Feb. 10, 1942. J. M. MANLEY 2,272,246

ELECTRICAL FREQUENCY CHANGER

Filed Sept. 17, 1940 2 Sheets-Sheet 1

FIG. 1

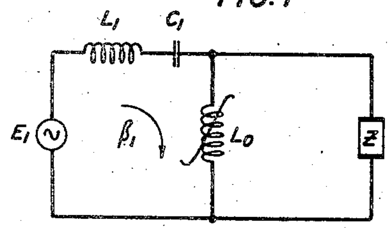


FIG. 2

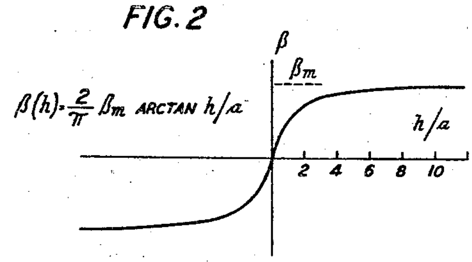


FIG. 3

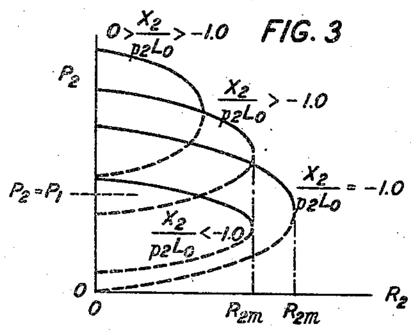


FIG. 4

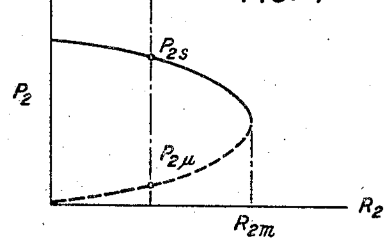


FIG. 5

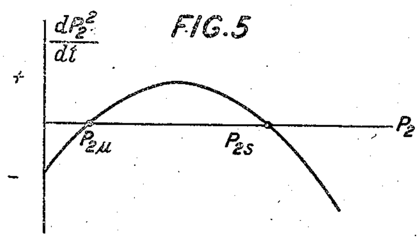


FIG. 6

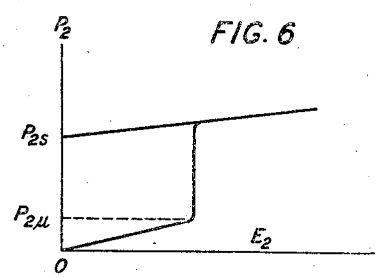


FIG. 7

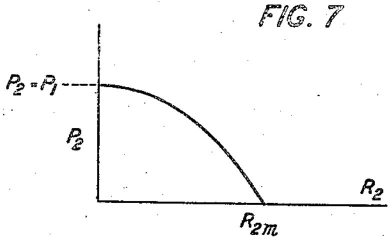


FIG. 8

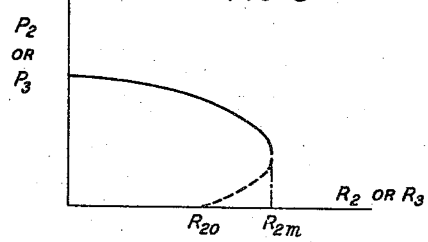
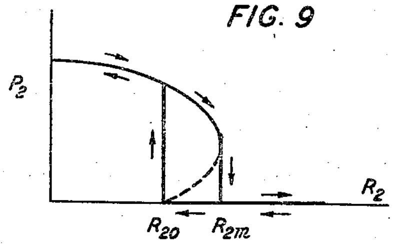


