

May 20, 1941.

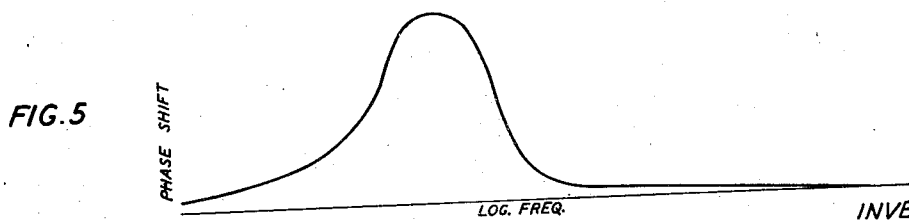
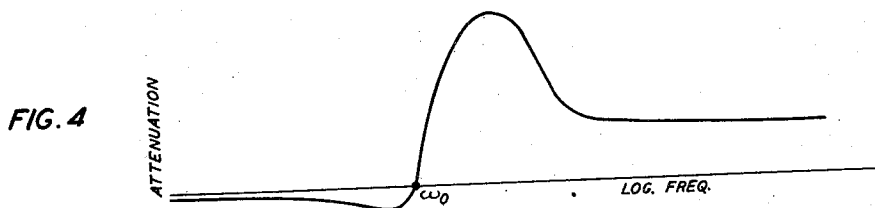
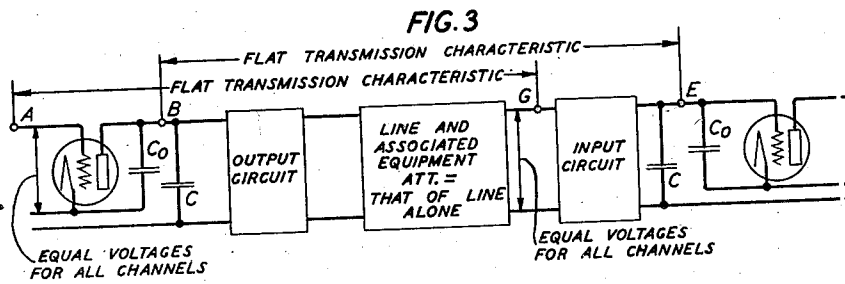
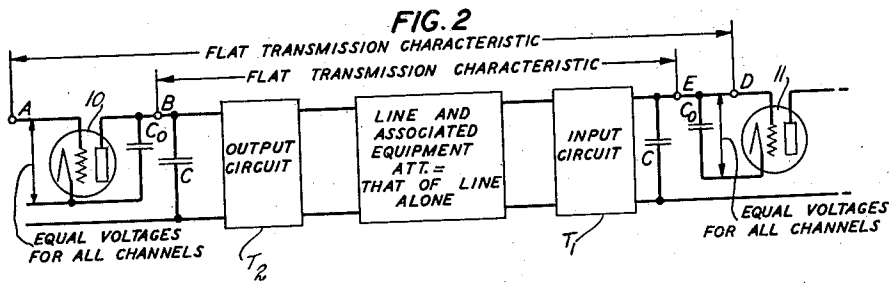
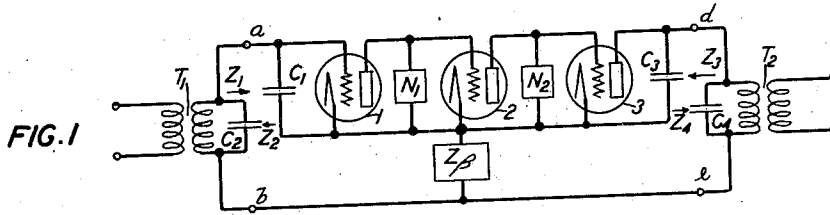
H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 1



INVENTOR
H. W. BODE
BY
H. A. Burgess
ATTORNEY

May 20, 1941.

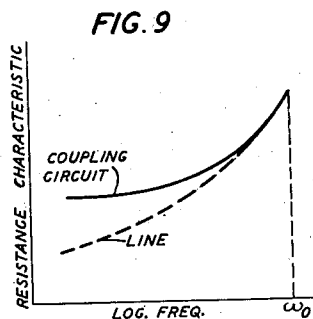
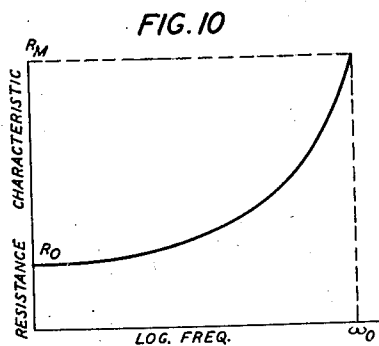
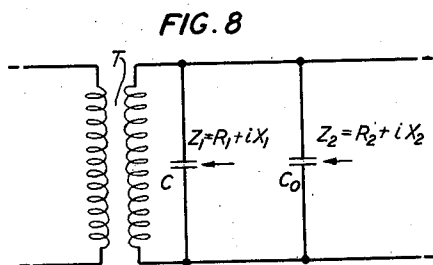
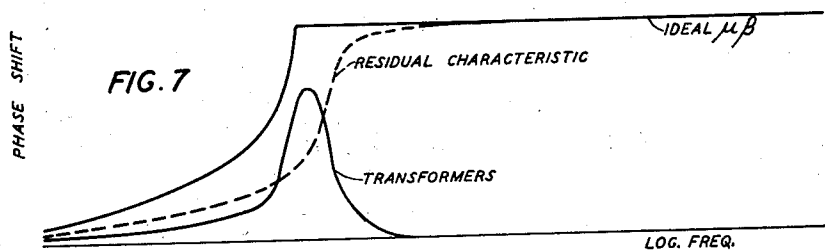
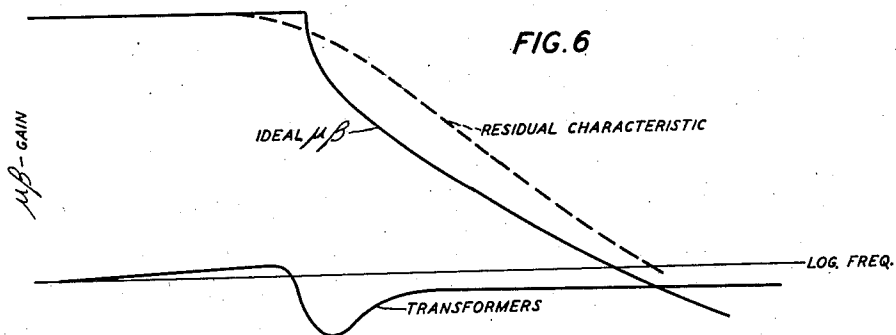
H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 2



INVENTOR
H. W. BODE
BY
H. A. Burgess
ATTORNEY

May 20, 1941.

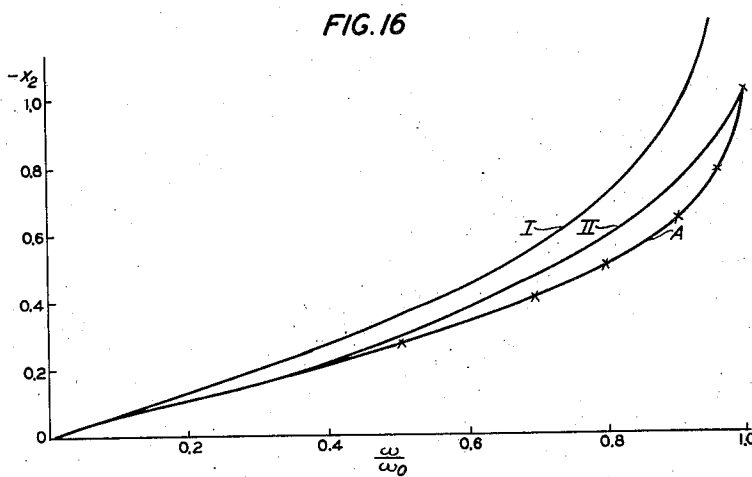
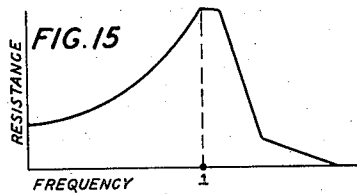
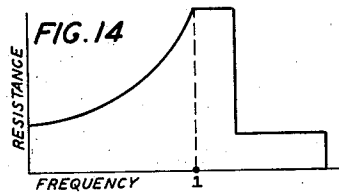
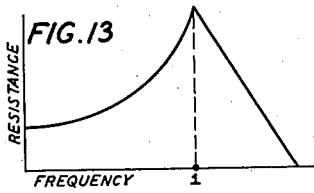
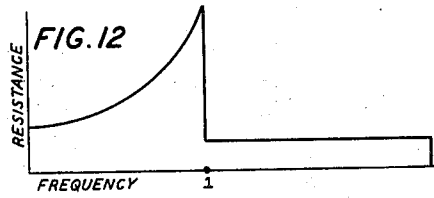
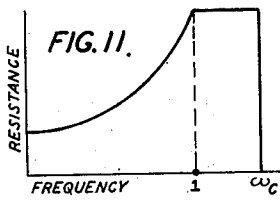
H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 3



INVENTOR
H. W. BODE
BY
H. A. BINGES
ATTORNEY

May 20, 1941.

