

June 18, 1940.

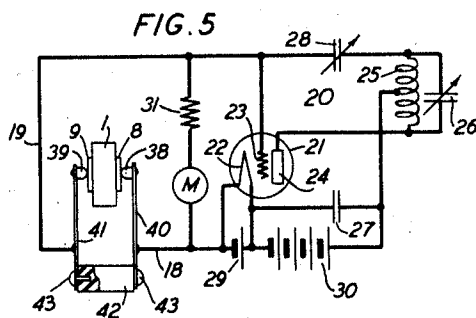
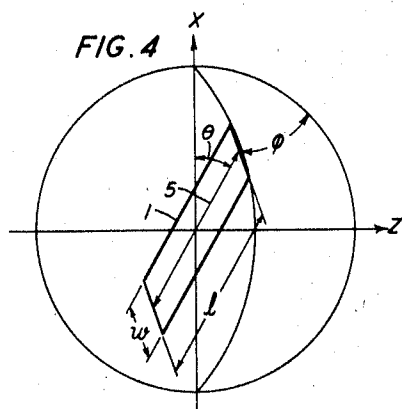
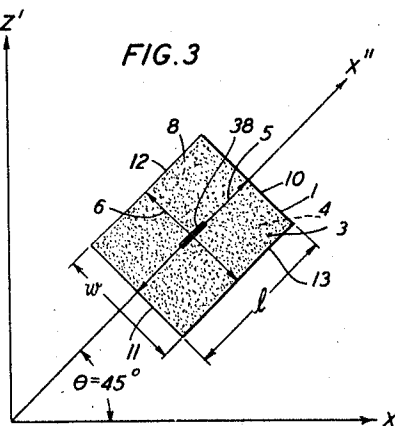
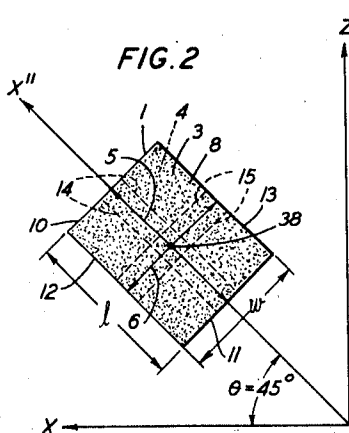
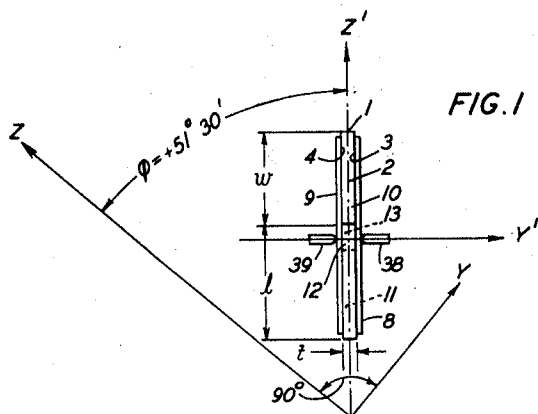
W. P. MASON

2,204,762

PIEZOELECTRIC CRYSTAL APPARATUS

Filed Dec. 21, 1937

2 Sheets-Sheet 1



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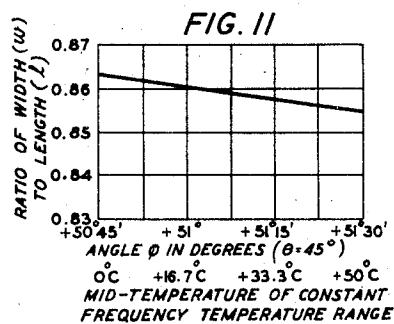
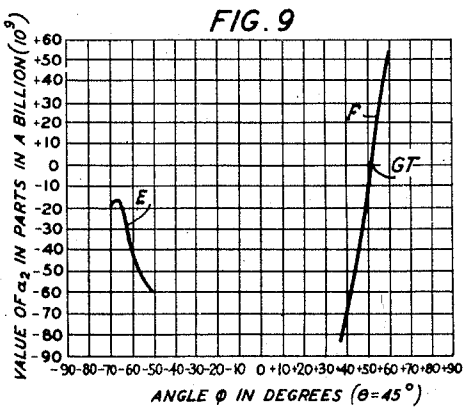
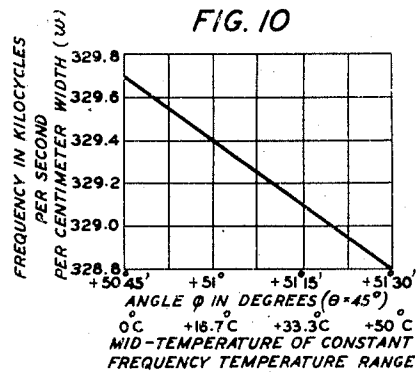
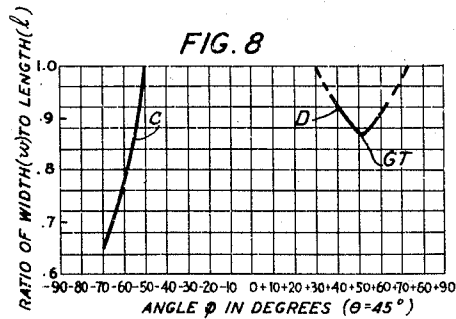
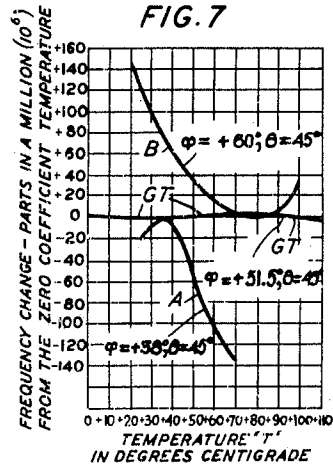
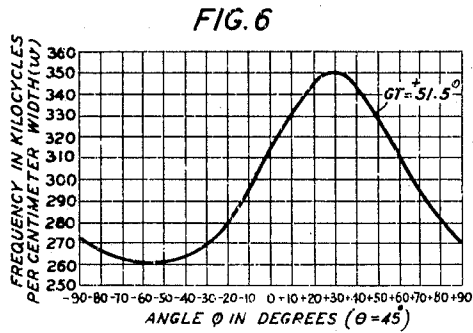
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PIEZOELECTRIC CRYSTAL APPARATUS

Filed Dec. 21, 1937

2 Sheets-Sheet 2



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UNITED STATES PATENT OFFICE

2,204,762

PIEZOELECTRIC CRYSTAL APPARATUS

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Application December 21, 1937, Serial No. 180,921

18 Claims. (Cl. 171-327)

This invention relates to piezoelectric apparatus and particularly to piezoelectric crystals suitable for use as frequency determining elements in vacuum tube oscillation generator systems and in electric wave filter systems or other networks, for example.

This application is a continuation in part of my application Serial No. 84,858 filed June 12, 1936.

One of the objects of this invention is to obtain a very constant frequency of vibration over a wide range of temperatures.

Another object of this invention is to provide piezoelectric apparatus that may have zero or other predetermined temperature coefficient of frequency.

Another object of this invention is to produce a piezoelectric body that may have a frequency substantially independent of changes in temperature over a substantial range of temperatures to permit temperature regulating apparatus to be simplified or eliminated and to permit a constant vibration frequency to be maintained.

Another object of this invention is to produce a piezoelectric crystal element that may have substantially zero or other desired predetermined temperature coefficient of oscillation frequency, either positive or negative, and that may have a relatively low frequency vibration in the range from 30 to 1000 kilocycles per second, for example.

It is well known that a quartz oscillator may produce an oscillation of fairly constant frequency and yet there may exist various factors affecting the constancy of oscillation frequency. Among them, the variation of the operating temperature of the crystal is an important factor. It is desirable, therefore, to obtain a crystal having its oscillation frequency independent of the temperature variation.

In accordance with this invention, a piezoelectric crystal element may be cut to have such multiple orientation angles with respect to the orthogonal crystallographic or X, Y and Z axes thereof and may have such selected shape or dimensional ratio of axes as to obtain improved operating characteristics therefor. The multiple orientation angles and the shape may be such as to obtain a low or substantially zero temperature coefficient of frequency for the crystal at selected temperatures within or over a very wide range of temperatures as, for example, from 0 degrees to 100 degrees centigrade.

