April 30, 1940. G. W. GILMAN 2,199,221

WAVE FILTER

Filed Aug. 20, 1938

FIG. 1

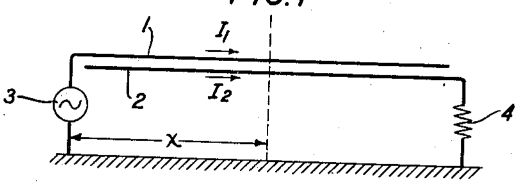


FIG. 1A

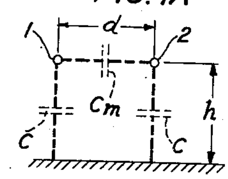


FIG. 2

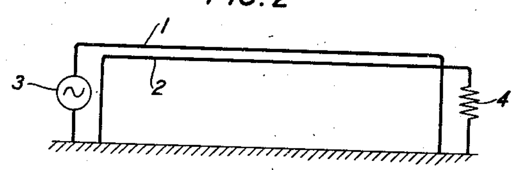


FIG. 3

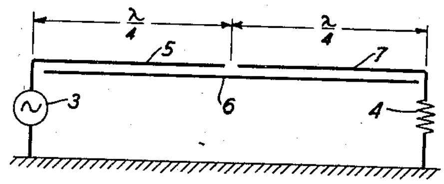


FIG. 4

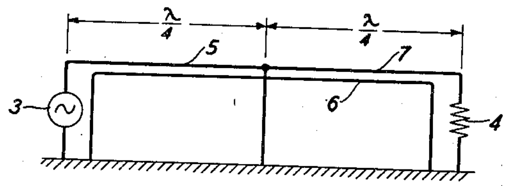


FIG. 5

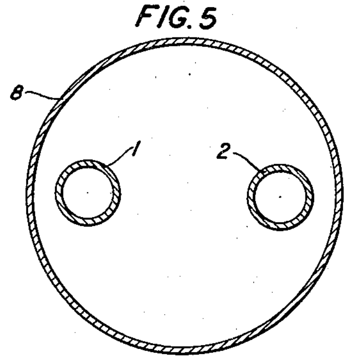


FIG. 6

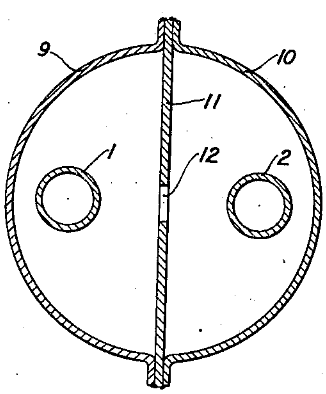
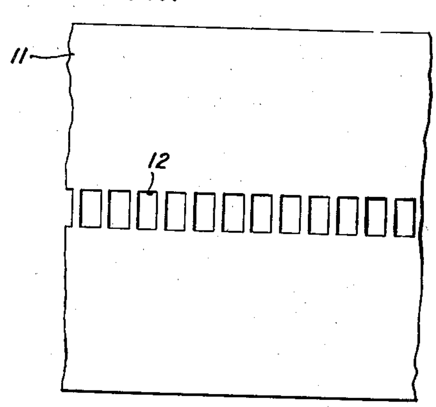


FIG. 7



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2,199,221

WAVE FILTER

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8 Claims. (Cl. 178—44)

This invention relates to wave filters and more particularly to wave filters for high frequencies in which the selective transmission properties are obtained by the use of sections of uniform transmission lines of low dissipation.

Heretofore it has been the practice in the construction of transmission line wave filters to use open-circuited or short-circuited line sections as two-terminal impedance elements, appropriate elements being combined in series-shunt or in other configurations in accordance with the kind of selective characteristic desired.

In the present invention, a novel configuration is employed according to which open-circuited or short-circuited line sections are electromagnetically coupled to each other along their lengths, whereby wave energy is transferred from the input terminals of one line to the output terminals of another with a band frequency selective characteristic which is determined by the character and degree of the coupling.

The lines may be open-wire types or they may be of the shielded conductor type and the coupling is preferably effected by running the conductors parallel to each other along their length. The degree of coupling may be modified by the interposition of partial electrostatic screens between the conductors or by proportioning their dimensions and separations.

The filters of the invention are characterized by multiple pass-bands spaced at harmonic frequency intervals. The width of the bands and the amount of attenuation in the intervening frequency ranges is controlled by the degree of coupling between the lines.

Other features of the invention and the principles underlying its operation will be more fully understood from the following detailed description and by reference to the attached drawing, of which

Figs. 1 and 1*a* are schematic diagrams illustrating one general form of the invention;

Fig. 2 shows an alternative general form of the invention;

Figs. 3 and 4 are schematic diagrams illustrating tandem arrangements of the forms shown in Figs. 1 and 2, respectively; and

Figs. 5, 6 and 7 are illustrative of mechanical constructions used in modifications of the invention.

