

May 5, 1942.

W. P. MASON

2,281,778

PIEZOELECTRIC CRYSTAL APPARATUS

Filed Oct. 19, 1940

2 Sheets-Sheet 1

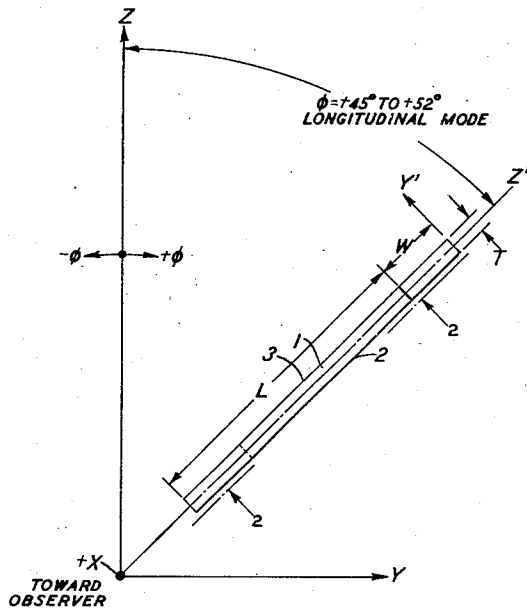


FIG. 1

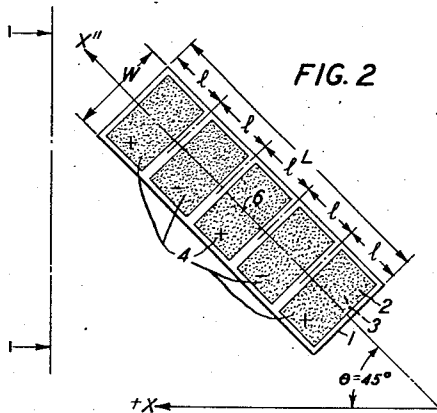


FIG. 2

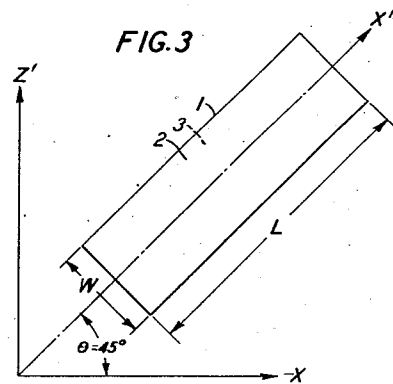


FIG. 3

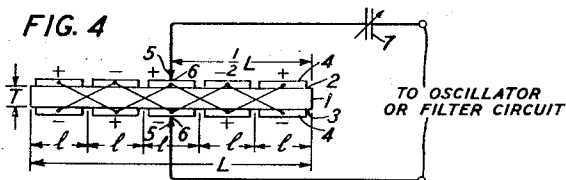


FIG. 4

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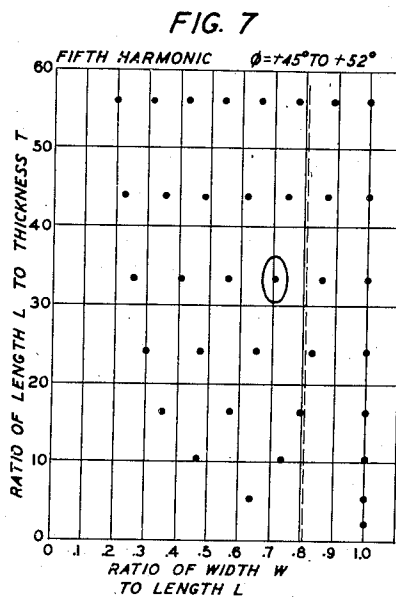
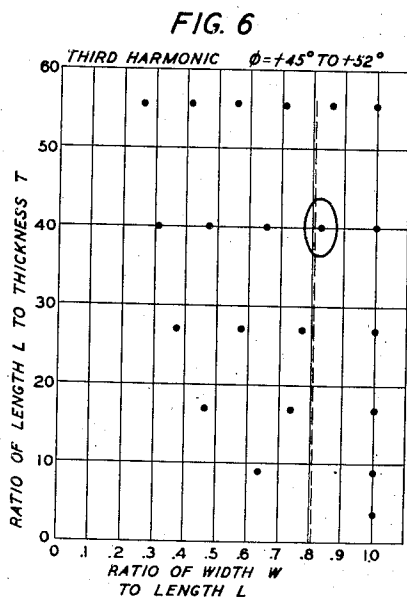
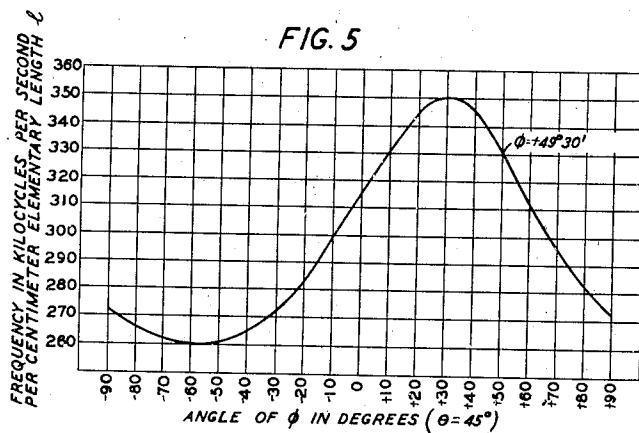
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PIEZOELECTRIC CRYSTAL APPARATUS

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2 Sheets-Sheet 2



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UNITED STATES PATENT OFFICE

2,281,778

PIEZOELECTRIC CRYSTAL APPARATUS

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11 Claims. (Cl. 171-327)

This invention relates to high frequency piezoelectric crystal apparatus and particularly to harmonic longitudinal mode piezoelectric quartz crystal elements adapted for use as circuit elements in such systems as electric wave filter systems and radio frequency oscillation generator systems, for example.