H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 4

FIG. 17

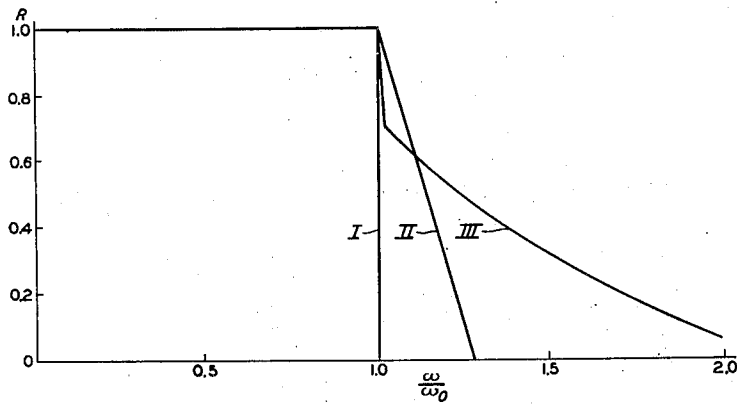


FIG. 18

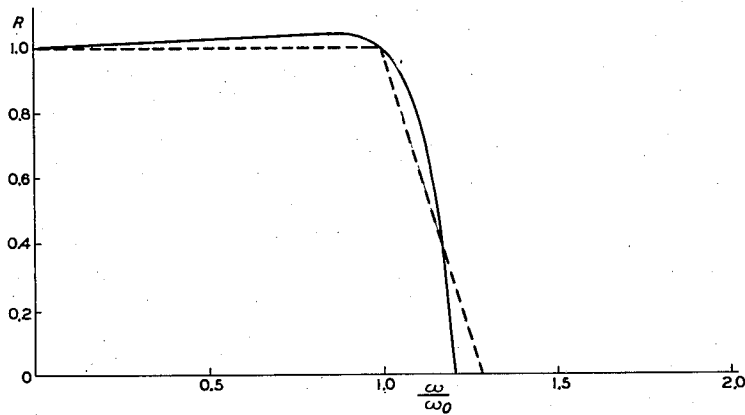
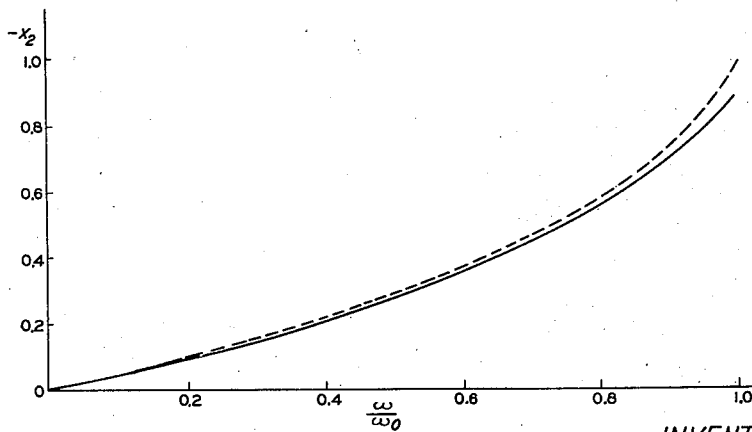


FIG. 19



INVENTOR
H. W. BODE
BY
H. A. Burgess
ATTORNEY

May 20, 1941.

H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 5

FIG. 20

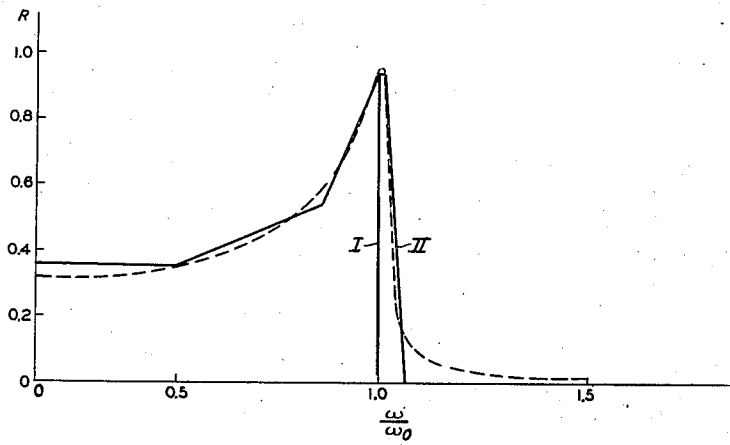


FIG. 21

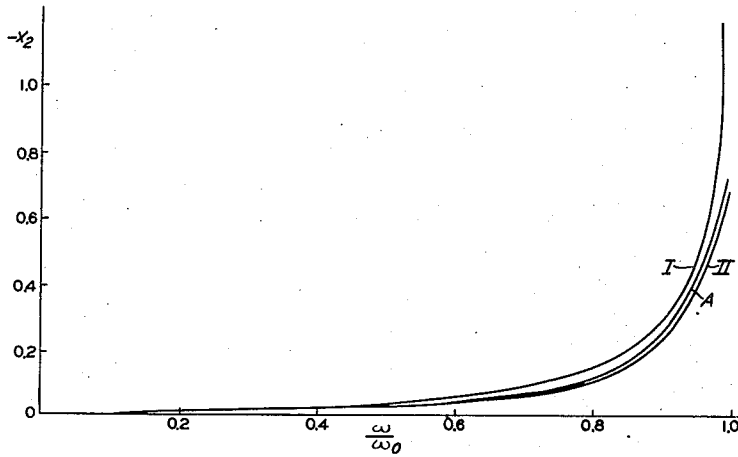


FIG. 22

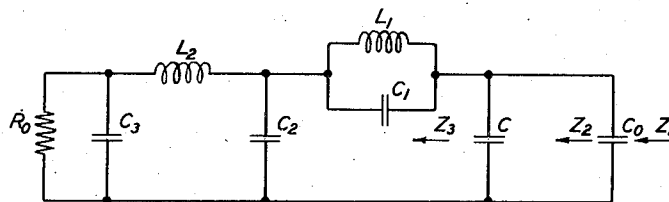
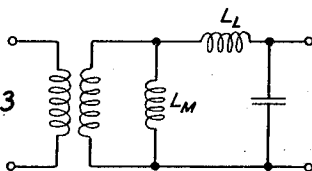


FIG. 23



INVENTOR
H. W. BODE
BY
H. A. Burgess
ATTORNEY

May 20, 1941.

H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 6

FIG. 24

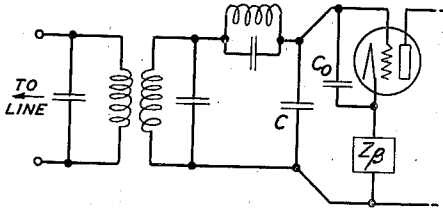


FIG. 25

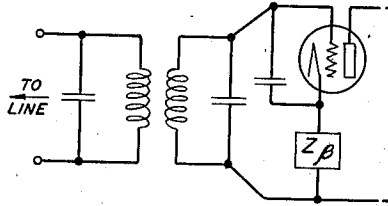


FIG. 26

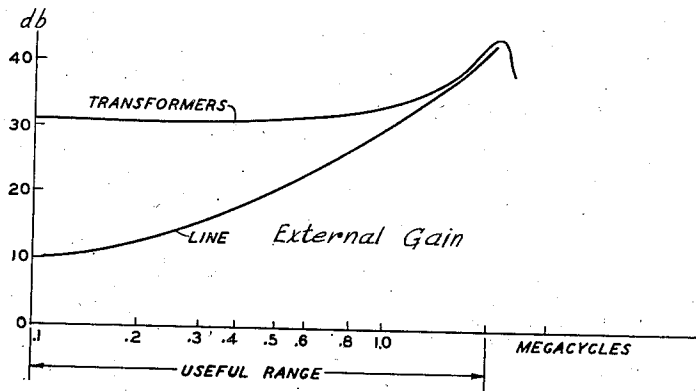
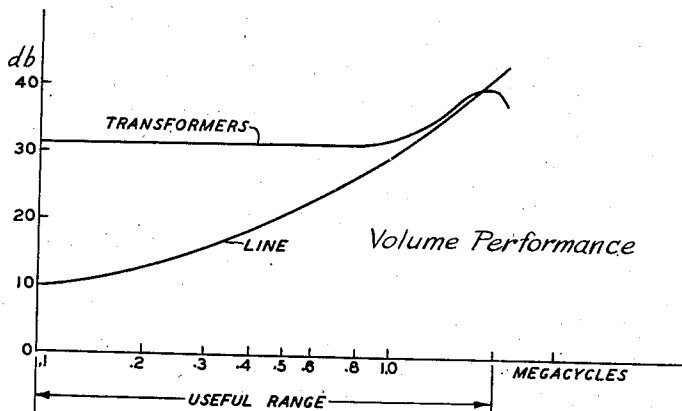


FIG. 27



INVENTOR
H. W. BODE
BY *H. A. Burgess*
ATTORNEY

May 20, 1941.