FIG. 9



INVENTOR
J. M. MANLEY
BY H. A. Burgess
ATTORNEY

Feb. 10, 1942. J. M. MANLEY 2,272,246

ELECTRICAL FREQUENCY CHANGER

Filed Sept. 17, 1940 2 Sheets-Sheet 2

FIG. 10

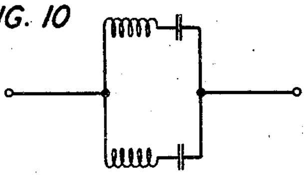


FIG. 11

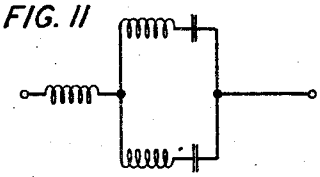


FIG. 12

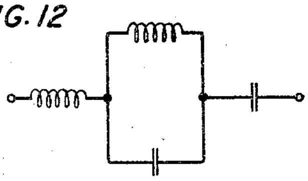


FIG. 13

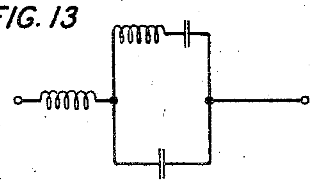


FIG. 14

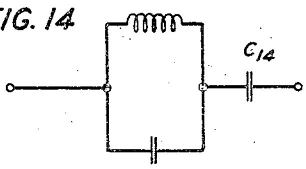


FIG. 15

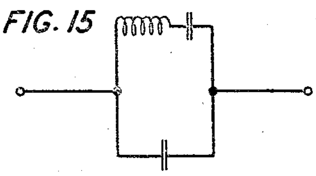


FIG. 16

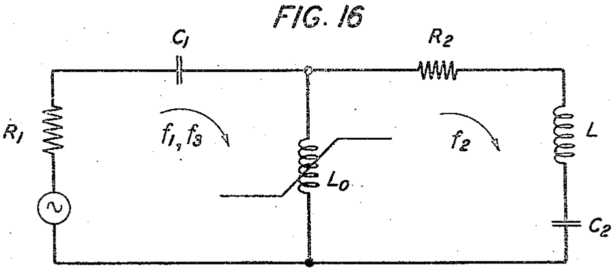
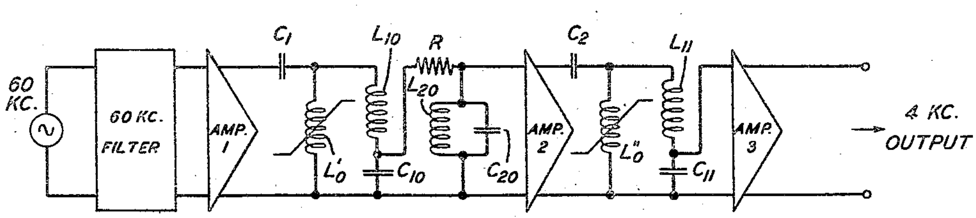


FIG. 17



INVENTOR
J. M. MANLEY
BY H. A. Burgess
ATTORNEY

Patented Feb. 10, 1942

2,272,246

UNITED STATES PATENT OFFICE

2,272,246

ELECTRICAL FREQUENCY CHANGER

Jack M. Manley, East Orange, N. J., assignor to Bell Telephone Laboratories, Incorporated, New York, N. Y., a corporation of New York

Application September 17, 1940, Serial No. 357,115

6 Claims. (Cl. 172—281)

This invention relates to frequency changing circuits and more particularly to means for obtaining a frequency other than that of a base frequency source.

The invention makes use of the well-known non-linear properties of magnetic coils and its object is to provide a reliable self-starting, self-maintaining subfrequency generator calling for no special shock producing device to initiate the generation of said subfrequencies.

The invention will be better understood by reference to the following specification in which the limiting conditions for self-starting are set forth and by reference to the accompanying drawings in which:

Fig. 1 shows a simple representative circuit including a non-linear magnetic element;

Fig. 2 is representative of a typical magnetization curve for the non-linear element;

Figs. 3 to 7 are curves explanatory of one manner of operation of my generator;

Figs. 8 and 9 are curves explanatory of another manner of operation of my generator;

Figs. 10 to 15 are electrical networks which may be used in connection with such a circuit as that of Fig. 1; and

Figs. 16 and 17 are circuits showing special applications of my invention.

The possibility of the existence in ferromagnetically coupled circuits of oscillations which are not odd harmonics of the applied frequency is well known in the art. Circuits illustrating this are disclosed in such patents as those to Fallou, 1,633,481, June 21, 1927; Heegner, 1,656,195, January 17, 1928; Stocker, 2,088,620, August 3, 1937; and Kalb, 2,112,533, March 29, 1938.