In my United States Patent No. 2,240,309 granted April 29, 1941 on application Serial No. 297,259, filed September 30, 1939, harmonic longitudinal mode quartz crystal elements of low or substantially zero temperature coefficient of frequency are described. In the present application, harmonic longitudinal mode quartz crystal elements are described which have much the same frequency-dimension characteristics but differ therefrom in orientation, and dimensional ratios.

One of the objects of this invention is to obtain relatively high frequency piezoelectric crystal elements of low impedance.

Another object of this invention is to obtain high frequency piezoelectric crystal elements substantially free from interfering vibrational modes and having fewer undesired extraneous or secondary resonances.

In electric wave filter systems such as wide band filters above 500 kilocycles per second for example, and other systems, it is often desirable to utilize crystal elements of relatively low impedance and having great freedom from extraneous or secondary resonances. For the relatively lower frequencies, this requirement is met by fundamental mode crystals of the type described in my United States Patent 2,204,762, dated June 18, 1940, but for the relatively higher frequencies, such fundamental mode crystals in some instances may be inconveniently small in size. It is the purpose of this invention to provide high frequency harmonic longitudinal mode crystal elements of low impedance and of fewer extraneous frequencies, that are capable of giving good stability for longitudinal mode frequencies up to 2 or more megacycles per second, the harmonic mode quartz crystal elements having such related orientation, dimensional ratios and vibrational mode as to obtain the desired low impedance and substantial freedom from undesired interfering modes within temperature ranges that occur in practice.

In accordance with this invention, a relatively thin piezoelectric quartz crystal plate of suitable orientation with respect to the X, Y and Z axes thereof, and of suitable dimensional ratios, may be subjected to a thickness direction or Y' elec-

tric field and vibrated at an odd or even harmonic resonance frequency dependent mainly upon the longest or major axis length dimension of the crystal plate in a mode of motion which consists of an harmonic longitudinal or extensional vibration along such length dimension giving the desired harmonic resonance frequency referred to. The orientation of the crystal plate may be any of several, the major axis or length dimension of the crystal plate being in every case inclined either about 45, or alternatively about 135 degrees, with respect to an electric axis X, and the major plane and major surfaces being in every case parallel or nearly parallel to such X axis and inclined with respect to the optic axis Z at any angle between about +45 and +52 degrees. Such quartz crystal plates when suitably proportioned as to relative thickness, width and length dimensions produce, for the desired harmonic longitudinal mode resonant frequency mentioned, a substantial freedom from extraneous frequencies at temperatures within ordinary temperature ranges. In a particular species where the major plane of the crystal plate is inclined at an angle about from +45 to +52 degrees with respect to the Z axis, and the width dimension of the harmonic mode crystal plate is related to the above-mentioned length dimension in the ratio of less than about 0.8, and the thickness dimension is related to the length and width dimensions in proper proportions, as given hereinafter, the desired longitudinal mode harmonic resonant frequency referred to is relatively free from interfering modes such as the undesired shear and flexure modes.

To reduce the temperature coefficient of frequency of such crystal elements operating at or near the desired resonant longitudinal mode frequency, a condenser of suitable capacity may be connected in series circuit relation with the crystal element, the condenser itself having a temperature coefficient of capacitance of such magnitude and sign with respect to that of the crystal element as to reduce or balance that of the resonant frequency of the crystal thereby reducing the over-all temperature coefficient of frequency of the combination.

For a clearer understanding of the nature of this invention and the additional advantages, features and objects thereof, reference is made to the following description taken in connection with the accompanying drawings, in which like reference characters represent like or similar parts and in which:

Figs. 1 and 2 are enlarged views of a harmonic

longitudinal mode piezoelectric quartz crystal plate embodying this invention, Fig. 1 being a projected edge view taken in the horizontal direction indicated by the arrows 1—1 of Fig. 2, and Fig. 2 being a major face view taken in the direction indicated by the arrows 2—2 of Fig. 1;

Fig. 3 is a major face view of a quartz plate similar to that of Fig. 2 but having an alternative 45-degree orientation angle with respect to the X axis;

Fig. 4 is an edge view of the crystal plate of Figs. 1 to 3 provided with electrodes and electrode connections for fifth harmonic mode operation;

Fig. 5 is a graph showing the calculated relation between the orientation angle and the desired longitudinal mode resonant frequency of quartz crystals in accordance with this invention; and

Figs. 6 and 7 are graphs showing by points thereon, preferred values of dimensional ratios of length, width, and thickness for third and fifth harmonic quartz crystal elements respectively, in accordance with this invention.

This specification follows the conventional terminology as applied to crystalline quartz which employs three orthogonal or mutually perpendicular X, Y and Z axes, as shown in the drawings, to designate an electric, a mechanical and the optic axes, respectively, of piezoelectric quartz crystal material, and which employs three orthogonal axes X', Y' and Z' to designate the directions of axes of a piezoelectric body angularly oriented with respect to such X, Y and Z axes thereof. Where the orientation is obtained by double rotations of the quartz crystal element 1, one rotation being in effect substantially about an electric axis X, and the other about the Y' axis of the piezoelectric body as illustrated in Figs. 1 and 2, the orientation angles ϕ and θ respectively, designate in degrees the effective angular position of the crystal plate 1 as measured from the optic axis Z and from the orthogonal electric axis X, respectively. The length or longitudinal axis X'' shown in Figs. 2 and 3 indicates the result of a second rotation.

Quartz crystals may occur in two forms, namely, right-handed and left-handed. A right-handed quartz crystal is one in which the plane of polarization of a plane polarized light ray traveling along the optic axis Z in the crystal is rotated in a right-hand direction, or clockwise as viewed by an observer located at the light source and facing the crystal. This definition of right-handed quartz follows the convention which originated with Herschel. Trans. Cam. Phil. Soc. vol 1, page 43 (1821); Nature vol. 110, page 807 (1922); Quartz Resonators and Oscillators, P. Vigoureux, page 12 (1931). Conversely, a quartz crystal is designated as left-handed if it rotates such plane of polarization referred to in the left-handed or counter-clockwise direction, namely, in the direction opposite to that given hereinbefore for the right-handed crystal.