H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 7

FIG. 28

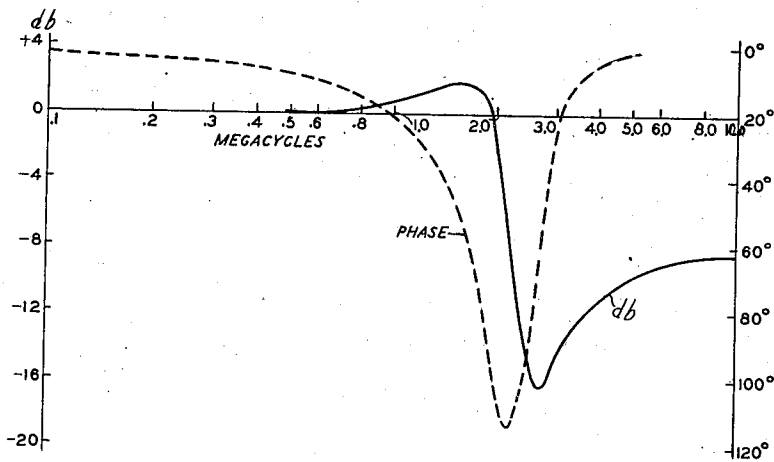


FIG. 29

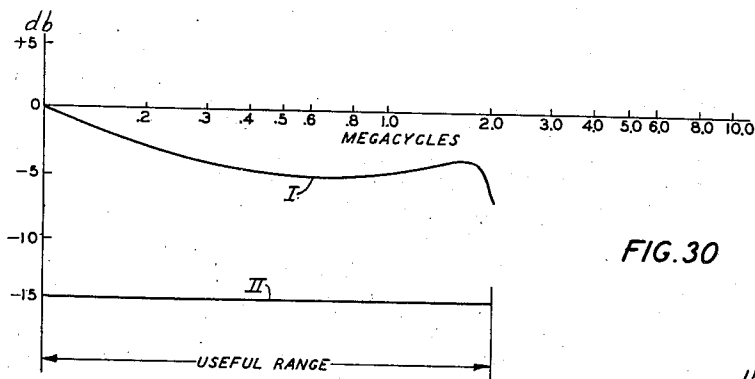
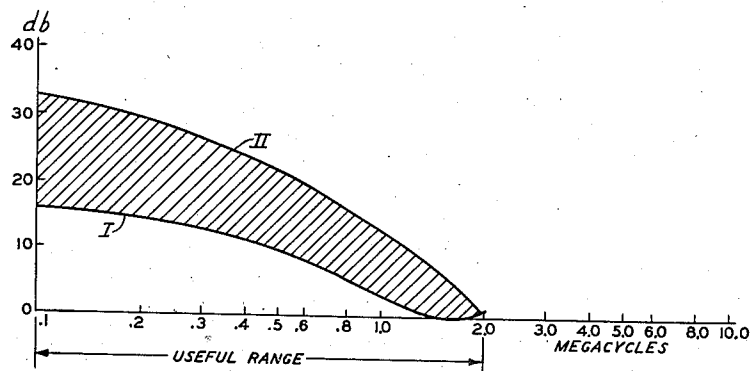


FIG. 30

INVENTOR
H. W. BODE
BY *H. A. Burgess*
ATTORNEY

May 20, 1941.

H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 8

FIG. 31

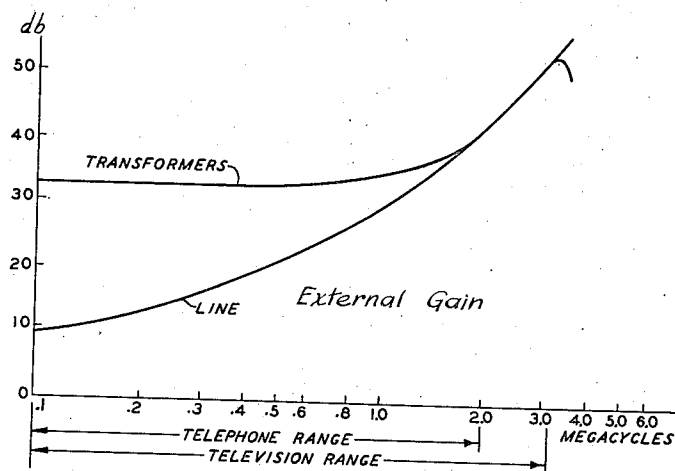


FIG. 32

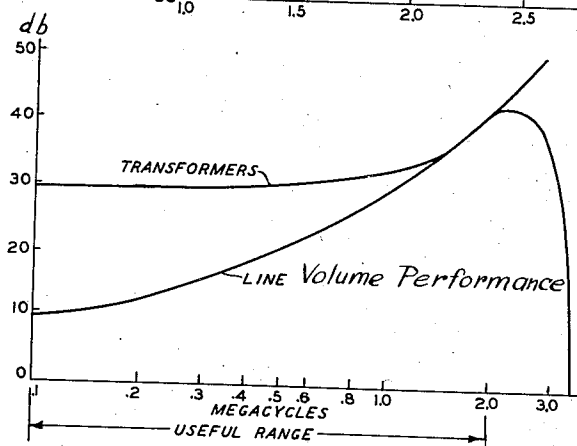
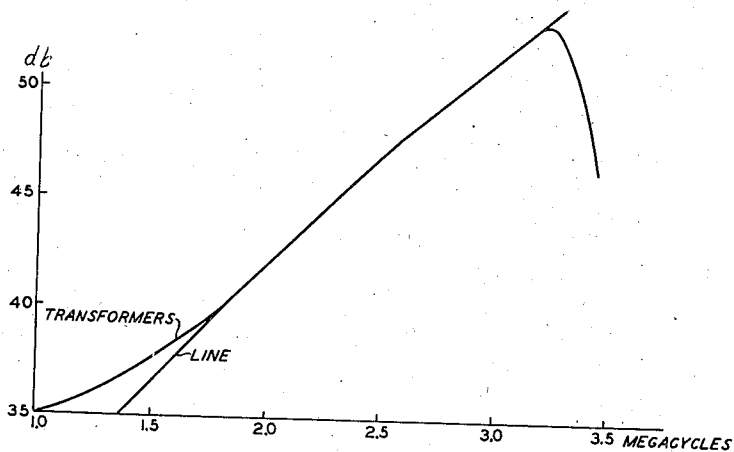


FIG. 33

INVENTOR
H. W. BODE
BY
H. A. Burgess
ATTORNEY

May 20, 1941.

H. W. BODE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Filed Sept. 29, 1939

9 Sheets-Sheet 9

FIG. 34

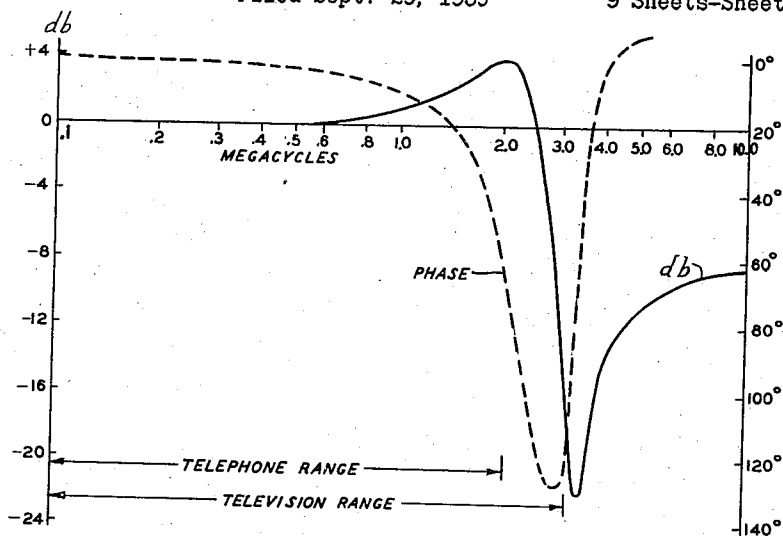


FIG. 35

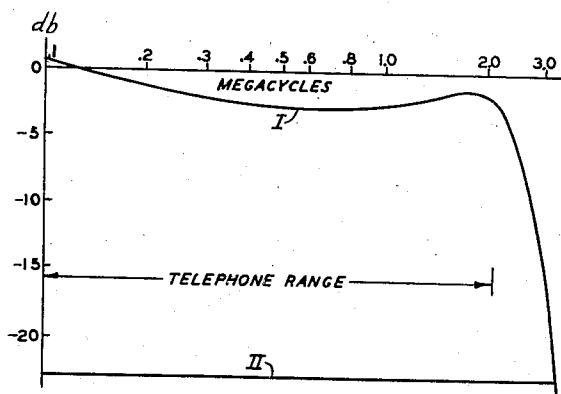
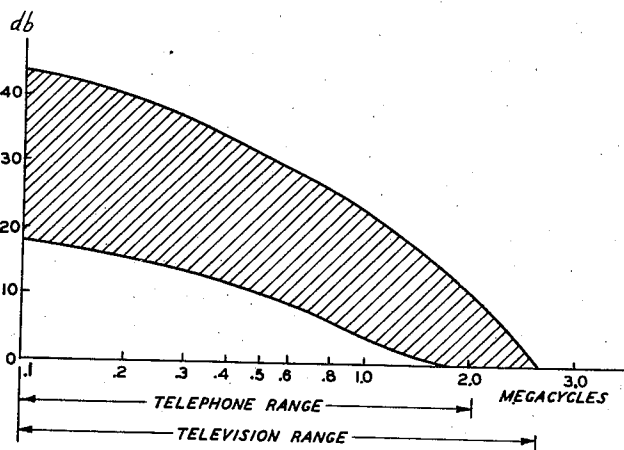


FIG. 36

INVENTOR
H.W. BODE
BY
H.A. Burgess
ATTORNEY

UNITED STATES PATENT OFFICE

2,242,878

DESIGN OF BROAD BAND REPEATERS

Hendrik W. Bode, New York, N. Y., assignor to
Bell Telephone Laboratories, Incorporated, New
York, N. Y., a corporation of New York

Application September 29, 1939, Serial No. 297,069

8 Claims. (Cl. 178-44)

The present invention relates to amplification and transmission of electrical waves covering a broad frequency band such as is used in multiplex carrier and television systems.