In a particular embodiment, the crystal element may be a quartz plate which has a substantially rectangular major plane of a selected dimensional ratio of width to length and a double orientation with respect to the orthogonal X, Y and Z axes thereof in order to obtain a low or substantially zero temperature coefficient of frequency over a wide range of temperatures. As applied to quartz, such dimensional ratio may be substantially 0.855

and such double orientation may include a rotation of the major plane of the quartz plate about an electric axis X thereof, to a position substantially $+51^{\circ} 30'$ with respect to the optic axis Z thereof, and another rotation of the body in either direction about the new mechanical or thickness axis Y' thereof to such angular position that the major or principal axis of the major plane is inclined substantially 45 degrees with respect to the orthogonal electric axis mentioned.

Such a quartz crystal plate, when subjected to an electric field in the Y' direction perpendicular to its major plane, has a desired fundamental resonance frequency which is dependent upon the width or minor axis dimension of its major plane and which has the advantage of a very constant frequency throughout a wide range of temperatures as, for example, throughout a 100 degree centigrade range of temperatures. The frequency of an oscillator controlled by such a crystal plate may not vary more than one cycle in a million and accordingly such an oscillator may be advantageously utilized as a portable or fixed frequency standard, for example. This constancy of frequency represents a substantial improvement in the order of the magnitude of the temperature range of crystals having a low or substantially zero temperature coefficient of frequency.

The crystal may also be utilized as a selective element in any suitable electric wave filter system or other network since the nearest resonant frequency thereof is about 17 per cent different in frequency from the desired vibration or fundamental frequency thereof. A simple tuned circuit or electrical network may be utilized to remove the effect of such nearest resonance frequency.

For a clearer understanding of the nature of this invention and the additional features and objects thereof, reference is made to the following description taken in connection with the accompanying drawings, in which like reference characters represent like or similar parts, and in which:

Figs. 1 and 2 are respectively an edge view and a projected major surface view of a piezoelectric quartz crystal plate embodying this invention;

Fig. 3 is a view similar to Fig. 2 but illustrating an alternative orientation thereof;

Fig. 4 is a perspective view illustrating the multiple orientation angles of the crystal plate shown in Figs. 1 to 3;

Fig. 5 is an illustrative diagram of a particular oscillator circuit and crystal holder arrangement;

Fig. 6 is a graph illustrating the relation between the angular orientation and the frequency constant of quartz crystals embodying this invention;

Fig. 7 is a graph illustrating the temperature-

frequency characteristics of quartz crystals constructed and operated in accordance with this invention;

Fig. 8 is a graph illustrating the dimensional ratios of quartz crystals in accordance with this invention;

Fig. 9 is a graph illustrating values of α_2 for quartz crystals made in accordance with this invention;

Fig. 10 is a graph showing the relation between the orientation angle, the temperature range, and the frequency of crystals embodying this invention; and

Fig. 11 is a graph showing the relation between the orientation angle, the temperature range, and the ratio of dimensions of crystals embodying this invention.

This specification will follow the standard terminology as applied to quartz which employs orthogonal X, Y and Z axes to designate the orthogonal electric, mechanical and optic axes respectively of the piezoelectric quartz crystal material and which employs X', Y' and Z' to designate the directions of axes or surfaces of a piezoelectric body angularly oriented with respect to the orthogonal X, Y and Z crystallographic axes thereof. Where the orientation is obtained by double rotations, one about an electric axis X and the other about another axis of the piezoelectric body as particularly illustrated in Figs. 1 to 4 herein, the orientation angles ϕ and θ respectively designate the effective angular position of the crystal in degrees as measured from the optic axis Z and from the orthogonal electric axis X, respectively. The axis X'' in Figs. 2 and 3 indicates the result of a second rotation.

Quartz crystals may occur in two forms, namely, right-hand and left-hand. A crystal is designated as right-hand if it rotates the plane of polarization of plane polarized light traveling along the optic or Z axis in the sense of a right-handed screw or in a clockwise direction when facing in the direction of propagation of the light and is designated as left-hand if it rotates the plane of polarization in the opposite or counter-clockwise direction. If a compressional stress be applied to the ends of the electric axis of a quartz crystal body and not removed, a charge will be developed which is positive at the positive end of the electric axis and negative at the negative end of the electric axis for either right-hand or left-hand crystals. The magnitude and sign of the charge may be measured with a vacuum tube electrometer, for example. In specifying the orientation of the right-hand crystal, the angle ϕ which the new axis Z' makes with respect to the optic axis Z as the crystal plate is rotated about the electric axis X is deemed positive when, with the positive end (by compression) of the X axis pointed toward the observer, the rotation is in a clockwise direction. A counter-clockwise rotation of such a crystal gives rise to a negative orientation angle ϕ . Conversely, the orientation angle ϕ of a left-hand crystal is positive when, with the positive end (by compression) of the electric axis X pointed toward the observer, the rotation is counter-clockwise, and is negative when the rotation is clockwise.

Referring to the drawings, Figs. 1 and 2 illustrate respectively projected edge and major surface views of a right-hand piezoelectric quartz crystal plate 1 of substantially rectangular parallelepiped shape. The major plane 2 and the

opposite parallel major surfaces 3 and 4 of the crystal 1 may be disposed substantially parallel to an electric axis X thereof and inclined at a selected acute angle of ϕ =substantially $+51^\circ 30'$ with respect to the optic axis Z thereof as illustrated in Fig. 1 where the electric axis X is perpendicular to the plane of the drawing with the compression positive end thereof pointed toward the observer and is orthogonally related to the optic axis Z and the mechanical axis Y.

The length dimension 1 and the principal or major axis 5 of the quartz plate 1 may be inclined at a selected acute angle of θ =substantially 45 degrees (or 135 degrees) with respect to the mentioned orthogonal electric axis X, as illustrated by the alternative orientations shown in Figs. 2 and 3.

The dimensional ratio of the width dimension w and the minor axis 6 of the crystal 1 with respect to the length dimension 1 and the major axis 5, respectively thereof, may be substantially 0.855 as shown in Fig. 8 at GT and in Fig. 11 to obtain a low or substantially zero temperature coefficient of frequency for the crystal 1 having the orientation angles ϕ and θ of substantially $+51^\circ 30'$ and 45 degrees, respectively, as illustrated in Figs. 1 to 3, when the crystal plate 1 is subjected to an electric field in the Y' direction perpendicular to the major faces 3 and 4 and vibrated at a frequency dependent upon its minor axis 6 or width w dimension, said frequency being substantially 328.8 kilocycles per second per centimeter of width dimension w as given by the curves of Figs. 6 and 10 for a ϕ angle of substantially $+51^\circ 30'$.

The crystal plate 1 may be cut to such values of selected double orientation and selected shape by first cutting a slab of suitable thickness t from the natural quartz, the slab having its major plane 2 parallel to an electric axis X and inclined at an acute angle ϕ =substantially $+51.5$ degrees measured from the optic axis Z, as illustrated in Fig. 1. From such slab, another slab or plate 1 may be cut having the major axis 5 or length dimension 1 thereof inclined at an acute angle θ =substantially 45 degrees measured from the electric axis X as illustrated in either Fig. 2 or Fig. 3. The ratio of dimensions of width w to length 1=about 0.855 may be obtained by selectively grinding the edge faces 10 to 13 of the crystal 1.

Electrodes 8 and 9, each of which may consist of a thin coating of chemically deposited silver, or aluminum or other suitable conductive material deposited by sputtering or evaporation and afterwards annealed to relieve strains therein, may be formed integral with or may be otherwise operatively disposed with respect to the opposite major or electrode surfaces 3 and 4, respectively, of the crystal 1 and may be utilized, when connected in circuit with a suitable system such as, for example, an oscillation generator system or an electric wave filter system, for exciting the crystal plate 1 at a vibration frequency determined mainly by the dimensions or area of the major surfaces 3 and 4 thereof to obtain substantially a constant vibration frequency that may not change more than from one to five cycles, for example, in a million cycles throughout a very wide range in temperatures such as, for example, a range extending from 0 to 100 degrees centigrade, as illustrated by the curve GT in Fig. 7.

