

June 16, 1942.

S. A. SCHELKUNOFF
DIRECTIVE ANTENNA SYSTEM

2,286,839

Filed Dec. 20, 1939

5 Sheets-Sheet 1

FIG. 1

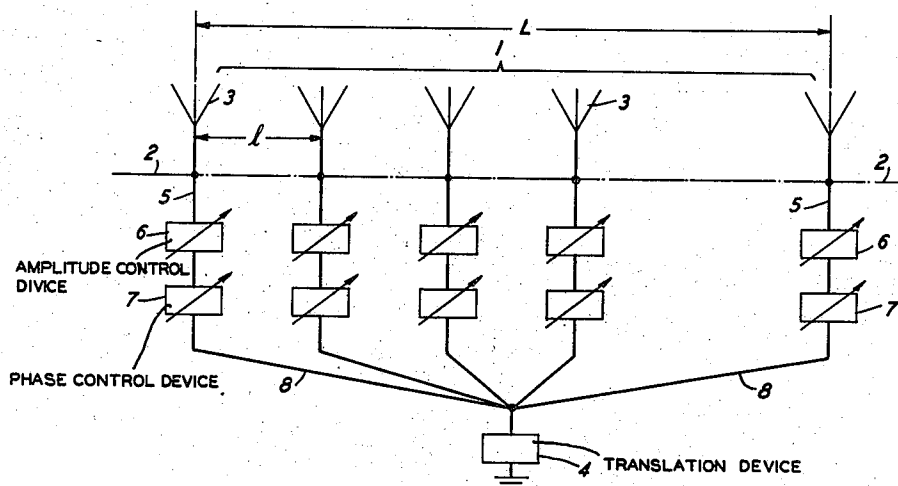


FIG. 2

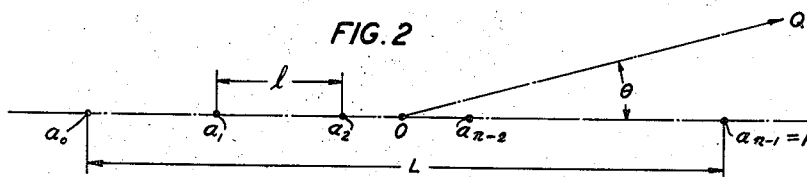
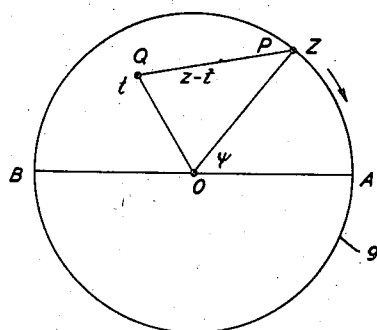


FIG. 3



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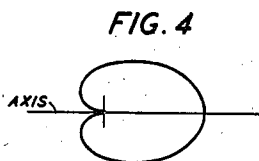


FIG. 5

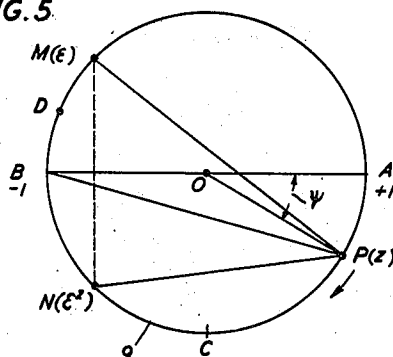
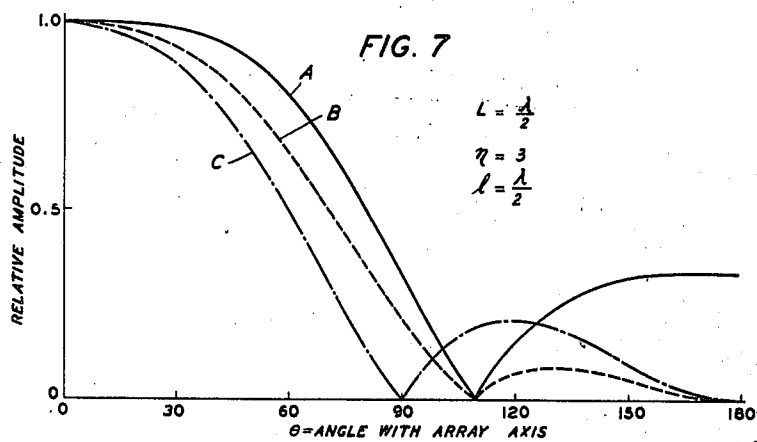
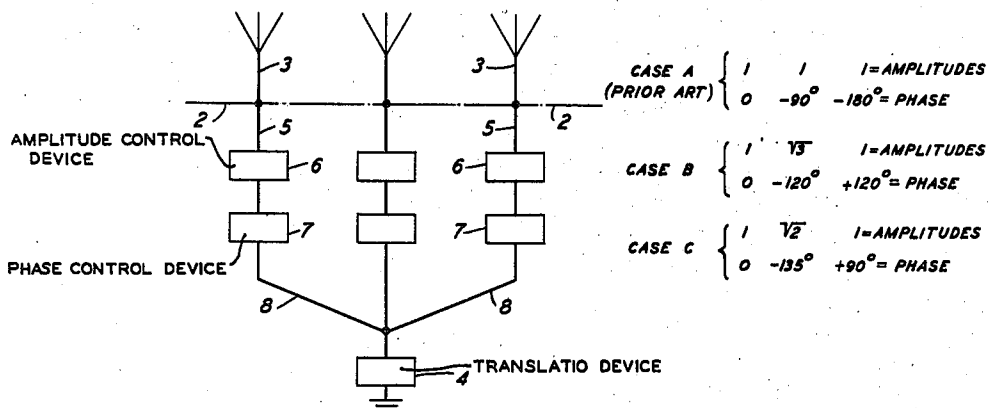


FIG. 6



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FIG. 8

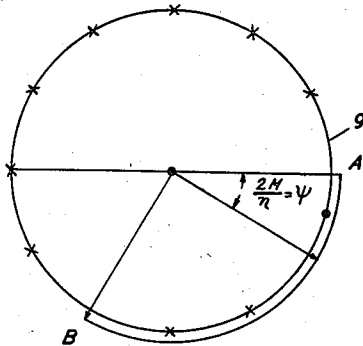


FIG. 9

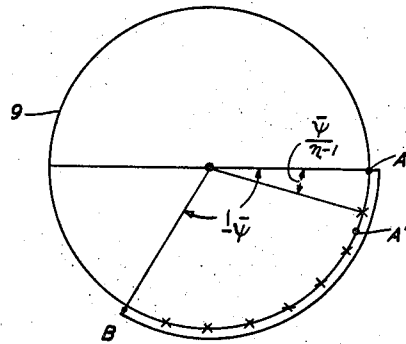
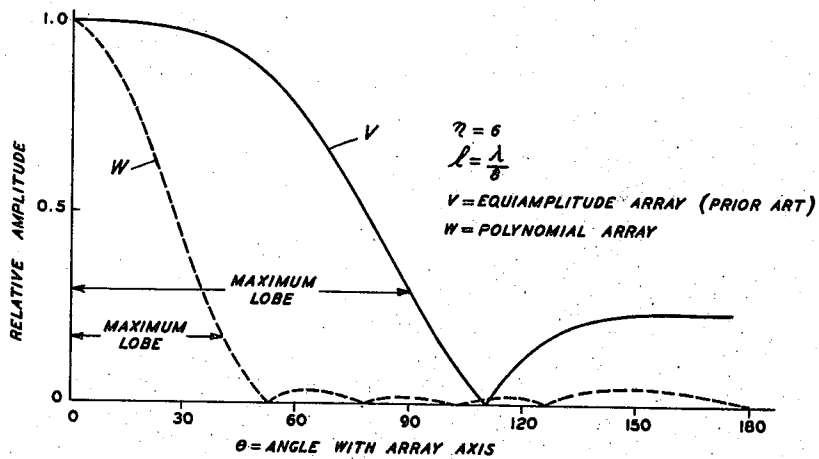


