

Aug. 16, 1938.

E. L. NORTON

2,126,915

WAVE TRANSMISSION NETWORK

Filed March 12, 1937

2 Sheets-Sheet 1

FIG. 1

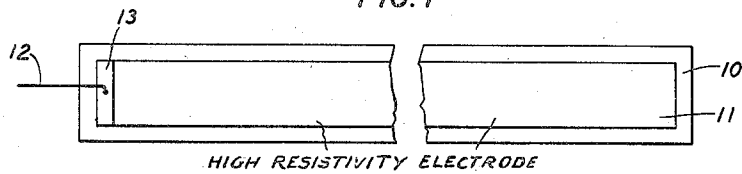


FIG. 2

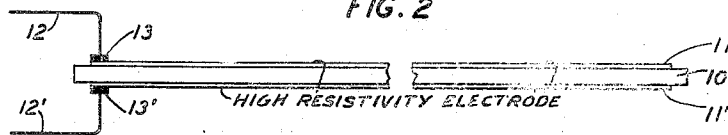


FIG. 3

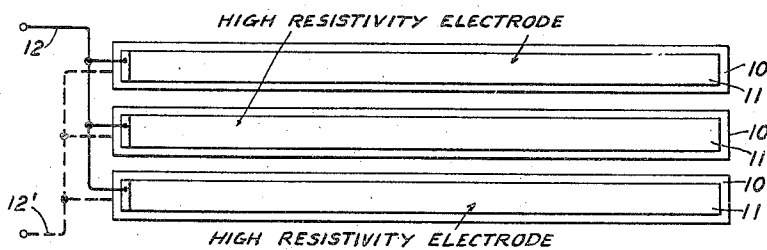


FIG. 5

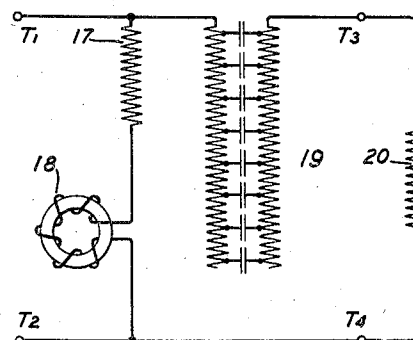


FIG. 4

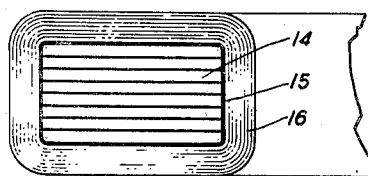


FIG. 4a.

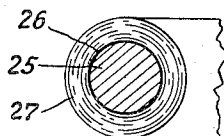
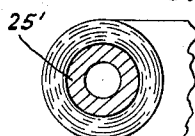


FIG. 4b.



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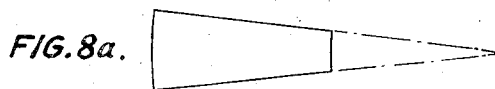
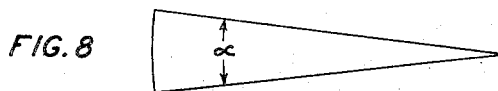
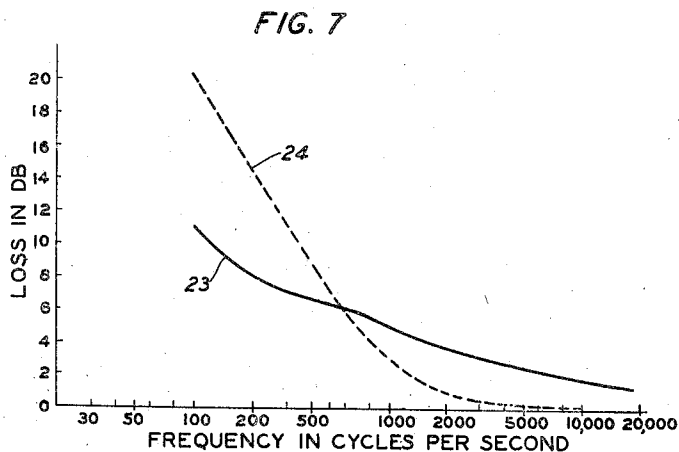
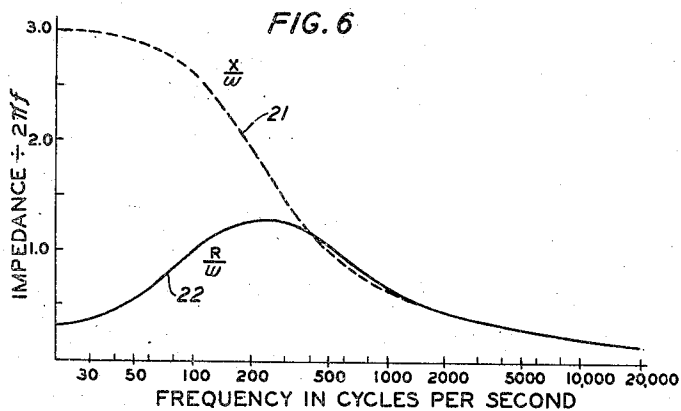
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2 Sheets-Sheet 2