Referring to Figs. 1 and 1*a*, elements 1 and 2 represent two elevated conductors of equal lengths lying parallel to each other at a height h above ground and separated by a distance d. Between one end of line 1 and ground is connected a high frequency generator 3 and a load resistance 4 is connected from the remote end of line 2 to ground. Conductors 1 and 2 and the common ground return constitute a pair of open-circuited transmission lines coupled electromagnetically to each other along their lengths by virtue of their juxtaposition. Transmission takes place in the system from the generator 3 to the load 4 through the distributed coupling and, because of the character of the coupling and the current and voltage distributions in the two lines. exhibits a definite band selective characteristic. The center of the first transmission band lies at the frequency for which the lengths of the two conductors are equal to a quarter of the propagation wave-length, that is, for which the conductors become quarter wave lines. The other transmission bands are located at certain equally spaced harmonic frequencies above the first band.

The manner in which the transmission bands are formed and the dependence of the selective characteristics upon the line parameters is explained by the following mathematical analysis.

It will be assumed that the effects of resistance in the two conductors and the return path are negligibly small so that the wave propagation velocity in the system may be taken as equal to the velocity of light. Fields other than radial fields may be disregarded and the steady distribution of oscillatory currents in the lines may be represented by the equations

$$I_{1(x)}=A_1e^{-\gamma x}-B_1e^{\gamma x}$$

and (1)

$$I_{2(x)}=A_2e^{-\gamma x}-B_2e^{\gamma x}$$

wherein $I_{1(x)}$ and $I_{2(x)}$ are the currents in conductors 1 and 2 respectively at points distant (x) from the generator, the A's and the B's are factors determined by the terminal or boundary conditions and the impressed voltage, and γ is the propagation constant of the individual lines. The quantity γ is a pure imaginary in the absence of resistance and has the value

$$\gamma=j\frac{\omega}{v} \qquad (2)$$

where ω is 2π times frequency and v is the velocity of light.

The voltages at any point along the conductors are related to the densities of the electric charges on the conductors by the equations

$$V_{1(x)}=p_{11}Q_1+p_{12}Q_2$$

and (3)

$$V_{2(x)}=p_{12}Q_1+p_{22}Q_2$$

where $V_{1(x)}$ and $V_{2(x)}$ represent the voltages at point x, Q_1 and Q_2, the charges per unit length on lines 1 and 2 respectively at the given point, and p_{11}, p_{12}, and p_{22} are the Maxwell potential coefficients of the system.

By means of the general steady state relationship

$$j\omega Q = -\frac{\partial I}{\partial x} \quad (4)$$

which states that the charge density is proportional to the rate of diminution of the current along the line, Equations 3 can be written in terms of the current differentials and, with the help of Equations 1 may be transformed to

$$V_{1(x)} = \frac{\gamma}{j\omega}[(p_{11}A_1 + p_{12}A_2)e^{-\gamma x} + (p_{22}B_1 + p_{12}B_2)e^{\gamma x}]$$

and (5)

$$V_{2(x)} = \frac{\gamma}{j\omega}[(p_{12}A_1 + p_{11}A_2)e^{-\gamma x} + (p_{12}B_1 + p_{22}B_2)e^{\gamma x}]$$

When the two conductors are similar and the system is symmetrical, the coefficients p_{11} and p_{22} are equal. Hereinafter, symmetrical systems will be assumed and the notations p and p_m will be used in place of p_{11}, p_{22} and p_{12}, respectively.

Since the system is symmetrical longitudinally, its general image parameters can be determined by solving Equations 1 and 5 for the particular case in which the load terminals are short-circuited. The establishment of the boundary conditions corresponding to this case permits the determination of the appropriate values of the A's and the B's and thereafter of the short-circuit input impedance at the generator end and the short-circuit transfer impedance from the source to the output terminals. These latter quantities suffice for the computation of the image impedance and the image transfer constant of the symmetrical system.