FIG. 10



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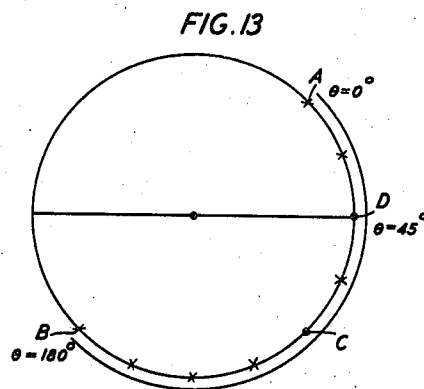
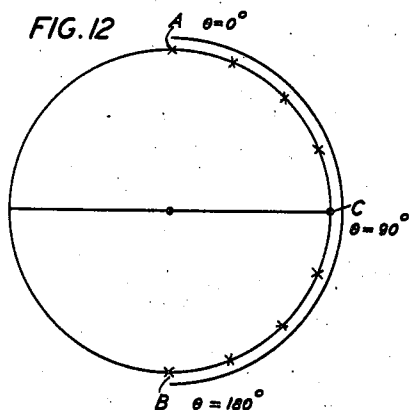
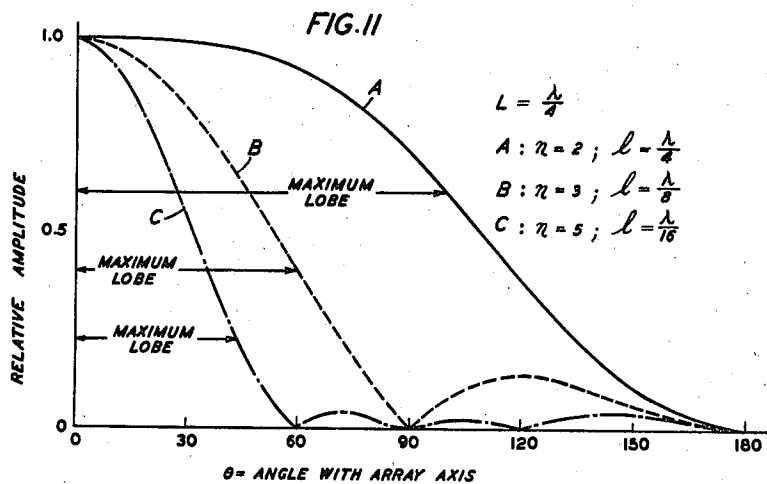
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DIRECTIVE ANTENNA SYSTEM

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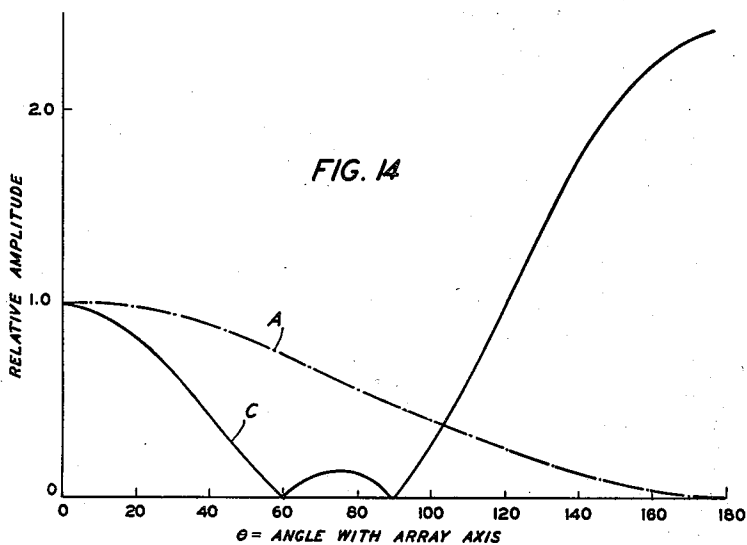


FIG. 15

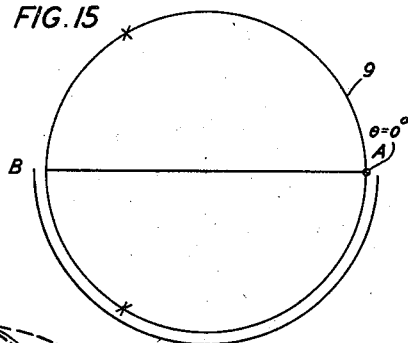
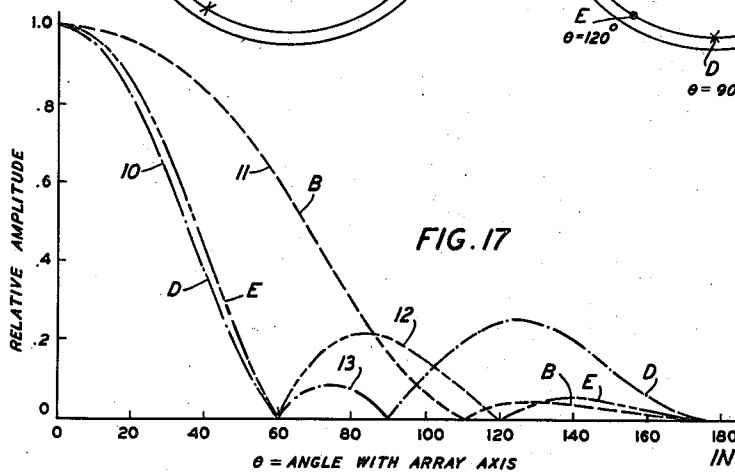
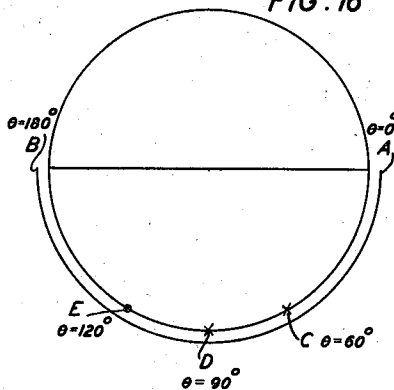


FIG. 16



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UNITED STATES PATENT OFFICE

2,286,839

DIRECTIVE ANTENNA SYSTEM

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Application December 20, 1939, Serial No. 310,130

7 Claims. (Cl. 250—11)

This invention relates to antenna systems and particularly to methods and means for obtaining optimum and/or prescribed directive action from a linear antenna array.

As is well known, linear arrays of the type disclosed in Patents 1,738,522, G. A. Campbell, December 10, 1929, and 2,041,600, H. T. Friis, May 19, 1936, and the paper "Certain factors affecting the gain of directive antennas" by G. C. Southworth, Proceedings of the Institute of Radio Engineers, September 1930, pages 1502 to 1536, are in general use in short wave communication. In this type of array a given plurality of evenly spaced, non-directive or directive, antenna units connected through separate lines are utilized and the currents supplied to, or received from, the antenna units, hereafter called "antenna currents," have equal or uniform amplitudes and either the same phase or a progressive phase difference which may be varied for the purpose of steering the direction of action.

The directive or space factor characteristic of the array is a function of the physical and electrical characteristics of the array. In particular, the sharpness of the maximum lobe of the characteristic, the principal radius of which ordinarily coincides with the desired direction of maximum, the angular spacing between the several directive lobes as measured in any plane containing the longitudinal axis of the array, the angular position of the zero or null directions of the characteristic, the ratio of radiant action in the direction of the principal radius of the primary or maximum lobe to that of the largest secondary lobe, and other features of the space factor characteristic, depend upon several factors including the length of the array, the number of antenna units, the spacing between units, and the phase relation of the equiamplitude currents. In one modification of this type of array, disclosed in Patent 1,643,323, J. S. Stone, September 27, 1927, the minor lobes are reduced and the ratio mentioned above increased, at some sacrifice in the sharpness and width of the maximum lobe, by proportioning the antenna currents in accordance with the coefficients of a binomial expansion. It now appears desirable to improve further the directive efficiency of a given linear array.

It is one object of this invention to secure, for a linear array, a prescribed space factor directive characteristic.

It is another object of this invention to secure a given directive action utilizing a linear array

of minimum length and comprising an optimum number of antenna units.

It is another object of this invention to obtain greater radiant action in a desired direction or range of directions, utilizing a linear array having a given length and comprising spaced antenna units, than heretofore achieved.

It is another object of this invention to suppress, completely and without substantially affecting the width of the maximum space factor lobe, the secondary space factor lobes of a linear array.

It is a further object of this invention to secure, for a linear array of relatively short length and comprising a small number of antenna units, a directive characteristic heretofore obtainable only with an expensive long array having a large number of units.

It is still another object of this invention to obtain an optimum resultant or over-all directive characteristic in a linear array comprising directive units.