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WAVE TRANSMISSION NETWORK

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Application March 12, 1937, Serial No. 130,459

8 Claims. (Cl. 178—44)

This invention relates to electrical transmission networks and more particularly to networks for equalizing or for simulating the attenuation of a uniform transmission line.

Heretofore, networks used for the purpose of equalizing the attenuation of a telephone transmission line have, for the most part, consisted of finite combinations of simple inductances, capacities and resistances, and the accuracy and frequency range of the compensation has been subject to the limitation of the range of characteristics obtainable with simple elements. This limitation arises in part from the fact that the line characteristics result from a continuous distribution of inductance, capacity, and resistance and are therefore essentially different from those obtainable from lumped systems.

The present invention provides new forms of equalizing networks in which the circuit elements have impedance characteristics dependent upon continuously distributed constants, but which are of convenient and compact structural form. Because of the special form of their impedance characteristics the use of these elements in transmission networks permits new attenuation characteristics to be obtained and simplifies the problem of line equalization. The design and construction of networks simulating the characteristic of a uniform transmission line is also simplified.

One form of impedance element employed in the networks of the invention consist of an inductance coil having a metallic core, preferably magnetic, in which eddy currents are permitted to flow to an extent that results in a marked modification of the coil impedance. The core may be solid or it may be laminated, provided the laminations are not so thin as to prevent the eddy currents having a substantial effect. The utility of such an element by itself is somewhat restricted unless there is available an element having an inversely related impedance characteristic. In the networks of the invention this is provided by a condenser, the electrodes of which are composed of highly resistive plates or films extended longitudinally in the form of ribbon-like conductors. Simple combinations of these inversely related elements are used to provide constant resistance equalizing networks of general types similar to those described in Zobel 1,603,305 issued October 19, 1926 and Stevenson 1,606,817 issued November 16, 1926.

A field in which the network of the invention have particular utility is the equalization of short transmission lines less than ten miles in length.

When the slope of the attenuation frequency characteristic is less than 6 decibels per octave, it becomes difficult to secure a uniform compensation over a wide frequency range by means of the equalizers heretofore employed unless a large number of impedance elements is used. By the present invention the compensation can be effected with very simple and inexpensive circuits.

The nature of the invention is explained more fully in the following detailed description and by the accompanying drawings of which:

Figs. 1, 2, 3, 4, 4a, and 4b are illustrative of structural features of impedance elements;

Fig. 5 is a schematic showing of a network of the invention;

Figs. 6 and 7 show performance characteristics of the networks of the invention, and

Figs. 8, 8a, and 9 illustrate features of modified structures used in the invention.

