gold, platinum, aluminum or other suitable metal or metals deposited upon the surfaces by painting, spraying, or evaporation in vacuum for example, or by other suitable process.

When the Rochelle salt crystal plate has an orientation angle of α =substantially 49° 56' with respect to the Y axis as illustrated in Fig. 1, or θ =substantially 42° 26' with respect to the X axis as illustrated in Fig. 2, the length L longitudinal mode of motion is uncoupled to the face shear mode of motion and may be used simultaneously, without coupling to the second flexural mode of motion bending the length dimensional L.

To obtain the flexural and longitudinal modes of motion, it is necessary that the crystal element have a piezoelectric constant which will generate longitudinal motion along the length dimension L.

In the case of the X-cut type Rochelle salt crystal element 1 illustrated in Fig. 1, the requirement of a suitable piezoelectric constant d'_{12} may be met when the length dimension L is inclined at any suitable angle α between the Y and Z axes of the YZ plane, the YZ plane being parallel or nearly parallel to the major plane and the major surfaces of the Rochelle salt crystal element 1 of Fig. 1.

As indicated in my hereinbefore-mentioned copending application Serial No. 392,688, filed May 9, 1941 (Case 65), referring to Equations 31 and 32 of my paper "A dynamic measurement of the elastic, electric and piezoelectric constants of Rochelle salt" published April 15, 1939, in Physical Review, vol. 55, page 775, the piezoelectric constant d'_{12} involved in the longitudinal or extensional mode vibrations along the length dimension L of the X-cut Rochelle salt crystal element 1 illustrated in Fig. 1 is equal to:

$$d'_{12} = \frac{d_{14}}{2} \sin 2\alpha \quad (1)$$

The constant d'_{12} reaches its maximum value when the angle α =45° or when the length dimension L of the X-cut type Rochelle salt crystal element 1 of Fig. 1 is inclined 45 degrees with respect to the Y and Z axes thereof, the major surfaces thereof being parallel to the plane of such Y and Z axes.

The angle α may be made of a value of substantially 49° 56' to provide for zero or substantially no coupling between the length L longitudinal mode and the major face shear mode, and thereby avoid a resulting mechanical coupling between the longitudinal length mode of motion along the length axis L and the flexural mode of motion bending the length axis dimension L. Since the flexure mode of motion is coupled only to the face shear mode but not to the longitudinal mode along the length dimension L, it is not necessary to provide for a zero coupling directly be-

tween the flexure mode of motion and the longitudinal mode of motion.

When the length dimension L of the X-cut type Rochelle salt crystal element 1 of Fig. 1 is inclined at an angle of $\alpha=49^\circ 56'$ with respect to the Y axis, the coupling coefficient S'_{24} representing the coupling between the major face shear mode and the length L longitudinal mode of motion becomes zero.

As indicated by Equation 33 of my Physical Review paper referred to, the equation for the value of S'_{24} is:

$$S'_{24} = \sin^2 2\alpha \left[\left(S_{23} + \frac{S_{44}}{2} - S_{22} \right) \cos^2 \alpha + \left(S_{33} - S_{23} - \frac{S_{44}}{2} \right) \sin^2 \alpha \right] \quad (2)$$

where

$$S_{23} = -1.03 \times 10^{-12}$$

$$S_{22} = 3.495 \times 10^{-12}$$

$$S_{33} = 3.34 \times 10^{-12}$$

$$S_{44} = 7.98 \times 10^{-12}$$

α = the angle between the length axis L and the Y axis as illustrated in Fig. 1.

Substituting these values in the above equation 2, the angle α for which S'_{24} becomes zero and vanishes is $\alpha=49^\circ 56'$, α being the angle between the length dimension L and the nearest Y axis as illustrated in Fig. 1.

While the maximum value of piezoelectric constant d'_{12} occurs when the angle $\alpha=45^\circ$, the angle of $\alpha=49^\circ 56'$ is near enough thereto to obtain good values of piezoelectric constant d'_{12} and at the same time obtain the longitudinal mode of motion along the length dimension L and the flexural mode of motion without interference or coupling with each other. Such an X-cut type Rochelle salt crystal element 1 of Fig. 1 also has a strong electromechanical coupling which varies somewhat with temperature change, and is easily driven by suitable electrodes in the longitudinal and flexural modes of motion.

As illustrated in Fig. 2, a Y-cut type Rochelle salt piezoelectric crystal element 2 having its length or longest dimension L inclined at an angle of θ = substantially $42^\circ 26'$ with respect to the X axis also may be used as a singly or doubly resonant crystal element. Such an element has an electro-mechanical coupling that does not vary much with temperature, and with the θ angle equal to substantially $42^\circ 26'$, the length L longitudinal mode of motion and the flexural mode of motion are substantially free from coupling with any other modes therein.

For the Y-cut type Rochelle salt crystal element 2 as illustrated in Fig. 2, the piezoelectric constant controlling the longitudinal mode vibration along the length dimension L is:

$$d'_{21} = -\frac{d_{25}}{2} \sin 2\theta \quad (3)$$

The coupling coefficient or constant S'_{15} for the longitudinal length mode is:

$$S'_{15} = \sin^2 2\theta \left[\left(S_{11} - S_{13} - \frac{S_{55}}{2} \right) \cos^2 \theta + \left(S_{13} - S_{33} - \frac{S_{55}}{2} \right) \sin^2 \theta \right] \quad (4)$$

where

θ = the angle between the length axis L along the X' axis and the X crystallographic axis.

$$S_{11} = 5.18 \times 10^{-12}$$

$$S_{33} = 3.34 \times 10^{-12}$$

$$S_{13} = -2.11 \times 10^{-12}$$

$$S_{55} = 32.8 \times 10^{-12}$$

Substituting these values in Equation 4, the angle θ for which S'_{15} becomes zero and vanishes is $\theta=42^\circ 26'$ as illustrated in Fig. 2.