In accordance with this invention an optimum space factor characteristic is obtained for a linear array, comprising a given number of units and a given spacing between adjacent units, by choosing proper values for the factors mentioned above and, especially, by selecting for the antenna currents both a proper amplitude distribution and a proper phase distribution. More specifically, the desired space factor characteristic is obtained for a linear array comprising a given number of antenna units having one or more elements, by spacing the adjacent units a distance less than a half operating wave-length, selecting in any plane containing the longitudinal axis of the array properly spaced directions for all the null directions of radiant action for the array, the number of null directions being one less than the number of elements, and securing antenna currents having an amplitude distribution and a phase distribution determined by the complex coefficients of a polynomial expansion representing a space factor of the array having the selected null directions of action. Conversely, the optimum array, considered from the standpoint of size and ground space requirements may be ascertained for securing a prescribed characteristic.

In addition, in accordance with this invention, a prescribed directive characteristic as, for example, the ideal stationary lobe for the subarray or antenna unit of a multiunit steerable array, the lobe having a given angular width and equal radii in polar coordinates, may be obtained, at

least approximately, by selecting a tentative antenna spacing of one-half wave-length, obtaining a Fourier series representing the prescribed characteristic, ascertaining from a curve approximating the series a polynomial expansion representing an approximate characteristic, utilizing a linear array having a plurality, equal to the number of terms in the polynomial, of antenna units, spacing the units in accordance with the powers of the polynomial terms, and controlling the amplitude and phase distributions of the antenna currents in accordance with the complex coefficients of the polynomial expansion.

The invention will be more fully understood from a perusal of the following specification considered in connection with the drawings on which like reference characters denote either elements of similar function or similar mathematical representations, and on which:

Fig. 1 illustrates a linear array designed in accordance with the invention;

Fig. 2 is a schematic representation of the array of Fig. 1;

Figs. 3, 5, 8, 9, 12, 13, 15 and 16 are Argand diagrams used in explaining the invention;

Figs. 4, 7, 10, 11, 14, and 17 are directive diagrams also useful for explaining the invention; and

Fig. 6 illustrates an array of three units designed in accordance with the invention.

Referring to Fig. 1, reference numeral 1 designates a linear array having a longitudinal axis 2 and comprising directive or non-directive antenna units 3. The units are connected to a translation device 4, which may be either a transmitter or receiver, by means of conductors 5, individual adjustable amplitude control devices 6, individual adjustable phase control devices 7 and lines 8. As will be hereinafter explained, the total length "L" of the array, the number "n" of antenna units 3 and the value of the spacing "l" between adjacent units are related to the amplitude distribution and phase distribution of the antenna currents, as determined by the adjustment of devices 6 and 7. The lines 5 and 8 are preferably, but not necessarily, of equal length and the translation device may be located at any convenient location as, for example, near one end of the array, instead of at the center, as illustrated.

Referring to Figs. 2 and 3, the manner of determining the array constants L, n and l, and the amplitude and phase relation, for securing the optimum space factor characteristic will now be explained. In Fig. 2 the angle θ represents the angle, in any plane containing the axis 2, between the direction of maximum radiation and the array axis. Thus if $\theta=0$ degrees the direction of maximum action, that is, the longest radius of the maximum space factor lobe, coincides with the array axis and the array is then a pure "end-fire" system, whereas if $\theta=90$ degrees the array is a broadside system. Now at a great distance from the array the amplitudes of the field intensities of the individual antennas are not affected by the actual separations between the antennas, but the phases are affected. Consider for example, two transmitting antennas a_0 and a_1 separated by a distance l. At a point Q lying in a direction given by the angle θ , the wave from one antenna (a_1 , by assumption) will be ahead of that from the other antenna (a_0). Since the phase change per wave-length (λ) is 2π ,

the phase at Q of the a_1 wave will be greater than that of the a_0 wave by the amount

$$\frac{2\pi l \cos \theta}{\lambda}$$

If the phase of the source a_1 is retarded with respect to that at a_0 by an amount or progressive phase change δ , then the total phase difference is

$$\frac{2\pi l \cos \theta}{\lambda} - \delta$$

which may be written $\beta l \cos \theta - \delta$, or simply represented by the letter " ψ ." By a proper choice of the time origin we can make the phase of the a_0 wave at Q equal to ωt . Thus the field intensities of the two waves will be $a_0 \cos \omega t$ and $a_1 \cos (\omega t + \psi)$. If the reference phase at a_0 is assumed to have zero phase and if the deviations at a_0 and a_1 from the assumed phases are represented, respectively, by δ_0 and δ_1 the resultant field intensity $\sqrt{\Phi_1}$ at Q will be $a_0 \cos (\omega t + \delta_0) + a_1 \cos (\omega t + \psi + \delta_1)$.

By applying the above principles to a linear array of n antennas we obtain for the instantaneous field strength amplitude $\sqrt{\Phi_1}$, at a particular point in space in the direction θ , the following Fourier series.

$$\begin{aligned} \sqrt{\Phi_1} = & A_0 \cos (\omega t + \delta_0) + A_1 \cos (\omega t + \psi + \delta_1) + \\ & A_2 \cos (\omega t + 2\psi + \delta_2) + \dots \\ & A_{n-2} \cos (\omega t + (n-2)\psi + \delta_{n-2}) + \cos (\omega t + (n-1)\psi) \end{aligned} \quad (1)$$

where

Φ_1 is proportional to the radiation intensity or power and where A_{n-1} , the coefficient of the last term equals unity by assumption. In other words, since the coefficients express the correlation of the amplitudes and phases the different amplitudes and phases may be referred to the amplitude and phase of the current in an "end" antenna element, the end antenna current having by assumption a unit amplitude and zero phase;

$\sqrt{\Phi_1}$ is field strength amplitude;

$A_0, A_1 \dots A_{n-1}$ are the relative amplitudes of the elements of the array;

δ is a progressive phase delay, from left to right, between the successive elements of the array, in the direction $\theta=0$;

$\delta_0, \delta_1, \delta_2, \delta_{n-2}, \delta_{n-1}$ represent the phase deviations from the above progressive phase delay and by assumption $\delta_{n-1}=0$;

$\beta=2\pi/\lambda$ is the phase constant

λ is the wave-length

$\psi=\beta l \cos \theta - \delta$ and is a function of the angle θ

ω is the angular velocity

t represents time in seconds

ωt is radians per second.

Forming another expression similar to (1) but with sines in the place of cosines, multiplying the result by

$$i = \sqrt{-1}$$

we have the following complex representation of the instantaneous value

$$\begin{aligned} \sqrt{\Phi_1} = & [A_0 e^{i\delta_0} + A_1 e^{i\psi + i\delta_1} + A_2 e^{2i\psi + i\delta_2} + \\ & \dots + A_{n-2} e^{i(n-2)\psi + i\delta_{n-2}} + e^{i(n-1)\psi}] e^{i\omega t} \end{aligned} \quad (4)$$

where e is the base of natural logarithms.

The true instantaneous value of the field strength is the real part of (4). Hence the amplitude

$$\sqrt{\phi}$$

of the field strength is the absolute value of (4).

Thus, for the array of n antennas, we have the following polynomial of degree $(n-1)$:

$$\sqrt{\Phi} = |a_0 + a_1 z + a_2 z^2 + \dots + a_{n-2} z^{n-2} + z^{n-1}| \quad (5)$$

where

$$z = e^{i\psi} \quad (6)$$

$$a_m = A_m e^{i\delta m} \quad (7)$$

In Equation 5 $a_0, a_1, a_2 \dots a_{n-2}, a_{n-1}=1$ are complex numbers representing the relative amplitudes of the elements of the array and the phase deviations of these elements from a given progressive phasing. Thus, if all the coefficients are real and positive, they represent the relative amplitudes of the elements of the array. If the progressive phase delay δ equals zero, the array is a broadside array, and if δ equals βl it is an end-on array. If the algebraic sign of a particular coefficient is reversed, the phase of the corresponding element is changed by 180 degrees; if some coefficient is multiplied by i or $-i$, the phase of the corresponding element is respectively advanced or delayed by 90 degrees; and in general the phase advance is equivalent to a multiplication by a unit complex number

$$e^{i\theta}$$

Some coefficients may be equal to zero and the corresponding elements of the array will be missing. In view of this possibility, we shall call "l" the "apparent" separation between the elements. When the elements are equispaced, the apparent separation is the actual separation. It follows that every linear array with commensurable separations between the elements may be represented by a polynomial and every polynomial may be interpreted as a linear array. If the separations are not commensurable the arrays are represented by an algebraic function with incommensurable exponents. The total length of the array is the product of the apparent separation between the elements and the degree of the polynomial. The degree of the polynomial is one less than the apparent number of elements, and the actual number of elements is at most equal to the apparent number. Stated differently, the instantaneous amplitude of the field strength

$$\sqrt{\Phi}$$

in a given direction, and hence the directive property of the linear array of non-directive antenna units 3 are represented by a polynomial depending upon a complex variable z . The general phase factor is represented by a unit complex number

$$e^{i\theta}$$

Since ψ is always real, referring to Equations 3 and 6, the absolute value of z equals unity and its powers can be represented by radius vectors terminating on the circumference of the circle 9 of unit radius as illustrated by Fig. 3, z itself always being on the circumference of the circle. As θ increases from 0 degrees to 180 degrees, ψ decreases and z moves in the clockwise direction. When $\theta=0$, $\psi=\beta l-\delta$; and when $\theta=180$ degrees $\psi=-\beta l-\delta$. Hence the range ψ described by z is

$$\psi=2\beta l \quad (8)$$

When the separation l between the successive elements of the array is equal to $\lambda/2$, the range of z equal 2π , and as θ varies from 0 degrees to