A classification of such oscillations may be given as: (1) submultiples or subharmonics of the applied wave, (2) even harmonics of the applied wave, (3) rational fractions, for which the ratio of frequencies of the generated wave and the applied wave is expressed by a rational fraction m/n, where neither m nor n is unity, and (4) incommensurables for which the frequency of oscillation is incommensurable with the applied frequency.

It is well known that the open circuit voltage across an unbiased magnetic coil which is magnetized by a sinusoidal current contains only odd harmonics of the magnetizing wave, where the exciting wave itself is considered as one harmonic. It is also well known that if a sinusoidal voltage is applied to the magnetic coil, the resulting current consists only of odd harmonics of the applied wave. It follows then that the open circuit voltage referred to does not contain frequencies which come under the above four classes and the question arises what special conditions must be fulfilled in order to start the oscillations of non-odd harmonics. The analysis of the conditions existing is given in this specification and forms the basis for my invention.

In this analysis it is assumed that there are present a few sinusoidal components of current through the coil or voltage across the coil and the conditions which must necessarily exist are then derived.

Referring to Fig. 1 there is shown a source of sinusoidal voltage E_1 of frequency f_1, current from which flows through a non-linear magnetic coil L_0. In the supply circuit there is included an inductance L_1 and condenser C_1 for tuning purposes as explained hereinafter. Across the terminal of the coil L_0 is connected a network Z, the particular characteristics of which will be discussed hereinafter. In considering this circuit it will be first assumed that the current through the coil L_0 consists of only two sinusoidal components. Under these conditions it is found that all the non-odd harmonic oscillations recited above, except those of class 4, can be maintained under suitable conditions, but that only the even harmonics can be self-starting.

Of the two sinusoidal components of the currents flowing in the coil, one is the applied current of frequency f_1, the other is the generated oscillation of frequency f_2. A fairly close approximation, ignoring hysteresis, to the magnetization or B—H curve of such a coil as is under consideration is shown in Fig. 2 and can be represented very closely by an arctangent function of the form

$$B=\frac{2}{\pi}B_m \text{ arc tan } h/a$$

where B_m is the saturation magnetization, h is the magnetizing force and a is a constant depending upon the dimensions and properties of the non-linear coil. One makes use of the voltage circuital equation that the vectorial sum of the components of voltage across the coil and voltage drop in the external circuit at the corresponding frequency must be zero at each frequency. Because of its non-linear properties it is well known that the voltage across the non-linear coil in this case will consist of sinusoidal components, having combination frequencies $mf_1 \pm nf_2$ (m and n integers) as well as the frequencies f_1 and f_2. If f_1 and f_2 are incommensurable, I find that the above voltage equations for the circuit cannot be satisfied unless the resistance of Z at f_2 is zero and there are no losses in the magnetic coil. If, however, the ratio of f_1 and f_2 can be expressed by a rational number, then some of the combination frequencies mf_1-nf_2 will coincide with f_2. The phase of the voltage across the non-linear coil, due to this modulation product is such that a component of this voltage is opposite to the voltage drop in the resistance of Z at f_2. Consequently, it is possible to satisfy the voltage equations of the circuit in this case even though R_2 (the resistance of Z) and the losses in the non-linear coil are greater than zero, provided R_2 and the losses are small enough. Because the voltage generated across the non-linear coil neutralizes part or all of the drop in the external resistance it may be said that the non-linear coil has a negative resistance at f_2. It may be noted that this negative resistance has the characteristics of the series type of negative resistance since any tendency toward oscillation can be stopped by increasing R_2.

The relation between Z and P_2 (the amplitude of the f_2 component of magnetizing force) as determined by the voltage circuit equations for a typical case, are shown in Fig. 3. P_2 is plotted against R_2 for several values of X_2, where X_2 is the reactance of Z at f_2 and must be negative. The phase of the f_2 component varies from point to point. From these curves of Fig. 3 it is seen that R_2 must be less than a certain value R_{2m}. Also it is seen that R_{2m} varies with X_2. R_{2m} is largest for that value of X_2 which approximately neutralizes the effective reactance of the non-linear coil L_0 for the particular values of P_1 and P_2 involved (P_1 being the amplitude of the f_1 component of magnetizing force). Whatever X_2 may be, R_{2m} is proportional to p_2L_0, p_2 being $2\pi f_2$ and L_0 the effective value of the non-linear inductance. For example, in the case of the 1/3 subharmonic, I find that the largest value of R_{2m}/p_2L_0 is about one-half. Since P_2 is not zero for $R_2=R_{2m}$, P_2 will suddenly drop to zero as R_2 increases and reaches R_{2m}. The ratio of R_{2m} to p_2L_0 decreases as the order of subharmonic increases. Throughout this specification, the expression "effective reactance of the non-linear coil" (p_aL_0) at some frequency f_2 is used to designate the ratio of amplitudes V_a/I_a, where V_a is the amplitude of the sinusoidal voltage component at frequency f_a across the coil in quandrature to the sinusoidal current component I_a through the coil at frequency f_a.