When the angle θ = substantially $42^\circ 26'$ as illustrated in Fig. 2, there is substantially no coupling between the longitudinal mode of motion along the length axis dimension L and either the flexural mode of motion bending the length axis dimension L or the major face shear mode of motion in the XZ plane of the Rochelle salt crystal element 2 of Fig. 2. This type of Y-cut doubly resonant crystal element 2 using the longitudinal length mode of motion and the second flexural mode of motion has an advantage over some other forms of Rochelle salt doubly resonant crystal elements in that the dimensions thereof are no larger and may be smaller than those of singly resonant crystals, with a resultant saving in material.

A Z-cut type Rochelle salt crystal element 3 as illustrated in Fig. 3 having its major surfaces perpendicular to the Z axis also may be used to generate flexural mode and longitudinal mode vibrations useful for a doubly resonant crystal element. However, in such a Z-cut type crystal element 3, the zero coupling angles involved in these two modes of motion cannot be used since the piezoelectric constants d'_{32} and d'_{31} controlling these modes of motion are both of zero value at the zero coupling angles of $\varphi=0^\circ$ or $\varphi=90^\circ$, the angle φ being measured between the length or longest dimension L of the crystal element 3 and the nearest X axis thereof as illustrated in Fig. 3.

For the Z-cut type Rochelle salt crystal element 3 of Fig. 3, the piezoelectric constants d'_{32} and d'_{31} are:

$$d'_{32} = -\frac{d_{36}}{2} \sin 2\varphi; \quad d'_{31} = \frac{d_{36}}{2} \sin 2\varphi \quad (5)$$

and the coupling constant S'_{26} for the length longitudinal mode and the face shear mode is:

$$S'_{26} = \sin^2 \varphi \left[\left(S_{22} - S_{12} - \frac{S_{66}}{2} \right) \cos^2 \varphi + \left(S_{12} - S_{11} + \frac{S_{66}}{2} \right) \sin^2 \varphi \right] \quad (6)$$

where

φ is the angle between the length axis dimension L along the X' axis, and the X crystallographic axis,

$$1 = 5.18 \times 10^{-12}$$

$$2 = -1.53 \times 10^{-12}$$

$$2 = 3.495 \times 10^{-12}$$

$$6 = 10.08 \times 10^{-12}$$

Inserting these values in Equation 6, the S'_{26} coupling constant vanishes only at $\varphi=0^\circ$ and $\varphi=90^\circ$. Since the piezoelectric constants d'_{32} and d'_{31} are both zero at both of these angles, the Z-cut type Rochelle salt crystal element having its length or longest dimension L disposed or inclined at an angle of $\varphi=0$ or 90° with respect to its major plane X axis is not useful at these angles as a doubly resonant crystal element involving the length longitudinal mode motion and the flexural mode of motion. However, such a Z-cut type Rochelle salt crystal element 3 may be utilized at φ angles other than those of 0 or 90° and particularly at the angle of $\varphi=45^\circ$ for a single longitudinal or flexural mode of motion.

The principal modes of interest that are particularly considered herein in connection with the crystal orientations illustrated in Fig. 1, 2 or 3

are the length longitudinal mode of motion along the L axis, and the flexural mode of motion, fundamental and harmonic, the latter bending the length axis L of the crystal element 1, 2 or 3 illustrated in Figs. 1, 2 and 3. The fundamental longitudinal length L mode of motion operates to alternately extend and shorten the length dimension L of the crystal element about a nodal line which extends across the center line width dimension W of the crystal element 1, 2 or 3. The second flexural mode of motion operates to alternately flex or bend the length dimension L of the crystal element about nodes located on the center line length dimension L of the crystal element 1, 2 or 3.

Since the nodes involved in the second flexural mode of motion are located on or near the center of the nodal line involved in the fundamental length longitudinal mode of motion, the crystal element 1, 2 or 3 of Figs. 1, 2 and 3 may be mounted there at or near the nodal point or points without damping or interfering with the simultaneous operation of either the second flexural vibration or the length L longitudinal mode vibration. In the case of the fundamental or first flexural mode vibration, the two nodal point regions on each of the major surfaces would be located on the center line length dimension L of the crystal element at points spaced about 0.224 of the length dimension L from each end thereof. Accordingly, at these nodal points the crystal element of Fig. 1, 2 or 3 may be mounted by rigidly clamping it there between two pairs of oppositely disposed clamping projections of small contact area which may be there placed or inserted in very small indentations or depressions provided at the four nodal points of the crystal element. Such small depressions may be cut in the major surfaces of the crystal element at the nodal points thereof and may have a depth of about 0.05 millimeter and a diameter of about 0.4 millimeter as measured on the major surfaces of the crystal element.