The frequency range of the crystal 1 may con-

veniently be from about 50 to 1000 kilocycles per second, for example, dependent upon the dimensions selected for the width w and the length l thereof.

Conductive connectors 18 and 19 may be utilized to electrically connect the electrodes 8 and 9 respectively of the crystal 1 in circuit with a vacuum tube oscillation generator 20 as illustrated in Fig. 5, for example, to control the frequency of oscillations thereof. The particular oscillator 20 includes a vacuum tube 21 having a cathode 22, a grid 23, and a plate electrode 24. The output circuit of the oscillator 20 may include a tuning coil 25 connected in parallel circuit relation with an adjustable condenser 26. A by-pass condenser 27 may connect the mid-point of the tuning coil 25 with the cathode 22. A feedback condenser 28 may feed back radio frequency voltage to the grid electrode 23. Suitable batteries 29 and 30, as illustrated, may energize the cathode 22 and the plate electrode 24 respectively. A grid leak resistance 31 and a milliammeter M may be connected between the grid 23 and the cathode 22.

It will be understood that the crystal 1 may be utilized to control the frequency of oscillations of any suitable oscillator, the particular oscillator 20 shown in Fig. 5 being an illustrative example only.

Similarly, the conductive connectors 18 and 19 may be utilized to operatively connect the crystal 1 in circuit with an electric wave filter system to form a selective element thereof, as illustrated, for example, in applicant's publication entitled "Electric wave filters employing crystals as elements" Bell System Technical Journal, page 433, July 1934.

As illustrated in Fig. 5, the crystal 1 having integral electrodes 8 and 9 may be rigidly nodally clamped along the longitudinal center line 5 of the major surfaces thereof by means of one or more pairs of coaxial metallic clamping projections illustrated generally by projections 38 and 39 which may be supported and resiliently controlled by metallic cantilever springs 40 and 41 secured at one end to an insulating block 42 by screws 43. Electrical connections with the crystal 1 may be established through the crystal electrodes 8 and 9, the clamping projections 38 and 39, the springs 40 and 41, and the conductor wires 18 and 19.

The crystal 1 adapted to vibrate in a longitudinal mode of motion along the width dimension w and having orientation angles of ϕ =substantially $+51.5$ degrees and θ =substantially 45 degrees as illustrated in Figs. 1 to 3, may be most easily and securely clamped along the central part of the nodal line 5, which runs parallel to the length dimension l and lengthwise along the center of the crystal 1 midway between the edges 12 and 13 and perpendicular to the width dimension w thereof. The crystal 1 may be clamped along such central nodal line 5 near the center thereof by a pair of opposite clamping projections 38 and 39, each of which may have a rectangular-shaped, flat clamping surface of narrow width covering about 1.5 per cent of the width dimension w and of length which may cover as much as 10 to 15 per cent of the length dimension l of the crystal 1 as illustrated at 38 in Fig. 3. It will be understood that such clamping length of each of the clamping projections 38 and 39 may cover the whole or a part of such lengthwise nodal part indicated at 38 in Fig. 3. Where single clamping surfaces are utilized on each side of the crystal

1, each may extend wholly or partly along that part of the nodal line 5 indicated by the length of 38 in Fig. 3; and where two coplanar clamping surfaces, as illustrated, for example, in Patent 2,032,865 to C. A. Bieling, are utilized on each side of the crystal plate 1 to cover the part 38 indicated in Fig. 3 of such lengthwise nodal line 5, the center of the crystal 1 may be disposed halfway between such two clamping surfaces.

While a particular arrangement for mounting and establishing electrical connections with the crystal 1 has been illustrated in Fig. 5, it will be understood that any suitable arrangement may be utilized for clamping or otherwise mounting the crystal 1 and that any suitable electrodes 15 and circuit arrangement may be utilized for exciting the crystal 1 in the desired mode of vibration.

The mode of vibration of a crystal cut at ϕ = $+51.5^\circ$ from the optic axis, as illustrated in Fig. 1, but with one of its principal axes along or parallel to the electrical axis X instead of being inclined at an angle of 45 degrees, as illustrated in Figs. 2 or 3, is a low frequency Z'X shear vibration which extends one set of oppositely placed corners and simultaneously contracts the other set. If we cut a crystal 1 such that its principal axis 5 lies at an angle of 45° from the electrical axis X, as illustrated in Figs. 2 or 3, then one pair of opposite edges such as the edge faces 12 and 13 will be extended while the other pair of edges 10 and 11 are contracted. As a result the mode of motion of this crystal 1 consists of two longitudinal vibrations coupled together, one vibration being along the length dimension l and the other along the width dimension w . For a square-shaped crystal plate, the two longitudinal vibrations will have the same frequency and, in view of the coupling between them, there will be two measured resonances, one above the natural frequency of resonance and the other below the natural frequency of resonance. If, however, one axis of the crystal 1, as the major axis 5, is made longer than the other or minor axis 6, the two frequencies will separate and will be less affected by the coupling between them. In the quartz crystal 1 illustrated in Figs. 1 to 3 having the double orientation angles of ϕ =substantially $+51.5^\circ$ and θ =substantially 45 degrees, and having the selected shape or dimensional ratio for the minor axis 6 with respect to the major axis 5 of about 0.855, the separation between the two resonances may be about 17 per cent. The higher resonance represented by the minor axis 6 or width dimension w is, in this instance, the stronger and may be utilized to obtain a constant frequency over a wide range of temperatures. The effect of the lower resonance represented by the major axis 5 in Figs. 2 and 3 may be removed by a suitable filter, if desired, which may consist of a fairly wide electrical band-pass filter, which passes the band of the crystal filter and introduces large attenuations 17 per cent below the pass-band of the crystal filter, thus removing the effect of the lower resonance. If the filter is used between vacuum tubes in a receiving or transmitting system, the tuned circuits between adjacent tubes of the system will automatically remove the effect of the secondary pass-band for they will pass the desired band and will attenuate strongly frequencies different from the desired frequency, provided that the difference amounts to 5 to 10 per cent on the frequency scale.

When the effect of the coupling hereinbefore

referred to is small, the frequency of the crystal, as the quartz crystal 1, may be given by the expression:

$$f_1 = \frac{1}{2l} \sqrt{\frac{1}{\rho s_{11}}} \quad (1)$$

where

ρ is the density of the crystal,
 l is the length of the crystal, and

s_{11} is Young's modulus in the directions of the longitudinal vibrations.

The value of s_{11} for a general orientation may be given by the expression:

$$s'_{11} = s_{11} (\cos^2 \theta + \sin^2 \theta \sin^2 \varphi)^2 + (2s_{13} + s_{44}) \sin^2 \theta \cos^2 \varphi (\cos \theta + \sin^2 \theta \sin^2 \varphi) + s_{33} \sin^4 \theta \cos^4 \varphi + 2s_{14} \sin^2 \theta \sin \varphi \cos \varphi (3 \cos^2 \theta - \sin^2 \theta \sin^2 \varphi) \quad (2)$$

where

s_{11} , s_{13} , s_{14} , s_{33} and s_{44} are known constants which, in the case of quartz, are given, for example, in a publication "Electric wave filters employing quartz crystals as elements," Bell System Technical Journal, July 1934, page 450, and where

θ and φ are the orientation angles having the significance shown in Figs. 1 to 4, where φ represents the angle of rotation of the major plane of the crystal about the electric axis X as measured from the optic axis Z, and θ represents the angle of rotation of the direction of the principal axis 5 of the crystal 1 as measured from the same electric axis X.