An object of the invention is to improve the signal transmission capabilities of a repeatered system, particularly by providing more effective coupling means between the repeater and the line.

A feature of the invention comprises a wave transforming circuit for associating an amplifier with a broad band transmission circuit, equalizing attenuation and producing favorable signal-to-noise ratio.

In accordance with this feature of the invention, a wave transforming and coupling circuit is provided which contributes to the total gain by a variable factor over the frequency band in such manner as to counteract the variable attenuation of the line over the frequency band and at the same time is substantially non-dissipative.

In a given type of system and for a given frequency band to be transmitted, the minimum permissible amplitude to which the signal may fall, hence the repeater spacing for a line of given over-all attenuation characteristic, is determined by the noise level of the system. Noise may originate in or find its way into the system at points between repeaters or in the repeaters themselves. In the present discussion it will be assumed that the repeaters are of the stabilized feedback type. In order to minimize noise originating in the repeater and for the sake of other advantages resulting from feedback it is desirable to use a large amount of feedback. While further discussion of the noise problem will be given later on, it is here pointed out that the wave transforming and coupling circuit of the invention permits full use of feedback with its accompanying advantages. By achieving attenuation equalization without energy dissipation, it contributes markedly to the signal-to-noise ratio.

While capable of general application, the invention will be disclosed herein for illustrative purposes as embodied in a coaxial line system for transmitting waves covering a broad band extending up to a top frequency in one typical case of two megacycles and, in another typical example, of three megacycles. Again, for illustrative purposes, the range up to two megacycles will be assumed devoted to multiplex carrier telephone transmission and the range up to three megacycles will be assumed devoted to television transmission.

The nature of the invention and its various features and objects will be more fully understood from the detailed description to follow together with the accompanying drawings:

In the drawings:

Fig. 1 is a simplified schematic diagram of a stabilized feedback amplifier with input and output transformers according to the invention;

Figs. 2 and 3 are block schematics of a complete repeater section to show typical noise conditions to be met;

Figs. 4 to 7, inclusive, show desirable characteristics for the transformer and repeater to have in practicing the invention;

Fig. 8 is a circuit diagram showing impedance relations to be realized in the circuit design;

Figs. 9 and 10 are characteristic curves to be referred to in the description;

Figs. 11 to 15, inclusive, show various shapes for a certain resistance characteristic to be made use of for design purposes;

Figs. 16 to 21 are plots between frequency and either resistance or reactance;

Figs. 22 to 25, inclusive, are detail circuit or impedance diagrams of coupling circuits designed in accordance with the invention;

Figs. 26 to 36 are plots between frequency and either attenuation (decibels) or phase.

Considering first the problem of noise, there are four principal sources, namely, (1) external interference, (2) modulation in the form of distortion which degrades the signal or in the form of cross-talk between channels, (3) resistance noise or thermal agitation, and (4) tube noise. The invention is immediately concerned with only the last three of these. External noise can be kept out by shielding the affected parts. Modulation can be reduced by feedback theoretically to any desired extent and practically to an extent determined by tube limitations so that any circuit elements entering into the amplifier design must be such as not to restrict the maximum use of feedback if the modulation is to be kept as low as possible. Nothing can be done, in general, about resistance noise except to avoid adding to it by introduction of any resistance that can be avoided. The effect of tube noise can be minimized by maintaining the highest possible margin of signal level above the level of the tube noise.

Attenuation equalizers in the line which depend upon dissipation of energy unequally over the frequency band add to the total resistance noise. It has been realized heretofore that equalization can be introduced into the trans-

formers used to couple the line to the repeater in the case of amplifiers of the non-feedback type and that in this way the equalization can be effected without raising the resistance noise level. For example, it is old to resonate the input transformer inductance with the tube input capacity above the upper frequency of the band, thus providing an input circuit characteristic which rises with frequency over the band.

When an attempt is made to do this in the case of a feedback amplifier, a more complicated problem is presented. In this case the transformer must meet three requirements: (1) It must have a characteristic such as to equalize the line or do its share in the equalization; (2) it must fit the design requirements imposed upon the amplifier-feedback loop since it is connected to this loop and affects its characteristics; this is important from the standpoint of permitting maximum use of feedback with its attendant advantages one of which is the reduction of modulation; and (3) the input transformer must be highly efficient in transferring voltage from the line to the grid in order to secure the greatest advantage in overriding tube noise, and the output transformer must be highly efficient in transferring power from the plate to the line in order to secure the greatest advantage in overriding noise from all sources.

While the invention is capable of general application, it will be illustrated and described as applied to the case of a series type of feedback amplifier, that is, one in which the high impedance terminals of the transformer are serially included in the feedback loop.

A simplified sketch of a repeater according to the invention is shown in Fig. 1. In this figure three stages of amplification comprising tubes 1, 2 and 3 are shown, with interstage circuits at N_1 and N_2 , input transformer at T_1 and output transformer at T_2 . The feedback impedance Z_β is shown at 5. The input capacity of tube 1 is C_1 , the capacity across the high side of transformer T_1 is C_2 , the output capacity of tube 3 is C_3 and the capacity across the high side of transformer T_2 is C_4 . The feedback impedance Z_β is assumed for simplicity to be resistive, and in practice this may be variable to permit control of the total amplifier gain such as for temperature compensation of the line. The gain of the amplifier itself is therefore flat with frequency. It will be understood that tubes 1, 2 and 3 will ordinarily be pentodes or tubes of any desired type rather than the simple triodes shown.

As regards the function which the transformers T_1 and T_2 have of transferring waves between the line and either end of the amplifier, there are two main considerations. The first has to do with the shaping of the waves to secure equalization, and the second relates to the effectiveness of the transformers in transferring voltage or energy between the line and the input grid or output plate of the amplifier. The reason that there are these two considerations may be seen from Fig. 1. Since the impedance Z_β is negligibly small compared with the impedance of C_1 the total impedance looking to the right at points a , b when the feedback is active may be considered to be the impedance of C_1 multiplied by the factor $(1+\mu\beta)$, which impedance can be regarded as practically infinite, the impedance of C_1 itself being quite large to begin with. This effect of feedback in raising the apparent impedance is disclosed more fully in patents to H. S. Black Nos. 2,102,671, December 21, 1937, and

2,131,365, September 27, 1938. The transmission characteristic of transformer T_1 is, therefore, that of the transformer terminated in capacity C_2 . The transformer is, therefore, to be designed to produce a given voltage across the points a , b , and its characteristics are to be such as to produce a voltage ratio varying from frequency to frequency over the band in such manner as to equalize the line attenuation to a suitable extent, to be further discussed. The problem still remains, however, of getting the largest possible amount of this voltage to appear across the grid and cathode of the tube 1 in order to produce the greatest margin of signal amplitude above the tube noise level existing at this point. The shunt capacity C_1 affects this transmission characteristic, so that in solving this part of the problem the transformer must be considered as terminated by the total capacity C_1+C_2 . Similar considerations apply to the output side. Due to the fact that the tube capacity C_3 is small, and that its impedance is raised by feedback, the impedance at points d , e is so great that the transformer T_2 may be considered as terminated only in its shunting capacity C_4 so far as its transmission characteristics as a four-terminal network are concerned. In considering the transmission characteristic of the circuit from the plate and cathode of tube 3 to the line, however, the effect of capacity C_3 enters and the transformer must be considered as terminated in the total capacity C_3+C_4 . The design is to be such as to permit maximum energy transfer from the plate of tube 3 to the outgoing line in the interest of efficiency and attainment of favorable signal-to-noise ratio. In a practical case the impedance relations are such that both transformers may be designed alike.

The problem can be put into more definite form by the aid of the simplified sketches of Figs. 2 and 3 showing complete repeater sections illustrating two typical cases as regards noise conditions.