The values of the resonance frequencies associated with the length L longitudinal mode of motion and with the flexural mode of motion in the crystal elements 1, 2 and 3 having the orientations illustrated in Figs. 1, 2 and 3 may be controlled by the value of the length dimension L and by the value of the dimensional ratio of width W with respect to length L. The fundamental of the length L longitudinal mode frequency for such crystal elements 1, 2 and 3 respectively has a frequency-dimension constant of about 160, 118, and 186, respectively, kilocycles per second per centimeter of the length dimension L, independent of the dimensional ratio of the width W with respect to the length L, as illustrated by curve A of Fig. 11 for the Y-cut type crystal element 2. The second overtone of the length L flexural mode of motion of the crystal element 1, 2 or 3 has a frequency-dimension constant of the same value, namely about 160, 118, or 186, respectively, kilocycles per second per centimeter of the length dimension L only when the value of the selected dimensional ratio of width W with respect to length L is of a proper value for the crystal elements 1 and 2, respectively, is in the region of about 0.24 and 0.22 respectively, the latter being as illustrated by the curve B of Fig. 11. The frequencies of these two modes of motion, namely that of the second flexural mode of motion, and that of the fundamental longitudinal length L mode of motion, approach each other when the ratio of the

width dimension W with respect to the length dimension L is in the region from about 0.2 to 0.25 and in this region of special interest, the resonances of these two modes of motion are substantially uncoupled, or only very loosely coupled, when the α angle of the X-cut Rochelle salt crystal element 1 of Fig. 1 has a value of substantially $49^\circ 56'$ as illustrated in Fig. 1, or the θ angle of the Y-cut type crystal element 2 of Fig. 2 has a value of substantially $42^\circ 26'$.

Accordingly, when the dimensional ratio of the width W with respect to the length L is in the region of roughly 0.2 and the orientation of the X-cut or Y-cut crystal element 1 or 2 is that of Fig. 1 or 2, the frequencies of the second flexural and of the fundamental length L longitudinal modes of vibration may be placed close together and yet be sufficiently uncoupled to provide simultaneously two independently controlled frequencies from the same Rochelle salt crystal element, which may be usefully employed in a filter system for example, to give conveniently frequencies of the order of 100 to 200 kilocycles per second, for example, within a range of frequencies from 50 or less to 500 or more kilocycles per second.

The fundamental frequency, expressed in kilocycles per second, of the longitudinal length L mode vibration in the X-cut, Y-cut, and Z-cut type crystal elements 1, 2 and 3, respectively, is given by the respective relations:

$$f = \frac{160}{L} (\text{X-cut}); f = \frac{118}{L} (\text{Y-cut}); f = \frac{186}{L} (\text{Z-cut})$$

where L is the length value of the length dimension L of the crystal element 1, 2 or 3 of Fig. 1, 2 or 3, expressed in centimeters.

The frequency expressed in kilocycles per second of the second flexural mode vibration is given by the same relations only when the ratio of the width dimension W with respect to the length dimension L of the crystal plate is of a suitable value in the region of roughly from 0.2 to 0.25.

As an example, the graph of Fig. 11 illustrates the measured resonance frequencies associated with the fundamental longitudinal length L mode of motion (curve A) and also associated with the second flexural mode of motion (curve B) in a Rochelle salt crystal element 2 having the θ angle of substantially $42^\circ 26'$ as illustrated in Fig. 2. The fundamental length L longitudinal mode frequency is represented by the two sections of the horizontal line curve A and has a frequency-dimension constant of about 118 kilocycles per second per centimeter of length dimension L which is independent of the dimensional ratio of the width W with respect to the length L. The second overtone flexural mode frequency is represented by the two sections of the oblique line curve B and has a frequency-dimension constant of from 110 to 140 kilocycles per second per centimeter of length dimension L dependent upon the value of the selected dimensional ratio of the width W with respect to the length L within the range from about 0.2 to 0.3. As shown by the curves A and B of Fig. 11, the frequency of the second flexural mode of motion and that of the fundamental longitudinal length L mode of motion approach each other when the ratio of the width dimension W with respect to the length dimension L is in the region of about 0.22, and they are substantially uncoupled when the θ angle of the Y-cut type Rochelle salt crystal element 2 of Fig. 2 has a value of substantially $42^\circ 26'$.

While the curves A and B of Fig. 11 apply particularly to a Y-cut type Rochelle salt crystal element 2 as illustrated in Fig. 2, the curve A of Fig. 11 also applies to the fundamental longitudinal length L mode frequency in the X-cut and in the Z-cut type Rochelle salt crystal elements 1 and 3 illustrated in Figs. 1 and 3, respectively, except that the frequencies are around 160 and 186 kilocycles per second, respectively, for crystals having a one centimeter length dimension L.

The curves for the second flexure mode frequency of the X-cut type Rochelle salt crystal element 1 of Fig. 1 and also for the Z-cut type Rochelle salt crystal element 3 of Fig. 3 are similar in form to the second flexure mode curve B of Fig. 11 for the Y-cut type crystal element 2 of Fig. 2 but are somewhat higher in frequency than the second flexure mode frequency of the Y-cut type crystal element 2 of Fig. 2 for a given dimensional ratio of the width W with respect to the length L, and approach the frequency of the longitudinal length L mode frequency, represented by the curve A of Fig. 11, at a ratio of the width dimension W with respect to the length dimension L in the region of about 0.20 to 0.25 for the X-cut type crystal element 1 of Fig. 1, and a ratio of about 0.20 to 0.25 for the Z-cut type crystal element 3 of Fig. 3, the frequency-dimension constants for the second flexural mode of motion in the X-cut type Rochelle salt crystal element of Fig. 1 being roughly from 150 to 171 kilocycles per second per centimeter of the length dimension L within a range of dimensional ratios of the width W with respect to the length L from about 0.20 to 0.25 and roughly from 0 to 200 kilocycles per second per centimeter of the length dimension L within a range for such dimensional ratios from about 0 to 0.3; and the frequency-dimension constants for the second flexural mode of motion in the $\phi=45$ degrees Z-cut type Rochelle salt crystal element 3 of Fig. 3 being roughly from 175 to 199 kilocycles per second per centimeter of the length dimension L within a range of dimensional ratios of the width W with respect to the length L from about 0.20 to 0.25 and roughly from 0 to 235 kilocycles per second per centimeter of the length dimension L within a range for such dimensional ratios from about 0 to 0.3.