Variations of the amplitude P_1 of the wave have two principal effects. First, the effective value of inductance L_0 decreases with increase of P_1, and hence R_{2m} decreases with increase of P_1 thus decreasing the range of R_2 over which oscillations can exist. Second, because of the variation of L_0 with P_1 the tuning of L_0 with X_2 is changed. This has the same effect as varying X_2 and causes a change to another curve as on Fig. 3. If the variation of P_1 is large enough the range over which oscillation is possible will be exceeded, and thus there are upper and lower limits for P_1 other conditions remaining the same.

In order then to maintain the oscillations at frequency f_2 it is necessary only that the impedance Z be a simple resonant circuit with negative reactance of such magnitude that the effective inductance L_0 is approximately nullified by it at f_2. It may be desirable to include as a portion of the network Z a simple anti-resonance at f_1 to prevent Z from shunting L_0 to an appreciable extent at f_1.

The considerations above relate primarily to the steady state conditions which exist if once arrived at. It has already been mentioned, however, that one of the purposes of this invention is to provide a circuit which is self-starting and it is, therefore, desirable to consider the stability of the circuit under the conditions given above.

To this end it may be noted on reference to Fig. 3 that for each pair of values of R_2 and X_2, there are two possible values of P_2. Both values of P_2 satisfy the circuital equations, but I find that only those values lying on the upper branch of the curve (solid line) are stable and hence physically realizable. This fact has a connection with the question of starting. In the considerations above and the curves of Fig. 3 it was assumed that P_2 was constant with respect to time. If, however, we allow P_2 to vary slowly as a function of time (i. e. slow with respect to the applied frequency) before the expressions for current are inserted in the differential equations, one attains results which depend not only on the amplitude P_2 but on the time derivative of amplitude dP_2^2/dt. I find under a given set of circumstances, this derivative turns out to be positive, in which case any disturbance, no matter how small, in the amplitude P_2 will cause this amplitude to increase. If the derivative is negative, any disturbance will cause the amplitude to decrease. In non-linear circuit problems this derivative is a function of the amplitude P_2. This function is plotted in Fig. 5 for the case for which the stationary state solution is as shown in Fig. 3. This plot is for constant values of R_2 and X_2 corresponding to the vertical position which intersects the unstable branch of Fig. 4 at $P_2=P_{2u}$ and the stable branch at $P_2=P_{2s}$. For a stationary state, the derivative dP_2^2/dt is zero. If the derivative is positive and the amplitude increases, then a value of P_2 will be reached for which dP_2^2/dt is zero and thereafter negative. Thus the increase is stopped and a stable value of P_2 is reached. It is now seen from Fig. 5 why the lower branch of the stationary state curve is unstable and the upper branch is stable. Any small change in P_2 that reduces it from P_{2u} causes P_2 to decrease further until $P_2=0$, while any change that increases P_2 from P_{2u} causes P_2 to be increased further until P_{2s} is reached. Just the reverse is true for P_{2s}. Any small change in either direction causes P_2 to return to P_{2s}. Thus P_{2s} is a stable stationary solution. $P_2=0$ is also a stable solution.

It is now possible to draw certain conclusions as to the question of starting. First, since $P_2=0$ is a stable condition, the oscillations will not arise spontaneously, i. e. from the very small disturbances that are always present in any circuit. Second, any initial value of P_2 less than P_{2u} will die away if there is no externally connected electromotive force of frequency f_2 to maintain it. Third, any initial value of P_2 between P_{2u} and P_{2s} will build up to P_{2s}. Hence in order to start the oscillation which has a steady state curve like those of Fig. 3 it is necessary to have an initial amplitude of P_2 greater than the value P_{2u} corresponding to the particular values of R_2 and X_2 and P_1 involved.