The angle θ may be either 45 degrees or 135 degrees depending on which of the directions illustrated in Fig. 2 or 3 is considered the principal axis 5. Since $\cos \theta$ and $\sin \theta$ enter into the Equation 2 only in even power, either $\theta=45$ degrees or $\theta=135$ degrees may be chosen as the principal axis and the value of $\sin^2 \theta$ and $\cos^2 \theta$ may be taken as one-half. With this simplification, the expression for s'_{11} becomes:

$$s'_{11} = \frac{s_{11}}{4} (1 + \sin^2 \varphi)^2 + \left(\frac{2s_{13} + s_{44}}{4} \right) \cos^2 \varphi (1 + \sin^2 \varphi) + \frac{s_{33}}{4} \cos^4 \varphi + \frac{s_{14}}{2} \sin \varphi \cos \varphi (3 - \sin^2 \varphi) \quad (3)$$

where the sign of φ is the same as that used in specifying the direction of rotation of the plane of the crystal.

Fig. 6 shows a frequency curve for quartz crystals based on values of frequency calculated from Equations 1 and 3 as a function of the angle φ , the angle θ being a constant and equal to either 45 degrees or 135 degrees. For any given angle of φ the curve of Fig. 6 gives the frequency constant corresponding thereto in terms of frequency in kilocycles per second per centimeter of width dimension w , or per centimeter of length dimension l as indicated by Equations 1 and 3 upon which the curve of Fig. 6 is based. As indicated hereinbefore, for any angle of φ the length dimension l represents the lower resonance frequency and the width dimension w the higher resonance frequency of the crystal plate 1. Since the frequency of such longitudinal mode vibrations varies inversely as the particular dimension of w or l involved, the value of w or l in centimeters corresponding to the frequency in kilocycles per second may be obtained directly from the frequency constant given by the curve of Fig. 6 for any value of the angle φ selected. The point marked GT on the curve of Fig. 6 corresponds to the orientation angle of φ substantially +51.5 degrees of the quartz crystal 1 shown in Figs. 1 to 3 and shows that the calcu-

lated frequency thereof is about 327 kilocycles per second per centimeter of width dimension w of the crystal 1 when the major plane 2 of the crystal 1 is rotated in effect about the electric axis X to the angular position of φ substantially +51° 30' as measured from the optic axis Z and the principal axis 5 thereof is rotated to a position of θ substantially 45 degrees with respect to the same electric axis X as illustrated in Fig. 2 or 3. Accordingly, where the angle φ is substantially +51.5 degrees and the angle θ is substantially 45 degrees, the width dimension w of such a quartz crystal plate may be made of a value to obtain the desired higher resonance frequency in accordance with the calculated frequency constant of about 327 kilocycles per second per centimeter of width dimension w as given by the curve of Fig. 6. For example, if a frequency of substantially 327 kilocycles per second is desired in a crystal plate 1 having such an orientation angle of φ equals substantially +51.5 degrees, the width dimension w thereof may be made substantially equal to one centimeter; and since the frequency varies inversely as the value of the width dimension w , any other frequency may be obtained by proportion in a known manner. Similarly, for any other angle of φ positive (+) or negative (-), the corresponding frequency constant for the longitudinal vibrational mode along either the width dimension w or along the length dimension l of the quartz plate may be directly obtained from the curve of Fig. 6. For the positive angles of φ such as, for example, any positive angle of φ between about +30 and +70 degrees as given by the curve D of Fig. 8, the higher frequency resonance dependent mainly upon the minor axis 6 or width dimension w is the preferred resonance. For the negative angles of φ such as, for example, any negative angle of φ between about -50 and -70 degrees as given by the curve C of Fig. 8, the lower frequency resonance dependent mainly upon the major axis 5 or the length dimension l is the preferred resonance. For any angle of φ , the mode of vibration of the quartz plates, the characteristics of which are given in Fig. 6 as well as in Figs. 7 to 11 to be described, is herein described as a coupled mode consisting of two modes of longitudinal motion, one along the minor axis 6 or width dimension w , and the other along the major axis 5 or length dimension l , the corresponding frequency constants of which are given in Fig. 6 for every angle of φ and also in Fig. 10 with greater precision for angles of φ in the region of substantially +51 degrees.

The calculated frequencies shown by the curve in Fig. 6 agree well with the values of the frequency constant for the lower and higher resonances as obtained by measurement for various values of the orientation angle φ . The lower resonances referred to correspond to the length l or major axis 5 dimension, and the higher resonances to the width w or minor axis 6 dimension of the crystal plates 1 of any angle of φ . The slight deviation that may exist between the frequencies as measured and those as calculated may be caused by the coupling of the two modes of longitudinal motion represented by a lower and a higher resonance referred to, which coupling may decrease the frequency of one mode and raise that of the other.

Fig. 7 illustrates the temperature-frequency characteristics of three quartz plates 1 each having a different orientation, the frequency constants for which are substantially given in Fig. 6

at $\varphi = +38^\circ$, $+51^\circ$ and $+60^\circ$ and also in Fig. 10 when φ is in the region of $+51^\circ$; and the dimensional ratios for which are given in Fig. 8 and also in Fig. 11 according to the value of φ selected. As illustrated by the curve A of Fig. 7, a substantially zero temperature coefficient of frequency occurs in the region of $+35^\circ$ C. when φ is $+38^\circ$ and θ is 45° ; in the region of $+80^\circ$ C. when φ is $+60^\circ$ and θ is 45° as illustrated by the curve B of Fig. 7; and occurs throughout the range from about 0° C. to 100° C. when φ is substantially $+51^\circ 30'$ and θ is substantially 45° as illustrated by the curve GT of Fig. 7.

Fig. 8 shows in the curves designated C and D thereof the orientation angles and the corresponding dimensional ratios that may be used in constructing quartz plates *l* in order to obtain a substantially zero temperature coefficient of frequency. The dimensional ratios referred to are given in terms of the width *w* with respect to the length *l* of the quartz plate *l*. The curve C of Fig. 8 gives the dimensional ratios and the corresponding negative angles of φ from -50° to -70° while the curve D of Fig. 8 gives the dimensional ratios and the corresponding positive angles of φ from about $+30^\circ$ to $+70^\circ$, the angle θ in every case being substantially 45° . The corresponding frequency constants for quartz plates *l* oriented and dimensioned in accordance with the curves C and D of Fig. 8 are substantially given by the curve of Fig. 6 at the intercept of the value of φ selected. For example, when φ is substantially $+51.5^\circ$, θ being substantially 45° , and the dimensional ratio of the width *w* to the length *l* of the quartz plate *l* is substantially 0.86 as indicated at the point marked GT on the curve D of Fig. 8, the frequency constant is approximately 328 kilocycles per second per centimeter width *w* as indicated at the point designated GT in Fig. 6 where φ is substantially $+51.5^\circ$; and the temperature-frequency characteristic is substantially as given by the curve GT of Fig. 7 which shows a substantially zero temperature coefficient of frequency throughout a temperature range from about 0° to $+100^\circ$ centigrade.

Fig. 9 illustrates the relation between certain orientation angles φ , θ being substantially 45° , and the corresponding values of a_2 , the second derivative of the frequency by the temperature about to be described.

The frequency of a crystal, such as the quartz crystal *l*, may be expressed as a function of the temperature by the series:

$$f = f_0 [1 + a_1(T - T_0) + a_2(T - T_0)^2 + a_3(T - T_0)^3 + \dots] \quad (4)$$

where

T_0 is any reference temperature.

T is the temperature of the crystal,

a_1 is the first derivative of the frequency by the temperature at the reference temperature T_0 .

a_2 is the second derivative of the frequency by the temperature at the reference temperature T_0 , and

a_3 is the third derivative of the frequency by the temperature at the reference temperature T_0 .

Differentiating f with respect to T , the expression becomes:

$$\frac{df}{dT} = f_0 [a_1 + 2a_2(T - T_0) + 3a_3(T - T_0)^2 + \dots] \quad (5)$$

For a zero temperature coefficient crystal, the change in frequency with temperature passes through zero at some temperature T_0 . Hence $a_1 = 0$. The frequency variation will then be:

$$f = f_0 [1 + a_2(T - T_0)^2 + a_3(T - T_0)^3 + \dots] \quad (6)$$

It will be noted from Equation 6 that where a_2 is much larger than the succeeding terms in Equation 6 a substantially parabolic curve for the frequency as a function of the temperature will be obtained, and that if a_2 is positive, as shown by the upper portion of the curve F of Fig. 9 above the point labeled GT, the frequency will increase for temperature values on either side of the zero coefficient temperature T_0 and will have a rising parabolic characteristic as illustrated by the curve B in Fig. 7; and that if a_2 is negative as shown in Fig. 9 by the curve E and by the lower portion of the curve F below the point labeled GT, the frequency will decrease for temperatures either side of the zero coefficient temperature T_0 and will have a falling parabolic characteristic as illustrated by the curve A in Fig. 7.