The impedance of a coil having a magnetic core may generally be taken as being equal to the part contributed by the magnetic core. If the winding is deep or if it is not closely applied to the core a part of the inductance will be contributed by the leakage flux which does not traverse the core, but usually this part is a very small fraction of the total inductance and may be neglected. The resistance of the coil winding is also usually quite small in comparison with the resistance introduced by eddy currents. An expression for the impedance of a coil having a laminated core has been given by Peterson and Wrathall: Eddy Currents in Composite Laminations, I. R. E. Proceedings February 1936. For a core having laminations of thickness $2b$, in centimeters, and of material having permeability μ and volume resistivity σ , the value of the impedance Z is given as

$$Z = \frac{\omega L_0}{\Phi} \left[\frac{\sinh \Phi - \sin \Phi}{\cosh \Phi + \cos \Phi} + j \frac{\sinh \Phi + \sin \Phi}{\cosh \Phi + \cos \Phi} \right] \quad (1)$$

where L_0 denotes the inductance at zero frequency, ω is 2π times frequency, j is the imaginary operator $\sqrt{-1}$ and Φ is an angle defined by

$$\Phi = 2b \sqrt{\frac{2\pi\mu\omega}{\sigma}} \quad (2)$$

An alternative expression for the impedance, which may be shown by standard mathematical processes to be equivalent to Equation (1), is

$$Z = j\omega L_0 \frac{\tanh \Theta}{\Theta} \quad (3)$$

where

$$\Theta = \frac{\sqrt{j\Phi}}{\sqrt{2}} = b \sqrt{\frac{j4\pi\mu\omega}{\sigma}} \quad (4)$$

Salient characteristics of the impedance are these: As the frequency increases, the reactance and resistance components converge towards a common value, the phase angle reaches a substantially constant value of approximately 45 degrees, and the effective inductance diminishes in a regular manner. Another property of the impedance may be seen by comparing it with the impedances of a section of uniform transmission line.

If the total values of the inductance, resistance, capacity, and conductance of a length of uniform line be denoted by L , R , C and G respectively, then the characteristic impedance K and the propagation constant P are given by the equation

$$K = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (5)$$

and

$$P = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (6)$$

The short-circuit impedance of a length of uniform line having only series inductance and shunt conductance has the value

$$Z_{sc} = \sqrt{\frac{j\omega L}{G}} \tanh \sqrt{j\omega LG} \quad (7)$$

where Z_{sc} denotes the short-circuit impedance. Since this expression has the same form as that for the coil impedance in Equation (3), it follows that the coil impedance corresponds to the impedance of a short-circuited uniform line having distributed series inductance and shunt conductance. By comparison of the two equations, the constants of the equivalent uniform line are found to be

$$L = L_0 \quad (8)$$

and

$$G = \frac{4\pi b^2 \mu}{\sigma L_0} \quad (9)$$

The propagation constant is θ , the value of which is given in Equation (4), and the characteristic impedance is given by

$$K = \frac{L_0}{2b} \sqrt{\frac{j\omega \sigma}{\pi \mu}} \quad (10)$$

To enable impedance of the above type to be used in constant resistance networks it is necessary to provide an impedance which has an inverse frequency variation. It turns out that an open-circuited section of uniform line having only series resistance and shunt capacity has the requisite type of impedance. The impedance Z_{oc} of such a line has the value

$$Z_{oc} = \frac{\sqrt{\frac{R}{j\omega C}}}{\tanh \sqrt{j\omega CR}} \quad (11)$$

and will have an inverse frequency variation to the coil impedance if the values of C and R are so proportioned that

$$CR = LG = \frac{4\pi b^2 \mu}{\sigma} \quad (12)$$

If the impedances of the coil and of the line section are to be inversely related with respect to a resistance of given value R_0 , the further relationship is required that

$$C = \frac{R_0^2}{L_0} \quad (13)$$

Equations (12) and (13) suffice for the determination of the constants of the resistance capacity line from the constants of the winding and core of a given coil.