Suppose that no oscillation is present, i. e., P_2 is zero and that R_2, X_2 and P_1 have values such that for the corresponding curve of Fig. 3, R_2 is less than R_{2m}. Now let an external generator at frequency f_2, having zero impedance and variable voltage, be connected in series with Z. If the voltage of this generator is gradually increased from zero a current of frequency f_2 will flow, and as long as the amplitude of the magnetizing force corresponding to this current is less than P_{2u} this current will be under the control of the external generator. That is, the current will increase or decrease as the voltage is increased or decreased and would die away entirely if the generator is removed. But if the voltage is increased until P_2 passes P_{2u}, then the oscillation is unstable and no longer under the control of the generator and will build up to a value near P_{2s}. If now the external generator is removed, P_2 does not die away but assumes the value P_{2o}. Experimentally determined curves of the form of Fig. 6 confirm these conclusions.

Steady state curves like those of Fig. 3 are obtained for classes 1 and 3 above, i. e., subharmonics and rational fractional products, when two sinusoidal components of current are allowed to flow. Consequently subharmonic and rational fracional oscillations are not self-starting under these conditions. In the case of even harmonics a different kind of curve from those of Fig. 3 is obtained. As shown in Fig. 7 the lower branch does not appear as in Fig. 3. In this case I find that $P_2=0$ is unstable when R_2 is less than R_{2m}. Hence this class of oscillation can arise spontaneously under the proper conditions of R_2 and X_2.

These results may be used to explain the starting action characteristics of some forms of subharmonic generators commonly used. These have but one degree of freedom besides that at the applied frequency and so conform to the assumptions made above. A relay switch to short-circuit a coil temporarily or some other means of applying a shock to the circuit is required for starting. The magnitude of the electrical transient resulting from such a shock depends on the phase of the applied wave at the time of shock. Since the time of application of the shock usually has no definite connection with the phase of the applied wave, the magnitude of the electrical transient will not necessarily be the same each time the shock is applied. The magnitude of the transient must be greater than a certain threshold value in order for starting to occur and hence the oscillation may or may not arise depending on whether or not the transient exceeds a certain threshold.

A more general and more interesting case is that in which an additional sinusoidal component is allowed to flow. In that case I find that any of the four classes of non-odd harmonic oscillations referred to above can exist and can be self-starting when the proper conditions are satisfied.

To consider this, let the impedance Z of Fig. 1 be so arranged that the circuit has two degrees of freedom so that two sinusoidal components besides the applied current are allowed to flow through the non-linear coil. In this case oscillations at two separate frequencies are produced and these have a definite relation to the applied frequency. If f_1 is again the applied frequency and f_2 and f_3 are the generated frequencies, the relation is $2mf_1=f_2+f_3$ where m is an integer. An analysis shows that the most favorable conditions for starting the oscillations occur when the reactance of the impedance Z nullifies the effective inductive reactance L_0 of the non-linear coil at the two frequencies f_2 and f_3. Complete annulment, i. e., precise tuning for these frequencies is not necessary, as will be pointed out hereinafter. A certain amount of departure from such tuning is permissible within limits which will be specified. Besides this condition I also find that the total resistances at both f_2 and f_3 of the circuit mesh formed by Z and L_0 must be less than a certain quantity which is proportional to the magnitude of the reactance p_2L_0. These two conditions turn out to be so connected that oscillations may arise when the reactance of Z is detuned from the above most favorable condition, provided the resistance is reduced from the above maximum value by the proper amount. The exact relation (derived on the basis of the arctangent function as given for Fig. 2) which must be satisfied in order that oscillations f_2 and f_3 shall arise spontaneously is

$$\sqrt{R_2^2+(X_2+p_2L_0)^2}\,\sqrt{R_3^2+(X_3+p_3L_0)^2}\leq p_2L_0p_3L_0b^2 \quad (2)$$

where R_2 and R_3 are the resistances of Z at f_2 and f_3 and X_2 and X_3 are the reactances of Z at f_2 and f_3 and where b^2 is a parameter depending only on the magnetic characteristic of the coil and the amplitude P_1 and is always less than unity, approaching unity as P_1 becomes very large.