180 degrees, z describes a complete cycle and returns to its original position. In this case there is a one-to-one correspondence between the points on the circumference of the unit circle 9 and the angular position in any plane containing the axis 2 of the array radiation cones. If the separation between the elements is less than $\lambda/2$, the range of z is smaller than 2π and z describes only a portion of the unit circle. Finally, if l is greater than $\lambda/2$, then the path of z overlaps itself. That is, since the radiation is a periodic function of ψ , the space factor of a given array will repeat itself when the separation between the elements is greater than $\lambda/2$.

From the theory of algebraic equations it is known that a polynomial of degree $(n-1)$ has $(n-1)$ zeros. Some of the zeros may be coincident and have to be counted in accordance with their order of multiplicity. Designating the zeros of Equation 5 by $t_1, t_2, \dots t_{n-1}$, we can write

$$\sqrt{\Phi} = |(z-t_1)(z-t_2)(z-t_3) \dots (z-t_{n-1})| \quad (9)$$

Thus, the space factor of the array is the product of the space factors of $(n-1)$ virtual couplets. The absolute value of a binomial $(z-t)$ is the length PQ in Fig. 3 of the line joining the points z and t . Hence the field strength of the array is represented by the products of the lines joining a typical point z on the unit circle 9 to the points representing the zeros of the polynomial or Equation 5. If none of the zeros is located on the unit circle 9, there is no direction in which the radiation is entirely absent. The converse is true only if z is permitted to move over the entire unit circle which is not always the case. The angle θ varies from zero to 180 degrees and its cosine from $+1$ to -1 . Hence, the exponent of z will change by an amount $2\beta l$. Point z will travel once around the circumference if the exponent changes by an amount 2π . Hence, if $\beta l=\pi$, that is, if the separation between the successive elements of the array is $\lambda/2$, then to every direction in space there will correspond one and only one point on the unit circle. If the separation is less than $\lambda/2$, z will be confined to a smaller arc, while if the separation is greater than $\lambda/2$, z will retrace some of its previous course. Consequently, when the distance between the adjacent units or elements of the array equals $\lambda/2$, to each zero on the unit circle 9 of the polynomial or Equation 9 there will correspond one and only one null direction, that is, a direction in which radiation is entirely absent. In general, there exists a one-to-one correspondence between null directions and the zeros, located on the path of z of the polynomial, each zero being counted anew as z happens to pass through it again while θ varies from zero to 180 degrees. Thus, if l , the spacing between adjacent elements is not greater than $\lambda/2$, the null directions of a linear array may be placed at will.

To illustrate the above let us assume an array comprising two non-directive antennas spaced a quarter wave-length apart and energized in quadrature but with equal amplitudes. As is known, the above two-element or two-unit array is the conventional antenna-reflector system which produces maximum direction in the front direction corresponding to $\theta=0$ degrees and zero radiation in the back direction, $\theta=180$ degrees, the axis being the line joining the two elements. The space characteristic for this array is the cardioid of Fig. 4.

Applying the method above, the spacing l and the progressive phase relation are assumed; and the problem is to determine the amplitude and phase distributions which will give the cardioid space factor characteristic. The space factor

$$\sqrt{\Phi} = |a_0 + z| \quad (10)$$

But, as assumed,

$$l = \frac{\lambda}{4} \quad (11)$$

$$\delta = \frac{\pi}{2} = \text{progressive phase delay} \quad (12)$$

also,

$$\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2} \quad (13)$$

$$\psi = \beta l \cos \theta - \delta = \frac{\pi}{2} (\cos \theta - 1) \quad (14)$$

In this case z travels clockwise from A to B, Fig. 5, over the lower half of the unit circle 9, as θ , in the plane of the array axis, varies from 0 degrees to 180 degrees. A, on Fig. 5, corresponding to $\theta = 0$ degrees, B to $\theta = 180$ degrees and C to $\theta = 90$ degrees. This is true since with $\delta = \pi/2$, at $\theta = 0$ degrees, $\psi = 0$ degrees; at $\theta = 90$ degrees, $\psi = \pi/2$, and at $\theta = 180$ degrees $\psi = -\pi$. Since the degree $n-1$ of the polynomial is 1, there is only one direction of null radiation and Equation 10 becomes

$$\sqrt{\Phi} = |z - t| \quad (15)$$

and we can place or fix t so that there will be no radiation in some one selected direction. If the back direction is to be the null direction

$$t = -1 \quad (16)$$

and

$$\sqrt{\Phi} = |z + 1| = 2 \left| \cos \left[\frac{\pi}{4} (\cos \theta - 1) \right] \right| \quad (17)$$

The chord BP, Fig. 5, represents the field strength. As indicated before, Equations 3 and 6, z is a function of θ . When $\theta = 0$, z is at A and the chord BP = BA, a maximum. The field strength in a direction perpendicular or broadside to the two-element array of non-directive units would be represented by the chord BP = BC. For the back direction $\theta = 180$ degrees, z is at B and BP = BB = 0.

With t placed at B the amplitude and phase distribution may be obtained from Equation 17. The fact that the coefficients in the polynomial 17 are equal to unity indicates the amplitudes are equal and that the phase deviations from the progressive phase delay, $\pi/2$, are equal to zero.

If the single null or zero t were placed at C, instead of at B, Fig. 5, the complex variable would still travel clockwise from A to B over one-half the circle 9. The array would be bidirectional and would function equally for directions $\theta = 0$ and $\theta = 180$ degrees, since the chord CB = the chord CA. The space factor characteristic would then be

$$\sqrt{\Phi} = |z + i| \quad (18)$$

Hence, the amplitudes are equal and the currents are 180 degrees out of phase.

Referring to Figs. 5, 6 and 7, we shall first consider a linear array of three non-directional elements with $\lambda/4$ spacing, a uniform amplitude distribution and an assumed original or initial uniform progressive phase delay of $\pi/2$. Then the

null points will be evenly spaced on the whole circumference 9, the space factor directive characteristic being represented by

$$\sqrt{\Phi} = |z^2 + z + 1| = |(z - \epsilon)(z - \epsilon^2)| \quad (19)$$

where ϵ is a primitive cube root of unity and is given by the following equation

$$\epsilon = e^{\frac{2\pi i}{3}} = \frac{-1 + \sqrt{3}}{2} \quad (20)$$

and

$$\epsilon^2 = \frac{-1 - i\sqrt{3}}{2} \quad (21)$$

Summing the geometric progression in Equation 19 we have

$$\sqrt{\Phi} = \left| \frac{1 - z^3}{1 - z} \right|$$

Substituting from Equation 6 we obtain

$$\begin{aligned} \sqrt{\Phi} &= \left| \frac{1 - e^{3i\psi}}{1 - e^{i\psi}} \right| \\ &= \left| \frac{e^{\frac{3i\psi}{2}} e^{-\frac{3i\psi}{2}} - e^{\frac{3i\psi}{2}} e^{-\frac{3i\psi}{2}}}{e^{\frac{i\psi}{2}} e^{-\frac{i\psi}{2}} - e^{\frac{i\psi}{2}} e^{-\frac{i\psi}{2}}} \right| \\ &= \frac{\sin \frac{3\psi}{2}}{\sin \frac{\psi}{2}} \text{ since } |e^{i\psi}| = 1 \end{aligned}$$