The frequency-dimension constants for the fundamental or first flexural mode of motion are somewhat lower than those given above for the second flexural mode of motion; and in the X-cut, Y-cut, and Z-cut type Rochelle salt crystal elements 1, 2 and 3 of Figs. 1, 2 and 3 are respectively, expressed in kilocycles per second per centimeter of the length dimension L, about from 0 to 80, from 0 to 60, and from 0 to 94, respectively, for dimensional ratios of the width W with respect to the length L in the range about from 0 to 0.3.

Figs. 4, 5 and 6 illustrate forms of electrode arrangements which may be utilized to drive any of the crystal elements 1, 2 or 3 of Fig. 1, 2 or 3. In Fig. 4, the two pairs of opposite electrodes 5, 6, 7 and 8 may be used to drive the crystal element 1, 2 or 3 in the first or fundamental flexural mode of motion which bends or flexes the length dimension L in the direction of the width dimension W about two nodal regions which are located on the center line length dimension L spaced about 0.224 from each end thereof. To drive the crystal element in such first flexural mode of motion, the electrodes 5, 6, 7 and 8 may be connected in circuit relation as illustrated, for

example, in Fig. 4, or as disclosed in Marrison U. S. Patent 1,823,329, dated September 15, 1931, or W. P. Mason U. S. Patent 2,259,317, granted October 14, 1941, on application Serial No. 344,892, filed July 11, 1940 (Case 63).

In Fig. 4, which is a perspective view of the crystal element 1, 2 or 3 of Fig. 1, 2 or 3, the two pairs of opposite electrodes 5, 6, 7 and 8 function to operate the crystal element 1, 2 or 3 of Figs. 1 to 3, in the fundamental or first flexural mode of motion at a frequency which depends upon the dimensional ratio of the width W with respect to the length L, the frequency being a value roughly between 0 and 100 kilocycles per second per centimeter of the length dimension L for ratios of widths W to lengths L between 0 and 0.35. The electrodes 5, 6, 7 and 8 may partially or wholly cover the areas of the major surfaces of the crystal elements 1, 2 or 3 and may be connected in circuit by a conductive member disposed in contact with each of the electrodes 5, 6, 7 and 8 at the nodes thereof. It will be understood that the X-cut and Y-cut type Rochelle salt crystal elements having the orientation angles α =substantially $49^{\circ} 56'$ or θ =substantially $42^{\circ} 26'$ illustrated in Figs. 1 and 2 respectively, and provided with electrodes of the type illustrated in Fig. 4 may be utilized to obtain a desired fundamental or first flexural mode vibration frequency that is substantially uncoupled with any other mode of motion therein. Where a harmonic or overtone of such desired first flexural mode of motion is used, the overtone flexure mode frequency is likewise substantially free from coupling to other modes of motion. The overtone flexural mode frequency may be of any desired order. The second flexural mode frequency may be obtained by the use of electrode arrangements as illustrated in Figs. 5 to 10.

As shown in Fig. 5, the second flexural mode of motion may be driven by means of four pairs of divided electrodes 10, 11, 12 and 13 which are oppositely placed on both of the major surfaces of the crystal element 1, 2 or 3 of Fig. 1, 2 or 3; and with suitable connections, the fundamental longitudinal length L mode of motion may be driven at the same time by one of the connected sets of electrode platings, with the result that the two useful and independently controlled resonance frequencies of the crystal element 1, 2 or 3 may be made to appear simultaneously. As illustrated in Fig. 5, the Rochelle salt crystal element 1, 2 or 3 of Fig. 1, 2 or 3 may be provided with eight equal-area electrodes 10, 11, 12 and 13, the two interconnected sections of each of the electrodes 10 and 11 being placed on one major surface of the crystal element with centrally located narrow transverse splits or gaps therebetween, and the two interconnected sections of each of the other two electrodes 12 and 13, being oppositely disposed and placed on the opposite major surface of the crystal element and separated with similar narrow and oppositely disposed splits or dividing lines therebetween, one set of the dividing lines extending generally in the direction of the length axis L and the other set extending generally in the direction of the width W axis of the crystal element, according to the value of the angle selected between the direction of the width W axis dividing line and the direction of the length dimension L. The gap or separation between the electrode coatings or platings on each of the major surfaces of the crystal element may be of the order of about 0.3 millimeter, the cen-

ter line of such splits in the platings on the opposite sides of the crystal plate being aligned with respect to each other.

Fig. 7 is a schematic diagram illustrating an example of balanced filter connections which may be used with the electrode arrangement of Fig. 5, in order to obtain a filter system comprising a single Rochelle salt crystal element 1, 2 or 3 having two independently controlled and simultaneously effective resonances which may be placed at desired frequencies, one of which may appear in the line branch of the equivalent lattice and the other in the diagonal branch thereof, as described more fully in connection with Figs. 2 and 3 of the Mason application Serial No. 303,757 referred to hereinbefore.