The boundary conditions when the load terminals are short-circuited are

$$V_1 = E \text{ and } I_2 = 0 \text{ when } x = 0$$

and

$$V_2 = 0 \text{ and } I_1 = 0 \text{ when } x = S$$

S denoting the whole length of the lines and E the terminal voltage of generator 3. The application of these boundary conditions to Equations 1 and 5 gives a new set of equations, namely,

$$A_1 e^{-\gamma S} - B_1 e^{\gamma S} = 0$$

$$A_2 - B_2 = 0$$

$$p(A_1 + B_1) + p_m(A_2 + B_2) = E\left(\frac{j\omega}{\gamma}\right) \quad (6)$$

$$(p_m A_1 + pA_2)e^{-\gamma S} + (p_m B_1 + pB_2)e^{\gamma S} = 0$$

These equations can be solved for the A's and B's, the values of which are found to be

$$A_1 = \frac{E}{2}\frac{j\omega}{\gamma}\frac{p \cosh \gamma S}{p^2 \cosh^2 \gamma S - p_m^2}e^{\gamma S} \quad (7)$$

$$A_2 = -\frac{E}{2}\frac{j\omega}{\gamma}\frac{p_m}{p^2 \cosh^2 \gamma S - p_m^2} \quad (8)$$

$$B_1 = \frac{E}{2}\frac{j\omega}{\gamma}\frac{p \cosh \gamma S}{p^2 \cosh^2 \gamma S - p_m^2}e^{-\gamma S} \quad (9)$$

and

$$B_2 = A_2 \quad (10)$$

The determination of the significant parameters of the complete system, namely, the image transfer constant and the image impedance requires only the knowledge of two impedances, namely, the input impedance at the generator end when the output of line 2 is shorted and the transfer impedance from the generator to the short-circuited output of line 2. Denoting the input end impedance by Z_C and the transfer impedance by Z_T, their values are found from Equations 1, with the help of Equations 7 to 10, to be

$$Z_C = -j\frac{2p_m\left[\left(\frac{p}{p_m}\cos\frac{2\pi S}{\lambda}\right)^2 - 1\right]}{pv \sin\frac{4\pi S}{\lambda}} \quad (11)$$

and

$$Z_T = -j\frac{p_m\left[\left(\frac{p}{p_m}\cos\frac{2\pi S}{\lambda}\right)^2 - 1\right]}{v \sin\frac{2\pi S}{\lambda}} \quad (12)$$

wherein

$$\lambda = \frac{v}{f} = j\frac{2\pi}{\gamma}$$

f being the frequency, and denotes the wavelength of the current in the line.

The image impedance K, which is the same for both ends of the system, is given by

$$K = \frac{Z_C Z_T}{\sqrt{Z_T^2 - Z_C^2}} \quad (13)$$

and has the value

$$K = \pm\frac{j}{v}\frac{p_m\left[\left(\frac{p}{p_m}\cos\frac{2\pi S}{\lambda}\right)^2 - 1\right]^{\frac{1}{2}}}{\sin\frac{2\pi S}{\lambda}} \quad (14)$$

The transfer constant θ is given by

$$\tanh \theta = \sqrt{1 - \left(\frac{Z_C}{Z_T}\right)^2} \quad (15)$$

and has the value

$$\tanh \theta = \frac{p_m}{p}\frac{\left[\left(\frac{p}{p_m}\cos\frac{2\pi S}{\lambda}\right)^2 - 1\right]^{\frac{1}{2}}}{\cos\frac{2\pi S}{\lambda}} \quad (16)$$

The transmission bands are located in the ranges for which K is real and $\tanh \theta$ imaginary and the band limits are determined as the frequencies at which these quantities change from reals to imaginaries. From the character of Equations 14 and 16 it is evident that the band limits are defined by the relationship

$$\frac{p}{p_m}\cos\frac{2\pi S}{\lambda} = \pm 1 \quad (17)$$

The potential coefficients p and p_m are of the nature of inverse capacities and are related to the capacities of the system as follows. As shown in Fig. 1*a*, the system of conductors has three significant capacities, namely, the direct mutual capacity between conductors 1 and 2, which is designated C_m, and the direct capacities of each wire to ground, which in the example illustrated are equal and are designated C. These capacities are the capacities per unit length of the line. The values of the potential coefficients in terms of the capacities are given by

$$p = \frac{1}{C}\frac{C + C_m}{C + 2C_m} \quad (18)$$

and

$$p_m = -\frac{1}{C}\frac{C_m}{C + 2C_m} \quad (19)$$

The lowest frequency band has its center at the frequency for which

$$S=\frac{\lambda}{4}$$

that is, for which the conductors are quarter wave-length lines. The other bands are centered at the odd harmonics of this frequency. The cut-off frequencies in terms of the line capacities are given by

$$\cos\frac{2\pi S}{\lambda}=\cos\frac{2\pi fS}{v}=\pm\frac{C_m}{C+C_m} \quad (20)$$

from which it is evident that the smaller C_m is with respect to C, the narrower will be the resulting band. A band width of about ten per cent of the mid-band frequency is obtained by proportioning the size and spacing of the conductors so that C_m is about one-twelfth of C.