Where the first and second derivatives a_1 and a_2 respectively of the frequency by the temperature are both zero, the crystal, as the quartz crystal *l* having the orientation angles $\varphi =$ substantially $+51.5$ degrees, and $\theta =$ substantially 45 degrees as shown in Figs. 1 to 3, and having a dimensional ratio of width *w* to length *l* equal to substantially 0.855 as given at the point GT on the curve D of Fig. 8 and by the curve of Fig. 11 will have a substantially flat frequency characteristic over a wide range in temperatures as illustrated by the curve GT in Fig. 7 when excited at its higher frequency longitudinal vibration along the width axis *w* as described hereinbefore. The third derivative a_3 as given in Equation 6 is a cubic term and appears to be so small that it has little effect on the constancy of frequency of this crystal.

More particularly, the quartz crystal *l* having orientation angles $\varphi =$ substantially $+51.5$ degrees and $\theta =$ substantially 45 degrees (or 135 degrees), and having a dimensional ratio of width *w* to length *l* substantially equal to about 0.855 as given at the point marked GT on the curve D in Fig. 8, has a zero value of a_2 (Equation 6) as shown at the point marked GT on curve F of Fig. 9 and has a temperature-frequency curve as shown by the curve marked GT in Fig. 7. It will be noted that the frequency of such crystal is substantially constant over the entire range of temperatures from 0 to 110 degrees centigrade as shown by the curve GT in Fig. 7 and may not vary more than one part in a million over such range of temperatures.

The curve GT of Fig. 7 gives the measured resonant frequency characteristics of the crystal *l*, when φ is substantially $+51^\circ 30'$, as measured in a so-called Pierce oscillator circuit with the crystal connected between the grid and filament electrodes of the electron tube utilized in such a circuit which is shown for example in a publication by G. W. Pierce entitled "Piezoelectric crystal resonators and crystal oscillators applied to the calibration of wave meters" Proceedings American Academy of Arts and Sciences, vol. 59, page 81, 1923. This circuit operates the crystal very near its resonant frequency. As indicated by the substantially flat or constant frequency portion of the curve GT in Fig. 7, the crystal *l* of Figs. 1 to 3 has when φ is substantially $+51^\circ 30'$ a nearly constant frequency

throughout the temperature range from substantially 0° centigrade to +100° centigrade, the mid-temperature of such constant frequency temperature range being substantially +50° centigrade as shown in Fig. 11. At temperatures below 0° centigrade, the crystal frequency rises or increases somewhat, and at temperatures above +100° centigrade the crystal frequency lowers or decreases somewhat while at all temperatures between 0 and 100° centigrade the crystal frequency remains substantially constant as indicated by the curve GT of Fig. 7 where the angle φ has a value of substantially +51° 30'.

The flat or constant frequency portion of the curve GT in Fig. 7 may be shifted from the 0° centigrade to 100° centigrade temperature range shown therein to some other selected temperature range, by a change in the angle φ . For a crystal plate cut at random, the frequency will change with temperature according to Equation 4 where a_1 , a_2 , a_3 , etc., are constants and f_0 is the frequency of the crystal at the temperature T_0 . For the crystal I having the temperature-frequency characteristic illustrated by the curve GT in Fig. 7, both a_1 and a_2 are zero and a_3 is the first constant given in Equation 4 that affects the frequency. The dependence of a_2 on the angle of cut φ is shown in Fig. 9. When the angle φ is changed, a_1 also changes but by changing or adjusting the dimensional ratio of axes w to l of the crystal to a suitable value, a_1 can be adjusted either positive or negative over a reasonable range, and a_3 will not change greatly for a small difference in angle φ since it is not a minimum point. Hence, if a small change in the angle φ is made, a_1 and a_2 will be finite and may be made to disappear for some other temperature range.

If Equation 4 be expanded in powers of T , it may be written:

$$f = f_0[1 + a_3 T^3 - T^2(3T_0 a_3 - a_2) + T(3T_0^2 a_3 - 2a_2 T_0 + a_1) - (T_0^3 a_3 - a_2 T_0^2 + a_1 T_0)] \quad (7)$$

If we have

$$3T_0 a_3 - a_2 = 3a_3 T_1 \text{ and } 3T_0^2 a_3 - 2a_2 T_0 + a_1 = 3a_3 T_1^2 \quad (8)$$

Equation 7 may be written in the form

$$= f_0[1 + a_3(T_0 - T_1)^3 + a_3(T - T_1)^3] = f_0[1 + a_3'(T - T_1)^3] \quad (9)$$

and hence terms which vary with T will be eliminated up to the cubic term. Furthermore the variation is about a different temperature T_1 . In order that this shall be true we must satisfy the relations

$$a_2 = 3a_3(T_0 - T_1); a_1 = 3a_3[T_1 - T_0]^2 \quad (10)$$

a_3 is negative and has a value of about

$$a_3 = -26 \times 10^{-12} \quad (11)$$

Hence if we wish to lower the mean temperature for the flat temperature range by 50° centigrade, the value of a_2 should be about

$$a_2 = -3 \times 50 \times 26 \times 10^{-12} = -3.9 \times 10^{-9} \quad (12)$$

As shown by curve F of Fig. 9, this would indicate that the angle of cut φ should be slightly less than +51 degrees in order that the mid-point of the flat part of the frequency-temperature curve may come at zero degrees centigrade.

Accordingly, by changing the value of angle φ and the dimensional ratio of the width axis w to the length axis l of the crystal I, the constant frequency temperature range throughout which the crystal has a flat frequency-temperature curve similar in form to the curve GT of Fig. 7 may be raised or lowered as desired.

The curves of Figs. 10 and 11 showing the frequency and the dimensional ratio as a function of the angle φ , illustrate values which may be used to obtain a constant frequency temperature range with the mean or mid-temperature thereof located at any temperature from 0 to +50° centigrade as shown in Fig. 11. The values given in Figs. 10 and 11 were obtained by tests using the crystal in a resistance bridge oscillator circuit as disclosed in U. S. Patent No. 2,163,403, granted June 20, 1939 on application Serial No. 151,564, filed July 2, 1937, by L. A. Meacham and in a paper by L. A. Meacham published in the Proceedings of the Institute of Radio Engineers, October 1938. This resistance bridge circuit is particularly insensitive to power supply and other circuit parameter variations and operates the crystal near its resonant frequency. For use with the Pierce circuit to which reference has been made, the data given by the curves of Figs. 10 and 11 are nearly the same since both of these circuits operate the crystal near its resonant frequency.

As shown in Figs. 10 and 11, where the angle φ is about +51° 30' and the dimensional ratio of axes w to l is about 0.855, the frequency of the crystal I in kilocycles per second per centimeter width w will be about 328.8 and the mid-temperature of the constant frequency temperature range will be about +50° centigrade giving a substantially constant frequency throughout a temperature range from about 0 to 100° centigrade. Where the angle φ is about +51° 7½' and the dimensional ratio of axes w to l is about 0.859, the frequency of the crystal I in kilocycles per second per centimeter width w will be about 329.2 and the mid-temperature of the constant frequency temperature range will be about +25° centigrade giving a substantially constant frequency throughout a temperature range from about -25° to +75° centigrade; where the angle φ is about +50° 45' and the dimensional ratio of axes w to l is about 0.863, the frequency of the crystal I in kilocycles per second per centimeter width w will be about 329.7 and the mid-temperature of the constant temperature range will be about 0° centigrade giving a substantially constant frequency throughout a temperature range from about -50° to +50° centigrade. Other values of the ratio of axes w to l and frequency constant for the crystal I as a function of the angle φ and the mid-temperature of the constant frequency temperature range may be obtained by interpolation of the values given by the curves of Figs. 10 and 11.