The balanced circuit of Figs. 5 and 7 may be converted into an unbalanced filter structure by interconnecting all sections of the two electrodes 12 and 13 which are disposed on one of the major surfaces of the crystal element. In this case, the four sections of the electrodes 12 and 13 of Figs. 5 and 7 may be replaced by a single electrode 15 as shown in Fig. 6, and the electroded crystal element of Fig. 6 may be connected in circuit as shown schematically in Fig. 8, and as described more fully in connection with Figs. 6 and 7 of the Mason application Serial No. 303,757 hereinbefore referred to.

It will be noted that in order to drive the electroded crystal element of Fig. 6 in the second flexural mode of motion, one half of the crystal plate is made of opposite polarity to that of the other half, as indicated by the + and - signs in Figs. 9 and 10, and that this may be accomplished by utilizing a crystal element having divided metallic coatings 10 and 11 placed on one of its major surfaces and connected in the form of a T network, for example, as illustrated in Fig. 8. Inductance coils may be added in the usual manner in series or in parallel with the network of Fig. 8 to produce broad-band low or high impedance filters for example. In order that the crystal impedance may appear in both arms of the lattice structure of Fig. 8, one mode is driven when the terminals 21 and 23 are both of the same polarity, and the other mode is driven when these terminals 21 and 23 are of opposite polarity. Since both modes are substantially uncoupled they may produce simultaneously two independently controlled resonances of predetermined frequencies of desired values.

In order to control the relative impedance levels and ratio of capacities of the two desired simultaneously effective crystal resonances, the divided crystal electrodes 10, 11, 12, 13 is associated with one half of the major surface or surfaces of crystal element 1, 2 or 3 of Figs. 5 and 6 may be extended to cover a portion of the other half thereof as illustrated in Fig. 10. This may be done, for example, by adjustment of the angular position of the electrode dividing line 17 obliquely with respect to the length dimension L as illustrated in Fig. 10. The angle may be any desired value over a wide range of angles. This adjustment does not materially affect the impedance of the longitudinal length L mode resonance, but with decreasing values for the 90 degree angle shown in Figs. 5, 6 and 9 will increase the impedance level and cut down or reduce the drive on second flexural mode resonance, without materially affecting the impedance of the length L longitudinal mode resonance. Thus, by changing the angle of inclination of the split

or division line 17 between the electrodes 10 and 11 with respect to the length dimension L of the crystal element illustrated by a 90 degree angle in Figs. 5 and 6 and a 45 degree angle in Fig. 10, the internal capacity associated with the second flexural mode of motion, which is nearly a maximum value when the angle equals 90 degrees as shown in Figs. 5, 6 and 9, may be varied and adjusted to a desired value without changing the internal capacity associated with longitudinal length L mode of motion.

Fig. 9 illustrates a method of reducing the drive on the longitudinal length L mode of motion without appreciably changing the drive on the second flexural mode of motion. In the arrangement of Fig. 9 the transverse division lines 13, 19 between the electrodes 10 and 11 is maintained generally in the direction of the width dimension W of the crystal element, and the relative separation therebetween, as illustrated by the shaded areas 18 and 19 in Fig. 9, may be adjusted to values in accordance with the desired amount of drive on the longitudinal length L mode of motion.

It will be understood that the circuits illustrated in Figs. 7 and 8 represent particular circuits. These and other forms of filter circuits, in which a doubly resonant crystal element may be utilized, are described in the W. P. Mason application Serial No. 303,757 hereinbefore referred to. If desired, mutual inductance may be used between the end coils of the crystal filter to obtain improved attenuation characteristics as described in U. S. Patent 2,198,684, granted April 30, 1940, to R. A. Sykes.

The electroded doubly resonant crystal elements of Figs. 5 to 10 may be mounted in any suitable manner, such as by supporting wires, or by clamping, or otherwise. Where the clamping form of mounting is used, opposite conductive clamping projections may resiliently contact the electroded crystal element at or near its nodal points only in order to support and to establish individual electrical connections therewith.

Alternatively, instead of being mounted by clamping, the electroded crystal plate, 1, 2 or 3 may be mounted and electrically connected by cementing, as by conductive cement such as dental amalgam for example, or by otherwise firmly attaching fine conductive supporting wires 20 directly to a thickened part of the electrodes of the crystal element at or near its nodal points only. Such fine supporting wires 20 secured to the central part of the major surfaces of the electroded crystal element as illustrated in Figs. 5, 6, 9 and 10 may extend horizontally from vertically disposed major surfaces of the crystal element and at their other ends be attached by solder, for example, to vertical conductive wires or rods carried by the press or other part of an evacuated or sealed glass or metal tube. The supporting wires and rods may have one or more bends therein to resiliently absorb mechanical vibrations. Also, bumpers or stops of soft resilient material such as mica may be spaced closely adjacent the edges, ends or other parts of the electroded crystal element in order to limit the bodily displacement thereof when the device is subjected to externally applied mechanical shock. Fig. 8, for example, of A. W. Ziegler U. S. Patent 2,275,122, granted March 3, 1942, on an application Serial No. 338,871, filed June 5, 1940 (Case 3), illustrates a suitable mounting of this type for the crystal element,

the horizontal supporting wires being spaced along the vertical rods to suit the nodal point region of the electroded crystal element. It will be understood that any holder which will give stability, substantial freedom from spurious frequencies and a relatively high Q or reactance-resistance ratio for the crystal element may be utilized for mounting the crystal element.