The image impedance is zero at the band edges and rises to the value K_m, given by

$$K_m=\frac{C_m}{vC(C+2C_m)} \quad (21)$$

at the center of the band. Absolute or c. g. s. units are assumed in all of the preceding formulae, the value of v being 3×10^{10} centimeters per second. The capacities C and C_m are the capacities per centimeter length of the conductors. If these capacities are measured in micro-microfarads per centimeter, Equation 21 becomes

$$K_m=\frac{100}{3}\,\frac{C_m}{C(C+2C_m)}\text{ ohms} \quad (22)$$

The modified filter circuit shown in Fig. 2 corresponds to that of Fig. 1 except that the free ends of the two conductors are short-circuited instead of being open-circuited. In both types the free ends of the conductors are terminated to provide full wave reflection. The transfer constant of the filter of Fig. 2 is the same as that of Fig. 1 and has the value given by Equation 16. The image impedance is the reciprocal of the expression in Equation 14 except for a constant multiplier. Its value becomes infinite at the edges of the transmission band and at the mid-band frequency drops to a minimum which is inversely proportional to the coupling as measured by p_m or C_m.

The open-circuit type shown in Fig. 1 has an impedance characteristic of the form exhibited by mid-series terminated filters and is characterized by a relatively low impedance in the transmission bands. The modified form shown in Fig. 2 resembles a mid-shunt terminated filter and is characterized by a relatively high image impedance in the transmission bands.

Since the filters are symmetrical longitudinally, similar sections may be connected in tandem to provide increased discrimination. The tandem connection of two sections of the type shown in Fig. 1 is illustrated in Fig. 3. The resulting structure comprises two quarter wave open-circuit wave conductors 5 and 7 paralleled by a half wave-length open-circuit conductor 6. A corresponding arrangement of two sections of the short-circuited type is shown in Fig. 4. Manifestly, additional sections may be added in any desired number.

Instead of using open wires with a common ground return, the filters of the invention may comprise a pair of conductors symmetrically disposed within a surrounding tubular conductor which constitutes the common return path. A cross-section of a line of this type is shown in Fig. 5. The line conductors 1 and 2 may be solid or may be tubular as shown and are disposed symmetrically within an enclosing cylindrical conducting tube 8, preferably of copper. The inner conductors are held in position by suitable insulators not shown. Tandem connections of multiple sections may be employed as shown in Figs. 3 and 4 by dividing the inner conductors or short-circuiting them to the shield at appropriate intervals.

To secure narrow transmission bands, the direct capacity between the inner conductors must be small relatively to the capacity of either to the shield. For this purpose, it is desirable to separate the conductors as widely as practicable and to keep their diameters small. The reduction of the direct capacity may, however, be secured more advantageously by the use of a partial shield interposed between the conductors, as shown in Fig. 6.

In Fig. 6 the outer shell is composed of two flanged channel sections 9 and 10 which are welded or soldered to a perforated copper screening plate 11 coextensive in length with the conductors. Plate 11 is solid except for a series of perforations such as shown at 12 in Fig. 7 extending along its axis. These perforations should be close together and uniform in size to maintain a substantially continuous longitudinal distribution of the mutual capacity. The size of the apertures may be determined experimentally to provide a desired relationship between the mutual capacity C_m and the shield or ground capacities C.

What is claimed is:

1. A band selective transmission system comprising a pair of similar transmission lines disposed parallel to each other and electrostatically coupled, a wave source connected to one end of one of said lines, and a load impedance connected to the remote end of the other of said lines, both of said lines being terminated at their free ends to produce full wave reflection and the lengths of the lines being equal to one quarter of the propagation wave length at the frequency of said wave source, whereby the system has a finite transmission band centered at said frequency.

2. A system in accordance with claim 1 in which the free ends of the two lines are open-circuited.

3. A system in accordance with claim 1 in which the free ends of the two lines are short-circuited.

4. A band-pass wave filter comprising a pair of parallel conductors, a common return path parallel thereto and disposed symmetrically with respect to said conductors, terminal means for connecting a wave source between one end of one of said conductors and said common return, and terminal means for connecting a load impedance between the remote end of the other of said conductors and said common return, said conductors being terminated at their free ends for full-wave reflection, and their lengths being equal to quarter wave-lengths at an assigned frequency whereby the system has a finite transmission band centered about said assigned frequency.

5. A wave filter in accordance with claim 4 in which the direct mutual capacity between said conductors is small in comparison with the capacity of each conductor to the return path, whereby a narrow transmission band is provided.

6. A wave filter in accordance with claim 4 in

which the return path comprises a closed tubular member surrounding said conductors.

7. A wave filter in accordance with claim 4 in which the return path is constituted by a conducting shield member enclosing said conductors and a partial electrostatic shield interposed between said conductors, whereby their coupling capacity is made small with respect to their direct capacities to the shield.

8. A system in accordance with claim 1 including a partial electrostatic shield interposed between the said lines.

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