If a compressional stress or a squeeze be applied to the ends of an electric axis X of a quartz body 1 and not removed, a charge will be developed which is positive at the positive end (+) of the X axis and negative at the negative end (-) of such electric axis X, for either right-handed or left-handed crystals. The magnitude and sign of the charge may be measured in a known manner with a vacuum tube electrometer for example. In specifying the orientation

of a right-handed crystal, the sense of the angle ϕ which the new axis Z' makes with respect to the optic axis Z as the crystal plate is rotated in effect about the X axis is deemed positive (+) when, with the compression positive end (+) of the X axis pointed toward the observer, the rotation is in a clockwise direction as illustrated in Fig. 1. A counter-clockwise rotation of such a right-handed crystal about the X axis gives rise to a negative orientation angle ϕ with respect to the Z axis. Conversely, the orientation angle of a left-handed crystal is positive when, with the compression positive end (+) of the electric axis X pointed toward the observer, the rotation is counter-clockwise, and is negative when the rotation is clockwise. The crystal material illustrated in Figs. 1 to 3 is right-handed as the term is used herein. For either right-handed or left-handed quartz, a positive (+) angle ϕ rotation of the Z' axis with respect to the Z axis as illustrated in Fig. 1 is toward parallelism with the plane of a minor apex face of the natural quartz crystal, and a negative (-) ϕ angle rotation of the Z' axis with respect to the Z axis is toward parallelism with the plane of a major apex face of the natural quartz crystal.

Referring to the drawing, Figs. 1 and 2 are respectively an edge view and a major face view of a right-handed relatively thin piezoelectric quartz crystal plate 1 of substantially rectangular parallelepiped shape having an over-all length dimension L, a width dimension W which is perpendicular to the length dimension L, and a thickness or thin dimension T which is perpendicular to the length dimension L and the width dimension W. As shown in Fig. 1, the major plane and the opposite major faces 2 and 3 of the crystal plate 1 may be parallel or nearly parallel to an electric or X axis of the quartz material and inclined with respect to the optic axis Z at a ϕ angle of about from +45 to +52 degrees as measured between the Z and Z' axes in a plane which is perpendicular to the X axis and to the major plane of the crystal plate 1. Small angle departures up to 5 degrees or more, for example, of the major faces 2 and 3, from parallelism with respect to the X axis do not greatly alter the ϕ angle and dimensional ratios required to obtain the result of substantial freedom from coupling to extraneous modes of vibration. Since the minor apex faces of the natural quartz crystal from which the quartz plate 1 is cut occur at a ϕ angle of about +38 degrees with respect to the optic axis Z, the positive (+) sense for the ϕ angle is equivalent to a rotation about the X axis from the Z axis toward parallelism with the plane of a minor apex face for either right-handed or left-handed quartz.

In Fig. 1, the X axis is perpendicular to the plane of the drawing with the compression positive end (+) of the X axis pointed towards the observer, and is also perpendicular to both the Y and Z axes. The over-all length dimension L of the crystal plate 1 lying along the major axis X'' as shown in Figs. 2 and 3 may be inclined at an angle θ of about 45 degrees with respect to the above-mentioned X axis in either direction as illustrated by the alternative θ angle orientations shown in Figs. 2 and 3. While the major axis length dimension L of the crystal plate 1 of Fig. 2 is inclined at a different 45-degree θ angle with respect to the X axis than that of Fig. 3, it will be understood that either

of these 45-degree positions for the angle θ may be used alternatively with any of the ϕ angles disclosed herein.

Suitable conductive electrodes such as the electrodes 4 of Figs. 2 and 4 may be placed on, adjacent or be formed integral with the opposite major faces 2 and 3 of the crystal plate 1 to apply electric field excitation to the quartz plate 1 in the direction of the thickness dimension T, and by means of any suitable circuit such as, for example, a filter or an oscillator circuit, the quartz plate 1 may be vibrated in the desired longitudinal mode of motion along the length dimension L at an odd or an even harmonic vibrational response frequency which depends mainly upon and varies inversely as the major axis length dimension L and the elementary length dimension l. It will be understood that the crystal plate 1 may be operated in any desired odd or even harmonic longitudinal mode along the length dimension L by means of a plurality of pairs of equal area opposite electrodes disposed adjacent the opposite major faces 2 and 3 of the crystal plate 1. The electrodes may be interconnected as illustrated schematically in Fig. 4 so that all positive (+) electrodes are connected together and all negative (—) electrodes are connected together but no positive electrode is connected with a negative electrode. The number of pairs of opposite electrodes to be used corresponds to the numerical order of the desired harmonic which may be any odd or even overtone of the fundamental mode. For example, to drive the crystal 1 in the fifth harmonic longitudinal mode, five pairs of equal area opposite electrodes 4 may be utilized as illustrated in Figs. 2 and 4; and similarly to drive the crystal plate 1 in third harmonic longitudinal vibrations along the length dimension L, three pairs of opposite electrodes which may partly or nearly wholly cover the equal elementary lengths l of the major faces 2 and 3 may be utilized. The odd harmonic longitudinal mode is of special interest since then the crystal plate 1 may be clamped at its geometrical center 6 at the centers of the middle pair of opposite electrodes 4 as illustrated in Figs. 2 and 4. Reference is made to my United States Patent No. 2,185,599 granted January 2, 1940 for examples of harmonic longitudinal mode electrode and electrode connection arrangements that may be utilized to drive any of the crystal plates described herein in harmonic mode longitudinal vibrations along the major axis length dimension L. These harmonic mode electrode and connection platings may be such as to leave three edges of the crystal body entirely free of any plating in order to make edge grinding adjustments of the frequency and the temperature coefficient of frequency.