The absolute maximum of the above value for $\sqrt{\Phi}$ occurs at $\psi = 0$ and is equal to 3. Since we are interested only in relative values, numerical values may be introduced, for convenience, and the equality sign may be replaced by a proportionality sign. To make the principal maximum of the directive characteristic equal to unity, the factor $1/3$ is introduced. Hence

$$\sqrt{\Phi} = |z - \epsilon| |z - \epsilon^2| \approx \frac{\left| \frac{\sin \frac{3\psi}{2}}{2} \right|}{3 \sin \frac{\psi}{2}} \quad (22)$$

where, as in Equation 14

$$\psi = \frac{\pi}{2} (\cos \theta - 1) \quad (23)$$

Referring to Fig. 5, the complex variable z travels clockwise, as before, over one-half of the circle and the field strength, for direction θ corresponding to z at point P, is given by the products of the chords MP · NP, the null point M(ϵ) being outside the range of z and the null point N(ϵ^2) being on the path of z . It is evident that the null point M(ϵ) is poorly placed. If our aim is to radiate as much as possible in the direction $\theta = 0$ represented by A, we could do much better by moving M to B and obtain the following array equation

$$\sqrt{\Phi} = |(z + 1)(z - \epsilon^2)| = |z^2 + (1 - \epsilon^2)z - \epsilon^2| \quad (24)$$

$$= \left| (z + 1) \left(z - e^{-\frac{2\pi i}{3}} \right) \right| = \left| 1 + z - \sqrt{3}e^{-\frac{i\pi}{6}} + z^2 e^{-\frac{i\pi}{3}} \right| \quad (25)$$

Substituting from (6)

$$\begin{aligned} \sqrt{\Phi} &= \left| (z + 1) \left(z - e^{-\frac{2\pi i}{3}} \right) \right| = \left| (e^{i\psi} + 1) \left(e^{i\psi} - e^{-\frac{2\pi i}{3}} \right) \right| \\ &= \left| e^{\frac{i\psi}{2}} \left(e^{-\frac{i\psi}{2}} + e^{\frac{i\psi}{2}} \right) e^{-\frac{i\psi}{2}} e^{-\frac{\pi i}{3}} \left(e^{-\frac{i\psi}{2}} + \frac{i\pi}{3} - e^{-\frac{i\psi}{2}} - \frac{i\pi}{3} \right) \right| \\ &= \left| 4 \cos \frac{\psi}{2} \sin \left(\frac{\psi}{2} + \frac{\pi}{3} \right) \right| \end{aligned}$$

The maximum value of the above expression oc-

curs at $\psi=0$ and is equal to $4 \sin \pi/3$. Hence, dividing the expression by $4 \sin \pi/3$, to get a maximum value of unity, we obtain

$$\sqrt{\Phi} \sim \left| \frac{\cos \frac{\psi}{2} \sin \left(\frac{\psi}{2} + \frac{\pi}{3} \right)}{\sin \frac{\pi}{3}} \right| \quad (26)$$

$$\psi = \frac{\pi}{2} (\cos \theta - 1) \quad (27)$$

The amplitude and resulting phase distribution would be, as indicated in Fig. 6, from left to right, 1, 1.73, 1 and 0 degrees, -120 degrees, +120 degrees, the final or resulting progressive delay being 120 degrees which differs, as explained in connection with Equation 1, by reason of the deviations δ_0 , δ_1 and δ_2 of different values from the originally assumed progressive phase delay of δ .

Curve A of Fig. 7 represents, in rectangular coordinates, the space factor for the uniform amplitude array with a 90-degree phase difference between adjacent elements. Curve B illustrates the characteristic of Equation 25. It will be seen that by proportioning the amplitudes and phases in accordance with this invention, a more narrow maximum lobe and a smaller secondary lobe, as compared with the uniform array of the prior art, are obtained. If desired, the major lobe of applicant's polynomial array may be made narrower. Thus, the M may be placed nearer to the direction $\theta=0$ degrees, the principal radius of the maximum lobe, for example, at point C, in which case

$$\sqrt{\Phi} = |(z+1)(z+i)| = |z^2 + (1+i)z + i| \quad (28)$$

$$= |1 - i(1+i)z - iz^2| = |1 + \sqrt{2}e^{-\frac{i\pi}{4}}z + e^{-\frac{i\pi}{2}}z^2| \quad (29)$$

$$\sim \left| \frac{\cos \frac{\psi}{2} \cos \left(\frac{\psi}{2} - \frac{\pi}{4} \right)}{\cos \frac{\pi}{4}} \right| \quad (30)$$

Equation 30 was derived, using Equation 6, in a manner similar to that indicated above for Equation 26, the expression

$$\frac{1}{4 \cos \frac{\pi}{4}}$$

being introduced to secure a maximum equal to unity.

From Equation 29 the amplitude distribution is 1, $\sqrt{2}$, 1 and the phase distribution is 0 degrees, -135 degrees, +90 degrees as shown by case C, Fig. 6, the final progressive delay being 135 degrees which differs by reason of the different deviations δ_0 , δ_1 and δ_2 from the assumed progressive phase delay δ . Curve C of Fig. 7 illustrates the space factor characteristic for this distribution. It will be noted that, as compared to curve B, the maximum lobe of C is accompanied by a more pronounced secondary or minor lobe.

Passing to the general case of n antennas let us consider end-fire arrays in which $l < \lambda/2$,

$$\psi = \beta l (\cos \theta - 1) \quad (31)$$

As θ passes from 0 degrees to 180 degrees, ψ varies from 0 to $-\psi = -2\beta l$.

If as shown in Fig. 8 we assume a range of

$z=AB$ and corresponding to $l=\lambda/6$ then ψ varies from 0 to $-2\beta l$, where

$$-2\beta l = \frac{(-2)(2\pi)(\lambda)}{(\lambda)(6)} = \frac{-2\pi}{3} \quad (33)$$

For the usual uniform amplitude distribution, we may obtain from Equation 6, in the manner explained above relative to Equations 19 and 22 wherein $\pi=3$, the following:

$$\sqrt{\Phi} = |1 + z + z^2 + \dots + z^{n-1}| = \left| \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right| \quad (34)$$

The width Δ of the maximum lobe is $2\theta_0$, where θ_0 is the direction of the null nearest the maximum and therefore the angle between the principal radius of the maximum lobe corresponding to $\theta=0$ and the lowest null direction. Hence ψ_0 corresponding to Δ ,

$$\psi_0 = \frac{-2\pi}{n}; \beta l (1 - \cos \theta_0) = \frac{2\pi}{n} \quad (35)$$

Hence,

$$\sin \frac{\theta_0}{2} = \sin \frac{\Delta}{4} = \sqrt{\frac{\pi}{n\beta l}} = \sqrt{\frac{\lambda}{2nl}} \quad (36)$$

If n is large, we have approximately

$$\Delta = 2\sqrt{\frac{2\lambda}{nl}} \quad (37)$$

The virtual couplets corresponding to those zeros of $\sqrt{\Phi}$ which are outside the range of z are comparatively non-directive. In order to increase the radiant action in the end-fire direction $\theta=0$, all these virtual couplets or, more accurately, the corresponding null directions of action are spaced equally in the range AB of z . Thus, referring to Fig. 9, the range AB is divided into $(n-1)$ equal parts and the extremities of these intervals are selected as null points for these virtual couplets, A being excepted. Thus,

$$\sqrt{\Phi} = |(z-t)(z-t^2) \dots (z-t^{n-1})| \quad (38)$$

where

$$t = e^{-\frac{i\psi}{n-1}} \quad (39)$$

and

$$\bar{\psi} = 2\beta l \quad (40)$$

or

$$\sqrt{\Phi} = 2^{n-1} \left| \sin^{\frac{1}{2}} \left(\psi + \frac{\bar{\psi}}{n-1} \right) \sin^{\frac{1}{2}} \left(\psi + \frac{2\bar{\psi}}{n-1} \right) \dots \sin^{\frac{1}{2}} \left(\psi + \frac{(n-1)\bar{\psi}}{n-1} \right) \right| \quad (41)$$

The angle of the major lobe in this case is

$$\psi_0' = -\frac{\bar{\psi}}{n-1}, \beta l (1 - \cos \theta_0') = \frac{2\beta l}{n-1} \quad (42)$$

$$\sin \frac{\theta_0'}{2} = \sin \frac{\Delta}{4} = \frac{1}{\sqrt{n-1}} \quad (43)$$

When n is sufficiently large so that θ_0' is small, we have approximately

$$\Delta' = \frac{4}{\sqrt{n-1}} \quad (44)$$

For this arrangement of null points, the angle of the major radiation lobe depends upon the number of elements.

From Equation 37 and Equation 44, we have

$$\frac{\Delta'}{\Delta} = \sqrt{\frac{2nl}{(n-1)\lambda}} = \sqrt{\frac{2l}{\lambda}} \quad (45)$$

75 when n is large.