It will be understood that the absolute temperature range given by the curve GT of Fig. 7 where $\varphi = +51° 30'$, may be similarly lowered or raised to other constant frequency temperature ranges as desired by a suitable adjustment in the magnitude of the angle φ . For example, where φ is about +50 degrees, the whole curve is shifted down or displaced about 100° centigrade from that for the crystal where $\varphi = +51° 30'$ as shown in Fig. 7. The dimensional ratio of the width w to length l for zero temperature coefficient of frequency is, for a $\varphi = +51°$ crystal I, about 0.86 as shown in Fig. 11 while for a $\varphi = +50°$ crystal, for example, it is about 0.865.

In order to obtain the frequency constancy indicated by the curve GT of Fig. 7 with the angles of φ of about +51 degrees as given in Fig. 11, it will be understood that any oscillating circuit may be employed which works the crystal I near its resonant frequency. Other types of

oscillating circuits do not give the same constancy of frequency for the angles φ of about ± 51 degrees mentioned but by changing the angle of φ , the crystal 1 may be adapted to any type of oscillator circuit to produce a substantially constant resonant frequency of a high order over a wide range of temperatures such as about one part in a million over about a 100° centigrade range of temperatures and about one part in ten million over about a 30° centigrade range of temperatures or $\pm 15^\circ$ centigrade from the mid-temperature.

The temperature-frequency characteristic of the crystal 1 depends to some extent on the type of oscillator circuit with which it may be used which indicates that the resonant and anti-resonant frequencies thereof do not change in quite the same manner with temperature. The anti-resonant frequency is determined by the elastic constants of the crystal while the resonant frequency depends also on the value of the piezoelectric constant. Hence, if the piezoelectric constant varies with temperature, the resonant and anti-resonant frequencies will change with respect to each other and will not have the same temperature coefficients of frequency. Accordingly, the crystal 1 does not give the same performance when used in oscillating circuits which operate them in different parts of their resonance characteristics. When used in a circuit where the crystal operates near its resonant point as in the Pierce type of circuit to which reference has been made, for example, the angle φ may be approximately as hereinbefore stated namely, an angle within the range from about $+50$ degrees to $+51^\circ 30'$ to obtain the substantially flat temperature-frequency characteristic, illustrated by the curve GT of Fig. 7. It will be understood, however, that by a suitable selection of the angle φ , the crystal 1 may have the substantially flat temperature-frequency characteristic for any type of circuit in which it may be used. For example, the crystal 1 may have orientation angles of $\varphi = \text{about } +45^\circ 30'$, $\theta = 45^\circ$ to obtain the substantially flat temperature-frequency characteristic in the mode of motion described when utilized in a low frequency inductively or coil-coupled oscillating circuit which may be similar to that illustrated in Fig. 5. With a ratio of the width dimension w to the length dimension l equal to about 0.89, such a crystal utilized in such a circuit has a very small change in frequency over the temperature range from about 0 to 100° centigrade.

It will be understood that the crystal 1 may have such values for the angle φ and for the dimensional ratios w and l slightly different from those particularly given herein as to obtain temperature-frequency curves that are substantially flat over any desired absolute temperature range.

In general, it will be noted that in this type of crystal, illustrated in Figs. 1 to 3 and referred to as the GT crystal, the shape or dimensional ratio of the width axis w to length axis l is utilized to produce the zero temperature coefficient of frequency at a given temperature and the double orientation angles φ and θ are utilized to control the horizontal slope of the curve and the range of temperatures throughout which the zero temperature coefficient of frequency is obtained in the longitudinal mode of motion described at a frequency as given by the curves in Figs. 6 and 10, and determined mainly by the width dimension w of the crystal 1.

It will be noted that the crystal 1 of the orien-

tation angles given in Figs 10 and 11 and illustrated in Figs. 1 to 3 may be utilized in filter systems as in narrow and moderately wide band crystal filters for stabilizing the pass band of the filter over wide temperature ranges. There is a small secondary resonance due to the vibration along the length l of the crystal 1 which occurs about 16 per cent below the main resonance vibration along the width w and which may be removed by the use of electrically tuned circuits if it is not desired for some other purpose. This type of crystal may also be used at the intermediate frequency in narrow band filters for selecting out the carrier in short wave radio systems.

A quartz crystal similar to the crystal 1 of Figs. 1 to 3 but having orientation angles of $\varphi = \text{substantially } +60$ degrees and $\theta = \text{substantially } 45$ degrees and having a dimensional ratio of width w to length l substantially equal to 0.9 as given by the curve D of Fig. 8, has a positive value of $a_2 = \text{about } +55.5 \times 10^{-9}$ as shown by curve F in Fig. 9 and has a temperature-frequency curve as shown by the curve B ($\varphi = +60$ degrees, $\theta = 45$ degrees) in Fig. 7. The frequency of such crystal increases on either side of the zero coefficient temperature T_0 which occurs at about $+80$ degrees centigrade as shown by curve B in Fig. 7.

A quartz crystal similar to the crystal 1 shown in Figs. 1 to 3 but having orientation angles of $\varphi = \text{substantially } +38$ degrees and $\theta = \text{substantially } 45$ degrees and having a dimensional ratio of width w to length l substantially equal to 0.94 as given by the curve D of Fig. 8, has a negative value of $a_2 = \text{about } -84 \times 10^{-9}$ as shown by curve F in Fig. 9 and has a temperature-frequency curve as shown by the curve A ($\varphi = +38$ degrees, $\theta = 45$ degrees) in Fig. 7. The frequency of such $+38$ degrees crystal decreases on either side of the zero coefficient temperature which occurs at about $+36$ degrees centigrade as shown by curve A in Fig. 7.

Other quartz crystals similar to the crystal 1, where the orientation angle θ is substantially 45 degrees (or 135 degrees) and the orientation angles φ are selected negative angles of orientation, for example, between -50 and -70 degrees as $\varphi = \text{substantially } -67.5$ degrees, may also be utilized to obtain a low or substantially zero temperature coefficient of frequency at a selected frequency. The ratio of the minor axis w with respect to the major axis l and the ratio of the width w with respect to the length l that may be utilized in such crystals having negative orientation angles of φ from about -50 to -70 degrees to obtain the low or substantially zero temperature coefficient of frequency therefor at temperatures within the temperature range mentioned in Fig. 7, is illustrated by curve C of Fig. 8 and the frequency constant thereof is illustrated by the curve in Fig. 6. For example, when the orientation angle θ is substantially 45 degrees with respect to an electric axis X, as illustrated in Fig. 2 or 3, and the orientation angle φ is substantially -67.5 degrees with respect to the optic axis Z, the ratio of width w to length l thereof may be substantially 0.67 as given by curve C in Fig. 8 to obtain a low or substantially zero temperature coefficient of frequency therefor, and the frequency constant may be about 260 kilocycles per centimeter length l as given by the curve in Fig. 6. For such crystals having a negative angle of orientation φ , it is the lower of the two coupled frequencies thereof, namely, the frequency of the vibration along the major axis 5 or length dimension l , that corresponds to the

shear frequency and which has the strongest resonance. The values of the second derivative of the frequency by the temperature a_2 for such crystals are illustrated by curve E of Fig. 9 and are all negative as illustrated. The minimum value of a_2 as given in curve E of Fig. 9 is about $a_2 = -15 \times 10^{-9}$ and occurs at $\varphi =$ substantially -67.5 degrees.

It will be noted from curve C of Fig. 8 that the crystals having the negative orientation angles of φ have, as compared with the positive angles of φ shown in curve D of Fig. 8, a smaller dimensional ratio of width w to length l to obtain the low or substantially zero temperature coefficient of frequency and therefore have a wider separation between the two coupled resonant frequencies dependent upon the length dimension l and the width dimension w respectively thereof, and may accordingly be advantageously utilized as selective elements of electric wave filter systems.

Other values of the dimensional ratio of width w to length l of the crystal l that may be utilized to obtain a low or substantially zero temperature coefficient of frequency at temperatures within the temperature range mentioned in Fig. 7 when the value of $\theta =$ substantially 45 degrees and the values of φ are between about -50 degrees and -70 degrees and between about $+30$ degrees and $+70$ degrees, may be obtained from curves C and D respectively of Fig. 8; and the corresponding values of a_2 therefor may be obtained from curves E and F respectively of Fig. 9. The corresponding frequency constants may be obtained from the curve of Fig. 6 for any corresponding angle of φ given by the curves C and D of Fig. 8.