The harmonic mode crystal plates 1 described herein may be mounted in any suitable manner such as, for example, by clamping the electroded crystal plate 1 between a pair of opposite conductive clamping projections 5 which may contact the electroded crystal plate 1 at opposite points of very small area designated 6 in Figs. 2 and 4. As an illustrative example, an evacuated holder of the type disclosed in United States Patent No. 2,203,486 granted June 4, 1940 to W. L. Bond may be utilized for this purpose.

Alternatively, the electroded crystal plate 1 may be supported by soldering or otherwise attaching one or more pairs of electrically conductive spring wires to one or more or any pair

such as the middle pair of the crystal electrodes 4, at the opposite points designated 6 in Figs. 2 and 4. Such conductive wires may support and hold the electroded crystal plate 1 in spring suspension.

It will be understood that any holder which will give stability and a relatively high reactance-resistance ratio, Q, may be used to mount these harmonic mode crystal elements 1.

The desired longitudinal mode resonant frequency of the harmonic mode crystal plate 1 is a function of the major axis length dimension L and of the several equal elementary or fundamental lengths l, the over-all length L being equal to the n times the elementary length dimension l where n is the numerical order of the harmonic as determined by the number of pairs of opposite electrodes 4 that are applied to the major faces 2 and 3 of the crystal plate 1 of any ϕ angle. Since the invention may be adapted to any order of harmonic operation, odd or even, the correlated values of longitudinal mode frequency and ϕ angle, are given in Fig. 5 in terms of the elementary length dimension l. It will be noted that in so far as the elementary areas are concerned, the related values of the orientation, and the frequency constant of the several elementary areas of the harmonic mode crystals of this application follow these values given for the fundamental mode crystals of the corresponding orientation, described in United States Patent 2,204,762 referred to.

Fig. 5 is a graph giving the calculated values of the desired longitudinal mode frequency in kilocycles per second per centimeter of elementary length dimension l of the crystal plate 1 for all angles of ϕ from -90 to $+90$ degrees, the angle θ being always equal to about 45 (or 135) degrees for every angle of ϕ . For any given angle of ϕ such as the ϕ angle of $+49^\circ 30'$, the curve of Fig. 5 gives the approximate frequency constant corresponding thereto in terms of frequency in kilocycles per second per centimeter of the elementary length dimension l of the crystal plate 1 or n times this value where n is the numerical order of the harmonic frequency such as the third or fifth harmonic, for example. Since the frequency of the longitudinal mode vibrations along the length l varies inversely as the particular length dimension of l involved, the value of l in centimeters corresponding to the resonant frequency in kilocycles per second may be obtained directly from the frequency dimension constant given by the curve of Fig. 5 for any value of the angle ϕ selected. The calculated values of the desired resonant frequency given in Fig. 5 approximate the measured values.

The graphs of Figs. 6 and 7 give for the orientation angles of ϕ from about $+45$ to $+52$ degrees, the angle θ being substantially 45 degrees, the corresponding dimensional ratios of width, thickness and length that may be used to construct third and fifth harmonic quartz plates 1 of rectangular parallelepiped shape to obtain the desired longitudinal mode frequency substantially free from undesired extraneous flexure mode frequencies. The preferred values of dimensional ratios therein substantially given by the points are in terms of the over-all length dimension L with respect to the thickness dimension T of the crystal plate 1, and also in terms of the width dimension W with respect to the overall length dimension L. Each of the elementary lengths l of the crystal plate 1 are equal to L/n where L is the major axis over-all length dimension of

the crystal plate l and n is the numerical order of the harmonic, namely, the third and the fifth harmonics covered in Figs. 6 and 7. The series of points of Fig. 6 give the preferred dimensional ratios for third harmonic longitudinal mode crystal elements l in terms of the width W , the length L and the thickness T for all of the positive angles of ϕ between about $+45$ and $+52$ degrees while the series of points of Fig. 7 give the preferred dimensional ratios for fifth harmonic longitudinal mode crystal elements l for all of the same positive angles of ϕ between about $+45$ and $+52$ degrees, the angle θ in every case being about 45 degrees as illustrated in either Fig. 2 or Fig. 3. The angles of ϕ from about $+45$ to $+52$ degrees covered in Figs. 6 and 7 represent the preferred range of ϕ angles.

The corresponding longitudinal mode frequency constants for the quartz plates l , oriented and dimensioned in accordance with the values given by the graphs of Figs. 6 and 7 are substantially given by the curve of Fig. 5 at the intercept of the particular value of ϕ selected. For example, when ϕ is substantially $+49^\circ 30'$, θ being substantially 45 degrees, the frequency constant is about 330 kilocycles per second per centimeter of elementary length dimension l or n times this value per centimeter of over-all length dimension L where n is the numerical order of the harmonic involved such as $3, 5$, etc. For example, a fifth harmonic quartz crystal plate l of such an orientation and of one centimeter over-all length L will have a desired longitudinal mode resonant frequency of about five times 330 or about 1650 kilocycles per second. Other values of corresponding orientation and frequency for quartz plates l may be obtained from the curve of Fig. 5 for any angle of ϕ , the preferred ϕ angles between about $+45$ and $+52$ degrees giving a frequency of about 328 to 333 kilocycles per second per centimeter of elementary length l .