If the distance l between the successive elements is $\lambda/6$, then

$$\frac{\Delta'}{\Delta} = \sqrt{\frac{1}{3}} = 0.577 \quad (46)$$

Thus the maximum lobe represented by the angle

$$\frac{\bar{\psi}}{n-1}$$

in Fig. 9 is 42.3 per cent narrower than the major lobe of the corresponding array with uniform amplitude distribution.

If $l = \lambda/8$, then

$$\frac{\Delta'}{\Delta} = .5 \quad (47)$$

The ratio X of the principal radii of the maximum lobe and the largest secondary lobe, for the array represented by Equation 38, will now be determined and compared to the corresponding ratio for the uniform amplitude array.

The maximum of the largest minor lobe is located at A' , Fig. 9, lying approximately half-way between the first two null points.

At A ,

$$\psi = 0 \quad (48)$$

and at A' ,

$$\psi = \frac{3\bar{\psi}}{2(n-1)} \quad (49)$$

Thus, from Equation 41, we have

$$X = \frac{\sin \frac{\bar{\psi}}{2(n-1)} \sin \frac{2\bar{\psi}}{2(n-1)} \sin \frac{3\bar{\psi}}{2(n-1)} \cdots \sin \frac{(n-1)\bar{\psi}}{2(n-1)}}{\sin \frac{\bar{\psi}}{4(n-1)} \sin \frac{3\bar{\psi}}{4(n-1)} \sin \frac{5\bar{\psi}}{4(n-1)} \cdots \sin \frac{(2n-5)\bar{\psi}}{4(n-1)}} \quad (50)$$

If eight antennas are placed within one-tenth of a wave-length,

$$l = \frac{\lambda}{70} \quad (51)$$

$$\bar{\psi} = 2\beta l = (2) \left(\frac{2\pi}{\lambda} \right) \left(\frac{\lambda}{70} \right) = \frac{4\pi}{70} \quad (52)$$

and

$$X = 36.6 \text{ approximately} \quad (53)$$

and the largest secondary lobe is lower than the maximum lobe by 31 decibels.

If $\bar{\psi} = \pi$ then

$$X = \frac{\sqrt{2(n-1)} \sin \frac{(2n-3)\pi}{4(n-1)}}{\sin \frac{\pi}{4(n-1)}} \quad (54)$$

and if n is large. Equation 54 follows from Equation 50 when the numerator of Equation 50 is simplified by means of Equation 18 on page 117 of the textbook "Treatise on Plane Trigonometry" (1928) by E. W. Hobson, and when the denominator is simplified by means of Equation 15 on page 115 of the same treatise.

$$X = \frac{4}{\pi} (n-1) \sqrt{2(n-1)}, \text{ approximately} \quad (55)$$

so that the subsidiary maximum or largest secondary lobe is

$$[20 \log_{10} (n-1) + 10 \log_{10} (n-1) + 5] \text{ decibels} \quad (56)$$

below the maximum lobe. For five units spread over a wave-length this is equal to 23 decibels. The largest secondary lobe is diminished by about 9 decibels each time the number of elements of the array is doubled. In the case of an array with equal amplitudes, the size of the secondary lobe is substantially independent of the number of elements.

The maximum lobe of the array designed in accordance with Fig. 9 is not only more narrow but also sharper or more pointed than the maximum lobe of the corresponding uniform array. Thus at the point lying half-way between A and the first null point or cross, Fig. 9, the ratio of the field strengths is

$$Y = \frac{\sin \frac{\bar{\psi}}{2(n-1)} \sin \frac{2\bar{\psi}}{2(n-1)} \cdots \sin \frac{(n-1)\bar{\psi}}{2(n-1)}}{\sin \frac{\bar{\psi}}{4(n-1)} \sin \frac{3\bar{\psi}}{4(n-1)} \sin \frac{5\bar{\psi}}{4(n-1)} \cdots \sin \frac{(2n-3)\bar{\psi}}{4(n-1)}} \quad (57)$$

For a quarter wave-length separation between the elements this ratio becomes $\sqrt{2(n-1)}$ so that the decrease in radiation intensity is $10 \log_{10} (n-1) + 3$ decibels. More specifically, Equation 57 is obtained by multiplying Equation 50 by the factor

$$\frac{\sin \frac{\bar{\psi}}{4(n-1)}}{\sin \frac{(2n-3)\bar{\psi}}{4(n-1)}}$$

Also, letting $\bar{\psi} = \pi$ and multiplying Equation 54 by the factor mentioned above, Equation 57 be-

comes equal to $\sqrt{2(n-1)}$. On the other hand, for a long uniform array, the corresponding drop is independent of n and is only 4 decibels.

As stated before, the amplitude and phase distribution for the array designed in accordance with Fig. 9 may be found by determining the value of the complex numbers a_0, a_1, a_{n-1} , constituting the coefficients of the polynomial of Equation 5. For our chosen null points, Fig. 9, Equation 5 becomes Equation 38 and, as indicated in the "Textbook of Algebra" by G. Crystal, vol. 2, page 340 (1926), this equation may be represented

$$\sqrt{\Phi} = |(1-t^{-1}z)(1-t^{-2}z) \cdots (1-t^{-n+1}z)| \quad (58)$$

$$= \left| 1 + \sum_{k=1}^{n-1} \frac{\sin \frac{(n-1)\beta l}{n-1} \sin \frac{(n-2)\beta l}{n-1} \cdots \sin \frac{(n-k)\beta l}{n-1}}{\sin \frac{\beta l}{n-1} \sin \frac{2\beta l}{n-1} \cdots \sin \frac{k\beta l}{n-1}} \times e^{\frac{ik\beta l}{n-1} + ik\pi} e^{ik\beta l} \cos \theta \right| \quad (59)$$

Hence, the total progressive phase delay from one antenna to the next is

$$\delta = \pi - \frac{\beta l}{n-1} \quad (60)$$

and the amplitude distribution is

$$1, \frac{\sin \beta l}{\sin \frac{\beta l}{n-1}}, \frac{\sin \beta l \sin \frac{(n-2)\beta l}{n-1}}{\sin \frac{\beta l}{n-1} \sin \frac{2\beta l}{n-1}}, \frac{\sin \beta l \sin \frac{(n-2)\beta l}{n-1} \sin \frac{(n-3)\beta l}{n-1}}{\sin \frac{\beta l}{n-1} \sin \frac{2\beta l}{n-1} \sin \frac{2\beta l}{n-1}}, \dots 1 \quad (61)$$

The amplitude of the elements equidistant from the ends of the array are equal.

The gain of the array designed in accordance with Fig. 9 over a single antenna unit is

$$G = 10 \log_{10} \frac{2}{\int_0^\pi \Phi \sin \theta d\theta} \text{ decibels} \quad (62)$$

and the gain of a uniform amplitude array is

$$G = 10 \log_{10} n \quad (63)$$

The gain of the array of the invention is greater than that of the corresponding uniform array. Thus, if $l = \lambda/4$ and $n = 3$, the gain of the array of Fig. 9 over the array of Fig. 8 is 2.6 decibels.

Referring to Fig. 10, curves V and W represent, respectively, the space factor characteristics for two arrays each comprising six non-directive units and having a uniform spacing of $\lambda/8$. The curve V represents the prior art equiamplitude array, having a progressive phase delay of $\delta = \pi/4$ and the curve W illustrates an array energized in accordance with applicant's invention. It will be observed from these curves that the maximum lobe of the array constructed in accordance with applicant's invention is, as compared to the prior art array, narrower and sharper, and the ratio of the principal radius of the maximum lobe to that of greatest secondary lobe is larger.

Fig. 11 illustrates the directive characteristics obtained when L is maintained constant and the values of n and l are varied. Thus, as indicated in this figure, increasing the number of elements decreases the width of the maximum lobe.