Fig. 2 shows a repeater section including the output tube 10 of one repeater coupled by means of output transformer T_2 to the line section leading to the next repeater station where the line is coupled by means of input transformer T_1 to the input stage 11 of the repeater at that station. It is assumed in this figure that the input and output tube capacities have the same value, C_0 , and that the transformers T_2 and T_1 are alike and are terminated in like capacity C . It is also assumed in this figure that tube noise is more important than resistance noise. It is further assumed that the output tube works at maximum efficiency if the signal level at A is the same for all channels. Since the tube noise at the grid D of a succeeding repeater is substantially the same at all frequencies the optimum design will evidently be realized if the transmission characteristic from A to D, including the intermediate line and all associated equipment, is perfectly flat over the frequency band and is at as high a level as possible. The ratio between the voltage at D and voltage at A will be maximum if there is no equalization or other attenuation added to the loss of the line itself. The complete transmission characteristic for the system as a whole, however, must also be flat with frequency over the band. Since there has been assumed no equalization in the β circuit of the amplifiers and, therefore, flat gain for the amplifiers themselves exclusive of transformers, one arrives at the resulting requirement that all of the line equalization

must be obtained in the transformer characteristics, regarding each transformer as a four-terminal transmission network. In other words, the transmission characteristic from B to E must also be flat with frequency. In this case each transformer equalizes one-half the line attenuation.

If the resistance noise is the predominant factor, the relations which should obtain in an ideal system are those shown in Fig. 3. This is the same as Fig. 2 except that the final signal-to-noise ratio is computed at G instead of at D. The reason for this is that the input circuit is non-dissipative so that there is no contribution to noise in Fig. 3 after the point G is passed so that the resistance (which is that of the line) and therefore the noise is here the same for all channels. Since the signal levels at A should be flat, a design giving the best signal-to-noise ratio over all channels would be flat from A to G.

In the further discussion the conditions represented by Fig. 2 will be considered as typical. There may be cases, however, in which the conditions represented by Fig. 3 are more nearly approached and the invention is applicable to both of these as well as to other situations. Further reference to Fig. 3 will be found at the end of this specification where some general statements are made as to the types of solutions applicable to the conditions assumed in Fig. 3.

As noted above, the transformer design also enters into the design of the feedback amplifier with which it is used. One of the problems in feedback amplifier design, especially to secure the advantages of large feedback, is the problem of so relating the phase and gain around the $\mu\beta$ loop as to stabilize the circuit against singing tendency up to the limit of the required degree of feedback. At very high frequencies, far higher than the utilized frequencies, the tubes lose their gain at a definite rate and the phase around the feedback loop is determined mainly by the shunt impedances which degenerate to capacities. As disclosed in my United States Patent No. 2,123,178, issued July 12, 1938, it is possible by proper design of the amplifier-feedback loop to control the manner and the rates at which these changes in gain and phase occur at the very high frequencies as to avoid the possibility of singing while using larger amounts of feedback than previously. The principles laid down in that patent are followed in the design of the amplifiers including their input and output transformers according to the present invention.

Referring to Fig. 1, the feedback voltage returned to the first tube is, broadly speaking, caused by the flow of plate current from the last tube through the output circuit and thence through the feedback impedance, the voltage drop across this impedance being the return voltage. It is apparent from the sketch, however, that the exact voltage which is fed back is modified from this in two respects: In the first place, a portion of the plate current is diverted by the capacity C_3 of impedance Z_3 , and returns to the output cathode without flowing through the feedback impedance. Since the feedback impedance is negligible in comparison with the impedance Z_4 of the output circuit, the fraction of the output or plate current I which actually flows through the feedback impedance is easily evaluated as

$$\frac{Z_1}{Z_3 + Z_4} I$$

In the second place, the two impedances, Z_1 and Z_2 , representing, respectively, the input capacity of the tube and the impedance of the input circuit, form a potentiometer so that only the fraction

$$\frac{Z_1}{Z_1 + Z_2}$$

of the voltage across the feedback impedance is actually applied to the grid of the input tube. The two factors

$$\frac{Z_3}{Z_3 + Z_4}$$

and

$$\frac{Z_1}{Z_1 + Z_2}$$

therefore, represent the modifications in total feedback produced in the input and output circuits. In present practical designs, these circuits are almost alike so that these factors can be regarded as identical.

These factors incorporate the impedances Z_2 and Z_4 of the input and output circuits and, therefore, depend upon the transformer designs. For the actual transformers under consideration, these impedances are much more variable functions of frequency than ordinary transformers would exhibit and the contributions of these terms to the complete feedback characteristic are correspondingly complicated.

From the general design point of view, the principal general criterion on the contribution of the transformers to the feedback circuit is furnished by the asymptotic values to which the two potentiometer terms reduce at extremely high frequencies. At a very high frequency, of course, the tube input and output circuit impedances reduce simply to the capacities C_2 and C_4 . These potentiometer terms, therefore, become

$$\frac{C_2}{C_1 + C_2}$$

and

$$\frac{C_4}{C_3 + C_4}$$

As the discussion in my patent above referred to shows, the fundamental criterion on the amount of feedback obtainable from the complete amplifier is obtained from the asymptotic behavior of the feedback loop at very high frequencies. It will be explained later on that in designing the transformers, the first step is to choose as large values for C_2 and C_4 as is consistent with the other requirements of the transformers, so that the asymptotic characteristics will be as favorable as possible.

Desirable types of curves for the transformer characteristics, in so far as they affect the feedback circuit, are such as those shown by Figs. 4 and 5. Within the useful transmission band, the $\mu\beta$ contribution is substantially constant and very nearly zero. This, as later discussion shows, is dictated by the other requirements on the transformers but it is also valuable for feedback purposes since in this range a nearly constant feedback is usually desired. Beyond the transmission band, however, the characteristic rises rather suddenly to a large loss and then reduces to a constant asymptotic loss at higher frequencies. This part of the characteristic is used to furnish the sharp selectivity near the edges of the bands required by the ideal $\mu\beta$ characteristics described in my patent above referred to.

The relations of the transformer characteristic

to such an ideal cut-off characteristic are exhibited by Figs. 6 and 7. The difference between the transformer and the complete cut-off characteristics must, of course, be supplied by the interstage networks N_1 and N_2 . By using the transformers to supply the sharply selective parts, relatively simple interstage networks without sharp tuning can be used.

From the standpoint of the insertion loss and phase shift of the transformers in the series amplifier-feedback loop, it is seen, therefore, that the capacities C_2 and C_4 across the high sides of the transformers must be large enough to introduce negligibly small attenuation in the band in order not to affect adversely the amount of permissible feedback. The shunt capacity together with the rest of the transformer introduces a peak of attenuation just above the band.

These are the main considerations as far as the insertion effects in the feedback loop are concerned. Other design criteria come in from the standpoint of through transmission between the line sections and the amplifier. In other words, the transformers have been considered in connection with Figs. 4 to 7 from the standpoint of their effects as two-terminal impedances in series in the feedback loop. It is also necessary to consider their design as four-terminal devices for coupling the line to the amplifier input or output.

The three requirements specified above which the transformer design must meet may, in the light of the foregoing considerations and with the conditions of Fig. 2 applicable, be somewhat more explicitly stated. The transformer, when regarded as a two-terminal impedance in series in the feedback loop, is to have attenuation and phase characteristics approaching those given in Figs. 4 to 7. As a four-terminal network terminated on its low side in the line and on its high side in capacity C (Fig. 2) it must have a voltage ratio varying over the band in such way as to provide the required degree of equalization. Again as a four-terminal network terminated on its high side in the total capacity $C+C_0$ the network must have both the required characteristic for line equalization and a maximum transmission characteristic over the prescribed band. It is convenient to refer to the characteristic of the transformer itself with its shunting capacity C , as the external gain characteristic and to refer to the transmission characteristic of the circuit including the transformer working into or out of the total capacity $C+C_0$ as the volume performance characteristic. It was described above that the impedance across points a , b or d , e of Fig. 1 is substantially infinite, so that in designing the transformers for their external gain characteristics the circuit can be thought of as severed at points a , b and d , e . The design must be such that when the circuit is completed at these points the transmission characteristic up to the grid of tube 1 or the plate of tube 4, that is, the volume performance, is maximum. The external gain characteristic should be equally high, as will be described more fully hereinafter.

Using the notation for the capacities as given in Fig. 2, the impedances Z_1 and Z_2 may be represented as in Fig. 8 where the transformer T is either T_1 or T_2 of Fig. 1 or Fig. 2. The impedance $Z_1=R_1+iX_1$ is the impedance looking into the capacity C or, in other words, it is the high side impedance of the transformer when it is terminated appropriately for the computation of external gain. Similarly, the impedance

$$Z_2=R_2+iX_2$$

represents the impedance looking into C_0 , or the high side impedance of the transformer where it is terminated appropriately for a computation of volume performance. Since the two impedances differ only by the parallel capacity C_0 , either can be computed from the other.

In typical repeater designs the relation between the input and output circuit characteristic and the line is that shown in Fig. 9. Since the line attenuation is greatest at the highest frequency, hence maximum amplification is required at the highest frequency, the input and output circuits simulate the line at the top of the band but leave some residual equalization to be done separately at lower frequencies. The corresponding general form of resistance characteristic must be that shown by Fig. 10. Under the Fig. 2 conditions this can be taken to represent either R_1 or R_2 since the external gain and volume characteristics are there supposed to be equal. The extreme values of R_0 and R_m are of special interest, the highest value, R_m , determining the volume performance at the top of the band and the lower value, R_0 , being equal to the line impedance multiplied by the nominal impedance ratio of the transformer.