In Figs. 6 and 7, the series of points represent the centers of the regions for the dimensional ratios which may be utilized to obtain the desired harmonic longitudinal mode vibrations in quartz crystal elements l substantially free from undesired flexural mode vibrations, Fig. 6 giving such information for the third harmonic longitudinal mode crystal elements l and Fig. 7 for the fifth harmonic longitudinal mode crystal elements l . As illustrated by the regions within the elliptical-shaped curves surrounding two of the points in Figs. 6 and 7, the preferred regions for the dimensional ratios may extend about a quarter of the way between any of the points illustrated. Accordingly, it will be understood that the graphs of Figs. 6 and 7 each comprehend a plurality of regions or spots each having as its center one of the points shown and while for optimum or most nearly ideal conditions, the dimensional relations of the piezoelectric element l may conform to one of the points, the advantages of the invention may be secured in varying degrees with any relationship indicated by the graph. In other words any position falling within an elliptical region centering at one of the points of Fig. 6 or Fig. 7 and which region may extend outwardly from the point not more than roughly a quarter of the distance to the nearest point, may be advantageously utilized. Such regions may be utilized for the dimensional ratios of the crystal elements l to separate the undesired flexural mode frequencies from the desired main length mode harmonic frequency. The location of such preferred regions free from

the undesired flexure mode frequencies is independent of the ϕ angle orientations from $+45$ to $+52$ degrees since both depend on the same elastic constant, namely the Young's modulus in the direction of propagation and hence their ratio is unchanged by the orientation ϕ angle. The same is nearly true for the undesired secondary shear frequencies since the shear constant varies in the same way with the orientation angle ϕ . In order to separate the undesired shear frequency from the desired main length mode harmonic frequency by at least 8 per cent, the ratio of the width dimension W with respect to the length dimension L may be less than about 0.8 which is indicated by the vertical broken lines in Figs. 6 and 7. Accordingly, third and fifth harmonic crystal elements l having dimensional ratios selected from Figs. 6 and 7 to the left of such vertical broken lines and near one of the series of points thereon may have the desired harmonic mode longitudinal frequency substantially free from interfering extraneous modes of the flexural type as well as of the shear type.

As an example, a fifth harmonic longitudinal mode crystal element l having a ϕ angle of substantially $+49^\circ 30'$, a thickness dimension T of 1 millimeter, a width dimension W of 23.02 millimeters, and a length dimension L which varies from 36 to 32 millimeters is illustrated by the dimensional ratios of length L to thickness T and of width W to length L given roughly within the ellipse-shaped curve of Fig. 7. At the point at the center of such ellipse-shaped curve of Fig. 7, the dimensional ratio of the length L with respect to the thickness T is about 33 and the dimensional ratio of the width W with respect to the length L is about 0.7 . Such a fifth harmonic crystal element l having a length dimension L of 33 millimeters has a desired longitudinal mode of fifth harmonic frequency of about 503 kilocycles per second which is substantially free from interfering modes such as the harmonics of flexure modes therein.

In order to avoid the undesired flexural harmonic modes referred to, it will be understood that in choosing the thickness dimension T of the crystal element l of any given or desired harmonic longitudinal mode frequency, it is desirable to avoid and preferably deviate as far as possible from any value of thickness dimension T that satisfies the following dimensional ratios; one should therefore use a thickness dimension T of a value other than and somewhere intermediate the values corresponding to these dimensional ratios, and preferably of a value midway or nearly midway between the nearest or adjacent values given by the dimensional relations:

$$(1) \quad \frac{L}{T} = \frac{m^2}{10.88n}$$

and

$$(2) \quad \frac{W^2}{LT} = \frac{m^2}{10.88n}$$

where L is the over-all length dimension of the crystal element l along the direction of the desired harmonic longitudinal vibration, W is the width dimension of the crystal element l perpendicular to the length dimension L , T is the thickness dimension measured along the Y' axis, and n is the numerical order of the selected longitudinal mode harmonic frequency which may be of any numerical order such as one of the values 2 to 5 for example. The values of m for the undesired even order harmonics of flexure

mode vibrations are given by the relation:
 $m_2=7.853$; $m_4=14.137$ and thereafter

$$m_i=\pi(i+1/2)$$

to seven decimal places, where i is any even order integer, such as 2, 4, 6, 8, etc. The foregoing equations have been used in obtaining the points plotted on Figs. 6 and 7, and may be similarly applied in connection with crystal elements l of other harmonic mode frequencies where n is a value according to the order of the harmonic. In the case of Fig. 6, the points plotted thereon represent optimum values of length L with respect to thickness T , and of width W with respect to length L , for third harmonic crystal elements l , and are obtained from the foregoing equations using a value of $n=3$, with assumed values of m and calculated values of

$$\frac{m^2}{10.88n}$$

roughly as follows:

m	$\frac{L}{T}$ - Equation 1 = $\frac{m^2}{10.88n}$
$m^2=\pi(2+\frac{1}{2})=7.85$	1.95
$m^4=\pi(4+\frac{1}{2})=14.14$	6.15
$m^6=\pi(6+\frac{1}{2})=20.5$	12.9
$m^8=\pi(8+\frac{1}{2})=26.8$	22.1
$m^{10}=\pi(10+\frac{1}{2})=33.0$	33.5
$m^{12}=\pi(12+\frac{1}{2})=39.3$	47.5
$m^{14}=\pi(14+\frac{1}{2})=45.5$	63.8

The seven values of

$$\frac{m^2}{10.88n}$$

given in the foregoing table represent values for L/T in Equation 1 that are to be avoided, as mentioned hereinbefore, and accordingly the six horizontal rows of points on Fig. 6 are placed about midway between such values. For example, the horizontal row of five points shown on Fig. 6 opposite the ratio of 40 thereon, are placed midway between the two values of 33.5 and 47.5 given in the foregoing table, the L/T ratio of 40 accordingly being one of the optimum L/T ratios for third harmonic crystal elements l , as shown on Fig. 6. Assuming a given length dimension L of a suitable value to give the desired third harmonic longitudinal mode frequency, the thickness dimension T of the crystal may be made of any value that avoids the

$$\frac{m^2}{10.88n}$$

values given in the foregoing table, and preferably is made of a value that is midway between any pair of such values, as, for example, the L/T ratio of 40 given on Fig. 6, which is about midway between the values 33.5 and 47.5 given in the foregoing tabulation.