Thus far end-on arrays only have been considered and the symbolic representation in Fig. 9 and the related equations are for the special case of an end-on array having maximum action in direction $\theta = 0$ degrees. The amplitude and phase distributions, however, may be arranged for maximum radiation in any given direction corresponding to any value of θ within the range of 0 degrees to 180 degrees. If greatest radiation is desired for $\theta = 90$ degrees we have the so-called broadside array. In this case, $\delta = 0$ and $\psi = \beta l \cos \theta$. If,

$$l = \frac{\lambda}{4} \quad (64)$$

then

$$\psi = \frac{\pi}{2} \cos \theta \quad (65)$$

and

$$\psi = \frac{\pi}{2} \text{ and } z = +i, \text{ when } \theta = 0^\circ \quad (66)$$

$$\psi = 0 \text{ and } z = 1, \text{ when } \theta = 90^\circ \quad (67)$$

$$\psi = -\frac{\pi}{2} \text{ and } z = -i, \text{ when } \theta = 180^\circ \quad (68)$$

The corresponding range for z , which range is determined only by l , is represented by the arc AB of Fig. 12, the range being one-half of the

circumference 9. In this case, the nulls are evenly spaced in each half of the arc AB, the point corresponding to $\theta = 90$ degrees being excepted. The maximum lobe for this broadside array with evenly placed nulls, assuming n is large, is twice as narrow as the corresponding prior art array with uniform amplitude distribution.

Again, if greatest radiation action is desired in the direction $\theta = 45$ degrees then, assuming $l = \lambda/4$,

$$\delta = \frac{\pi}{2\sqrt{2}} \quad (69)$$

15 and

$$\psi = \beta l \left(\cos \theta - \frac{1}{\sqrt{2}} \right) = \frac{\pi}{2} \left(\cos \theta - \frac{1}{\sqrt{2}} \right) \quad (70)$$

when

$$\theta = 0, \psi = \frac{\pi(\sqrt{2}-1)}{2\sqrt{2}}, \text{ and } z = e^{i\frac{\pi(\sqrt{2}-1)}{2\sqrt{2}}} \quad (71)$$

$$\theta = 45, \psi = 0, \text{ and } z = 1 \quad (72)$$

$$\theta = 135, \psi = -\frac{\pi}{\sqrt{2}}, \text{ and } z = e^{-\frac{i\pi}{\sqrt{2}}} \quad (73)$$

$$\theta = 180, \psi = -\frac{\pi(\sqrt{2}+1)}{2\sqrt{2}}, \text{ and } z = e^{-\frac{i\pi(\sqrt{2}+1)}{2\sqrt{2}}} \quad (74)$$

The range of z for this arrangement is illustrated by the arc AB of Fig. 13. In order to secure additional gain over the equiamplitude array in the direction $\theta = 45$ degrees, the nulls should be placed on the arc AB; for example, they may be evenly spaced as indicated by the crosses in this figure.

In the case of the array arranged for broadside operation, the nulls being placed as shown in Fig. 12,

$$\sqrt{\Phi} = |(z-t^m)(z-t^{m-1}) \dots (z-t)(z-t^{-1}) \dots (z-t^{-m})| \quad (75)$$

where

$$t = e^{\frac{i\beta l}{m}} \quad (76)$$

and

$$n = 2m + 1 \quad (77)$$

where m is the number of null directions between $\theta = 0$ degrees and $\theta = 90$ degrees.

In the case of the oblique array, that is, when the greatest direction of radiation action is at $\theta = 45$ degrees, and the nulls are spaced as illustrated on Fig. 13,

$$\sqrt{\Phi} = |(z-t_1)(z-t_1^2) \dots (z-t_2^m)(z-t_2)(z-t_2^2) \dots (z-t_2^k)| \quad (78)$$

where

$$t_1 = \exp \left[\frac{i\beta l}{m} \left(1 - \frac{1}{\sqrt{2}} \right) \right] \quad (79)$$

$$t_2 = \exp \left[-\frac{i\beta l}{k} \left(1 + \frac{1}{\sqrt{2}} \right) \right] \quad (80)$$

$$n = m + k + 1 \quad (81)$$

where m is the number of null directions between $\theta = 0$ and $\theta = 45$ degrees and k the number of null directions between $\theta = 45$ degrees and $\theta = 180$ degrees.

The amplitude and phase distribution for the broadside arrangement would be obtained by expanding Equation 75 and for the oblique array by expanding Equation 78. Thus, the space factor characteristic of a given array may be steered for optimum operation in a desired direction θ by adjusting devices 6 and 7 so that the amplitude and phase distributions are in accordance with those corresponding to the optimum null positions and range of z for the desired direction.

Thus far we have considered arrays comprising non-directive units and in which directivity is secured solely by means of the spacial arrangement of the elements. If the units are directive, the resultant characteristic is the product of the space factor characteristic and the unit characteristic, and only a part of the space factor characteristic is important. In accordance with this invention the space factor characteristic may be designed to fit the unit characteristic. Thus the null points may be placed so that a maximum space factor lobe is secured having a shape, width and a sharpness suitable for use in a steerable system.

Referring to Figs. 14, 15, 16 and 17, assume an array comprising three unidirectional antennas spaced a quarter wavelength apart, assume further that curve A, Fig. 14, represents the directive characteristic of the unit antenna. Since $l=\lambda/4$, the range z , Fig. 15, will be one-half of the circle 9. If the antenna currents are of the same amplitude and if the progressive phase delay is $\pi/2$, the two nulls will be placed as shown by the crosses, Fig. 15. Since one null point is outside the range of z the maximum space factor lobe will be relatively wide and, the resultant characteristic illustrated by curve B, Fig. 17, will not be the optimum. If on the other hand, the two nulls are placed, as indicated by the crosses $\theta=60$ and $\theta=90$ degrees, Fig. 16, within the range z , an array space factor illustrated by curve C, Fig. 14, and a resultant characteristic shown by curve D, Fig. 17, will be obtained. It will be noted that the maximum lobe 10 of characteristic D, obtained in accordance with the present invention, is sharper and narrower than the corresponding maximum lobe 11 of characteristic B obtained with the prior art uniform amplitude array. If the null $\theta=90$ degrees is shifted to $\theta=120$ degrees, Fig. 16, the secondary lobe 13 of curve D is expanded to the lobe 12 of curve E, widening of the maximum lobe being accompanied by a slight decrease in the radiation in directions 120 degrees to 180 degrees. For the array corresponding to curves C and D, the amplitude distribution is 1, 1.85 and 1, and the phase distribution is 0 degrees, 157.5 degrees and 45 degrees. For the array represented by curve E the amplitude distribution is 1, $\sqrt{2}$, 1 and the phase distribution is 0 degrees, 180 degrees and 0 degrees.

Thus far the method and means for obtaining an optimum space factor and, when directive units are utilized, for obtaining an optimum resultant characteristic, have been explained, the number of units, the spacing therebetween and the array length being assumed. Also, directive characteristics obtained with arrays having different number of units, the array length being the same, have been compared. The method of ascertaining the array length, number of units and unit spacing, this spacing being less than $\lambda/2$, for securing a prescribed space factor characteristic given by an arbitrary

function of ψ or $F(\theta)$ will now be explained. Ordinarily the number of required units will be infinite, but the desired space factor may be approximately, and for most practical purposes, satisfactorily, obtained with a critically selected finite number of units.

Let $n=2m+1$, so that n is odd. Then, since the modulus of z is unity the polynomial, Equation 5, can be divided by z^m without affecting $|\sqrt{\Phi}|$; thus

$$\sqrt{\Phi} = |a_0 z^{-m} + a_1 z^{-m+1} + \dots + a_m + a_{m+1} z + \dots + a_{2m} z^m| \quad (82)$$

If we now assume that the coefficients equidistant from the ends of the polynomial are conjugate complex, the polynomial is real and we can drop the bars. Thus, setting

$$a_m = A_0 \quad (83)$$

$$a_{m+k} = A_k - iB_k, \quad k > 0 \quad (84)$$

$$a_{m-k} = A_k + iB_k \quad (85)$$

where the A and B are real and the asterisk designates the conjugate complex number, we have

$$a_{m+k} z^k + a_{m-k} z^{-k} = 2A_k \cos k\psi + 2B_k \sin k\psi \quad (86)$$

Consequently, Equation 82 becomes

$$\sqrt{\Phi} = \sum_{k=0}^m \epsilon_k (A_k \cos k\psi + B_k \sin k\psi) \quad (87)$$

where ϵ_k is the Neumann number. Since $\sqrt{\Phi}$ is to be a prescribed function $f(\psi)$ of the variable ψ , we expand this function in a Fourier series

$$\sqrt{\Phi} = f(\psi) = \sum_{k=0}^{\infty} \epsilon_k (p_k \cos k\psi + q_k \sin k\psi) \quad (88)$$

and approximate it with any desired accuracy by means of the finite series, Equation 87. To illustrate, let $f(\psi)$ be defined by

$$f(\psi) = 0 \text{ when } 0 < \psi < \pi \quad (89)$$

$$f(\psi) = 1 \text{ when } \pi < \psi < 2\pi \quad (90)$$

If $l=\lambda/2$ the range of ψ is 2π . Expanding Equations 89 and 90 into a Fourier series we have

$$f(\psi) = \frac{1}{2} - \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{\sin (2k-1)\psi}{2k-1} \quad (91)$$