It will be observed that the first factor on the left-hand side of the expression is the impedance Z_2 of the circuit at f_2 and the second factor is the impedance Z_3 for the circuit at f_3. On the right-hand side of the expression it will be observed p_2L_0 is the reactance X_2' of the coil at f_2 and p_3L_0 is the reactance X_3' of the coil at f_3. Since b^2 is less than unity the expression above may be put in the more condensed form

$$Z_2Z_3\leqslant X_2'X_3'b^2 \quad (3)$$

At first sight it might appear that since Z_2 includes X_2' the above expression would not be possible but it is to be observed that in Equation 2 the quantity $X_2+p_2L_0$ may be quite small, even approaching zero. Under this special condition Equation 2 would reduce to

$$R_2R_3\leqslant X_2'X_3'b^2 \quad (4)$$

A curve for this case of three sinusoidal components and corresponding to the curves of Fig. 3 is shown in Fig. 8. It will be seen that the essential difference of this curve from those of Fig. 3 is that the lower branch intersects the R axis at a value of R greater than zero. This point of intersection is obtained from the above condition for oscillation by using the equality sign in (4) above. Thus, if both reactances are brought to the most favorable condition, i. e., are tuned to zero, we have for the intersection point for R_2

$$R_{20}=b^2X_2'X_3'/R_3 \quad (5)$$

Investigation shows that both time derivatives dP_2^2/dt and dP_3^2/dt are positive for infinitesimal values of P_2 and P_3 when the condition of Equation 2 is satisfied. Thus in this condition the situation of zero amplitude for P_2 and P_3 is unstable when the resistance is to the left of the intersection point of the steady state curve of Fig. 8 corresponding to the other parameters of the circuit, and so the oscillations at f_2 and f_3 will build up until a stable steady state is reached. Or suppose that oscillations are not present, i. e., P_2 and P_3 are zero and that the resistance is greater than R_{2m}. Then as R_2 is decreased nothing happens until the point R_{20} is reached. As soon as this point is passed, the two time derivatives of amplitude become positive, the condition of zero values for amplitudes P_2 and P_3 is unstable and the oscillations at f_2 and f_3 will automatically build up, having been started off by the small disturbances present in any circuit. This set of conditions is shown in Fig. 9 in which a sudden building up of oscillations is shown when R_2 is lowered to R_{20}. For further decrease of R_2 the amplitude of oscillation will increase slowly. Increase of resistance above R_{20} will still permit the oscillations to continue until R_2 reaches the value R_{2m} whereupon the oscillations die down very quickly.

It is possible to trace the reasons for the starting performance of this circuit to the modulating properties of the magnetic coil, this time under magnetization of three sinusoidal components. In the two-component case above it was noted that in order to have self-starting oscillations it is necessary to have a difference combination frequency (lower side-frequency) coinciding with the frequency to be generated and also that the amplitude of the generated voltage at this frequency must be proportional to the first power of amplitude of the current to be generated. The first of these is necessary in order to have a portion of the generated voltage opposite in phase to the voltage drop in the external resistance, and the second is necessary for self-starting since it implies that the negative resistance corresponding to the generated voltage does not approach zero as the amplitude of the current becomes very small. It will be found upon substitution of the assumed magnetizing force function due to the three components into the polynomial expression for flux density that these requirements will be met only if we have combination frequencies of the form $2mf_1-f_2=f_3$ and $2mf_1-f_3=f_2$ with voltage amplitudes proportional to

$$P_1^{2m}P_2 \text{ and } P_1^{2m}P_3$$

respectively. It turns out that the negative resistances in this case do not approach zero as P_2 and P_3 become very small. Hence the negative resistances retain the power to nullify the external resistance even when P_2 and P_3 are very small. Experiment verifies the obvious conclusion that in obtaining these oscillations $m=1$ is the most favorable value since third order modulation products are used and these give higher voltages than do the higher orders of modulation products.