In the example given, with the length L of the crystal determined by the desired third harmonic frequency and the L/T ratio determined by Equation 1 and selected as 40, the W/L ratios to be avoided are given by Equation 2, using the same

$$\frac{m^2}{10.88n}$$

values as were used in connection with Equation 1 and will be found by calculation, using Equation 2, to be as follows for the third harmonic

crystal element having an L/T dimensional ratio of 40:

$\frac{m^2}{10.88n}$	$\frac{W}{L}$ - Equation 2
1.95	.22
6.15	.39
12.9	.56
22.1	.75
33.5	.91
47.5	1.1

It will be noted that the horizontal row of the five points, shown opposite the L/T ratio of 40 on Fig. 6 and representing optimum W/L ratios for an optimum L/T ratio of 40, are located midway between the six calculated values of .22, .39, .56, .75, .91, and 1.1 given in the above tabulation. Similarly, the optimum W/L ratios for the other optimum L/T ratios given by the points on Fig. 6 are obtained from the foregoing equations.

In the case of Fig. 7, the optimum L/T and W/L ratios for fifth harmonic crystal elements are similarly obtained from the foregoing equations using a value of $n=5$ where n is the numerical order of the harmonic frequency.

Accordingly, in such crystals, the length L value being determined by the desired frequency, the thickness T value is determined by Equation 1 and the width W value by Equation 2 for any order of harmonic, and crystal elements having their dimensions so proportioned may be rendered substantially free from the undesired flexural mode frequencies referred to. The remaining undesired resonances are probably due to coupling with shear modes of motion and may be minimized by selecting a suitable ratio of the width dimension W with respect to the length dimension L less than about 0.8 after selecting the proper ratio for the width dimension W to avoid the undesired flexural even order harmonic modes referred to. While the graphs of Figs. 6 and 7 illustrate the preferred dimensional ratios for only the third and fifth harmonic longitudinal mode crystal elements l having ϕ angles from $+45$ to $+52$ degrees, the θ angle being about 45 degrees in every case, the preferred dimensional ratios for other harmonics of the desired longitudinal mode may be similarly obtained from the dimensional ratio equations hereinbefore given.

The harmonic longitudinal mode crystal elements l having ϕ angles within the range from $+45$ to $+52$ degrees, the θ angle always being substantially 45 degrees, have a ratio of capacities of the order of 300 to 600, the $+45$ degree ϕ angle crystal element l giving the lower value and the $+52$ degree ϕ angle crystal element l giving the larger value of ratio of capacities.

The temperature coefficient of the desired harmonic longitudinal mode frequency of such crystal elements l having ϕ angles from about $+45$ to $+52$ degrees and having dimensional ratios selected to avoid the undesired harmonic flexural modes is of the order of 15 to 30 parts per million per degree centigrade, the $+45$ degree ϕ angle crystal element l giving the higher value and the $+52$ degree ϕ angle crystal element l giving the lower value of temperature coefficient of frequency. A crystal element l having a ϕ angle in the region of $+49^\circ 30'$ has a ratio of capacities that is nearly as good as that of a $+45$ degree ϕ angle crystal element l and a temperature coefficient of frequency that is

much lower. For a thickness dimension T of less than 0.5 millimeter, the equivalent inductance of such a crystal element I is around 0.3 to 0.5 henry.

While the temperature coefficient of frequency of such crystal elements I is of the order of 15 or more parts per million per degree centigrade, this change in frequency due to temperature change results only in a relatively small change in frequency in a filter system. For example, a filter at 1,800,000 cycles per second having a temperature coefficient of frequency of 15 parts in a million per degree centigrade will change its frequency only 27 cycles per degree centigrade. For a room temperature change from 60° fahrenheit to 110° fahrenheit, for example, this would result in a shift in the filter characteristic of ± 370 cycles in the edge of the pass band. If 5 kilocycles are allowed for the filter loss to come from the edge of the band pass to the high attenuation point, a cut-off of 0.27 per cent results, which is a relatively small change in frequency. When the dimensioned crystal element I is utilized in a filter system, the extraneous resonances may be placed so that they do not come at low impedance points in the equivalent lattice. Illustrative examples of filter circuits in which the crystal element I may be utilized are shown in W. P. Mason Patent 1,967,250, dated July 24, 1934, Fig. 4, and in W. P. Mason Patent 2,212,840, dated August 27, 1940, Fig. 1, for example.

The frequency of any of these longitudinal mode crystal plates I may be adjusted to relatively precise values by edge grinding on the edges along the width W of the crystal plate. Since the frequency is controlled mainly by the major axis length dimension L, the frequency of a slightly oversize crystal plate I may be increased by grinding on either of the edges that are perpendicular to the length dimension L thereby reducing the length dimension L and increasing the frequency to a desired value. By grinding on either of the longest edges perpendicular to the width dimension W, the width dimension W may be uniformly reduced thereby changing the dimensional ratio of the width W with respect to the length L of the crystal plate I until the desired value is obtained. By the process of edge grinding, the frequency of the crystal plate I may be ultimately adjusted to the correct or desired value.

These harmonic mode crystals may be used in wide band filters for example at radio frequencies up to 2 megacycles per second or more with substantial freedom from troublesome subsidiary or extraneous resonances. Wide band quartz crystal filters have heretofore been limited to about 500 kilocycles per second due to the extra resonances existing in high frequency crystals. In carrier systems, it is often desirable to have high and low-pass filters of very sharp selectivity for the purpose of dropping off supergroups at intermediate points and hence to have crystals which can be used in wide band filters at frequencies up to 2 megacycles per second or more and which have relatively low impedance, and vibrate at high frequencies with the desired frequency of vibration separated as much as possible from the frequencies of other modes. Any nearby undesired resonance that is present may, if caused by a flexure mode determined by the thickness dimension T of the crystal plate I, be removed by changing the ratio of the thickness dimension T with respect

to the length dimension L, without changing the desired resonance frequency or the temperature coefficient of the crystal. The thickness dimension T of the crystal plate I may ordinarily be of the order of 1 millimeter more or less for example, and of a value to suit the impedance or other requirements of the particular circuit with which it may be associated. Either the third harmonic or fifth harmonic crystal elements I, for example, may be used to advantage in oscillators or moderately high impedance filters.