Having given the problem of securing a general resistance-frequency relationship of the type shown in Fig. 10 by means of a circuit of the general form shown in Fig. 8, or of this form to start with, it is advantageous to make use of a certain general resistance-capacity relation. It can be shown that any minimum reactance physical network impedance which vanishes at infinite frequency like a capacity C_∞ must have a resistance component related to the capacity by:

$$\int_0^\infty R d\omega = \frac{\pi}{2C_\infty} \quad (1)$$

This equation obviously applies directly to the calculation of the maximum step-up obtainable from input or output transformers terminated in a prescribed capacity on the high impedance side. Since the high side resistance is proportional to the square of the voltage or current gain through the circuit, and since R cannot be negative at any frequency, it is possible to represent the maximum response through the transformer over any prescribed band extending from ω_1 to ω_2 in the form

$$\int_{\omega_1}^{\omega_2} e^{2a} d\omega \leq \frac{\pi}{2R_1 C_\infty} \quad (2)$$

where R_1 is the resistance of the line and the equals sign applies only in the limiting case when the resistance is concentrated entirely between ω_1 and ω_2 .

As an example of these relations we may suppose that the desired transmission characteristic is flat over the frequency range between ω_1 and ω_2 . Equation 2 then becomes

$$e^{2a} \leq \frac{\pi}{2(\omega_2 - \omega_1) C_\infty R_1} \quad (3)$$

This is the same as saying that the maximum flat gain obtainable from the best possible transformer terminated in the prescribed capacity C_∞ is the same as that which would be obtained from an ideal transformer whose high side impedance is equal to

$$\frac{\pi}{2}$$

times the impedance of the capacity for the

band width in question. In most of this discussion, the concern will actually be with transformers whose transmission characteristics vary with frequency according to some prescribed law. To study such a case, suppose that the gain α is written as $\alpha_0 + \alpha_1$ where α_1 represents the desired variation in the transmission characteristic and α_0 is a constant specifying the general level of transmission. The maximum obtainable level can then be determined from the equation

$$\alpha_0 \leq \frac{1}{2} \log_e \left[\frac{\pi}{2C_{\infty} R_1 \int_{\omega_1}^{\omega_2} e^{2\alpha_1} d\omega} \right] \quad (4)$$

where the integral in the right-hand side can be evaluated numerically in any particular case, since the variation of α_1 with frequency is supposed to have been prescribed.

In the present problem the capacity C_{∞} is evidently C for the R_1 or external gain characteristic and $C+C_0$ for the R_2 or volume characteristic. Obviously, therefore, the resistance integral for R_1 must be appreciably greater than that for R_2 . Since the two resistance characteristics are identical in the useful band, when working under the conditions of Fig. 2, a considerable surplus resistance may be expected beyond the useful band for the R_1 characteristic and a much smaller surplus for R_2 . The fact that the surplus for R_2 must be relatively small makes it possible to simplify the analysis somewhat, since the assumption can be made that the characteristic drops to zero rather rapidly outside the band. Consequently, attention will be confined henceforth to this one of the two resistances.

Evidently, the maximum possible volume performance, for a given value of C_0+C , is realized if the complete resistance integral is consumed by the characteristic in the useful band. This, however, must require an infinite number of elements, since it corresponds to an abrupt break in the resistance characteristic at the band edge. Moreover, if attempt is made to approach the limit too closely the apparent gain in performance may be lost by parasitic dissipation in the elements. The actual ratio between the resistance integral in the band and the integral over the complete spectrum will be called the "resistance efficiency" or simply the "efficiency" of the circuit. For illustrative purposes the efficiency will be taken as 80 per cent, which corresponds to a degradation of 1 decibel per transformer from the ideal. In the general design process, however, it is intended to be an arbitrary parameter chosen in advance by the design engineer. By choosing a high value, he obtains high performance, while by choosing a lower value he is led to an easier design problem and a simplified circuit, at the penalty of somewhat poorer performance.

With the foregoing general relations in mind, the first step toward actual circuit design is the determination of a suitable value or range of values for the capacity C . Evidently, the resistance integral will be largest, and therefore the best volume performance will be secured, if C is very small. But it was seen from the preliminary considerations above that the value of capacity C also affects the feedback. The maximum feedback is obtained if C is made large. This suggests a solution to find the optimum value for capacity C . The capacity C_0 is assumed fixed by the tube structure. The volume performance varies in some inverse manner with

respect to $C+C_0$ but this sum is subject to control only to the extent of varying C .

It can be shown from the teaching of my prior patent above referred to that a degradation in the high frequency characteristic of the feedback loop of A neper will reduce the available feedback by

$$\frac{12(l-y)A}{k}$$

where k is the asymptotic slope of the feedback loop in decibels per octave and y is a small fraction which determines the phase margin in the circuit. In the present instance, since two transformers are involved in the complete loop, the loss A is readily found to be

$$2 \log_e \frac{C_0 + C}{C}$$

On the other hand, it is noted that the transformer high side resistance, which is proportional to the square of the voltage step-up, must be inversely proportional to the terminating capacity $C+C_0$. Under the conditions specified in Fig. 2, this effect must be evaluated both for the output transformer of one repeater and the input transformer of the succeeding repeater. For the two together it becomes $\log_e (C_0+C)$.

It will be assumed that the best value of C is the one for which a slight change in C in either direction degrades one of these factors as much as it improves the other. Upon adding the two quantities together and differentiating with respect to C , we have

$$\frac{1}{C_0+C} - \frac{24y}{k} \frac{C_0}{C(C_0+C)} = 0 \quad (5)$$

which reduces to

$$C = \frac{24y}{k} C_0 \quad (6)$$

In the coaxial repeaters, where k is 18, this means that C should be about 1.2 C_0 . Under the conditions represented by Fig. 3, the analysis is similar except that since the input transformer is no longer of interest, the coefficient of the term representing the degradation in volume performance is reduced by one-half. The corresponding value of C is then twice as great as that given by Equation 6.

This analysis holds when the desired ratio between R_M and R_0 in Fig. 10 is not too great. For example, taking as the unit of frequency the frequency, ω_0 , of the upper edge of the band and taking

$$\frac{1}{\omega_0 C_0}$$

as the unit of impedance, if $C=1.2C_0$ and the efficiency is 80 per cent, the resistance integral over the useful range is 0.57. This evidently will not lead to a value of $R_M > 1$ as long as the prescribed resistance characteristic is either flat or only moderately tilted. The significance of this statement will appear later on in considering Equation 7. Depending somewhat upon the curvature of the prescribed characteristic, a reasonable assumption to make is that the limiting case will be represented by $R_M=1$ and R_0 —about 0.4. For the two transformers together, this corresponds to a selectivity of 8 decibels over the useful range.

Values of R_M/R_0 which exceed this limit can evidently be accommodated by assuming a lower resistance efficiency for the circuit. In general, however, it is preferable to choose $R_M=1$ and determine a suitable value for C from the resulting area under the resistance characteristic. For

reasonable selectivity, this may lead to values of C in the range 1.5–2.0 C_0 .

The actual designs, values of C in the neighborhood of 1.5 C_0 or 1.6 C_0 have been used. This compromises reasonably well between the two values determined above. It has the additional advantage over the value 1.2 C_0 that it somewhat favors feedback at the expense of noise, which is desirable since there are other reasons than modulation suppression for desiring a sufficient amount of feedback.

Thus a determination of a suitable value for capacity C in terms of capacity C_0 has been arrived at from a general consideration of the relation of total capacity to the resistance integral and to maximum gain and volume performance on the one hand, and the relation of C to the asymptotic behavior of the feedback amplifier on the other hand.

The next step in the preliminary design is to specify either Z_1 or Z_2 (Fig. 8) in a form for which the reactance and resistance are physically consistent with each other and with the prescribed high side capacities C and $C+C_0$.

Since the two impedances Z_1 and Z_2 of Fig. 8 differ only by the parallel capacity C_0 , a set of simple relations must exist between their resistances and reactances. In accordance with the working assumption that the conditions are those given in Fig. 2, R_1 and R_2 are to be taken as equal in the useful band. The two reactances are then

$$X_1 = -X_2 = \frac{1}{\omega C_0} [1 - \sqrt{1 - \omega^2 C_0^2 R^2}] \quad (7)$$

where R represents either R_1 or R_2 .

It should be noted that this expression is a frequency-by-frequency specification which needs to be satisfied only in the useful band. Outside the band the resistance-reactance relations may vary from the Equation 7 relation and it is this fact which enables a physical solution to be realized.