As a rule, the required resistance of the resist-

ance capacity line will be quite high and special constructions are necessary to permit the high resistance to be obtained. Preferably the line is constructed in the form of a condenser having long ribbon-like electrodes of high resistance material. The general form is illustrated in Figs. 1 and 2 which show the condenser in plan and elevation respectively. The electrodes are designated 11 and 11' and the dielectric is indicated by a plate or strip 10. Leads 12 and 12' connected to the electrodes at one end of the condenser serve as the condenser terminals. To provide a high resistance the electrodes may consist of strips of condenser paper coated with a thin film of graphite obtained by painting the paper with a colloidal solution of graphite in water and then drying. The solution known commercially as "Aquadag" is suitable for this purpose. The resistance may be graded by applying as many coats of the solution as necessary. Alternatively, the electrodes may be prepared by coating the paper with a very thin film of high resistance metal by sputtering electrically in a vacuum or by other suitable process. The dielectric 10 may consist of a number of layers of condenser paper.

With the construction indicated above, a number of adjustments are available to control the relationship of the resistance and the capacity in accordance with the requirements of Equations (12) and (13). In addition to controlling the surface resistivity of the electrodes by altering their thickness, as already described, the width of the electrodes may be varied, thereby varying the ratio of the capacity to the resistance, and the capacity may be varied independently by changing the thickness, or the number of sheets, of the dielectric. By the manipulation of all three of the variables, a desired impedance characteristic can be obtained with substantial accuracy.

If the electrodes are made very wide, the high resistivity of the conducting film may cause some change in the impedance characteristic because of the slow spreading of the current away from the terminals. This may be obviated by making contact with the electrodes through strips of copper foil 13 and 13' extending across the end of the electrode. An alternative plan is to divide the electrodes longitudinally into a plurality of narrow strips and to connect them in parallel at one end as shown in Fig. 3.

The construction of the coil core is shown schematically in Fig. 4. Numeral 14 designates the laminated core, 15 is a layer of insulation around the core, and 16 is the coil winding. In order that Equations (1) and (3) may represent the coil impedance accurately, it is desirable that the width of the laminations should be large compared with the thickness. Preferably the width should be at least 10 times the thickness. A toroidal core form is preferred, but other closed magnetic circuit structures may be used and in certain cases a straight core of considerable length may be permissible. Other forms of coil construction which may be used in modified forms of the invention along with condensers of appropriate configuration are shown in Figs. 4a and 4b. These will be described in detail later.

At high frequencies the coil impedance has substantially equal resistance and reactance components, giving a substantially constant phase angle of 45 degrees. This condition obtains accurately for all values of the angle ϕ in Equations (1) and (2) greater than π and with fair ac-

curacy for values of Φ as low as 2.3. To take advantage of the unique impedance characteristic the coil core should be so proportioned that the angle Φ reaches the value π at a frequency somewhere near the lower end of the range in which the coil, or the network in which it is employed, is to be used. From Equation (2) it will be seen that the value of this frequency is determined by the thickness of the laminations for any given core material.

Fig. 5 shows a simple form of constant resistance equalizer in accordance with the invention. The network has the configuration of one of the types shown in Zobel's U. S. Patent 1,603,305, October 19, 1926. The input terminals are designated T_1 and T_2 and the output terminals T_3 and T_4 . The network consists of two branches, a shunt branch connected between the input terminals including a resistance 17 and a metallic core coil 18, and a series branch containing a resistance capacity line 19. The resistance 17 may be assumed to include the fixed resistance of the coil winding. The load into which the network operates is shown as a resistance 20. The impedances of coil 18 and line 19 are proportioned so that their product is equal to the square of the value of resistance 17. Under this condition, the attenuation factor of the network, denoted by Γ , is given by

$$\Gamma = 1 + \frac{R_0}{Z} \quad (14)$$

where Z is the coil impedance and R_0 the value of resistance 17 including the direct current resistance of the coil winding. The characteristic impedance at terminals T_1 and T_2 is equal to R_0 and if the network be connected between a source and a load, each of resistance R_0 , the insertion loss will be the same as the attenuation factor given above.