By using an electrical element which varies its impedance with temperature change, the temperature coefficient of frequency of the crystal circuit may be adjusted and over-all variations in the frequency of an oscillator that may be associated therewith may be considerably reduced. For this purpose, as illustrated in Fig. 4, a condenser 7 may be connected in series circuit relation with the electroded crystal plate I. The condenser 7 may have a temperature coefficient of capacitance of such magnitude and sign as to balance that of the crystal plate I and thereby reduce the over-all temperature coefficient of frequency of the combination crystal element and condenser.

A small trimmer condenser of suitable capacity for example, of the order of 20 micro-microfarads more or less may be connected in parallel circuit relation with the electrode terminals 5 of the crystal element I to adjust the frequency thereof to the desired final value. As an illustrative example, such as condenser may consist of a thin mica sheet having conductive material such as silver or other suitable conductive material deposited in a known manner upon the opposite major surfaces thereof. The adjustment of capacitance may be made by scraping off or otherwise removing part of the silver coating until the desired values of capacitance and frequency are obtained.

Although this invention has been described and illustrated in relation to specific arrangements, it is to be understood that it is capable of application in other organizations and is, therefore, not to be limited to the particular embodiments disclosed, but only by the scope of the appended claims and the state of the prior art.

What is claimed is:

1. A piezoelectric quartz crystal vibratory body having its opposite substantially rectangular major faces substantially parallel to an X axis and inclined at an angle of substantially $\pm 49^\circ 30'$ with respect to the Z axis as measured in a plane substantially perpendicular to said major faces, the major axis over-all length dimension and the width dimension of said major faces being inclined substantially 45 degrees with respect to said X axis, said over-all length dimension being in effect divided into a plurality of equal length elementary lengths corresponding to the numerical order of a desired harmonic frequency, said width having a dimensional ratio of less than substantially 0.8 with respect to said over-all length dimension, the thickness dimension between said major faces being one of the values substantially midway between the nearest values given by the relations:

$$\frac{L}{T} = \frac{m^2}{10.88n}$$

and

$$\frac{W^2}{LT} = \frac{m^2}{10.88n}$$

where L, T and W are respectively said over-all

length, thickness, and width dimensions expressed in the same units, n is the numerical order of said harmonic frequency, and m is equal to $\pi(i+1/2)$ where i is an even order integer that is one of the integers 2, 4, 6, 8, 10, 12, 14, 16 and 18.

2. A piezoelectric quartz crystal vibratory body having its opposite substantially rectangular major faces substantially parallel to an X axis and inclined at one of the angles substantially from +45 to +52 degrees with respect to the Z axis as measured in a plane substantially perpendicular to said major faces, the major axis over-all length dimension and the width dimension of said major faces being inclined substantially 45 degrees with respect to said X axis, said over-all length dimension being in effect divided into a plurality of equal length elementary lengths corresponding to the numerical order of a desired harmonic frequency to form a plurality of elementary areas the number of said elementary lengths being one of the integers 2 to 5, the thickness dimension between said major faces being a value substantially intermediate the values given by the relations:

$$\frac{L}{T} = \frac{m^2}{10.88n}$$

and

$$\frac{W^2}{LT} = \frac{m^2}{10.88n}$$

where L, T and W are respectively said over-all length, thickness, and width dimensions expressed in the same units, n is the numerical order of said harmonic frequency, and m is equal to $\pi(i+1/2)$ where i is an even order integer that is one of the integers 2, 4, 6, 8, 10, 12, 14, 16 and 18.

3. A piezoelectric quartz crystal vibratory body having its opposite substantially rectangular major faces substantially parallel to an X axis and inclined at one of the angles substantially from +45 to +52 degrees with respect to the Z axis as measured in a plane substantially perpendicular to said major faces, the major axis over-all length dimension and the width dimension of said major faces being inclined substantially 45 degrees with respect to said X axis, said over-all length dimension being in effect divided into a plurality of equal length elementary lengths corresponding to the numerical order of a desired harmonic frequency to form a plurality of elementary areas, the number of said elementary lengths being one of the integers 2 to 5, the thickness dimension between said major faces being a value intermediate the values given by the relations:

$$\frac{L}{T} = \frac{m^2}{10.88n}$$

and

$$\frac{W^2}{LT} = \frac{m^2}{10.88n}$$

where L, T and W are respectively said over-all length, thickness, and width dimensions expressed in the same units, n is the numerical order of said harmonic frequency, and m is equal to $\pi(i+1/2)$ where i is an even order integer, said width having a dimensional ratio of less than substantially 0.8 with respect to said over-all length dimension, and electrodes adjacent each of said elementary areas of said major faces of said crystal body.

4. A piezoelectric quartz crystal vibratory body having its opposite substantially rectangular

major faces substantially parallel to an X axis and inclined at one of the angles substantially from +45 to +52 degrees with respect to the Z axis as measured in a plane substantially perpendicular to said major faces, the major axis over-all length dimension and the width dimension of said major faces being inclined substantially 45 degrees with respect to said X axis, said over-all length dimension being in effect divided into a plurality of equal length elementary lengths corresponding to the numerical order of a desired harmonic frequency to form a plurality of elementary areas, the number of said elementary lengths being one of the integers 2 to 5, the thickness dimension between said major faces being a value substantially midway between the nearest values given by the relations:

$$\frac{L}{T} = \frac{m^2}{10.88n}$$

and

$$\frac{W^2}{LT} = \frac{m^2}{10.88n}$$

where L, T, and W are respectively said over-all length, thickness, and width dimensions expressed in the same units, n is the numerical order of said harmonic frequency, and m is equal to $\pi(i+1/2)$ where i is an even order integer, and means including electrodes formed integral with said elementary areas of said major faces for operating said crystal body in said mode of motion consisting substantially of harmonic longitudinal vibrations along said major axis length dimension.