Fig. 11, as already mentioned hereinbefore, is a graph illustrating the values of the two measured second flexural and first longitudinal mode resonant frequencies in a Y-cut type Rochelle salt crystal element as illustrated in Fig. 2 with a θ angle of about $42^\circ 30'$ between the length dimension L and the X axis. The two resonant frequencies represented by the curve A and B of Fig. 11 are given as a function of the ratio of width W to length L dimensions, and at the dimensional ratio of about 0.22 come within a kilocycle per second of each other, a separation which is small enough for use in filters of 60 kilocycles per second or less. In that region with the frequency-dimension constant of about 118 kilocycles per second per centimeter of the length dimension L, the crystal element even at the relatively lower frequencies may be comparatively small in size.

For such a fully plated Y-cut type crystal element 2 connected as shown in Fig. 5, the ratio of capacities of the longitudinal length L mode of motion illustrated by the curve A of Fig. 11 is of the order of 15. When the second flexural mode frequency represented by the curve B of Fig. 11 is spaced therefrom by several kilocycles per second, the ratio of capacities is of the order of 30. When the two frequencies are closer together, the ratio of capacities thereof may differ somewhat from the values given of 15 and 30 due probably to a residual coupling.

As an example, some general considerations are given as follows for applying two double resonant Rochelle salt crystal elements of the Y-cut type illustrated in Fig. 2 for example, to form a filter section of the type shown in Fig. 24 of the W. P. Mason application Serial No. 303,757 hereinbefore referred to. From the general filter equations given in a former paper entitled "Resistance compensated band pass crystal filters for unbalanced circuits," W. P. Mason, Bell System Technical Journal, October 1937, Table I, it can be shown that for a single crystal element in each arm of a lattice, the ratio of the static capacity of the crystal element and associated condensers to the dynamic capacity of that crystal element is, for a filter having a given attenuation characteristic, inversely proportional to the square of the band width, or

$$r = K_1 \frac{f_M^2}{(f_2 - f_1)^2} \quad (7)$$

The value given by Equation 7 determines the lowest value of mean frequency f_M which can be used for a crystal element with a given value of r and a given band width $(f_2 - f_1)$.

Similar, it can be shown that the absolute value of the dynamic capacity C_D of the single crystal element varies with frequency and band width, as shown by:

$$C_D = \frac{K_2 (f_2 - f_1)^3}{Z_0 f_M^4} \quad (8)$$

The same relations as those given in Equations 7 and 8 hold for two crystals in each arm of a lattice corresponding to the circuit of Fig. 24 of

the W. P. Mason application Serial No. 303,757 referred to.

Based on calculations, the approximate evaluation of the constant K_1 in Equation 7 is 2.2 and that of K_2 in Equation 8 is 0.068. Using this value of K_1 in Equation 7 and assuming a ratio of capacities r of 15 to 1 in one mode of motion and 45 to 1 in the other mode of motion, the band width ratio can be as great as

$$\frac{f_2 - f_1}{f_M} = .442$$

Hence, the ratio of capacities r existing in the Y-cut Rochelle salt crystal element 2 of Figs. 2, 5 and 6, will allow band widths suitable for filters of as low as 8 kilocycles per second.

The lower frequency limit will be determined by the size of crystal element and the limiting value of the impedance of the filter. Since the frequency-dimension constant of the Y-cut type Rochelle salt crystal element 2 is about 118 kilocycles per second per centimeter of length dimension L, a length L of about 7.0 centimeters could be used to build a 16 to 20 kilocycle per second filter channel.

The impedance range for filters of this particular type may be calculated from Equation 8. For a 16 to 20 kilocycle per second filter for example, the lowest frequency Rochelle salt crystal element 2 of the Y-cut type illustrated in Figs. 2, 5 and 6 would have a length L of about 7.1 centimeters and a width W of about 1.63 centimeters. With a thickness T of 1 millimeter, the static capacity of the complete electrode crystal element would be about 128 micro-microfarads, one half of which would appear across each arm of the filter. For this example, assuming a ratio of capacities r value of 15 for one mode of motion and of 45 for the other mode of motion, the value of C_D in Equation 8 would then be about 5.68 micro-microfarads. Inserting this value in Equation 8, and taking K_2 as .068, the mean frequency f_M as 18.1 kilocycles per second, and the band width $f_2 - f_1$ as 3,655 kilocycles per second, the impedance Z_0 is about 5,400 ohms. If the thickness dimension T is kept constant at 1 millimeter, the value of C_D will vary inversely as f_M^2 and hence the impedance for a given band width will vary inversely as the square of the mean frequency f_M . At another frequency, as 106 kilocycles per second for example, the impedance Z_0 would be of the order of 150 ohms. The variation in impedance over a frequency range may be made less by varying the thickness dimension T of the crystal element used. Where a number of filters are used together, the transformer connecting them to the line may have a number of taps to match the impedance of the various crystal filters placed in parallel with each other.

Although this invention has been described and illustrated in relation to specific arrangements, it is to be understood that it is capable of application in other organizations and is therefore not to be limited to the particular embodiments disclosed, but only by the scope of the appended claims and the state of the prior art.

What is claimed is:

1. An X-cut type low frequency flexural mode piezoelectric Rochelle salt type crystal element having its substantially rectangular major surfaces substantially parallel to the plane of a Y axis and the Z axis, the major axis length dimension of said major surfaces being inclined at an angle of substantially $49^\circ 56'$ with respect to said Y axis, the dimensional ratio of the width dimen-

sion of said major surfaces with respect to said length dimension thereof being one of the values between substantially 0.05 and 0.5, said length dimension and said dimension ratio being related values in accordance with the value of said frequency.