The frequency of the crystal l illustrated in Figs. 1 to 3 is a function of the temperature gradient between the surface and the inside thereof. When the surface temperature is higher than the inside temperature of the crystal, the frequency of the crystal is lowered, whereas when the surface temperature is lower than the body temperature of the crystal, the frequency is raised. As a result, if the temperature of the crystal is changed too rapidly, its frequency changes correspondingly but gradually comes back to the same value as the temperature of the quartz approaches equilibrium and, when the crystal l is taken relatively suddenly through a complete temperature cycle of wide temperature limits, the ascending and descending temperature-frequency curves are not exactly the same. If the temperature be changed more slowly, the ascending and descending temperature frequency curves converge more closely. Since the ambient temperature change will ordinarily be very slow, the frequency of the crystal l having the values of φ and θ , as shown in Figs. 1 to 3, and the ratio of axes $=$ substantially 0.855 may not vary more than about five parts in a million from 0 degrees centigrade to $+110$ degrees centigrade as illustrated by the curve GT in Fig. 7. By heat insulating the crystal so that a sudden change in the outside ambient temperature may produce only a very slow temperature change in the crystal, this temperature gradient effect can be eliminated.

When the crystal is operated in a closed box or container and the temperature therein is changed, the frequency of the crystal may vary cyclically by as much as three parts in a million due to a change in resonance conditions in the enclosing box caused by temperature changes. Such change in acoustic resonance conditions is

probably due to the change in velocity of air waves with temperature change and may be completely eliminated by evacuating the container for the crystal since then the air waves which produce the effect will be removed. This effect of acoustic resonance may be eliminated also by surrounding the crystal with acoustic damping material such as a layer or layers of felt or other porous damping materials which may be placed in some or all of the innermost or inside walls of the metal container which usually encloses the crystal. The holes in such felt or in other porous material that may be so used act or function as acoustic resistances to provide good acoustic damping and eliminate acoustic resonances especially at the higher frequencies. Such felt damping or sound insulation material may also function to provide heat insulation between the crystal and the outside atmosphere to prevent a too sudden temperature change in the crystal and thereby minimize the temperature gradient effect to which reference has been made. The effect of humidity change on the frequency of the crystal may be eliminated or minimized by utilizing a quantity of calcium chloride or other suitable drying agent placed within the container adjacent the crystal, or by evacuating the container, or by passing dry air through the container and sealing the container with the dry air enclosed therein. When the crystal is mounted in a vacuum container, it is protected by the vacuum from humidity, barometric and acoustic resonance changes which may cause changes in frequency and energy dissipation in the crystal is eliminated. With an etched and plated electrode crystal of the type shown in Figs. 1 to 3, a "Q" or ratio of reactance to resistance as high as 330,000 may be obtained with the crystal mounted in a suitable holder and disposed in an evacuated container.

The frequency and also the temperature coefficient of frequency of the crystal l may be adjusted to relatively precise values by a slight amount of grinding of the edge faces 10 to 13 thereof. Grinding either of the edge faces 10 and 11 of the crystal l having orientation angles as shown in Figs. 1 to 3 reduces the length l and changes the ratio of dimensions and hence changes the temperature coefficient of frequency without affecting the frequency very much, since the frequency thereof is controlled mainly by the width w of the crystal l . Accordingly, to obtain a zero temperature coefficient crystal at a selected frequency, the crystal may be cut slightly oversize and the frequency thereof then adjusted to a value a few cycles under the desired frequency by grinding either of the edge faces 12 or 13 to decrease the width w and the minor axis dimension 6. Then the length l may be adjusted, as by grinding either of the edge faces 10 or 11, until the dimensional ratio of width w to length l is such as to obtain the zero temperature coefficient of frequency which, in the crystal l illustrated in Figs. 1 to 3 having the orientation angles $\varphi =$ substantially $+51.5$ degrees and $\theta =$ substantially 45 degrees, is obtained when the ratio of dimensions of width w to length l is about 0.855. The frequency will have been raised slightly by this last step of reducing the length l . If the frequency is still too low it may be adjusted slightly higher by reducing the width w , and then the length l may thereafter be readjusted to obtain the zero temperature coefficient of frequency. By this process of edge face

grinding, both the frequency and the temperature coefficient of frequency of the crystal 1 may be adjusted to the correct or desired values. In case the frequency desired is overshot, the frequency of the crystal 1 may be lowered a desired amount by slightly concaving either or both of the major surfaces 3 and 4 thereof along the central major axis length dimension 5 in the space between the pair of parallel dotted lines 14 of Fig. 2. Such concaving will have only a slight effect on the temperature coefficient of frequency. But if the temperature coefficient of frequency has thereby become too high, it may be lowered by concaving either or both of the major faces 3 and 4 along the central minor axis width dimension 6 in the space between the pair of parallel dotted lines 15 of Fig. 2. This last mentioned step will make the temperature coefficient of frequency more negative without changing the frequency appreciably.

While the crystals herein have been particularly disclosed as being excited in the fundamental mode of vibration, it will be understood that a harmonic or overtone as, for example, the third harmonic of the strongest resonance may be excited in a known manner by a suitable number of pairs of interconnected electrodes for example, and utilized to control the frequency of an oscillation generator or an electric wave filter system, for example, where it is desired to obtain a high frequency with a moderate size crystal. It will be understood that the dimensional ratio of axes for such harmonic crystals to obtain the zero temperature coefficient of frequency thereof and that the multiple orientation angles φ and θ for which the first and second derivatives of the frequency by the temperature a_1 and a_2 are zero, may be obtained in accordance with the principles herein disclosed for the fundamental vibration. Harmonic mode crystals are disclosed and claimed in my copending application for Piezoelectric crystal apparatus, Serial No. 297,259, filed September 30, 1939.

While the crystal 1 herein has been disclosed as being excited and vibrated at a frequency determined mainly by the larger dimensions l and w thereof, it will be understood that a crystal having a desired first and second derivative of the frequency by the temperature may be obtained for vibrations in the thin dimension t thereof. By cutting the boundary surfaces of the crystal with respect to the three possible orientation angles thereof, a whole surface of zero temperature coefficient crystals may be obtained as disclosed and claimed in my copending application for Piezoelectric crystal apparatus, Serial No. 112,685, filed November 25, 1936. On this surface, a line may be located for which the second derivative a_2 is zero and several points may be located at which the first, second and third derivatives a_1 , a_2 and a_3 as given in Equation 6 are all zero resulting in a crystal having a very constant frequency over a wide temperature range.

Although this invention has been described and illustrated in relation to specific arrangements, it is to be understood that it is capable of application in other organizations and is, therefore, not to be limited to the particular embodiment disclosed, but only by the scope of the appended claims and the state of the prior art.

What is claimed is:

1. A piezoelectric quartz crystal element having a major plane of substantially rectangular shape

disposed substantially parallel to an electric axis and inclined at a selected acute angle with respect to the optic axis thereof as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, the minor axis width dimension of said major plane having a selected ratio with respect to the major axis length dimension thereof, said angle and said dimensional ratio being substantially those given by the curve of Fig. 11 to obtain a substantially constant resonant frequency within substantially five parts in a million throughout substantially a 100 degree centigrade temperature range.

2. A piezoelectric quartz crystal element of substantially rectangular parallelepiped shape having the major plane thereof substantially parallel to an electric axis and inclined at a selected acute angle with respect to the optic axis thereof as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, the minor axis of said major plane having a selected dimensional ratio with respect to said major axis, said angle and said dimensional ratio being substantially those given by the curve of Fig. 11 to obtain a substantially constant resonant frequency for said crystal element when subjected to an electric field in a direction perpendicular to said major plane and vibrated at a frequency determined mainly by said minor axis dimension in a mode of motion consisting substantially of longitudinal vibrations along said major and minor axes, said frequency being substantially that given by the curve of Fig. 10.