It should be noted that no restrictions are put on the frequencies f_2 and f_3 other than the one given above, i. e., $2mf_1=f_2+f_3$. Thus, f_2 and f_3 may be incommensurable with f_1 or they may be rationally related. Any two frequencies satisfying this relation can be established, the oscillations being self-starting so long as the cricuit is within the range of conditions specified above. Furthermore, having been established they will then be maintained over a finite range of reactance and resistance variation.

In the description which has been given thus far no definite specification has been given as to the nature of the impedance Z other than that taken with L_0 it shall in the two-component case have one degree of freedom and in the three-component case have two degrees of freedom. A number of circuits which are appropriate in the latter case are shown in Figs. 10 to 13. Each of these circuits has two degrees of freedom, i. e., in combination with the inductance, L_0 may be tuned to two frequencies f_2 and f_3. The arrangement of Figs. 14 and 15 although not equivalent to the other four may be used satisfactorily since the resonance at two frequencies f_2 and f_3 in the loop of L_0 may be maintained when the presence of L_0 is taken into account. The resistance load may be added in series with any of these networks. For the circuits of Figs. 10 to 13 a resistance may be included in each branch to provide a separate load at each frequency. If only one of the components is desired as output a selective filtering circuit containing the load may be connected across a part or all of the network.

For these circuits also it is desirable to have an anti-resonance at f_1 to prevent Z from shunting L_0 to an appreciable extent at f_1. The need for an anti-resonance at f_1 may be avoided by using two balanced magnetic cores connected oppositely with respect to the applied wave so that f_1 is balanced out of the winding connected to Z.

In all of these arrangements the circuits are not perfectly selective and other components besides those having frequencies f_1, f_2 and f_3 will flow. If f_1, f_2 and f_3 are rationally related then the other components will be additional terms in a Fourier series, whose fundamental term has a frequency which is the greatest common divisor of f_1, f_2 and f_3. If two of the frequencies f_1, f_2 and f_3 are incommensurable the other components will be additional terms in a double Fourier series. The flow of these components produces various effects and particularly if the term with the upper side frequency $2f_1+f_2$ flows, it will absorb energy from the generated component of frequency f_2 and so reduce the value of R_{2m}, i. e., it introduces a positive resistance instead of a negative one. But analysis shows further that the flow of this component $2f_1+f_2$ will also introduce negative resistances at other frequencies because of higher order modulation products. For example, its flow will introduce a negative resistance at f_3 due to fifth order modulation. The circuits in which the air inductances can be made as large as is convenient will be more selective than those in which this inductance has a fixed relation to L_0. The circuits of Figs. 10 to 13 are of the former type while those of Fig. 14 are of the latter. The flow of the additional components in small quantities does not invalidate the analysis given above or the conclusions therefrom.

Other forms of circuit may be used to specify the condition for self-starting although not usually in the most favorable manner. Fig. 16 shows one of these in which Z is a resonant circuit with fairly high impedance tuned in series with the effective value of L_0 at f_2. No tuning coil is used in the input so that this mesh is partially tuned at both f_1 and f_3 or at least so adjusted that the starting conditions are satisfied. Such a circuit, for example, was set up to generate a one-third subfrequency, i. e., $f_2=1/3\ f_1$ and $f_3=5/3\ f_1$. These frequencies were found present in the two portions of the circuit. When a simple anti-resonance at f_3 was connected in series with the condenser C_1 then oscillations would not arise but would start only upon removal of the anti-resonance or sufficient detuning of it to either side of f_3. This circuit may be modified by the insertion of an inductance in series with C_1 to tune this branch of the circuit to f_3.

Fig. 17 shows one particular application of the circuit of Fig. 16. A base frequency of 60 kilocycles was impressed on the non-linear coil L'_0, the associated circuits of which were adjusted to give the subharmonic $1/3\ f_1$. The output at this frequency across a condenser was then impressed on the input of an amplifier thus supplying 20-kilocycle current to the non-linear coil L''_0. The circuits associated therewith, which are of the same form as that of Fig. 16, were set to give the fifth subharmonic of 20 kilocycles thus giving an output frequency of 4 kilocycles.