2. A Y-cut type low frequency flexural mode piezoelectric Rochelle salt type crystal element having its substantially rectangular major surfaces substantially parallel to the plane of an X axis and the Z axis, the major axis length dimension of said major surfaces being inclined at an angle of substantially $42^{\circ} 26'$ with respect to said X axis, the dimensional ratio of the width dimension of said major surfaces with respect to said length dimension thereof being one of the values between substantially 0.05 and 0.5, said length dimension and said dimensional ratio being related values in accordance with the value of said frequency.

3. A Z-cut type low frequency flexural mode piezoelectric Rochelle salt type crystal element having its substantially rectangular major surfaces substantially parallel to the plane of an X axis and the Y axis, the major axis length dimension of said major surfaces being inclined at an angle of substantially 45 degrees with respect to said X axis, the dimensional ratio of the width dimension of said major surfaces with respect to said length dimension thereof being one of the values between substantially 0.05 and 0.5, said length dimension and said dimensional ratio being related values in accordance with the value of said frequency.

4. A piezoelectric Rochelle salt type crystal element having its substantially rectangular major surfaces substantially parallel to the plane of two of the three mutually perpendicular X, Y and Z axes thereof, the major axis length dimension of said major surfaces being in said plane and intermediate said two of said three X, Y and Z axes, the dimensional ratio of the width dimension of said major surfaces with respect to said length dimension thereof being one of the values between substantially 0.05 and 0.5, and means including a plurality of sets of functionally independent electrodes adjacent said major surfaces for operating said element simultaneously at a plurality of independently controlled frequencies dependent upon different sets of said major surface dimensions, one of said frequencies being dependent upon the second flexural mode vibration controlled by said width and length dimensions.

5. A piezoelectric Rochelle salt type crystal element having its substantially rectangular major surfaces substantially parallel to the plane of two of the three mutually perpendicular X, Y and Z axes thereof, the major axis length dimension of said major surfaces being in said plane and intermediate said two of said three X, Y and Z axes, the dimensional ratio of the width dimension of said major surfaces with respect to said length dimension thereof being one of the values substantially in the region from 0.05 to 0.3, and means including a plurality of sets of functionally independent electrodes adjacent said major surfaces for operating said element simultaneously at a plurality of independently controlled frequencies dependent upon said major surface dimensions, one of said frequencies being dependent upon the fundamental of the longitudinal or extensional mode vibration along said length dimension, and another of said frequencies being dependent upon the second

harmonic of the flexural mode vibration controlled by said length and width dimensions.

6. An X-cut type piezoelectric Rochelle salt type crystal element adapted to vibrate simultaneously at desired independently controlled longitudinal mode and flexural mode frequencies respectively, said longitudinal mode frequency being dependent mainly upon the length dimension only and said flexural mode frequency being dependent upon the width and the length dimensions of its substantially rectangular major surfaces, said length dimension being substantially in the plane of a Y axis and the Z axis and inclined at an angle of substantially $49^{\circ} 56'$ with respect to said Y axis, said major surfaces being substantially parallel with respect to said YZ plane, the ratio of said width dimension of said major surfaces with respect to said length dimension thereof being one of the values within the region substantially from 0.15 to 0.3, said length dimension expressed in centimeters being a value of substantially 160 divided by the value of said longitudinal mode frequency expressed in kilocycles per second.

7. A Y-cut type piezoelectric Rochelle salt type crystal element adapted to vibrate simultaneously at desired independently controlled longitudinal mode and flexural mode frequencies respectively, said longitudinal mode frequency being dependent mainly upon the length dimension only and said flexural mode frequency being dependent upon the width and length dimensions of its substantially rectangular major surfaces, said length dimension being substantially in the plane of an X axis and the Z axis and inclined at an angle of substantially $42^{\circ} 26'$ with respect to said X axis, said major surfaces being substantially parallel with respect to said XZ plane, the ratio of said width dimension of said major surfaces with respect to said length dimension thereof being one of the values within the region substantially from 0.15 to 0.3, said length dimension expressed in centimeters being a value of substantially 118 divided by the value of said longitudinal mode frequency expressed in kilocycles per second.

8. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said length and width dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of a Y axis and a Z axis thereof, and said length dimension being inclined at an angle of substantially $49^{\circ} 56'$ with respect to said Y axis to obtain said desired flexural mode frequency substantially free from coupling with other modes of motion in said crystal element.

9. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said length and width dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of a Y axis and a Z axis thereof, and said length dimension being inclined at an angle of substantially $49^{\circ} 56'$ with respect

to said Y axis to obtain said desired flexural mode frequency substantially free from coupling with other modes of motion in said crystal element, said desired flexural mode frequency being a fundamental or first flexural mode frequency, said length dimension expressed in centimeters being equal to substantially one of the values from 0 to 80 divided by said desired frequency expressed in kilocycles per second, and the ratio of said width dimension with respect to said length dimension being one of the values from 0 to 0.3.

10. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said length and width dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of a Y axis and a Z axis thereof, and said length dimension being inclined at an angle of substantially 49° to 56° with respect to said Y axis to obtain said desired flexural mode frequency substantially free from coupling with other modes of motion in said crystal element, said desired flexural mode frequency being a second overtone flexural mode frequency, said length dimension expressed in centimeters being equal to substantially one of the values from 0 to 200 divided by said desired frequency expressed in kilocycles per second, and the ratio of said width dimension with respect to said length dimension being one of the values from 0 to 0.3.

11. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said length and width dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of an X axis and a Z axis thereof, and said length dimension being inclined at an angle of substantially 42° 26' with respect to said X axis to obtain said desired flexural mode frequency substantially free from coupling with other modes of motion in said crystal element.

12. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said length and width dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of an X axis and a Z axis thereof, and said length dimension being inclined at an angle of substantially 42° 26' with respect to said X axis to obtain said desired flexural mode frequency substantially free from coupling with other modes of motion in said crystal element, said desired flexural mode frequency being a fundamental or first flexural mode frequency, said length dimension expressed in centimeters being equal to substantially one of the values from 0 to 60 divided by said desired frequency expressed in kilocycles per second, and the ratio

of said width dimension with respect to said length dimension being one of the values substantially from 0 to 0.3.

13. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said length and width dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of an X axis and a Z axis thereof, and said length dimension being inclined at an angle of substantially 42° 26' with respect to said X axis to obtain said desired flexural mode frequency substantially free from coupling with other modes of motion in said crystal element, said desired flexural mode frequency being a second overtone flexural mode frequency, said length dimension expressed in centimeters being equal to substantially one of the values from 110 to 140 divided by said desired frequency expressed in kilocycles per second, and the ratio of said width dimension with respect to said length dimension being one of the values substantially from 0.2 to 0.3 substantially as given by the curve B of Fig. 11.

14. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said width and length dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of an X axis and a Y axis thereof, and said length dimension being inclined at an angle of substantially 45 degrees with respect to said X axis, said desired flexural mode frequency being a fundamental or first flexural mode frequency, said length dimension expressed in centimeters being equal to substantially one of the values from 0 to 94 divided by said desired frequency expressed in kilocycles per second, and the ratio of said width dimension with respect to said length dimension being one of the values substantially from 0 to 0.3.

15. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said width and length dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of a Y axis and an X axis thereof, said length dimension being inclined at an angle of substantially 45 degrees with respect to said X axis and said Y axis, said desired flexural mode frequency being a second overtone of the fundamental flexural mode frequency, said length dimension expressed in centimeters being equal to substantially one of the values from 0 to 235 divided by said desired frequency expressed in kilocycles per second, and the ratio of said width dimension with respect to said length dimension being one of the values substantially from 0 to 0.3.

16. A piezoelectric Rochelle salt type crystal

element in accordance with claim 10, said element being adapted to vibrate simultaneously at another frequency which is an independently controlled longitudinal mode frequency dependent mainly upon said length dimension of its substantially rectangular major surfaces, said length dimension expressed in centimeters being substantially equal to 160 divided by the value of said longitudinal mode frequency expressed in kilocycles per second, the ratio of said width dimension of said major surfaces with respect to said length dimension thereof being a value of substantially 0.24, said width dimension and said length dimension being a set of corresponding values in accordance with the values of said plurality of frequencies.

17. A piezoelectric Rochelle salt type crystal element in accordance with claim 13, said element being adapted to vibrate simultaneously at another frequency which is an independently controlled longitudinal mode frequency dependent mainly upon said length dimension of its substantially rectangular major surfaces, said length dimension expressed in centimeters being substantially equal to 118 divided by the value of said longitudinal mode frequency expressed in kilocycles per second, the ratio of said width dimension of said major surfaces with respect to said length dimension thereof being a value of substantially 0.22, said width dimension and said length dimension being a set of corresponding values in accordance with the values of said plurality of frequencies.

18. A piezoelectric Rochelle salt type crystal element in accordance with claim 10, and means including two functionally independent sets of opposite electrodes cooperating with said major surfaces of said element for vibrating said element simultaneously at two independently controlled desired frequencies, one of said frequencies being said flexural mode frequency and the other being a longitudinal mode frequency dependent mainly upon said length dimension of the rectangular major surfaces of said element.

19. A piezoelectric Rochelle salt type crystal element in accordance with claim 13, and means including two functionally independent sets of opposite electrodes cooperating with said major surfaces of said element for vibrating said element simultaneously at two independently controlled desired frequencies, one of said frequencies being said flexural mode frequency and

the other being a longitudinal mode frequency dependent mainly upon said length dimension of the rectangular major surfaces of said element.

20. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said length and width dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of a Y axis and a Z axis thereof, and said length dimension being inclined at an angle of substantially $49^{\circ} 56'$ with respect to said Y axis to obtain said desired flexural mode frequency substantially free from coupling with other modes of motion in said crystal element, said desired flexural mode frequency being a second overtone flexural mode frequency, said length dimension expressed in centimeters being equal to substantially one of the values from 150 to 171 divided by said desired frequency expressed in kilocycles per second, and the ratio of said width dimension with respect to said length dimension being one of the values from 0.2 to 0.25.

21. A Rochelle salt type piezoelectric crystal element adapted to vibrate at a desired flexural mode frequency, said crystal element having substantially rectangular shaped major surfaces, said major surfaces having a length or longer dimension and a width or shorter dimension, said width and length dimensions being made of values in accordance with the value of said desired frequency, said major surfaces and said length and width dimensions being substantially parallel to the plane of a Y axis and an X axis thereof, said length dimension being inclined at an angle of substantially 45 degrees with respect to said X axis and said Y axis, said desired flexural mode frequency being a second overtone of the fundamental flexural mode frequency, said length dimension expressed in centimeters being equal to substantially one of the values from 175 to 199 divided by said desired frequency expressed in kilocycles per second, and the ratio of said width dimension with respect to said length dimension being one of the values substantially from 0.2 to 0.25.

WARREN P. MASON.