3. A quartz piezoelectric element having a major electrode face of substantially rectangular shape disposed substantially parallel to an electric axis, said electrode face being inclined at an acute angle with respect to the optic axis, having its major axis inclined substantially 45 degrees with respect to said electric axis, and having a predetermined ratio of dimensions of its minor axis to said major axis, said angle and said dimensional ratio being substantially those given by the curve D of Fig. 8 to produce substantially zero temperature coefficient of resonant frequency when vibrating at a frequency determined mainly by one of said dimensions of said electrode face in a mode of motion consisting substantially of two coupled longitudinal vibrations along said major and minor axes, said frequency being substantially that given by the curve of Fig. 6 for said angle.

4. A piezoelectric quartz crystal element having a major plane of substantially rectangular shape, said crystal element being adapted for longitudinal vibrations along and at a frequency determined substantially by the minor axis or width dimension of said major plane, said major plane being substantially parallel to an electric axis and inclined at a selected angle between +35 and +60 degrees with respect to the optic axis thereof as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, and the dimensional ratio of the width of said major plane with respect to the length of said major axis thereof being between substantially 0.85 and 1.0, said selected angle and said dimensional ratio having such relative values as to produce substantially zero temperature coefficient of frequency.

5. A piezoelectric quartz crystal element having a major plane of substantially rectangular shape, said crystal element being adapted for longitudinal vibrations along and at a frequency determined substantially by the major axis dimension of said major plane, said major plane being substantially parallel to an electric axis thereof and inclined at a selected angle between -50 and -70 degrees with respect to the optic axis thereof as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, and the dimensional ratio of the width of said major plane with respect to the length of said major axis thereof being between substantially 0.64 and 1.0 , said selected angle and said dimensional ratio having such relative values as to produce substantially zero temperature coefficient of frequency.

6. A piezoelectric quartz crystal element having a major plane substantially parallel to an electric axis and inclined substantially -67.5 degrees with respect to the optic axis thereof as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, the dimensional ratio of the width of said major plane with respect to the length of said major axis thereof being substantially 0.67 .

7. A piezoelectric quartz crystal element adapted to vibrate at a frequency determined substantially by its width or minor axis dimension of its major plane, said element having its major plane substantially parallel to an electric axis and inclined substantially $+38$ degrees with respect to the optic axis thereof as measured in a plane perpendicular to said electric axis, the major length axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, and the dimensional ratio of the width axis to said length axis of said major plane being substantially $.94$ to produce substantially zero temperature coefficient of frequency at a given temperature.

8. A quartz piezoelectric element of low temperature coefficient of frequency adapted to vibrate in a mode of motion consisting substantially of two coupled transverse longitudinal vibrations, one along the length or major axis and the other along the width or minor axis of the major plane thereof and at a frequency determined by the dimension along one of said axes as given by the curve of Fig. 6 for the angle corresponding to the angle given by the curves of Fig. 8, said major plane being of substantially rectangular shape, disposed substantially parallel to an electric axis and inclined at an acute angle with respect to the optic axis, said major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, the magnitude and sense of direction of said angle and the corresponding dimensional ratio of said minor axis to said major axis being substantially those values given by the curves of Fig. 8.

9. A quartz piezoelectric element adapted to vibrate in a mode of motion consisting substantially of two transverse coupled longitudinal vibrations, one along the length or major axis and the other along the width or minor axis of the major plane thereof and at a frequency determined substantially by the dimension along said width or minor axis as given by the curve of Fig. 10, said major plane being of substantially

rectangular shape, disposed substantially parallel to an electric axis and inclined at an acute angle with respect to the optic axis, said major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, the magnitude and sense of direction of said acute angle and the corresponding dimensional ratio of said minor axis with respect to said major axis being substantially those values given by the curve of Fig. 11.

10. A piezoelectric quartz crystal plate of substantially rectangular parallelepiped shape, having the major plane thereof substantially parallel to an electric axis and inclined at an angle of substantially $+51$ degrees with respect to the optic axis as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, and the ratio of dimensions of the minor axis or width of said major plane with respect to said major axis or length of said major plane being substantially 0.86 .

11. A piezoelectric quartz crystal plate of substantially rectangular parallelepiped shape, having the major plane thereof substantially parallel to an electric axis and inclined substantially $+51$ degrees with respect to the optic axis thereof as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, and the ratio of dimensions of the minor axis or width of said major plane with respect to said major axis or length of said major plane being substantially 0.86 , and means including electrodes operatively disposed with respect to the opposite major surfaces of said crystal plate for exciting said crystal plate near its resonant frequency determined mainly by said width dimension of said major plane to obtain a substantially zero temperature coefficient of frequency throughout a relatively wide temperature range.

12. A quartz piezoelectric element adapted to vibrate at a frequency determined substantially by the minor axis or width dimension of the major plane thereof, said major plane being of substantially rectangular shape, disposed substantially parallel to an electric axis and inclined with respect to the optic axis substantially $+51$ degrees as measured in a plane perpendicular to said electric axis, the major axis or length dimension of said major plane being inclined substantially 45 degrees to said electric axis, the dimensional ratio of said minor or width axis with respect to said major or length axis being substantially 0.86 .

13. A quartz piezoelectric element adapted to vibrate at a frequency determined substantially by the minor axis or width dimension of the major plane thereof, said major plane being of substantially rectangular shape, disposed substantially parallel to an electric axis and inclined with respect to the optic axis substantially $+46$ degrees as measured in a plane perpendicular to said electric axis, the major axis or length dimension of said major plane being inclined substantially 45 degrees with respect to said electric axis, and the dimensional ratio of said minor or width axis with respect to said major or length axis being substantially 0.89 .

14. The method of adjusting to desired values the frequency and the temperature coefficient of frequency of a piezoelectric crystal plate which includes the steps of decreasing the width or

minor axis dimension of a major surface until the frequency is increased to a value slightly under the desired frequency, and decreasing the length or major axis dimension of a major surface until the dimensional ratio of the width to the length is such as to obtain the desired temperature coefficient of frequency while simultaneously increasing the frequency.

15. The method of adjusting to desired values the frequency and the temperature coefficient of frequency of a piezoelectric crystal plate which includes the steps of decreasing the width or minor axis dimension of a major surface sufficiently until the frequency is increased to a value slightly under the desired frequency, decreasing the length or major axis dimension of a major surface until the dimensional ratio of the width to the length is such as to obtain the desired temperature coefficient of frequency while simultaneously increasing the frequency, and concaving a major surface centrally along the length thereof until the frequency is decreased to a desired value.

16. The method of adjusting to desired values the frequency and the temperature coefficient of frequency of a piezoelectric crystal plate which includes the steps of concaving a major surface centrally along the major axis length dimension thereof until the frequency is decreased to a desired value, and concaving a major surface centrally along the minor axis width dimension thereof until the temperature coefficient of frequency is decreased to a desired value.

17. A piezoelectric quartz crystal plate of substantially rectangular parallelepiped shape, having the major plane thereof substantially parallel to an electric axis and inclined at an angle

of substantially +51 degrees with respect to the optic axis as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, and the ratio of dimensions of the minor axis or width of said major plane with respect to said major axis or length of said major plane being substantially 0.86, electrodes adjacent the opposite major faces of said crystal plate, and means including projections for rigidly clamping said electroded crystal plate therebetween at regions on said major faces of relatively small area with respect to the area of said major faces to hold said electroded plate against bodily movement out of a predetermined position between said projections.

18. A piezoelectric quartz crystal plate of substantially rectangular parallelepiped shape, having the major plane thereof substantially parallel to an electric axis and inclined at an angle of substantially +51 degrees with respect to the optic axis as measured in a plane perpendicular to said electric axis, the major axis of said major plane being inclined substantially 45 degrees with respect to said electric axis, and the ratio of dimensions of the minor axis or width of said major plane with respect to said major axis or length of said major plane being substantially 0.86, said minor axis or width dimension being reduced until the frequency is of substantially the desired value, and said major axis or length dimension being reduced until said dimensional ratio of said minor axis with respect to said major axis is such as to obtain the desired temperature coefficient of frequency and the desired frequency.

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