Circuits of the same general character as described above can also be used to obtain 20-cycle current from a 60-cycle supply system, the 20-cycle current having possible application in telephone system for ringing purposes. Here again the self-starting in the way outlined above over a considerable range of variations is readily obtained so long as one can get within the range given by the limitations already set forth. Thus a form of Z network given in Fig. 14 was used in the circuit of Fig. 1 and variations from 15 to 20 microfarads was allowable in the series condenser C_{14} and from 45 to 55 volts in the generator voltage E_1 and the derived frequency remained constant throughout such variations.

It may be noted in this connection that if the reactance at f_3 in the circuit of Fig. 16 is detuned from zero sufficiently by adding a large inductance in series in the input mesh, we approach the form of circuit required for the two-component case and so this latter is a special case of the more general one. A better idea is obtained of what happens as the change is made from one case to the other by observing the changes that occur in the amplitude resistance curve of Fig. 8. Suppose this curve is P_2 plotted against R_2 and that the reactance at f_3 is being detuned from zero. Then according to the condition for starting given in Equation 2 the intersection point R_{20} of the curve moves to the left as the detuning from zero increases. Ultimately R_{20} will become zero and the curve will have changed into one like those of Fig. 3.

In the analysis, as given above, a single valued magnetic characteristic was assumed. Hence the results do not take into account explicitly the losses which occur in the magnetic core. It is to be understood, however, that such losses are reflected in the windings as an equivalent resistance and such resistance is to be understood as being included in and as being a part of the external resistances as considered above.

What is claimed is:

1. In a circuit for generating a non-harmonic component of a base frequency f_1 comprising a non-linear inductance, an input circuit including a source of sinusoidal waves of said base frequency connected across said inductance, and an output circuit for the generated frequency connected across said inductance, means to make said generating circuit self-starting and self-sustaining for the generated non-harmonic frequency component comprising an impedance network in said output circuit making the reactances X_2 and X_3 of said output circuit at the frequencies f_2 and f_3, respectively, such as to substantially neutralize the reactance X_2' and X_3' of said inductance at these respective frequencies and making the total resistance R_2 and R_3 of said output circuit and said inductance at these respective frequencies less than a critical value R_{2m} which is a function of the effective reactance of said inductance, as defined in the specification, one of the two frequencies f_2 and f_3 being the non-harmonic frequency component to be generated and the two frequencies being related to the base frequency f_1 by the relation $2mf_1=f_2+f_3$ where m is an integer.

2. The combination of claim 1, in which said critical value of resistance is determined by the relation $R_2R_3\leqq X_2'X_3'$.

3. A circuit for obtaining a non-harmonic component of a base frequency f_1 comprising a non-linear inductance, a source of sinusoidal waves of said base frequency, an input circuit connecting said source across said non-linear inductance and an output circuit for the generated frequency connected across said inductance, said output circuit as a whole having two degrees of freedom at the frequencies f_2 and f_3, respectively, one of which is the frequency of the non-harmonic component to be generated and which are related to said base frequency in accordance with the equation $2mf_1=f_2+f_3$ where m is an integer, the total impedances Z_2 and Z_3 of said output circuit with said inductance at the frequencies f_2 and f_3, respectively, being reduced to substantially a minimum and coming within the relation $Z_2Z_3\leqq X_2'X_3'$, where X_2' and X_3' are the reactances of said inductance at the frequencies f_2 and f_3, respectively.

4. The combination of claim 3 in which said non-linear impedance is a saturable magnetic core coil and said output circuit comprises at least two parallel impedance branches tuned with said coil to provide said two degrees of freedom.

5. The combination of claim 3 in which said non-linear inductance is a saturable magnetic core coil and said output circuit comprises two parallel branches each containing linear inductance and capacitance in series, one of said branches being tuned with said coil to the frequency f_2 and the other being tuned with said coil to the frequency f_3.

6. A self-starting even harmonic generator for generating a harmonic of frequency f_2 and amplitude P_2 from a base frequency f_1, comprising an unbiased saturable magnetic core coil, a sinusoidal voltage source of frequency f_1 and amplitude P_1 for exciting said coil, and an output circuit for the generated harmonic, connected in parallel with said exciting voltage source, across said coil, said output circuit having a reactance such as to approximately neutralize the reactance of said coil at the frequency f_2 for the particular values of P_1 and P_2 involved, the combined resistance of said output circuit and said coil being less than a critical value R_{2m} which is a function of the effective inductance of said coil.

JACK M. MANLEY.