It is shown in my prior patent and in my application, Serial No. 260,682, filed March 9, 1939, that in any ordinary physical network the reactance component of the impedance can be determined from the resistance component by means of the relation

$$X(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dR}{du} \log \coth \frac{|u|}{2} du \quad (8)$$

where

$$u = \log_e \frac{\omega'}{\omega}$$

ω' being the variable of integration. A solution to be physically realizable must, therefore, satisfy Equation 8. Equation 8 it will be noted takes account of the resistance characteristic over the complete frequency spectrum. This fact offers the possibility, necessary to a practical solution, of proportioning the resistance outside the useful band to influence the shape of the characteristic within the band, and, specifically, to simulate the required reactance characteristic as computed from Equation 7. Since there must be some surplus resistance outside the band to manipulate, the entire resistance integral cannot be taken up by the characteristic in the useful band and the resistance efficiency must be less than 100 per cent even theoretically. The assumed value of 80 per cent, however, allows more than enough margin for this purpose.

The steps in the procedure are, therefore, to compute the reactance within the band from Equation 7 on the assumptions that $R_1 = R_2 = R$.

The total resistance characteristic is then derived, in a manner which will more fully appear later on, and such a shaping of it is made outside the band as will give the required reactance within the band as computed. After the complete resistance characteristic has been determined, the reactance characteristic outside the band can be found, thus completing the total reactance characteristic for the entire frequency spectrum. This procedure provides the specifications of either Z_1 or Z_2 in a form that is physically realizable.

It is convenient in attacking the problem of determining the reactance characteristic to consider the frequency range as divided into two portions, a region extending over the lower two-thirds or so of the band and the remaining region near the upper band edge. The high frequency characteristic is usually the larger and more critical. It is found in practice that approximate methods of design suffice in the case of the lower frequency portion and that the necessary reactance characteristic in this region is automatically provided with sufficient accuracy for practical purposes if the resistance characteristic is of a reasonable type.

The problem of determining the reactance in the frequency region near the top of the band such as to meet the requirements of Equation 7 is facilitated by the method of successive approximation, guided by the general relations between resistance and reactance as worked out in my prior patent and application referred to. It is there shown how the variations in the resistance component over one portion of the total frequency range influence the shape of the reactance characteristic in an adjacent portion of the band. By observing the general way in which the two characteristics are related, much time and effort can frequently be saved in solving a design problem. In a practical design there will always be an excess of resistance integral R_2 above the band which allows of manipulation to adjust the high-end reactance, since otherwise a theoretically infinite number of elements would be required to give the infinitely sharp break in the characteristic. Further, as a practical matter a determination of the reactance at one frequency is usually sufficient and for convenience this frequency will be taken at the upper band-edge.

As a first assumption, referring to Fig. 10, let $R_M = 1$. Then the reactance required at the band-edge from Equation 6 will also be unity. An appropriate resistance characteristic to furnish this reactance will lie somewhere between two extremes. One extreme, for example, is obtained by continuing the resistance characteristic at unity up to some point ωc and then allowing it to drop abruptly to zero, as shown by Fig. 11. Such a characteristic may be studied in terms of Equation 8 by representing the sharp drop at ωc as an extremely large negative value of

$$\frac{dR}{du}$$

concentrated in a very narrow range of values of u . Since

$$\frac{dR}{du}$$

is otherwise zero beyond the useful range the complete reactance at ωc can then be determined by combining the reactance which would correspond to this component of

$$\frac{dR}{du}$$

in accordance with Equation 8, with the reactance corresponding to the

$$\frac{dR}{du}$$

characteristic representing the resistance variation in the useful band. Considering only the first of these, we find from Equation 8 that the reactance at ω_0 is negative and infinite when ω_0 coincides with ω_0 . As ω_0 is moved to higher values, however, the absolute value of the reactance diminishes rapidly and becomes less than unity when $\omega_0 > 1.1\omega_0$. Since

$$\frac{dR}{du}$$

in the band, and therefore the reactance to which it corresponds, is positive, the inclusion of this component still further diminishes the result. The resistance integral beyond the band, however, will normally be greater than 0.1 so that if the proportions of Fig. 11 are followed, ω_0 must be greater than $1.1\omega_0$. These proportions therefore lead to too small a reactance at the band-edge.

The other extreme is that illustrated by Fig. 12. These proportions give an infinite reactance at the band-edge. As noted, a suitable characteristic must be intermediate between these extremes. Possible characteristics, for example, are shown in Figs. 13, 14, and 15. Since a rather smooth curve is most easily approximated in the final design, the last of these would undoubtedly be preferred in practice.

As an example of the process of reactance simulation, a flat transformer will be assumed whose resistance characteristic is equal to 1 in the useful range. A plot of the corresponding required value of X_2 as determined from Equation 6 is given by curve A of Fig. 16. If we first suppose that the resistance characteristic drops abruptly beyond the band to some negligibly small value as shown by curve I of Fig. 17, the corresponding reactance characteristic is that given by curve I in Fig. 16. It will be seen that the reactance is somewhat too high throughout and reaches an infinite value at the edge of the band because of the discontinuity in the resistance characteristic. Both of these phenomena merely indicate, however, that the assumed change in resistance was too sharp. If we assume, for example, that R_2 varies, as shown by curve II of Fig. 17, the reactance characteristic is that given by curve II of Fig. 16, while if we assume the more elaborate characteristic shown by curve III, Fig. 17, we secure the almost perfect match indicated by the crosses of Fig. 16.

It may be interesting to notice that the results shown by Figs. 16 and 17 can be approximately verified by a simple network. Let it be supposed that the transformer is an ideal constant-k low-pass filter terminated in a fractional shunt branch. Consider the terminations of $0.4 C_k$ to represent Z_1 of Fig. 8 and $1.6 C_k$ to represent Z_2 of that figure, where C_k is the capacity which would be appropriate for a mid-shunt termination. Clearly then $0.4 C_k$ represents the C of Fig. 8 and $1.2 C_k$ the C_0 . Their ratio is 0.33, which in accordance with Equation 1 gives an area under the R_2 curve of 1.19. This is substantially intermediate between the areas under curves II and III of Fig. 17. By building out a mid-shunt termination by the addition of the extra capacity $0.6 C_k$, however, there is added an $M=0.6$ half section, except that the required series branch is not included. The resistance component after this addition must therefore be the same as the mid-

series image impedance of such a half section, while the reactance component is the negative of the missing series arm. When the capacity $0.6 C_k$ of the mid-shunt termination is subtracted to produce the fractional termination of $0.4 C_k$, on the other hand, there is secured this same resistance but a reactance of opposite sign. Resistance and reactance characteristics for this case as compared with those shown by Figs. 16 and 17 can therefore be plotted from ordinary filter theory. Upon making an appropriate change in the frequency scale, to take account of the fact that the admittance of the C_0 of this example is 1.2 at the cut-off, the results appear in the form shown by Figs. 18 and 19. The broken lines represent the characteristics shown by curves II in the preceding figures.

The application of the technique to a circuit with tilted characteristics is shown by Figs. 20 and 21. The maximum resistance is taken as slightly less than unity in order to avoid the very sharp characteristics which would result if this limit were attained. As in the previous figures, curve A (of Fig. 21) gives the required reactance corresponding to the prescribed resistance characteristics in the useful band. Curve I gives the result if the resistance is first supposed to vanish abruptly beyond the band, and curve II the result if the resistance disappears more gradually. In a practical design, of course, the resistance characteristic actually realized would be of the smoothly varying type exemplified by the broken line in Fig. 20.

The area under curve II of Fig. 20 is 0.50. This corresponds to a resistance efficiency of slightly less than 90 per cent. If the broken line characteristic is used, this value is diminished. In either case, the total area corresponds to a value for C equal to about $2C_0$.

The principal control of the feedback characteristic exercised in the design process is that obtained through the choice of the asymptotic potentiometer ratio determined by C and C_0 . In addition, however, it is possible to control the details of the $\mu\beta$ characteristic to some extent. This results from the fact that in general the high frequency resistance characteristic required to produce a sufficiently accurate reactance at the band-edge is not unique. By varying the resistance characteristic, one can indirectly control the $\mu\beta$ characteristic. In general, as might be expected, spreading the resistance out above the band makes the $\mu\beta$ characteristic less sharply selective, and vice versa.

The foregoing discussion illustrates the procedure that may be followed in any given situation for determining the resistance and reactance characteristics of the impedance Z_1 (or Z_2). It remains to show how a network may be constructed to possess this impedance.

This is evidently a problem in filter theory and the various resources of filter design technique are available for its solution. For example, if we regard the input or output circuit of the repeater as a filter whose image impedance at the line end matches the line resistance very closely, the resistance characteristic on the other end will depend upon the type of image impedance assigned and can be controlled in the usual fashion by the use of appropriate impedance controlling factors.

A technique may be used that is described broadly in my U. S. Patent No. 1,814,238. It is shown there that if a ladder line is constructed

of alternate series inductances and shunt capacities terminated in the resistance R_0 , the input resistance of the structure can be written in the general form

$$R = \frac{R_0}{1 + A_1\omega^2 + A_2\omega^4 + \dots + A_n\omega^{2n}} \quad (9)$$

where the coefficients $A_1 \dots A_n$ depend upon the particular values assigned to the elements of the network. Conversely, knowing these coefficients, the elements themselves can be determined. Since it is clear from the equation that with this type of structure the ratio of R_0 to R depends simply upon a polynomial, use can be made of elementary mathematical methods to construct a suitable polynomial in any given case and from this determine an appropriate network.