Consequently,

$$A_0 = \frac{1}{2}, \text{ and } A_k = 0 \text{ if } k \neq 0 \quad (92)$$

$$B_k = 0, \text{ if } k \text{ is even} \quad (93)$$

and

$$B_k = -\frac{1}{k\pi}, \text{ if } k \text{ is odd} \quad (94)$$

A satisfactory approximation by the finite series (87) gives

$$\sqrt{\Phi} = \frac{1}{\pi} \left[\frac{1}{9} + \frac{1}{7} z^2 + \frac{1}{5} z^4 + \frac{1}{3} z^6 + z^8 - \frac{i\pi}{2} z^9 - z^{10} - \frac{1}{3} z^{12} - \frac{1}{5} z^{14} - \frac{1}{7} z^{16} - \frac{1}{9} z^{18} \right] \quad (95)$$

The total length L of this array is

$$(n-1) \frac{\lambda}{2} = 2m \frac{\lambda}{2} = 9\lambda \quad (96)$$

Since the odd powers of z except z^9 are missing, a total of eleven units are required and all units except the three central ones are separated

by one wave-length. The amplitude distribution for the eleven units is $1/9, 1/7, 1/5, 1/3, 1, \pi/2, 1, 1/3, 1/5, 1/7, 1/9$, and the phase distribution is 0 degrees, 0 degrees, 0 degrees, 0 degrees, 0 degrees, -90 degrees, 180 degrees, 180 degrees, 180 degrees, 180 degrees, 180 degrees.

Thus, in accordance with the invention, assuming the use of an array of a given number of units and a given spacing between units, a sharper and narrower maximum space factor lobe, and a larger maximum-to-minor lobe ratio, is secured than is obtainable with the corresponding prior art equiamplitude arrangement or the prior art binomial amplitude arrangement disclosed in the J. S. Stone Patent 1,643,323 mentioned earlier herein. Conversely, a polynomial array of the invention, designed to secure a maximum lobe of given width and sharpness, is exceedingly short and comprises a very small number of antenna units, and hence requires a small number of transmission lines, as compared to one utilizing either the equiamplitude or the binomial amplitude distribution and designed to secure the same maximum lobe. Stated differently, in the equiamplitude and binomial systems, the lobe sharpness increases with, and primarily depends upon, the length and number of units utilized, whereas in the array of the invention these qualities are dependent upon, and controlled by, the polynomial amplitude and phase distributions. It follows that, in accordance with the invention, a less expensive and less complicated antenna system than heretofore employed may now be used to secure a given directive result.

Although the invention has been explained in connection with certain specific embodiments, it is to be understood that it is not limited to these embodiments since other structure and arrangements may be utilized without exceeding the scope of the invention.

What is claimed is:

1. In combination, a linear array comprising three antenna units, the spacing between adjacent units being one-quarter wave-length, a translation device, individual lines connecting said units to said device for conveying currents between said units and device, each line including an adjustable amplitude control device and an adjustable phase control device, said control devices being adjusted so that said currents have a 1, 1.41, 1 amplitude distribution and a 0 degree, 135 degree, 90 degree phase distribution.

2. A method of securing an exceedingly sharp maximum directive lobe for a linear array comprising more than two of evenly spaced antenna units connected through separate lines to a translation device, utilizing means for adjusting the amplitude and phase of the currents, which comprises spacing the adjacent units less than a half wave-length, evenly spacing the null directions of action for the array and adjusting the amplitudes and phases of the line currents to agree with the coefficients of the polynomial

$$\sqrt{\Phi} = |a_0 + a_1 z + a_2 z^2 + \dots + a_{n-2} z^{n-2} + z^{n-1}|$$

where Φ is the radio action along a direction in said plane, n is the number units, z is a complex variable dependent upon the unit spacing and the coefficients $a_0, a_1, a_2 \dots a_{n-2}$ are complex numbers representing the amplitude distribution of said currents and the phase deviations $\delta_0, \delta_1, \delta_2, \dots \delta_{n-2}$ of said currents from a given phase distribution having a progressive phase delay δ .

3. A method of obtaining, in a linear array

having a given number of units, an optimum space factor characteristic, said characteristic comprising a primary lobe having a relatively long principal radius and a prescribed angular width and a relatively small secondary lobe which comprises spacing the adjacent units a particular distance less than a half wave-length, selecting a null direction of array action adjacent to the primary lobe, selecting evenly spaced directions of action for the remaining array null directions, and adjusting the amplitude and phase distributions of the antenna currents to agree with the coefficients of the polynomial

$$\sqrt{\Phi} = |a_0 + a_1 z + a_2 z^2 + \dots + a_{n-2} z^{n-2} + z^{n-1}|$$

where Φ is the radio action along a direction in said plane, n is the number of units, z is a complex variable dependent upon the unit spacing and the coefficients $a_0, a_1, a_2, \dots a_{n-2}$ are complex numbers representing the amplitude distribution of said currents and the phase deviations $\delta_0, \delta_1, \delta_2 \dots \delta_{n-2}$ of said currents from a given phase distribution having a progressive phase delay δ .

4. A method of securing an optimum resultant directive characteristic in a given plane for a linear array of more than two evenly spaced directive antenna units connected through separate lines to a translation device and having a common null direction of action, utilizing means for adjusting the amplitude and phase distributions of the antenna currents conveyed by said lines, which comprises spacing the adjacent units a distance l less than a half wave-length, correlating all of the null directions $t_1, t_2, t_3, \dots t_{n-1}$ of action of said array with identified directions in said plane other than the said common null direction, and utilizing amplitude and phase distributions corresponding coefficients of the polynomial

$$\sqrt{\Phi} = |a_0 + a_1 z + a_2 z^2 + \dots + a_{n-2} z^{n-2} + z^{n-1}|$$

where Φ is the radio action along a direction in said plane, z is a complex variable dependent upon l and the coefficients $a_0, a_1, a_2, \dots a_{n-2}$ are complex numbers.

5. A method of securing in any given plane containing the array axis a sharp maximum directive lobe for a linear array of n , where n is greater than two, evenly spaced antenna units connected through separate lines to a translation device, utilizing means for adjusting the amplitude and phase of the line currents, which comprises spacing the adjacent units a distance l less than a half wave-length whereby the $n-1$ directions $t_1, t_2, t_3 \dots t_{n-1}$ of null radiant action for the array correspond to the zero values of the polynomial

$$\sqrt{\Phi} = |a_0 + a_1 z + a_2 z^2 + \dots + a_{n-2} z^{n-2} + z^{n-1}|$$

where z is a complex variable dependent upon l , the coefficients $a_0, a_1, a_2 \dots a_{n-2}$ are complex numbers and Φ is the instantaneous radiant action of the array in said plane, correlating said directions of null action with identified directions in said plane and adjusting the amplitude and phases of the antenna currents conveyed by said lines to agree with the coefficients of said polynomial.

6. In combination, a linear array comprising n antenna units having a given unit spacing l less than a half wave-length, a translation device connected to said units, the null directions of action for said array in a plane containing the array axis being preselected and corresponding

to the zero values $t_1, t_2, t_3, \dots, t_{n-1}$ of the equation

$$\sqrt{\Phi} = |(z - t_1)(z - t_2)(z - t_3) \dots (z - t_{n-1})|$$

where Φ is the radio action of the array along a direction in said plane and z is a complex variable having a value dependent upon l .

7. In combination, a linear array for utilizing waves having a wave-length λ , said array comprising n antenna units spaced a given distance l in wave-lengths, a translation device connected through said lines to said units, said lines including means for controlling the amplitude and phase distributions of the currents conveyed between said units and said device, the null di-

rections of action for said array in a plane containing the array axis being preselected and corresponding to the zero values $t_1, t_2, \dots, t_{n-1}$ of the equation

$$\sqrt{\Phi} = |a_0 + a_1 z + a_2 z^2 + \dots + a_{n-2} z^{n-2} + z^{n-1}|$$

where Φ is the radio action of the array along a direction in said plane, z is a complex variable dependent upon the distance l and the coefficients $a_0, a_1, a_2, \dots, a_{n-2}$ are complex numbers representing the amplitude distribution of said currents and the phase deviations $\delta_0, \delta_1, \delta_2, \dots, \delta_{n-2}$ of said currents from a given phase distribution having a progressive phase difference of δ .

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