5. A piezoelectric quartz crystal body adapted to vibrate at a harmonic frequency dependent mainly upon its major axis length dimension, the major faces of said body being substantially parallel to an X axis and inclined at one of the angles substantially from +45 to +52 degrees with respect to the Z axis as measured in a plane perpendicular to said major faces, said length dimension and the width dimension of said major faces being inclined substantially 45 degrees with respect to said X axis, said frequency being substantially 330 kilocycles per second per centimeter of said length dimension arithmetically multiplied by the numerical order of said harmonic frequency, the thickness dimension between said major faces being a value substantially intermediate the values given by the relations:

$$\frac{L}{T} = \frac{m^2}{10.88n}$$

and

$$\frac{W^2}{LT} = \frac{m^2}{10.88n}$$

where L, T and W are respectively said over-all length, thickness and width dimensions expressed in the same units, n is the numerical order of said harmonic frequency, and m is equal to $\pi(i+1/2)$ where i is an even order integer that is one of the integers 2, 4, 6, 8, 10, 12, 14, 16 and 18.

6. A piezoelectric quartz crystal plate having substantially rectangular major faces and means including a plurality of pairs of electrodes disposed adjacent said major faces for operating said crystal plate at a harmonic frequency dependent mainly upon the major axis dimension of said major faces, said pairs corresponding in number to the numerical order of said harmonic and being disposed along the equal elementary lengths of said major axis dimension, said major

faces being substantially parallel to an X axis and inclined substantially $+49^{\circ} 30'$ with respect to the Z axis as measured in a plane perpendicular to said major faces, said major axis of said major plane being inclined substantially 45 degrees with respect to said X axis, the dimensional ratio of the width dimension with respect to said major axis dimension of said major faces being less than substantially 0.8, the thickness dimension between said major faces being a value substantially midway between the nearest values given by the relations:

$$\frac{L}{T} = \frac{m^2}{10.88n}$$

and

$$\frac{W^2}{LT} = \frac{m^2}{10.88n}$$

where L, T, and W are respectively said over-all length, thickness, and width dimensions expressed in the same units, n is the numerical order of said harmonic frequency, and m is equal to $\pi(i+1/2)$ where i is an even order integer.

7. A piezoelectric quartz crystal body adapted to vibrate at a third harmonic frequency dependent mainly upon each of the fundamental or elementary length dimensions along the major axis of said body multiplied by said numerical order of said harmonic frequency, the major plane of said body being substantially rectangular and disposed substantially parallel to an X axis and inclined at an angle from $+45$ to $+52$ degrees with respect to the Z axis, said major axis of said major plane being inclined substantially 45 degrees with respect to said X axis, the dimensional ratios of length, width and thickness dimensions of said body being substantially a set of values given by the graph of Fig. 6, and the frequency for each centimeter of said elementary length dimensions being given by the curve of Fig. 5 substantially at the intercept for said angle.

8. A piezoelectric quartz crystal body, the major axis length dimension of said body being divided in effect into three equal elementary lengths in accordance with the numerical order of the harmonic selected to obtain third harmonic mode vibrations along said major axis dimension, the major faces of said body being substantially rectangular, substantially parallel to an X axis and inclined at an angle from $+45$ to $+52$ degrees with respect to the Z axis, said major axis length dimension being inclined substantially 45 degrees with respect to said X axis, the dimensional ratios of the length, width and thickness of said body being related values substantially as given by the graph of Fig. 6.

9. A piezoelectric quartz crystal body adapted

to vibrate along the major axis length dimension at a fifth harmonic mode frequency dependent upon the elementary length dimension equal to said major axis length dimension arithmetically divided by said numerical order of said harmonic, said frequency being given by the curve of Fig. 5 for an angle from $+45$ to $+52$ degrees, the major plane of said body being of substantially rectangular shape, disposed substantially parallel with respect to an X axis and inclined at an angle from $+45$ to $+52$ degrees with respect to the Z axis, said major axis of said major plane being inclined substantially 45 degrees with respect to said X axis, the dimensional relation between the length, width and thickness dimensions of said body being substantially one of the sets of values given by the graph of Fig. 7.

10. A piezoelectric quartz crystal body adapted for longitudinal vibrations along and at a third harmonic frequency dependent mainly upon the elementary lengths of the major axis length dimension of its substantially rectangular major plane, said major plane being substantially parallel to an X axis and inclined at an angle between substantially $+45$ and $+52$ degrees with respect to the Z axis as measured in a plane perpendicular to said major plane, said major axis being inclined substantially 45 degrees with respect to said X axis, the dimensional ratio of the width with respect to the over-all length dimension of said major plane being less than substantially 0.8, the dimensional ratios of length, width, and thickness of said body having relative values substantially as given by one of the points of Fig. 6 to provide substantial freedom from extraneous modes near said third harmonic frequency.

11. A piezoelectric quartz crystal body adapted for longitudinal vibrations along and at a fifth harmonic frequency dependent mainly upon the elementary lengths of the major axis length dimension of its substantially rectangular major plane, said major plane being substantially parallel to an X axis and inclined at an angle between $+45$ and $+52$ degrees with respect to the Z axis as measured in a plane perpendicular to said major plane, said major axis being inclined substantially 45 degrees with respect to said X axis, the dimensional ratio of the width dimension of said major plane with respect to said major axis length dimension being less than substantially 0.8, the dimensional ratios of length, width, and thickness of said body having relative values substantially as given by one of the points in Fig. 7 to produce substantial freedom from extraneous modes near said fifth harmonic frequency.

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