In a particular example, the coil had the following structure and dimensions: The core was toroidal in form and comprises 19 laminations of thickness 0.114 centimeter, width 1.75 centimeters and outside diameter 6.03 centimeters. The core material was a nickel iron alloy known as 45 Permalloy, the ratio of permeability to resistivity for this material being 0.045. The winding consisted of 850 turns of No. 32 gauge copper wire. The direct current resistance of the coil was 35 ohms and its initial, or zero frequency, inductance was equal to 3.03 henries. The product LG corresponding to the core dimensions is .001835 giving a total distributed conductance of .00608 mho. The measured impedance characteristics of the coil are shown in Fig. 6 in which curve 21 represents the reactance divided by ω , or the effective inductance, and curve 22 the effective resistance divided by ω . The two components become equal at about 400 cycles per second and remain substantially equal at all higher frequencies.

The network was designed to have a characteristic impedance equal to 5350 ohms. Resistance 17 was therefore equal to this value less the direct current resistance of the coil, or 5315 ohms. The constants of the capacity resistance line 19 to give the proper inverse relationship to the coil impedance are found from Equations (12) and (13) to be

$$CR = LG = .001835$$

$$C = \frac{R_0^2}{L} = .105 \text{ microfarad} \quad (15)$$

and

$$R = 17400 \text{ ohms}$$

These constants were obtained in a rolled paper

condenser having two plates of waxed paper, each 154 centimeters in length and 6.35 centimeters wide, coated on both sides with Aquadag solution to give a surface resistivity when dried, of 363 ohms per square centimeter. The dielectric consisted of four layers of untreated condenser paper .001 centimeter thick. The accuracy of the condenser was tested by measuring the characteristic impedance of the complete network at different frequencies between 100 cycles per second and 20,000 cycles per second. The measured reactance was less than 50 ohms in all frequencies and the resistance did not vary more than 50 ohms from the desired value 5350 ohms.

The insertion loss characteristic of the complete network when operating between resistive terminations of 5350 ohms is shown by the full line curve 23 in Fig. 7. The slope of the characteristic on the logarithmic frequency scale is very nearly uniform over the five octave ranges from 200 to 6400 cycles per second, the value of the slope being about 1.5 decibels per octave. The wide range uniformity and the small slopes obtainable with the networks of the invention make them particularly suitable for the equalization of short lengths of transmission line of the order of a few miles. They may be also used in combination with other networks to provide an equalizer for long lines adjustable in small steps. The effect of the special coil and condenser constructions is shown by a comparison of curve 23 with curve 24 which shows the insertion loss obtained when the dissipative coil and condenser are replaced by simple inductance and capacity.

The networks of the invention may have any of the well-known circuit configurations giving constant resistance characteristics. A bridged-T configuration is shown in Stevenson Patent 1,606,817, November 16, 1926, and other forms are illustrated in an article by O. J. Zobel, Distortion Correction in Electrical Circuits with Constant Resistance Recurrent Networks, Bell System Technical Journal, Vol. VII, No. 3, July 1928. These alternative forms give substantially similar loss characteristics but in certain cases may extend the frequency range of linear loss variation by about an octave or more.

The dissipative coils and condensers may also have other forms than those described above. The coil core may be solid instead of laminated and of circular cross section or it may take the form of a hollow tube. A coil having a solid core of circular cross-section is illustrated diagrammatically in Fig. 4a, in which 25 designates the core, 26 a layer of insulation thereon, and 27 the coil winding. The modified form using a tubular core is shown in Fig. 4b and is similar to that of Fig. 4 except that the cross-section of the core 25 is annular instead of solid. Theoretically, the core may have any arbitrary cross-sectional shape and for each form an appropriate form of resistive condenser may be found. However, except for the flat lamination and the circular section cores, the condenser construction is likely to become impracticable. The character of the condenser in the general case may be determined from the following considerations:

The differential equation of the magnetic force distribution in a solid core of arbitrary cross section and resistivity σ_1 is

$$\frac{\partial^2 H}{\partial (m_1 x_1)^2} + \frac{\partial^2 H}{\partial (m_1 y_1)^2} + H = 0 \quad (16)$$

where x_1 and y_1 are the coordinates in the plane of the cross section, H is the magnetic force at

the point (x, y) and m_1 is the quantity given by

$$m_1^2 = j \frac{4\pi\mu\omega}{\sigma_1} \quad (17)$$

The flux variation in each elemental area, δs , of the cross section of the core will produce a contribution δE to the back electromotive force generated in the coil, the value of which is given by

$$\delta E = j\omega \frac{L_0}{S_1} \frac{1}{4\pi n} H \delta s \quad (18)$$

where L_0 is the zero frequency inductance of the coil, S_1 the cross-sectional area of the core and n is the number of turns of the winding per unit length of the core. The total back electromotive force and hence the impedance of the coil is obtained by integrating this expression over the whole area of the core with the help of Equation (16).

Consider now a condenser made up of one plate of zero resistance and a second plate having a surface resistivity per unit area equal to σ_2 , the two plates being separated by a uniform dielectric. Let it be assumed that connection is established to the resistance plate by a low resistance conductor around its edge so that all points along the edge are at the same potential. The differential equation of the distribution of the potential difference E between the two plates is, then,

$$\frac{\partial^2 E}{\partial(m_2 x_2)^2} + \frac{\partial^2 E}{\partial(m_2 y_2)^2} + E = 0 \quad (19)$$

where x_2 and y_2 are the coordinates in the plane of the plates and m_2 the value given by

$$m_2^2 = j\omega C\sigma_2 \quad (20)$$

C being the capacity per unit area of the plates. The total current flow into the capacity, and hence the admittance of the condenser, is obtained by integrating the currents in all of the elemental areas of the dielectric with the help of Equation (19).

Because of the similarity of Equations (16) and (19) it follows that, if the resistive condenser plate and the cross section of the coil core are of the same shape and are of such relative sizes that the quantities $m_1 x_1$ and $m_1 y_1$ are respectively equal to $m_2 x_2$ and $m_2 y_2$ for similarly chosen pairs of coordinates, the contours of equal magnetic force in the one case will correspond to the contours of equal potential difference in the other and the impedance of the coil will have the same character and frequency variation as the admittance of the condenser.

If the core section is circular the condenser plate may be circular, in which case the condition for the correspondence of the coil impedance and the condenser admittance characteristics reduces to

$$m_1 r_1 = m_2 r_2 \quad (21)$$

or

$$\frac{4\pi\mu r_1^2}{\sigma_1} = C\sigma_2 r_2^2 \quad (22)$$

where r_1 and r_2 are the radii of the core and the condenser plate respectively. If both plates are made of the same high resistance material, a value of the surface resistivity half as great as that required by Equation (22) may be used. In that case, the condition of correspondence may be transformed to

$$\frac{\mu S_1}{\sigma_1} = \frac{C_0 \sigma_2}{2\pi} \quad (23)$$

where S_1 is the area of the core section and C_0 is the total capacity of the condenser.

In most cases the use of circular condenser plates would require either an electrode material of extremely high resistivity or else a condenser of very large physical dimensions. This difficulty may be avoided by making the condenser in the form of a very narrow sector of a very large circle, as illustrated in Fig. 8, the voltage being applied at the arc of the sector. Since the lines of current flow are radial, the same characteristics are obtained using a sector as for a whole circle, the radius and the arc of the sector being chosen so that the electrode area is great enough to provide the desired total capacity. Under this condition Equation (23) becomes

$$\frac{\mu S_1}{\sigma_1} = \frac{C_0 \sigma_2}{\alpha} \quad (24)$$

where α is the angle of the sector in radians. For most purposes, the angle α may be very small so that the condenser electrodes take the form of long narrow tapered ribbons. For practical purposes, it is simpler to use a stepped tapered form as shown in Fig. 9, the approximation to the required characteristic being very close when ten or more uniform steps are used.

If the coil core is a hollow cylinder, the condenser electrodes will take the form of a truncated section with the inner and outer radii in the proportions of the cylinder radii. As the cylinder wall becomes very thin, the core becomes equivalent to a flat lamination of twice the thickness of the cylinder wall. The form of the condenser plates corresponding to a hollow cylindrical core of the type shown in Fig. 4b is illustrated in Fig. 8a.