The design procedure can best be explained by means of an example. Fig. 22 shows a preliminary design configuration for the input and output circuit for a 3-megacycle amplifier. C_0 and C have their previous significance. The condenser C_1 is an allowance for parasitic capacity about the inductance L_1 and would not necessarily appear if this inductance were perfect. Knowing either Z_1 or Z_2 , the first step in the procedure is to compute the impedance Z_3 by subtracting the capacities C and C_0 . In making this computation, allowance should be made for the fact that C must be slightly less than the true asymptotic capacity assumed in the preliminary design because of the existence of the parasitic path through C_1 and C_2 . This path is practically the same as C_1 since C_2 is relatively much larger than C_1 .

The elements L_1 and C_1 do not affect the resistance component of Z_3 and can be disregarded for the moment. This resistance is then matched by means of Equation 9. It appears by trial that three terms of the denominator are sufficient for a suitable match, which explains the existence of the three elements C_2 , L_2 , and C_3 . In other circumstances this network could be either extended or diminished as the particular resistance characteristic requires. The reactance characteristic furnished by this part of the network is next computed and subtracted from the total reactance required from Z_3 . The difference is then simulated by L_1 and C_1 .

In order to convert this design into the final structure, use may be made of the fact that a physical transformer is approximately equivalent to the ideal transformer and associated elements indicated by Fig. 23. Since this is essentially a high frequency design, the mutual inductance L_M can be ignored. The leakage inductance L_L is then identified with the coil L_2 and the high side parasitic capacity is absorbed as part of the condenser C_2 . The ideal transformer takes account of the fact that the R_0 of Equation 9 is not the same as the actual line impedance. This resistance and the capacity C_3 are assumed to appear on the low side of the transformer and their values changed in accordance with the impedance ratio of the structure. The final circuit is shown by Fig. 24.

As a first example of specific design, input and output transformers for a 2-megacycle coaxial system will be considered. Their schematic is shown by Fig. 25. The external gain characteristic and volume performance characteristic are shown on Figs. 26 and 27. In each case the curves are doubled to take account of the fact that there are two transformers in each repeater. The plot of the line attenuation shown in each

figure indicates the extent to which the characteristic is equalized by the transformers. The contribution of the transformers to the $\mu\beta$ loop is shown by Fig. 28.

Upon subtracting the external gain characteristics of the transformers from the line characteristic, the required preequalization shown by curve I of Fig. 29 is obtained. Curve II of this figure shows the amount of preequalization which would be required from a perfectly flat amplifier with flat transformers. The difference, of course, measures the general improvement in volume performance which can be expected from the system through the introduction of tilt characteristics. In order to utilize this improvement effectively, however, the volume performance curves of the transformers must, of course, be tilted as well as their external gain characteristics. Curve I of Fig. 30 represents the net volume performance characteristic for the system when the amount of preequalization indicated by curve I of Fig. 29 is added to the line loss. Curve II of Fig. 30 represents a very rough estimate of the volume performance which might be achieved with conventional flat transformers having the same asymptotic $\mu\beta$ characteristics.

The next specific design example comprises input and output circuits of a 3-megacycle system intended primarily to transmit telephone messages in the band up to 2 megacycles with no utilization of frequencies beyond this point. Stringent requirements on noise modulation are assumed to exist only for the telephone band. The actual transmission band, however, is extended to somewhat beyond 3 megacycles to allow the system to be used for 441 line television transmission rather than telephone, if desired. The external gain characteristics must therefore be satisfactory up to 3 megacycles or more.

This dual requirement makes it necessary to use a more elaborate design than was adequate for the 2-megacycle system example just considered. Since even residual preequalization in the frequency band near 2 megacycles would be detrimental to the volume performance, the external gain characteristics are assumed required to match the line quite accurately from 3 megacycles down to about 1 megacycle. At the same time, of course, it is desirable to concentrate the volume response as much as possible in the range below the 2 megacycles.

The schematic of these networks has already been shown in Fig. 24. Their external gain characteristic is shown by Fig. 31 in comparison with the attenuation of the line. Fig. 32 gives an enlarged version of the top of the characteristic to show the accuracy of line match which can be achieved by calculation. This accuracy, of course, is not easily obtained directly from the design technique sketched above especially when allowance is made for parasitic dissipation. The technique should be accurate enough, however, to permit final corrections to be made by simple first order approximations.

The volume characteristic and $\mu\beta$ characteristic are given by Figs. 33 and 34. The preequalization requirement, found in a manner analogous to that of Fig. 29, is shown by Fig. 35 and the net volume performance characteristic or the system as a whole by Fig. 36.

Reverting now to the conditions assumed in Fig. 3, where resistance noise is assumed to be of greater importance from a design standpoint than tube noise, it has already been pointed out that a design giving the best signal-to-noise ra-

tio over all channels would result in a transmission characteristic that is flat from A to G of Fig. 3.

One possible solution is obtained by letting the output circuit equalize the complete line in both external and volume performance. This is obviously the same as the design problem actually considered above except that the tilt is doubled. The input circuit must then be flat, at least in external gain, in order not to upset the line equalization.

The problem may also be solved by allowing the output circuit to equalize the line completely in its volume performance but with the external gain equalization divided between input and output circuits. This might be superior in practice as requiring less average tilt per transformer. It would be necessary to compute X_1 and X_2 directly without using the relationship given in Equation 7, however, since R_1 and R_2 can no longer be considered as equal to each other.

The invention is not to be construed as limited to the specific circuits that have been disclosed nor to the quantitative data given, these being illustrative rather than limiting; the scope of the invention is indicated in the claims which follow.

What is claimed is:

1. A repeater section for broad band transmission comprising an amplifying repeater at each end of the section, with an intervening line of variable attenuation over the band, each repeater having a stabilizing feedback path, said repeater section as a whole having a transmission characteristic that is flat over a significant portion of the band, the means which renders the transmission characteristic flat comprising in part a non-unity ratio transformer between one of said repeaters and the line, having its high side connected to the repeater and its low side connected to the line, said transformer being terminated on its high side in a capacity serially included in said feedback path.

2. A combination according to claim 1 in which said capacity is of a predetermined value which is substantially optimum both as regards the phase characteristic of said repeater in the region of the high frequency gain cut-off and as regards the voltage transfer between the repeater and the line in said significant portion of the band.

3. In a space discharge tube amplifier circuit, a transmission circuit, means coupling said space discharge tube amplifier to said transmission circuit comprising a transformer, effective capacity C in shunt across the terminals of said transformer facing said amplifier, effective capacity C_0 across the terminals of the space discharge tube facing said transformer, and a stabilizing feedback circuit around said amplifier including in series the capacity C , the ratio of capacity C to capacity C_0 being between unity and 2.0.

4. In an amplifier circuit for waves of a broad band of frequencies, said amplifier having a negative feedback for reducing distortion and stabilizing gain, an input and an output transformer for connecting said amplifier between two sections of line, each transformer having effective

shunt capacity across its high side terminals, said transformers having an attenuation characteristic which rises steeply to a large value just above the band and then falls to a substantially constant asymptotic value at higher frequencies whereby said transformers contribute the principal part of the sharp selectivity near the upper edge of the band required by the ideal $\mu\beta$ characteristic.

5. A repeater section for a line transmitting waves of a broad band of frequencies, including in the order of their occurrence: the output stage of a first stabilized feedback repeater, output transformer having a capacity across its high side, section of line, input transformer having a capacity across its high side, input stage of next stabilized feedback repeater, said transformers having a sloping gain characteristic over a significant portion of the band such that the transmission characteristic from the output electrode of the first-mentioned repeater stage to the input electrode of the second-mentioned repeater stage is substantially flat over said portion of the band.

6. In a repeater circuit for a line transmitting waves of a broad band of frequencies, a stabilized feedback type of repeater, a circuit for coupling said repeater to a section of line, including a two-winding transformer, a capacity across the high side of said transformer, said capacity being serially included in the feedback path, said transformer being proportioned with respect to the line such that when terminated only in said capacity its impedance conforms to that of the line over at least the upper portion of the transmitted frequency band, said coupling circuit, when connected to said repeater and line, including means causing its impedance to conform to that of the line over at least the upper portion of the band, including means to adjust the reactance near the upper edge of the band by controlling the resistance characteristic above the band.

7. In transformer design for broad band transmission line systems having varying attenuation over the band, the method of making both the external gain characteristic and volume performance characteristic have substantially the same shape within the band and conform to the line characteristic over at least the high frequency portion of the band comprising controlling the rate of variation of the resistance integral of the external gain characteristic with frequency in the region immediately above the band.

8. In a system for the transmission of waves of a broad band of frequencies over a line whose attenuation varies over the band, a repeater comprising one or more space discharge tubes for amplifying the transmitted waves, said repeater having a gain-reducing feedback path, and a coupling circuit comprising a non-unity ratio transformer for coupling said repeater to said line, said coupling circuit having external gain and volume performance characteristics (as defined in the specification) that are high and substantially equal and that conform to the attenuation characteristic of said line over a significant portion of the transmitted range.

HENDRIK W. BODE.