The foregoing theory is based on the assumption that the flux in the coil core is everywhere normal to the plane of the cross section of the core. When the permeability of the core is high, this is substantially true for all forms of closed magnetic circuit. When the core material is non-magnetic or of very low permeability, the above condition may be realized by using a ring-shaped core with a uniformly distributed winding.

The frequency at which the phase angle of the coil impedance becomes equal to 45 degrees depends upon the ratio of the permeability to the resistivity of the core material. For coils operating at low frequencies or for coils of large inductance, it is preferable to use magnetic cores of high permeability. For coils of low inductance, or coils for use at high frequencies such as those employed in carrier telephony, cores of non-magnetic metal such as copper may be used with advantage.

What is claimed is:

1. In a wave transmission network having a constant resistance characteristic impedance and a frequency dependent attenuation, a pair of inversely related impedances, the ratio of which determines the attenuation and the product of which determines the characteristic impedance, one of said impedances comprising a coil having a metallic core and a winding thereon, said core having a solid cross section of area such that the effective resistance produced by eddy current flow is substantially equal to the effective reactance of the coil at frequencies above a preassigned value determining the lower limit of the operating range of the network, and the other of said impedances comprising a condenser having electrodes of low conductivity material, the shape of said electrodes conforming to the shape of the solid cross section of said coil core and the

surface resistivity of said electrodes being proportioned in relation to the permeability and the volume resistivity of the material of said core to provide the inverse relationship of said impedances throughout the operating range of frequencies of the network.

2. A network in accordance with claim 1 in which the coil core comprises flat laminations and in which the condenser electrodes are of rectangular shape with terminals at adjacent short edges of the rectangles.

3. A network in accordance with claim 1 in which the coil core comprises flat laminations and in which the condenser electrodes consist of narrow rectangular films of colloidal graphite with terminals at adjacent short edges of the rectangles.

4. A network in accordance with claim 1 in which the coil core is of circular cross section, and in which the condenser electrodes are substantially narrow sectors of a circle with terminals at the circular arcs.

5. A network in accordance with claim 1 in which the coil core is of circular cross section, and in which the condenser electrodes are shaped substantially in the form of narrow tapering wedges and are provided with terminal connections at their wide ends.

6. A network in accordance with claim 1 in which the coil core has an annular cross section and in which the condenser electrodes are shaped substantially in the form of a narrow annular sector having radii proportionally related to the radii of the core section.

7. In a wave transmission network having a frequency dependent attenuation, a pair of inversely related impedances, the ratio of which determines the attenuation and the product of which determines the characteristic impedance of the network, one of said impedances comprising a coil having a metallic core and a winding

thereon, said core comprising flat laminations, and the other of said impedances comprising a condenser having electrodes of low conductivity material and of ribbon-like form, the total resistance of said electrodes having substantially the value given by the equation

$$R = \frac{4\pi\mu b^2}{\sigma c}$$

where R denotes the resistance of the condenser electrodes, c the capacity of the condenser, μ and σ the permeability and the volume resistivity of the material of the coil core, and b half the thickness of the laminations, all quantities being in c. g. s. units.

8. In a wave transmission network having a frequency dependent attenuation, a pair of impedance elements having inversely related impedances, the ratio of which determines the attenuation, and the product of which determines the characteristic impedance of the network, one of said impedances comprising a coil having a metallic core and a winding thereon, said core having a circular cross section, and the other of said impedances comprising a condenser having electrodes of low conductivity material and in the form substantially of narrow circular sector, the surface resistivity of said electrodes having the value given by the equation

$$\sigma_2 = \frac{\alpha\mu S_1}{\sigma_1 C_0}$$

where σ_2 is the surface resistivity, α is the angle in radians of the circular sector represented by the electrodes, C_0 is the total capacity of the condenser, μ and σ_1 are the permeability and volume resistivity respectively of the core material, and S_1 is the cross-sectional area of the coil core, all quantities being in c. g. s. units.

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